Modeling Bamboo as a Functionally Graded Material

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Abstract. Natural fibers are promising for engineering applications due to their low cost. They are abundantly available in tropical and subtropical regions of the world, and they can be employed as construction materials. Among natural fibers, bamboo has been widely used for housing construction around the world. Bamboo is an optimized composite material which exploits the concept of Functionally Graded Material (FGM). Biological structures, such as bamboo, are composite materials that have complicated shapes and material distribution inside their domain, and thus the use of numerical methods such as the finite element method and multiscale methods such as homogenization, can help to further understanding of the mechanical behavior of these materials. The objective of this work is to explore techniques such as the finite element method and homogenization to investigate the structural behavior of bamboo. The finite element formulation uses graded finite elements to capture the varying material distribution through the bamboo wall. To observe bamboo behavior under applied loads, simulations are conducted considering a spatially-varying Young's modulus, an averaged Young's modulus, and orthotropic constitutive properties obtained from homogenization theory. The homogenization procedure uses effective, axisymmetric properties estimated from the spatially-varying bamboo composite. Three-dimensional models of bamboo cells were built and simulated under tension, torsion, and bending load cases.
Keywords: Bamboo, functionally graded material, homogenization, finite element analysis, graded elements.

INTRODUCTION

Biological systems such as plant and tree stems, animal bones and other biological hard tissues tend to be optimized for the loading conditions to which they are subjected. Biological structures are usually made of composite materials which are multifunctional and have living organisms which provide adaptability. This occurs due to the fact that biological systems must be able to perform a variety of functions well, and thus, they are optimized for multifunctional purposes. As a consequence, biological structures are complicated and non-uniform, which makes their modelling difficult and involved.

Among biological structures, the natural fibers are very interesting for engineering applications due to their low cost and availability. They grow abundantly in tropical and subtropical regions of the world, and they can be usefully employed as construction materials [1,2,3,4]. Examples of natural fibers are bamboo, coconut fibers, sisal, etc...

Bamboo is a tree-like plant that belongs to the subfamily *Bambusoideae* of the grass family *Poaceae*. Bamboo stalks are optimized composite materials that naturally exploit the concept of Functionally Graded Materials (FGMs) [1, 5-7]. FGMs are materials that

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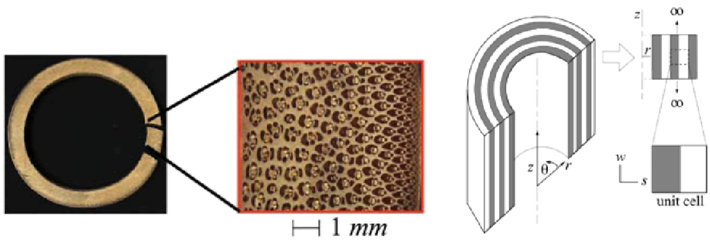
possess continuously-graded properties and are characterized by spatially-varying microstructures created by nonuniform distributions of the constituent phases. In these materials, the role of reinforcement and matrix (base) material interchanges in a continuous manner [8]. The smooth variation of properties may offer advantages such as reduction of stress concentration and increased bonding strength [9, 10].

The bamboo culm is an approximately cylindrical shell that is divided periodically by transversal diaphragms at nodes. Between 20-30% of the cross-sectional area of the culm is made of longitudinal fibers that are distributed non-uniformly through the wall thickness, the concentration being most dense near the exterior (see Fig. 1(a)). The orientation of these fibers makes bamboo an orthotropic material with high strength along, and low strength transverse to fibers [3, 11].



(a) (b)

Figure 1. (a) Cross section of culm showing radial distribution of fibers through the thickness (photo courtesy of [5]); (b) An axisymmetric composite and its corresponding unit cell.

Most work in the literature that characterizes bamboo is experimental, dedicated to estimating strength and stiffness properties [11-13]. Few works treating the modeling of natural fibers have been found in the literature, and these deal primarily with simplified analytical models.

Considering that biological structures, such as bamboo, have complicated shapes and material distribution inside their domain, the use of numerical methods such as the finite element method (FEM) [14, 15] and multiscale methods, such as homogenization [17] can be useful tools for understanding the mechanical behavior of these materials. The objective of this work is to explore computational techniques, including the FEM and a multi-scale method based on homogenization, to investigate the structural behavior of bamboo. The homogenization method enables the computation of effective properties of a composite material. With these properties, it is possible to model the composite structure as an equivalent homogeneous medium, allowing the use of traditional FEM codes or simple analytical models for numerical modeling.

MODELING BAMBOO WITH GRADED FINITE ELEMENTS

When modeling a functionally graded material using the FEM, the continuous variation of material properties within the domain must be taken into account. In the traditional finite element formulation [14], the properties are assumed to be constant inside each element. To model an FGM material using this traditional formulation, a continuous material distribution is approximated by piecewise-constant elements. This generates an artificially discontinuous stress field, however, which may not adequately simulate the true conditions. An FEM formulation better-suited for FGMs employs graded elements that incorporate actual material properties at integration points [16]. This procedure using graded elements incorporates continuous material distribution into the numerical simulation, and leads to smoothly-varying and more accurate stresses [16].

HOMOGENIZATION THEORY FOR FUNCTIONALLY-GRADED AXISYMMETRIC COMPOSITES

A periodic composite material is a composite whose microstructure exhibits a periodic repetition of a representative substructure called a unit cell [17]. The homogenization method enables the estimation of effective properties of a complex periodic composite material with a known unit-cell topology. It is a general method for computing effective properties and has no limitations regarding volume fraction or shape of the composite constituents. The primary assumptions are that the unit cell is periodic and that the scale of the composite part is much larger than the microstructural dimensions [17]. An important consideration that arises when computing effective properties of composite materials is the effect of the specimen scale with respect to the scale of the unit cell.

Bamboo can be considered an axisymmetric composite material. An axisymmetric composite and its corresponding two dimensional unit cell is illustrated in Fig. 1(b). Considering the standard homogenization procedure for elastic materials [17], the unit cell is defined as $Y = [0,Y_1]\times[0,Y_2]$ and the elastic property function is considered to be a Y-periodic function, that is, $\mathbf{E}^{\varepsilon}(\mathrm{x},\mathrm{y})=\mathbf{E}(\mathrm{x},\mathrm{y}+\mathbf{Y})$, and $\mathrm{y}=\mathrm{x}/\varepsilon$, where $\varepsilon > 0$ is the composite microstructure scale that represents the scale at which the material properties are changing. Coordinates $x = (r, z)$ and $y = (s, \omega)$ are associated with the composite macro- and micro-dimensions, respectively (see Fig. 1b). The first step in the homogenization procedure is to expand the displacement **u** inside the unit cell as [17]

$$\mathbf{u}^{\varepsilon}=\left\{u_r^{\varepsilon} \quad u_z^{\varepsilon}\right\}^T=\mathbf{u}_0(\mathrm{x})+\varepsilon\mathbf{u}_1(\mathrm{x},\mathrm{y}) , \tag{1}$$

where only the first-order variation $(\mathbf{u}_1(\mathrm{x},\mathrm{y}))$ is taken into account, and $\mathbf{u}_1$ is Y-periodic. Displacements $\mathbf{u}_0(\mathrm{x})$ and $\mathbf{u}_1(\mathrm{x},\mathrm{y})$ correspond to the composite specimen scale and the unit cell scales, respectively [17].

By substituting Eq. (1) in the equilibrium equations, we obtain distinct microscopic and macroscopic equations, respectively. Due to the linearity of the elastic problem, and assuming the separation of variables for $\mathbf{u}_1(\mathrm{x},\mathrm{y})$, we obtain [17]:

$$\mathbf{u}_1=\chi(x,y)\varepsilon(\mathbf{u}_0(x)) \text{ and } \partial_y\mathbf{u}_1(x,y)=\partial_y\chi(x,y)\partial_x(\mathbf{u}_0(x)) \tag{2}$$

where $\chi(x,y)$ is the characteristic displacement of the unit cell, which is also Y-periodic, which corresponds to the periodicity condition in the unit cell. Then, substituting Eq. (2) into the unit-cell (microscopic) equations, and then into the specimen (macroscopic) equations, the definition of the effective properties can be obtained [17]

$$\frac{1}{|Y|}\int_Y \left[\left(\mathbf{I}+\partial_y\chi(x,y)\right):\mathbf{E}(x,y):\partial_y\delta\mathbf{u}_1(x,y)\right]dY=0,\quad \forall\delta u_1\in H_{per}\left(Y,R^3\right), \tag{3}$$

$$E^H=\frac{1}{|Y|}\int_Y \left[\left(\mathbf{I}+\partial_y\chi(x,y)\right):\mathbf{E}(x,y):\left(I+\partial_y\chi(x,y)\right)\right]dY, \tag{4}$$

FINITE-ELEMENT ANALYSES OF BAMBOO

In geometry, bamboo is essentially a hollow cylinder with periodic stiffeners called diaphragms, located at positions called nodes (see Fig. 2(a)). A bamboo cell is the section of culm between two diaphragms. In addition, the diameter of the culm is tapered, being largest near the ground. Figure 2(b) illustrates the geometry of the modeled cell. The angle of taper is neglected. For the load cases of tension and torsion, one bamboo cell was modeled. For bending, two cells were modeled in order to include one diaphragm in the interior of the domain away from applied loads and support conditions.

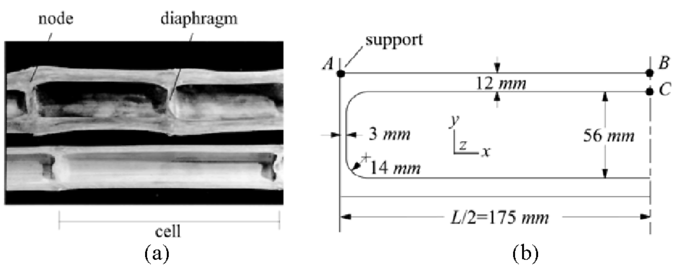


Figure 2. (a) Cross section of bamboo culm showing internal structure (photo courtesy of [5]); (b) Section view of one-half of cell showing dimensions adopted for finite-element meshes.

The elastic properties considered in this study employ the Young's moduli obtained by Nogata and Takahashi through detailed experiments [1]. The estimated variation of Young's modulus through the bamboo thickness is given by the expression [1]

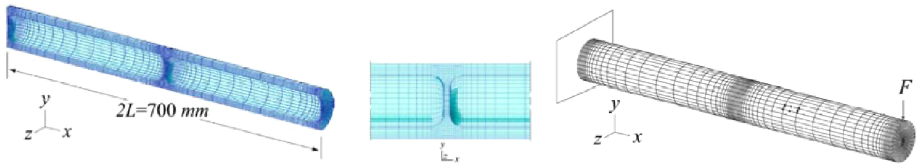
$$E(r)=3.75e^{(2.2r/t)} \tag{5}$$

where r denotes position through the thickness of the cell wall starting at the inner surface, and t is the thickness of the cell wall.

To study bamboo behavior using different material models, three types of material models were considered for each of the load cases of tension, torsion and bending. The first model considers a homogeneous isotropic structure with a bulk Young's modulus determined by integrating Eq. (5) from 0 to t which yields $E_b = 13.68$ *GPa*. The second material model considers the continuous gradation of Young's modulus through the thickness of the cell wall as described by Eq. (5). Finally, the third material model studied here is a homogeneous orthotropic material whose elastic stiffness matrix is obtained using the homogenization method described in Section 3.

These properties will be considered in a homogeneous orthotropic FEM model of the bamboo structure. The objective of comparing these material models is to illustrate the differences in displacements and stresses that are caused by material gradation.

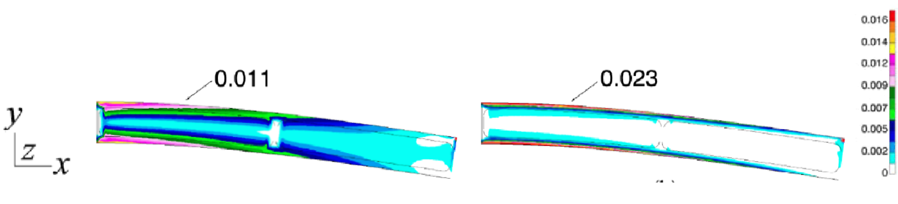
As example, the mesh employed to simulate the bending load case is shown in Fig. 3(a), and Fig. 3(b) shows a detailed view of the mesh in the region of the interior node. A schematic of the boundary conditions for bending is shown in Fig. 3(c).



(a) (b) (c)

Figure 3. Bamboo discretization for the bending load case: (a) Cross section of two-cell FEM model. Full mesh includes 14,760 20-noded brick finite elements and 6,417 nodes; (b) Detail of mesh at interior bamboo node region; (c) Schematic of boundary conditions and applied load for bending case.

The simulations showed that the second material model is stiffer than the first model. The axisymmetric model based on homogenized orthotropic material is the stiffest of the three cases. There is close agreement between axial deformations in the FGM and homogeneous isotropic model indicating that the use of averaged (or bulk) properties is consistent for estimating elongation. The homogenized orthotropic material model leads to displacements that differ from the other two models, thus indicating that the presented homogenization procedure affects the axial stiffness of the cell wall. For torsion and bending loading, comparison between the FGM and homogeneous material models indicates that the homogeneous approximation leads to a more flexible structure. This is expected because the FGM model places the stiffest material farthest away from the neutral axis.



(a) (b)

Figure 4. Von Mises stress distributions for bending. (a) Homogeneous isotropic; (b) FGM material. Stresses are normalized by the maximum stress value that occurs among the two models.

Simulations showed that stresses near the interior of the wall are much lower in the actual material as represented by the FGM, than they are in the homogeneous case. Stresses near the outside of the cell wall are much higher in the FGM case than in the homogeneous case. Thus, the FGM leads to a remarkable stress redistribution in the bamboo and the stress response of (inhomogeneous) FGMs differs substantially from those of their homogeneous counterparts (c.f. [16]). Figure 4 show fringe plots of the von Mises stress in the model under bending loads. The highest stresses appear near support boundary conditions or applied nodal loads. The plots demonstrate that material gradients through the cell wall have a strong influence on local cell-wall stresses.

CONCLUSIONS

By using the graded finite element concept the continuous change of bamboo properties along the thickness could be taken into account, and its influence in the bamboo mechanical behavior was shown. By using the homogenization method for graded material, the effective properties of an axisymmetric bamboo composite were computed. By means of these homogenized properties it is possible to model bamboo as a homogeneous orthotropic medium. Given the additional computational effort of the homogenization procedure, it seems that a simple averaged elastic modulus obtained from a rule of mixtures or an averaged modulus obtained from the FGM variation will provide suitable numerical accuracy for capturing the "global" deflection/response of a bamboo structure. To estimate local features, however, such as stresses near supports, pin connections or holes etc., it is necessary to employ a numerical procedure that accurately models material gradients through the cell wall. This work illustrates modern numerical analysis techniques that lend special insight into the structural and mechanical response of bamboo as a naturally-graded fiber composite.

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