Materials Science Forum Vols. 631-632 (2010) pp 47-52
© (2010) Trans Tech Publications, Switzerland
doi:10.4028/www.scientific.net/MSF.631-632.47

Asphalt Pavement Aging and Temperature Dependent Properties through a Functionally Graded Viscoelastic Model, Part-I: Development, Implementation and Verification

Eshan V. Dave[1,a], Glaucio H. Paulino[1,b] and William G. Buttlar[1,c]

[1]Department of Civil and Environmental Engineering, University of Illinois, 205 N. Mathews Avenue, Urbana, IL 61801, United States

[a]edave@illinois.edu, [b]paulino@illinois.edu, [c]buttlar@illinois.edu

Keywords: viscoelasticity, functionally graded materials, numerical simulations, correspondence principle, finite-element method

Abstract. Asphalt concrete pavements are inherently graded viscoelastic structures. Oxidative aging of asphalt binder and temperature cycling due to climatic conditions are the major cause of such graded non-homogeneity. Current pavement analysis and simulation procedures either ignore or use a layered approach to account for non-homogeneities. For instance, the recently developed Mechanistic-Empirical Design Guide (MEPDG) [1], which was recently approved by the American Association of State Highway and Transportation Officials (AASHTO), employs a layered analysis approach to simulate the effects of material aging gradients through the depth of the pavement as a function of pavement age. In the current work, a graded viscoelastic model has been implemented within a numerical framework for the simulation of asphalt pavement responses under various loading conditions. A functionally graded generalized Maxwell model has been used in the development of a constitutive model for asphalt concrete to account for aging and temperature induced property gradients. The associated finite element implementation of the constitutive model incorporates the generalized iso-parametric formulation (GIF) proposed by Kim and Paulino [2], which leads to the graded viscoelastic elements proposed in this work. A solution, based on the correspondence principle, has been implemented in conjunction with the collocation method, which leads to an efficient inverse numerical transform procedure.

This work is the first of a two-part paper and focuses on the development, implementation and verification of the aforementioned analysis approach for functionally graded viscoelastic systems. The follow-up paper focuses on the application of this approach.

Introduction and Motivation

Functionally graded materials (FGMs) are characterized by spatially varying constitutive properties that are generally contributed through non-uniform microstructure. In broader sense, the FGMs could be classified as engineered FGMs and non-engineered FGMs. Engineered FGMs have broad range of applications including for example, biomechanical, automotive, aerospace, mechanical, civil, nuclear, and naval engineering [3][4]. Extensive research has been carried out in fields of designing and optimizing the material distribution and properties as well as manufacture of the engineered FGMs [5]. Non-engineered FGMs include materials that naturally exhibit graded microstructure and properties (for example, bamboo[6]) as well as man-made materials and structures showing graded behavior due to other factors, such as construction practices, aging of materials and temperature dependent material properties. Asphalt concrete pavements are examples of the last category whereby effects of aging and temperature dependent material properties make them functionally graded structures.

Apart from exhibiting continuously graded properties, asphalt concrete exhibits viscoelastic material behavior at most service temperatures. In order to perform accurate and efficient analysis and design of asphalt pavement systems, it is important to utilize an approach that encompasses both viscoelastic (temporal) and functionally graded (spatial) material variations. This paper describes the development of the finite element based formulation for analysis of viscoelastic FGMs using correspondence principle. The formulation details along with implementation and verification are presented in this paper. The companion paper (Part-II) details the application of this research in context of analysis of asphalt pavements.

Formulation

This section describes the formulation developed in the current research. The subsections present, (a) viscoelastic constitutive relations, (b) elastic-viscoelastic correspondence principle, and (c) finite-element formulations.

Viscoelastic Constitutive Relations. Hilton [7] and Christensen [8] provided detailed overview on the constitutive relationships for viscoelastic materials. For non-homogeneous isotropic viscoelastic materials, the stress-strain relationship are written as:

$$\sigma_{ij}{}^{d}\left(x,t\right)=2\int_{t^{'}=-\infty}^{t^{'}=t} G_{ijkl}\left(x,\xi\left(t\right)-\xi\left(t^{'}\right)\right)\varepsilon_{kl}{}^{d}\left(x,t^{'}\right)dt^{'} . \tag{1}$$

$$\sigma_{kk}\left(x,t\right)=3\int_{t^{'}=-\infty}^{t^{'}=t} K_{kkll}\left(x,\xi\left(t\right)-\xi\left(t^{'}\right)\right)\varepsilon_{ll}\left(x,t^{'}\right)dt^{'} . \tag{2}$$

Here σ_{ij} are the stresses, ε_{kl} are the strains, super-script d represents the deviatoric (shear) components at any location x. These parameters G_{ijkl} and K_{ijkl} are the shear and bulk moduli and $\xi^{'}$ is the integration variable. Subscripts $(i,j,k,l=1,2,3)$ refer to Einstein summation convention. ξ is the reduced time, which is related to real time t and temperature T through time-temperature superposition principle given by:

$$\xi(t)=\int_{0}^{t} a\left(T\left(t^{'}\right)\right)dt^{'} . \tag{3}$$

Stress and strain components are related through,

$$\sigma_{ij}{}^{d}=\sigma_{ij}-\frac{1}{3}\sigma_{kk}\delta_{ij} . \tag{4}$$

$$\varepsilon_{ij}{}^{d}=\varepsilon_{ij}-\frac{1}{3}\varepsilon_{kk}\delta_{ij} . \tag{5}$$

δ_{ij} is the Kronecker delta.

For a non-homogeneous viscoelastic body in quasi-static condition with boundary conditions imposed as, displacement u_i on volume Ω_u and traction P_i on surface Ω_σ. Using Eq. (1), (2) and (4), one obtains the stress-strain relationship as:

$$\sigma_{ij}\left(x,t\right)=2\int_{t^{'}=-\infty}^{t^{'}=t} G_{ijkl}\left(x,\xi\left(t\right)-\xi\left(t^{'}\right)\right)\varepsilon_{kl}{}^{d}\left(x,t^{'}\right)dt^{'}+\int_{t^{'}=-\infty}^{t^{'}=t} K_{iikk}\left(x,\xi\left(t\right)-\xi\left(t^{'}\right)\right)\varepsilon_{kk}\left(x,t^{'}\right)\delta_{ij}\,dt^{'} . \tag{6}$$

Equilibrium (assuming no body forces) and strain-displacement relationships (for small deformations) are as shown in Equations (7) and (8).

$$\sigma_{ij,j}=0 . \tag{7}$$

$$\varepsilon_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right). \tag{8}$$

where, u is displacement and $\left(\square\right)_{,j}=\partial\left(\square\right)/\partial x_j$.

Elastic-Viscoelastic Correspondence Principle. The correspondence between elastic solutions and transformed viscoelastic solutions can be found, for example, in the work by Read [9]. It has been extensively utilized for obtaining a variety of viscoelastic solutions and formulation for various engineering problems ranging from beam problems [10] to fracture [11]. The applicability

of the correspondence principle in case of non-homogeneous materials is restrictive as established by several authors [12-14]. Here the constitutive properties are assumed to have separable spatial and temporal portions.
The Laplace transform of the equilibrium equation (Eq. (7)) is,

$$\begin{aligned}\tilde{\sigma}_{,j}(x,s) &= 2\tilde{G}(x,s)\tilde{\varepsilon}_{,j}^{\,d}(x,s)+2\tilde{G}_{,j}(x,s)\tilde{\varepsilon}^{d}(x,s)\\ &\quad+\tilde{K}(x,s)\tilde{\varepsilon}_{,j}(x,s)+\tilde{K}_{,j}(x,s)\tilde{\varepsilon}(x,s).\end{aligned} \tag{9}$$

Here s is the transformation variable and the symbol tilde (~) on top of the variables represents transformed variable. The Laplace transform of any function $f(t)$ is given by,

$$L[f(t)] = \tilde{f}(s) = \int_0^\infty f(t)Exp[-st]dt\,. \tag{10}$$

Inspection of Eq. (9) shows that the transformed non-homogeneous viscoelastic problem description has form identical with that of non-homogeneous elastic problem; hence the elastic solution could be readily utilized for the viscoelastic problem. Similar correspondence could be utilized for developing finite-element formulation for solving non-homogeneous viscoelastic problems.
Finite-Element Formulation. Variational principles have been established for linear viscoelastic problems [15], the first variation forms the basis for finite-element formulation. The variational principle proposed by Taylor et al. [16] for thermo-viscoelastic boundary value problem has been utilized in the current paper. For a body with volume Ω_u with tractions P_i imposed on surface Ω_σ, the first variation is given by,

$$\begin{aligned}\delta\Pi=&\int_{\Omega_u}\int_{t'=-\infty}^{t'=t}\int_{t''=-\infty}^{t''=t-t'}\left\{C_{ijkl}\left[x,\xi_{ijkl}(t-t'')-\xi'_{ijkl}(t')\right]\frac{\partial}{\partial t'}\left(\varepsilon_{ij}(x,t')-\varepsilon^{*}_{ij}(x,t')\right)\frac{\partial\delta\varepsilon_{kl}(x,t'')}{\partial t''}\right\}dt'\,dt''\,d\Omega_u\\ &-\int_{\Omega_\sigma}\int_{t''=-\infty}^{t''=t}P_i(x,t-t'')\frac{\partial\delta u_i(x,t'')}{\partial t''}dt''\,d\Omega_\sigma=0.\end{aligned} \tag{11}$$

Here, u_i are the displacements and C_{ijkl} are space and time dependent material constitutive properties. Moreover, ε_{kl} are the mechanical strains and ε_{kl}^{*} are the thermal strains and ξ is the reduced time related to real time t and temperature T through time-temperature superposition principle given in Equation (3).

The element displacement vector u_i is related to nodal displacement degrees of freedom q through the isoparametric shape functions N_{ij},

$$u_i(x,t) = N_{ij}(x)q_j(t)\,. \tag{12}$$

Differentiation of Equation (12) yields the relationship between strain ε_i and nodal displacements q_j and derivatives of shape functions B_{ij},

$$\varepsilon_i(x,t) = B_{ij}(x)q_j(t)\,. \tag{13}$$

Eqs. (11), (12) and (13) provide the equilibrium equation for each finite element, i.e.

$$k_{ij}(x,\xi(t))q_j(0)+\int_{0^+}^{t}k_{ij}\left(x,\xi(t)-\xi(t')\right)\frac{\partial q_j(t')}{\partial t'}dt' = f_i(x,t)+f_i^{th}(x,t)\,. \tag{14}$$

Here k_{ij} is the element stiffness matrix, f_i is the mechanical force vector and f_i^{th} is the thermal force vector, which are described as following expressions:

$$k_{ij}\left(x,t\right)=\int_{\Omega_u} B_{ik}^{T}\left(x\right)C_{kl}\left(x,\xi\left(t\right)\right)B_{lj}\left(x\right)d\Omega_u \,. \tag{15}$$

$$f_i\left(x,t\right)=\int_{\Omega_\sigma} N_{ij}\left(x\right)P_j\left(x,t\right)d\Omega_\sigma \,. \tag{16}$$

$$f_i^{th}\left(x,t\right)=\int_{\Omega_u}\int_{-\infty}^{t} B_{ik}\left(x\right)C_{kl}\left(x,\xi(t)-\xi\left(t^{'}\right)\right)\frac{\partial \varepsilon^{*}{}_{l}\left(x,t^{'}\right)}{\partial t^{'}}dt^{'}\,d\Omega_u \,. \tag{17}$$

$$\varepsilon^{*}{}_{l}\left(x,t\right)=\alpha\left(x\right)\Delta_T\left(x,t\right). \tag{18}$$

Here α is the coefficient of thermal expansion and Δ_T is the temperature change with respect to initial conditions.

On assembly of the individual finite element contributions for the given problem domain, the global equilibrium equation can be obtained as:

$$K_{ij}\left(x,\xi\left(t\right)\right)U_j\left(0\right)+\int_{0^+}^{t} K_{ij}\left(x,\xi\left(t\right)-\xi\left(t^{'}\right)\right)\frac{\partial U_j\left(t^{'}\right)}{\partial t^{'}}dt^{'}=F_i\left(x,t\right)+F_i^{th}\left(x,t\right). \tag{19}$$

Here K_{ij} is the global stiffness matrix, U_i is the global displacement vector, and F_i and F_i^{th} are the global mechanical and thermal force vectors. The solution to the problem requires solving the convolution shown above to determine nodal displacements. The direct integration for solving hereditary integral has significant computational cost. In order to apply correspondence principle the Laplace transform of the equilibrium equation (Eq.(19)) leads to:

$$\tilde{K}_{ij}\left(x,s\right)\tilde{U}_j\left(s\right)=\tilde{F}_i\left(x,s\right)+\tilde{F}_i^{th}\left(x,s\right). \tag{20}$$

Notice that the Laplace transform of hereditary integral (Eq.(19)) led to an algebraic relationship, (Eq.(20)), with form identical to that of elastic formulation. Thus solution to non-homogeneous viscoelastic problem obtained using correspondence principle based finite-element implementation, which is broadly itemized into the following steps:

- Define force vectors, $F(x,t)$ and $F^{th}(x,t)$, and stiffness matrix, $K(x,t)$
- Perform Laplace transform to obtain, $F(x,s)$, $F^{th}(x,s)$ and $K(x,s)$
- Solve linear system of equation to obtain transformed solution, $U(s)$
- Perform inverse Laplace transform to obtain solution, $U(t)$.

The numerical inversion of Laplace transform has been extensively studied and several methods have been proposed, for example, Sutradhar et al. [17] used Stehfest's algorithm for boundary element analysis of heat flow in FGMs. In the current research collocation methods proposed by Schapery [18] is utilized.

Graded Finite-Elements. In the conventional finite-element analysis method a single set of properties are assigned to an element, which makes it an unattractive method of choice for simulation of graded materials. Graded elements allow for non-homogeneous material distribution within an element. Lee and Erdogan [19] and Santare and Lambros [20] have used graded elements with direct Gaussian integration. This type of formulation involves selection of material properties directly at the Gauss integration points. Kim and Paulino [2] proposed graded elements with generalized isoparametric formulation (GIF). In case of GIF, the constitutive material properties are selected at each nodal point and interpolated back to the Gauss-quadrature points (Gaussian integration points) using isoparametric shape functions which makes it an appropriate approach for capturing material gradation.

In case of GIF, material properties are interpolated to the integration points as:

$$C_{\text{Int.Point}}=\sum_{i=1}^{m} C_i N_i \,. \tag{21}$$

Here N_i are the shape functions corresponding to node i, and m is the number of nodal points in the element. This concept is illustrated in Fig. 1. The figure shows the sampling of the material properties at element nodes. The non-homogeneous material properties are shown by the shaded plane and property sampling is indicated by the arrows in z-direction. The bold faced (red-colored) arrows show the interpolation of material properties from nodal points to one of the Gaussian integration points (shown by plus marks). Paulino and Kim [21] have demonstrated the importance of GIF to account for two length scales for simulation of non-homogeneous problems, (a) element size, and (b) scale associated with material non-homogeniety. Proper consideration of both is important to obtain accurate results.

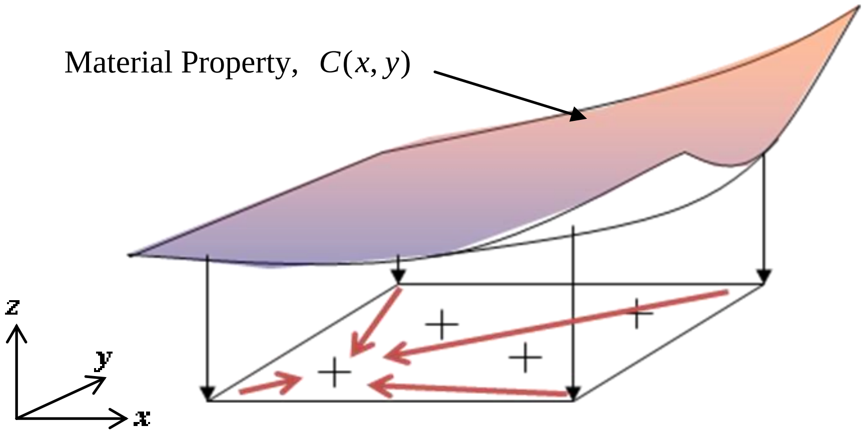

Fig. 1 Generalized Isoparametric Graded Finite Element

Implementation and Verification

The formulation presented in the previous section has been implemented for two-dimensional analysis using plane-strain, plane-stress or axisymmetric assumptions. The implementation has been performed for linear and quadratic isoparametric shape functions. Verification of the graded elements was performed in similar manner as reported by Kim and Paulino [2]. The viscoelastic analysis portion of the implementation was verified by simulation of creep in a viscoelastic bar and by comparing simulation results with analytical solution.

For further verification and to make comparison with conventional finite element procedure, a viscoelastic beam in three-point bending configuration is simulated using the proposed procedure as well as using the commercially available analysis code *ABAQUS*. The graded viscoelastic properties at the top and bottom of the beam are shown in Fig. 2(a). In case of *ABAQUS* simulations the graded problem was transformed to layered problem with varying degrees of refinement, where each layer was assumed to have average properties sampled at the midpoint. Fig. 2(b) shows selected results with comparison between FGM and conventional analysis approaches.

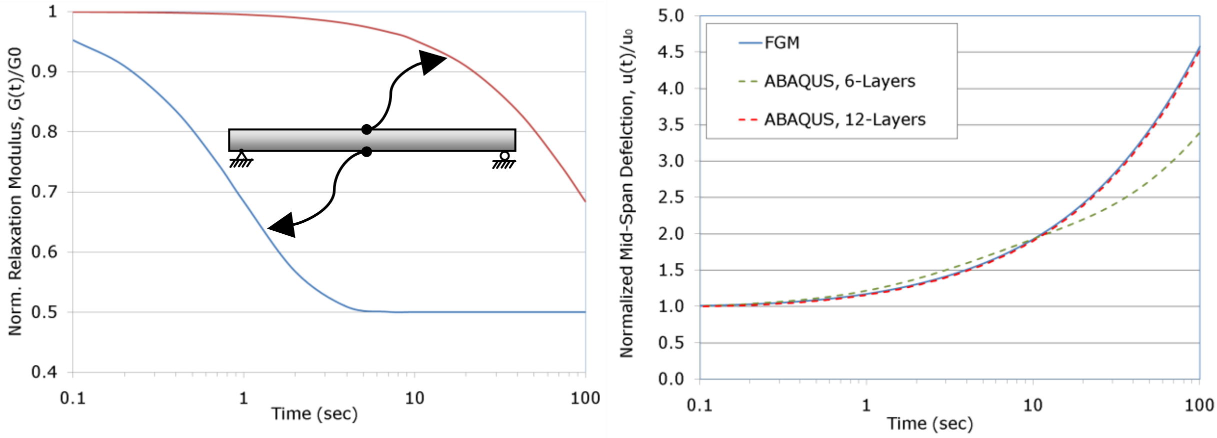

Fig. 2 Comparison of Graded Viscoelastic Analysis with Commercial Software (*ABAQUS*)

Summary

A variety of engineering problems require analysis of functionally graded viscoelastic systems. The current research proposes a formulation for finite element analysis of viscoelastic functionally graded problems using the elastic-viscoelastic correspondence principle. The formulation is implemented and verified. The application examples in context of asphalt concrete pavements are discussed in the companion paper titled “Asphalt Pavement Aging and Temperature Dependent Properties through a Functionally Graded Viscoelastic Model, Part-II: Applications”.

References

[1] ARA Inc., ERES Consultants, "Guide for Mechanistic-Empirical Design of New and Rehabilitated Pavement Structures," National Cooperative Highway Research Program, NCHRP Project 1-37A Final Report, 2002.

[2] J. Kim, and G.H. Paulino: Journal of Applied Mechanics Vol. 69-4 (2002) p. 502-14

[3] Suresh, S., and Mortensen, A., "Functionally Graded Materials," The Institute of Materials, IOM Communications Ltd., London, 1998.

[4] Y. Miyamoto, W.A. Kaysser, B.H. Rabin, eds., "*Functionally Graded Materials: Design, Processing and Applications*," Kluwer Acedemic, Dordrecht, The Netherlands, 1999.

[5] G.H. Paulino, M.-J. Pindera, R.H. Dodds, F.A. Rochinha, E.V. Dave, and L. Chen eds.: *Proceedings of the Multiscale and Functionally Graded Materials 2008 Conference*, American Institute of Physics, Melville, NY, 2008.

[6] E.C.N. Silva, M.C. Walters, and G.H. Paulino: J.Mater.Sci. Vol. 41-21(2006) p. 6991-7004

[7] Hilton, H.H., "Viscoelastic Analysis," *Engineering Design for Plastics,* Reinhold, New York, 1964, pp. 199.

[8] Christensen, R.M., "Theory of Viscoelasticity," Dover Publications, Inc., Mineola, New York, 1982.

[9] W.T. Read Jr.: J.Appl.Phys. Vol. 21 (1950) p. 671-674

[10] Hilton, H.H., and Piechocki, J.J.: Shear Center Motion in Beams with Temperature Dependent Linear Elastic or Viscoelastic Properties, *Proceedings of the Forth U.S. National Congress of Applied Mechanics*, The American Society of Mechanical Engineers, New York, 1962, pp. 1279-1289.

[11] G.H. Paulino, and Z. Jin: Journal of Applied Mechanics Vol. 68-2(2001) p. 284-93

[12] G.H. Paulino, and Z. Jin: Journal of Applied Mechanics Vol. 68-1(2001) p. 129-32

[13] S. Mukherjee, and G.H. Paulino: Journal of Applied Mechanics Vol. 70-3(2003) p. 359-63

[14] L. Khazanovich: Int.J.Solids Structures Vol. 45-17(2008) p. 4739-4747

[15] M.E. Gurtin: Archives of Rational Mechanics and Analysis Vol. 36(1963) p. 179-185

[16] R.L. Taylor, K.S. Pister, and G.L. Gordou: International Journal of Numerical Methods in Engineering Vol. 2-1(1970) p. 45-59

[17] A. Sutradhar, G.H. Paulino, and L.J. Gray: Eng.Anal.Boundary Elements Vol. 26-2(2002) p. 119-132

[18] R.A. Schapery: Franklin Institute Journal Vol. 279-4(1965) p. 268-289

[19] Y. Lee, and F. Erdogan: Int.J.Fract. Vol. 69-2(1995) p. 145-65

[20] M.H. Santare, and J. Lambros: Transactions of the ASME.Journal of Applied Mechanics Vol. 67-4(2000) p. 819-22

[21] G.H. Paulino, and J. Kim: Journal of the Brazilian Society of Mechanical Sciences and Engineering Vol. 29-1(2007) p. 63-81