Graded Viscoelastic Approach for Modeling Asphalt Concrete Pavements

Eshan V. Dave[a], William G. Buttlar[a], Glaucio H. Paulino[a] and Harry H. Hilton[b]

[a]Department of Civil and Environmental Engineering
[b]Department of Aerospace Engineering and National Center for Supercomputing Applications
University of Illinois at Urbana-Champaign

Abstract. Asphalt concrete pavements exhibit severely graded properties through their thickness due to oxidative aging effects, which are most pronounced at the surface of the pavement and decrease rapidly with depth from the surface. Most of the literature to date has focused on use of layered-elastic models for the consideration of age stiffening. In the current work, a graded viscoelastic model has been implemented within a numerical framework for the simulation of asphalt pavement responses under thermal and mechanical loading. The graded viscoelastic work is extension of the previous work by Paulino and Jin [1], Mukherjee and Paulino [2], and Buttlar et al. [3]. A functionally graded generalized Maxwell model has been used in the development of a constitutive model for asphalt concrete considering aging and temperature gradients. The aging gradient data from laboratory test results reported by Apeagyei [4] is used for obtaining material properties for the graded viscoelastic model. Finite element implementation of the constitutive model incorporates the generalized iso-parametric formulation (GIF) proposed by Kim and Paulino [5], which leads to the graded viscoelastic elements used in this work.

Keywords: Viscoelasticity, Correspondence Principle, Asphalt Concrete, Aging, Finite element method.

INTRODUCTION AND MOTIVATION

The aging of asphalt concrete pavements is well established and has been studied by numerous researchers including Apeagyei [4], and Mirza and Witczak [6]. The studies have shown that the asphalt concrete material undergoes stiffening because of oxidative hardening. The effect of aging creates graded material properties due to variation in the amount of age stiffening across the depth of pavement. The material on surface undergoes most aging due exposure to higher temperatures and air (oxygen). Depending on factors including (but not limited to) density of mixture, aggregate gradation, chemical composition of asphalt binder, the amount of aging gradually reduces with depth. Apart from aging, the graded behavior in asphalt concrete pavements is also exhibited due to varying temperatures across the thickness. Fig. 1 illustrates the effect of aging as a function of depth as predicted by Global Aging Model of Mirza and Witczak [6]. The plot shows the aging properties of a

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typical asphalt concrete pavement located in Central Illinois after two years in service. Figure 2 shows the temperature variation in the asphalt concrete overlay.

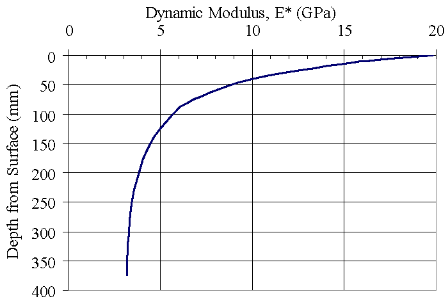


FIGURE 1. Aged Asphalt Concrete Properties after Two-Years in Service

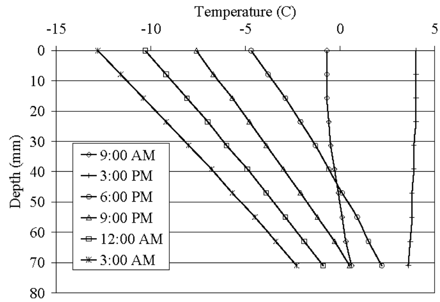


FIGURE 2. Temperature Variation through the Depth of Asphalt Concrete Pavement

Asphalt concrete exhibits viscoelastic material behavior at most of its service temperatures. The temperature dependent properties can be taken into account by means of Time-Temperature Superposition Principle (TTSP) proposed by Williams et al. [7].

This paper is organized in four main sections, (1) Theoretical Background, (2) Material Properties and Constitutive Model, (3) Finite-Element Implementation, and (4) Closure.

BACKGROUND

Numerous authors have shown the benefits of using correspondence principle in viscoelasticity, which can be readily used to solve viscoelastic problems using the corresponding solutions for elastic problems. Paulino and Jin [1] and Mukherjee and Paulino [2] demonstrated the limitations of the correspondence principle especially in terms of usage for various material models. They demonstrated that only a certain class of viscoelastic material models can be used directly in conjunction with the correspondence principle. These classes of models are known as "separable models." The separable models are characterized by their time and space dependent portions, where the material parameters have either spatial and time dependence, but not both. For example the generalized Maxwell model can be presented in separable and non-separable forms as:

Non-separable:

$$G(\mathbf{X},t)=\sum_{h=1}^{n} G_h(\mathbf{X})Exp\left[-t/\tau(\mathbf{X},t)\right] \quad (1)$$

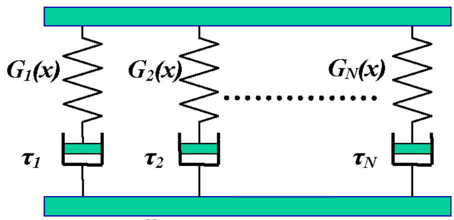
Separable:

$$G(\mathbf{X},t)=\sum_{h=1}^{n} G_h(\mathbf{X})Exp\left[-t/\tau(\mathbf{X})\right] \quad (2)$$

where, G is relaxation modulus at time t and location $\mathbf{X}$, G_h is spring coefficient and τ_h is relaxation time of h Maxwell unit.

Hilton [8] has discussed the limitations and distinctions between analyses for various non-homogeneous viscoelastic models, along with other discussion on functionally graded viscoelastic designer materials. The variation in temperature with space is acceptable but temperature variation with time also causes similar problems as inseparable model when using correspondence principle.

It is common practice to use Prony series type model for representing viscoelastic material properties of asphalt concrete. Fig. 3 shows an illustration of Generalized Maxwell model which is used in this study. Notice that the illustrated model is a separable model. The equations accompanying the figure show the properties for Generalized Maxwell Model.



$$G(\mathbf{X},t) = \sum_{h=1}^{N} G_h(\mathbf{X}) Exp[-t/\tau_h] \qquad (3)$$

FIGURE 3. Generalized Maxwell Model

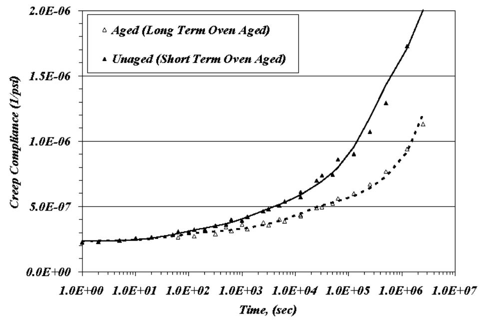


FIGURE 4. Creep Compliance Master Curves with Fitted Generalized Kelvin Model (markers indicate laboratory data and lines show fitted models)

MATERIAL PROPERTIES AND CONSTITUTIVE MODEL

Asphalt concrete viscoelastic properties are commonly determined by running creep tests on a cylindrical specimen in indirect tensile modes. Typically tests are run at three different temperatures to establish the time-temperature superposition principle and generate a master curve. The AASHTO T322 [9] test specification provides the details on test procedure and analysis of test results. The test results in this study were analyzed in terms of deviatoric and volumetric components of material properties.

Apeagyei [4] has extensively studied the effects of short-term and long-term aging on the behavior of asphalt concrete properties. The laboratory data from Apeagyei was used to verify whether the separable form of Generalized Maxwell model can be used to model the aging gradient in asphalt concrete. Creep tests were performed for short term and long term aged asphalt concrete samples. The short-term aging typically represents the asphalt concrete material with little or no aging in the field conditions whereas long term aging tends to simulate the aging of asphalt concrete pavements over a 20-year service life. Thus these two stages represent the most extreme aging gradient that is anticipated in an asphalt concrete pavement. This hypothetical extreme condition is similar to aging gradient that was shown in Fig. 1.

The generalized Maxwell model was fitted to the data. In this case the spring (G_h) and dashpot (τ_h) parameters were limited to spatial dependency. Thus the theory underlying the separable models can be used. The master curves with lab data and the fitted model are shown in Fig. 4.

FINITE-ELEMENT IMPLEMENTATION

The generalized quasi-static viscoelastic variational principle described by Gurtin [10] forms the basis for the finite element implementation. The formulation is provided for non-homogeneous and time dependent conditions, and thus all the material properties depend on their location described through **X** at time *t*. The formulations have been derived to be used with transformation methods; in other words, they were derived to represent separable material model.

The variational principle for body with volume Ω and surface S as proposed by Gurtin [10]:

$$\delta\Pi = \int_{\Omega}\int_{-\infty}^{t}\int_{-\infty}^{t-t'} C_{ijkl}\left[\xi_{ijkl}(t-t'') - \xi'_{ijkl}(t')\right]\frac{\partial}{\partial t'}\left(\varepsilon_{ij}(\mathbf{X},t')\right)\frac{\partial\delta\varepsilon_{kl}(\mathbf{X},t'')}{\partial t''}dt'\,dt''\,d\Omega$$
$$-\int_{S}\int_{-\infty}^{t} P_i(\mathbf{X},t-t'')\frac{\partial\delta u_i(\mathbf{X},t'')}{\partial t''}dt''\,dS = 0 \qquad (4)$$

where, $\underline{\underline{C}}$ is constitutive properties, $\underline{P}$ is traction, $\underline{u}$ is displacement, $\underline{\underline{\varepsilon}}$ is mechanical strain, and $\underline{\underline{\xi}}$ is reduced time (defined later). The displacement is related to nodal displacement degrees of freedom $\underline{q}$ via shape functions $\underline{\underline{N}}$:

$$u_i(\mathbf{X},t) = N_{ij}(\mathbf{X})q_j(t) \qquad (5)$$

The element equilibrium equation can be written as

$$k_{ij}^{e}(\mathbf{X},\xi_{ij})q_j(0) + \int_{0}^{\xi} k_{ij}^{e}(\mathbf{X},\xi_{ij} - \xi'_{ij})\frac{\partial q_j(\xi')}{\partial \xi'}d\xi' = f_i^{e}(x,\xi) \qquad (6)$$

where, k^e is element stiffness matrix, and f^e is the traction force vector.

The reduced time ξ is related to temperature T and time t through time-temperature superposition given by,

$$\xi_{ijkl}(\mathbf{X},t) = \int_{0}^{t} a_{ijkl}(T(\mathbf{X}))\,dt' \qquad (7)$$

where, a is the experimentally determined shift-factor.

The constitutive properties for the generalized Maxwell model are

$$C_{ijkl}(\mathbf{X},t) = \sum_{h=1}^{N} E_{ijkl_h}(\mathbf{X})Exp\left[-t/\tau_{ijkl_h}(\mathbf{X})\right] \qquad (8)$$

On basis of previous formulations, the element stiffness matrix, k_{ij}^{e} and force vector, f_{i}^{e} can be written as:

$$k_{ij}^{e}\left(\mathbf{X},\xi_{ij}-\xi'_{ij}\right)=\sum_{h=1}^{N}\int_{\Omega}B_{im}(\mathbf{X})\,E_{mn_h}(\mathbf{X})Exp\left[-(\xi_{ij}-\xi'_{ij})/\tau_{mn_h}(\mathbf{X})\right]B_{nj}(\mathbf{X})d\Omega \tag{9}$$

$$f_{i}^{e}(\mathbf{X},t)=\int_{S}N_{ij}(\mathbf{X})\,P_{j}(\mathbf{X},t)dS \tag{10}$$

On assembly we get final form of equilibrium for the body,

$$K_{ij}(\mathbf{X},\xi)U_{j}(0)+\int_{0}^{\infty}K_{ij}(\mathbf{X},\xi-\xi')\frac{\partial U_{j}(\xi')}{\partial\xi'}d\xi'=F_{j}(\mathbf{X},\xi) \tag{11}$$

Transformation of above yields,

$$\begin{gathered}K^{E}(\mathbf{X})\,s\tilde{f}(\mathbf{X},s)\,\tilde{U}(s)=\tilde{F}(\mathbf{X},s)\\ where,\quad \tilde{f}(\mathbf{X},s)=\sum_{h=1}^{N}\frac{\tau_{h}(\mathbf{X})}{\tau_{h}(\mathbf{X})s+1}\\ K^{E}(\mathbf{X})=K(\mathbf{X},t=0)\end{gathered} \tag{12}$$

s = Transformation Variable

Verification: Elastic and Viscoelastic Solutions

As mentioned earlier, the generalized iso-parametric formulation proposed by Kim and Paulino [5] was used in the finite-element implementation. Three and six-node triangular elements were implemented. The verification for these specialized elements was carried out by comparing tensile loading, bending, and fixed grip displacement results with analytical exact solutions. Fig. 5 shows the comparison between exact solution and FE results for bending. The results for other tests also showed very close agreement between analytical and FE results.

The viscoelastic solution in the FE framework was obtained using the Fourier transformation method by implementing the formulations presented earlier. The implementation was performed in Matlab, and the available fast Fourier transform (FFT) scheme was utilized for transformations. To verify the viscoelastic analysis analytical solution for tensile loading (creep) and fixed grip displacement (relaxation) were compared with FE solution. Results for creep displacement under a constant tensile load are shown in Fig. 6.

CONCLUSIONS

On basis of the data analyses and finite element implementations described in this paper it can be concluded that non-homogeneous aging gradients in asphalt concrete pavements can be represented by a functionally graded generalized Maxwell model. Further it was verified that finite-element implementation with transformation technique can be used to solve functionally graded viscoelastic problems, as long as material properties are governed through "separable models." The implemented method can be used to analyze asphalt concrete pavements with non-homogeneous aging and temperature gradients

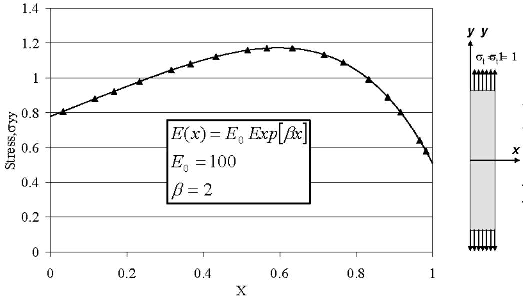


FIGURE 5. Comparison of Finite Element and Exact Solution for Functionally Graded Bar under Tension

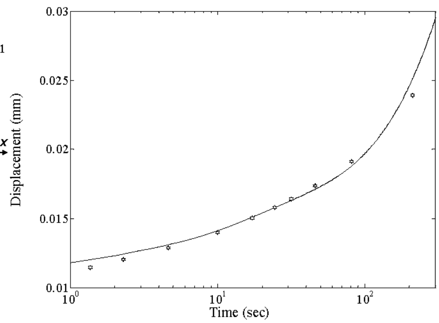


FIGURE 6. Comparison of Finite Element and Exact Solution for Viscoelastic Bar under Tension

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