Topology Optimization Applied to the Design of Functionally Graded Piezoelectric Bimorph

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Abstract. Functionally Graded Materials (FGMs) possess continuously graded material properties and are characterized by spatially varying microstructures. The smooth variation of properties may offer advantages such as reduction of stress concentration and increased bonding strength. Recently, this concept has been explored in piezoelectric materials to improve properties and to increase the lifetime of bimorph piezoelectric actuators. Usually, elastic, piezoelectric, and dielectric properties are graded along the thickness of a piezoceramic FGM. Thus the gradation law of piezoceramic properties can influence the performance of piezoactuators. In this work, topology optimization has been applied to find the optimum gradation variation in piezoceramic FGMs to improve actuator performance measured in terms of output displacements. A bimorph type actuator design is considered. Accordingly, the optimization problem is posed as finding the optimized gradation variation of piezoelectric properties that maximizes output displacement or output force in the tip of bimorph actuator. The optimization algorithm combines the finite element method with sequential linear programming (SLP). The finite element method applied is based on the graded finite element concept where the properties change smoothly inside the element. This approach provides a continuum approximation of material distribution (CAMD), which is appropriate to model FGMs. The alternative FGM modelling using traditional FEM formulation and discretizing the FGM into layers gives a discontinuous stress distribution, which is not compatible with FGM behavior. The present results consider gradation between two different piezoceramic properties and consider two-dimensional models with plane stress assumption.

Keywords: Micromachines, nanopositioners, piezoelectric actuators, topology optimization, FGM.

INTRODUCTION

Piezoelectric microdevices have a wide range of applications in precision mechanics, nanopositioning and micromanipulation fields. Functionally Graded Materials (FGMs) are advanced materials that possess continuously graded properties and are characterized by spatially varying microstructures created by nonuniform distributions of the reinforcement phase as well as by interchanging the role of reinforcement and matrix (base) materials in a continuous manner [1]. The smooth variation of properties may offer advantages such as local reduction of stress concentration and increased bonding strength. Recently, this concept has been explored in piezoelectric materials to improve properties and to increase the lifetime of bimorph piezoelectric actuators [2]. These actuators have attracted significant

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attention due to their simplicity and reliability. Usually, elastic, piezoelectric, and dielectric properties are graded along the thickness of an FGM bimorph piezoactuator. Studies conducted on FGM bimorph actuators [4-6] have shown that the gradation of piezoceramic properties can influence the performance of bimorph piezoactuators, such as generated output displacements. This suggests that optimization techniques can be applied to take advantage of the property gradation variation to improve FGM piezoactuators.

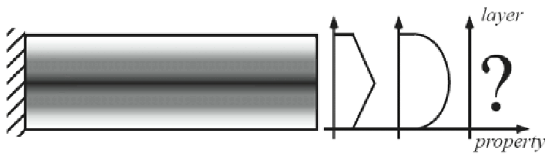

FIGURE 1. Finding the optimum gradation variation in piezoceramics FGMs.

Thus, in this work, topology optimization is applied to find the optimum gradation variation in FGM piezoceramics to improve piezoactuator performance measured in terms of output displacements (see Figure 1). Accordingly, the optimization problem is posed as finding the optimized gradation variation of piezoelectric properties that maximizes output displacement or output force at the tip of the bimorph actuator. The optimization algorithm combines the finite element method (FEM) with sequential linear programming (SLP). The FEM is based on the graded finite element concept. The material model is implemented based on the solid non-piezoelectric material with penalization (SIMP) [7] where fictitious densities are interpolated at each finite element. This approach provides a continuum approximation of material distribution (CAMD) [8], which is appropriate to model FGMs.

FUNCTIONALLY GRADED PIEZOELECTRIC FINITE ELEMENT MODEL

In this work, FGM piezoelectric actuators considered for design operate in quasi-static or low-frequency (inertia effects are neglected). The material properties continuously change inside the piezoceramic domain, which means that they can be described by some continuous function of position of the piezoceramic domain.

As a consequence, according to the mathematical definition finite element matrices, these material properties remain inside the matrices integrals and are integrated using the graded finite element concept [9] where properties are continuously interpolated inside each finite element based on property values at each finite element node. Approximation of the continuous change of material properties by a stepwise function, where a property value is assigned for each finite element, results in undesirable discontinuity of the stress field [9].

Because a non-piezoelectric conductor material and a piezoceramic material may be distributed in the piezoceramic domain, the electrode positions are not known “a priori”. Thus, the electrical excitation will be given by an applied electric field. In this case, all electrical degrees of freedom are prescribed in the FEM problem, and as a consequence the dielectric properties will not influence the design.

TOPOLOGY OPTIMIZATION FORMULATION

Topology optimization is a powerful structural optimization technique that combines the FEM with an optimization algorithm to find the optimal material distribution inside a given domain (extended fixed domain), bounded by supports and applied loads, that contains the unknown structure [7].

In the current topology optimization formulation the design variables are defined for each element node and a continuous distribution of the design variable inside the finite element is interpolated using some continuous function. This formulation, known as CAMD (Continuous Approximation of Material Distribution) [8] is robust and it is also fully compatible with the FGM concept and philosophy [10]. We are interested in a continuous distribution of piezoelectric materials in the design domain, and thus, the following material model is proposed based on a simple extension of the well-known SIMP model [7]:

$$\mathbf{C}^{H} = \rho\ \mathbf{C}_1 + (1-\rho)\mathbf{C}_2 \qquad \mathbf{e}^{H} = \rho\ \mathbf{e}_1 + (1-\rho)\mathbf{e}_2 \tag{1}$$

where ρ (ρ = 1.0 denotes piezoelectric material type 1 and ρ = 0.0 denotes piezoelectric material type 2) are pseudo-density functions describing the amount of material at each point of the domain. The design variables can assume different values at each finite element node. $\mathbf{C}^{H}$ and $\mathbf{e}^{H}$ are stiffness and piezoelectric tensor properties, respectively, of the mixture. $\mathbf{C}_1$ and $\mathbf{e}_1$ are tensors related to the stiffness and piezoelectric properties for piezoelectric material type 1, respectively, and $\mathbf{C}_2$ and $\mathbf{e}_2$ are the corresponding properties for piezoelectric (or non-piezoelectric, such as Aluminum) material type 2. These are the properties of basic materials that will be distributed in the piezoceramic domain to form the FGM piezocomposite. For a discretized domain into finite elements Equations (1) are considered for each element node, and the material properties inside each finite element are given by a function $\rho = \rho(\mathbf{x})$, according to the CAMD concept. However, in this problem, the piezoceramic domain is not fixed and the piezoceramic electrodes are not known "a priori". Thus, to circumvent this problem an electric field is applied to the domain as electrical excitation.

Essentially, the objective function is defined in terms of generated output displacements (u_1) for a certain applied electric field to the design domain (see Figure 2a and 2b). The piezoactuator must resist to reaction forces (in region Γ_{t2}) generated by a body that the piezoactuator is trying to move or grab. Therefore, the mean compliance must be minimized to provide enough stiffness (see Figure 2c), and for its calculation, the electric field is kept null inside the medium. Finally, the undesired generated displacement must be minimized [10]. A multi-objective function is constructed to find an appropriate optimal solution that can incorporate all design requirements. Thus, the final optimization problem is defined as:

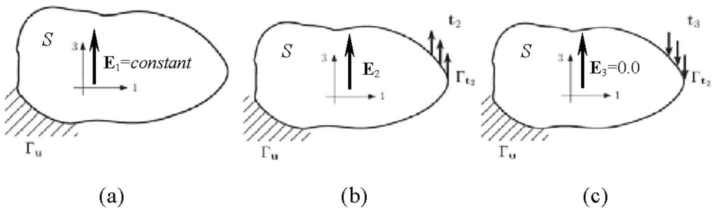

FIGURE 2. Load cases used for calculation of the desired and undesired output displacements (a), and mean compliance (c). Here, $\mathbf{E}_i = -\nabla\phi_i$ denotes the electrical field associated with load case i.

$$\begin{aligned}
&\underset{\rho(x)}{Maximize}: F(\rho) = w * \ln\left[L_2\left(u_1,\phi_1\right)\right] - \frac{1}{2}(1-w)\ln\left[L_3\left(u_3,\phi_3\right)^2 + \beta L_2\left(u_1,\phi_1\right)^2\right] \\
&subject\ to: \text{ Equilibrium equations for different load cases} \\
&\qquad 0 \le \rho(x) \le 1 \\
&\qquad \Theta(\rho) = \int_S \rho dS - \Theta_1 \le 0 \\
&\qquad 0 \le w \le 1
\end{aligned} \tag{2}$$

where S is the design domain, Θ is the volume of piezoceramic type 1 material in the design domain, Θ_1 is the upper-bound volume constraint defined to limit the maximum amount of material type 1. In this work, the continuous distribution of design variable $\rho(\mathbf{x})$ is given by function [8]

$$\rho(x) = \sum_{I=1}^{nd} \rho_I(x) N_I \tag{3}$$

where ρ_I is a nodal design variable, N_I is the finite element shape function, and n_d is the number of nodes at each finite element. The design variable ρ_I can assume different values at each finite element node.

Sequential Linear Programming (SLP) is applied to solve the optimization problem. The linearization of the problem at each iteration requires the sensitivities (gradients) of the multi-objective function and constraints in relation to ρ_I. Suitable moving limits are introduced to assure that the design variables do not change by more than 5-15% between consecutive iterations.

RESULTS

The design of a bimorph piezoactuator will be presented to illustrate the FGM piezoelectric actuator design using the proposed method. The design domain for this problem is shown in Figure 3 and it has 10500 finite elements. The bimorph is essentially a piezoelectric cantilever type actuator. The design domain is divided into 21 horizontal layers and a design variable (pseudo density ρ_I) is considered for each layer interface. Thus, there are 22 design variables. Piezoelectric material and Gold are distributed. The center layer is made of Gold and it is kept fixed during optimization (non-optimized region). The mechanical and electrical boundary conditions are shown in the same figure.

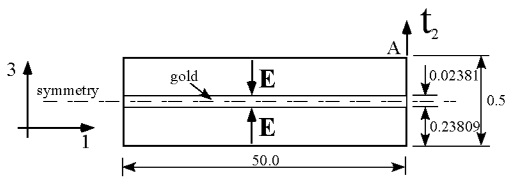

FIGURE 3. Bimorph design domain divided in horizontal layers. A design variable is defined for each layer interface.

The piezoelectric material properties used in the simulations for all examples are obtained from Carbonari *et. al.* [15]. The Young's modulus and Poisson's ratio of Gold are equal to 83 GPa and 0.44, respectively. Two-dimensional isoparametric finite elements under plane-stress assumption are used in the finite element analysis.

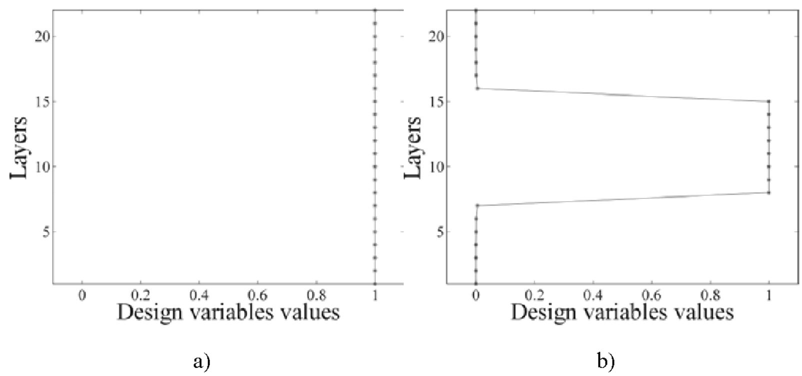

FIGURE 4. Bimorph optimal topology designs; a) w=0.5; b) w=1.0.

The electric field applied to the design domain is equal to 4200V/mm (see Figure 3). Two designs were obtained considering the value of w coefficient equal to 0.5 and 1.0, respectively. The displacement coupling constraint was not activated, thus, the coefficient β is equal to zero in both cases. The volume constraint for piezoelectric material Θ_1 is set equal to 50%. The initial value for design variables ρ_I is equal to 0.15 in both cases. The results are shown by plotting the pseudo-density gradation variation along the layers. Through material models described in Equations (1) the property gradation variation can be obtained. The topology optimization results are shown in Figure 4. The optimization algorithm concentrates the piezoceramic in the upper and lower layers of the design domain. The property gradation variation is symmetric in both cases. The optimization finished with the constraint Θ_1 active in both designs. Displacements generated by bimorph described in Figures 4 (a) and (b) are equal to 7.12mm and 12.85mm, respectively.

CONCLUSIONS

A topology optimization formulation was proposed which allows the search for an optimal gradation of piezoelectric material properties in the design of FGM piezoelectric actuators. The optimization problem allows the simultaneous distribution of two piezoelectric materials or a non-piezoelectric (such as Gold) and piezoelectric materials in the design domain. The design of an FGM bimorph actuator is presented to illustrate that the actuator performance can be improved by finding the optimal gradation of FGM piezoelectric material properties in the actuator. Considering the maximization of output displacement, the optimization algorithm comes up with an optimized gradation variation of properties which consists of a distribution of piezoceramic material in the upper and lower layers and gold in the central layers.

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