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On The Modeling of Structural Dynamics of Risers Composed of Functionally Graded Materials

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Abstract: This work aims to provide a numerical framework for the dynamic behavior representation of riser structures, considering the use of functionally graded materials (FGM). In this respect, a new corotational finite element formulation for the numerical representation of such risers is considered, including the effects of geometric presented to show the numerical model capabilities on representing the important kinematics of a riser structure in dynamics.

Keywords: risers, functionally graded material, dynamic behavior representation, large displacement motions

NOMENCLATURE

u, v, w = displacement vector coordinates, m
x, y, z = Cartesian coordinates, m
A = element cross section area, m2
E = material Young's modulus, N/m^2
G = material shear modulus, N/m^2
I_y, I_z = cross section axial moment of inertia, m^4
J = cross section polar moment of inertia, m^4
I, J = element node numbers, dimensionless
L = element length, m
U = element strain energy
r = position vector, m
u = displacement vector, m
R = frame transformation matrix, dimensionless

Greek Symbols

$\overline{\vartheta}$ = cross-section rotation vector referred to local coordinates
$\kappa_x, \kappa_y, \kappa_z$ = curvatures w.r.t. x,y,z
ε = center line linear strain component
γ = shear strain component
ξ = local radial coordinate

Subscripts

C relative to corotational coordinates
G relative to global coordinates
T relative to convective coordinates
0 relative to undeformed (initial) coordinates

Superscripts

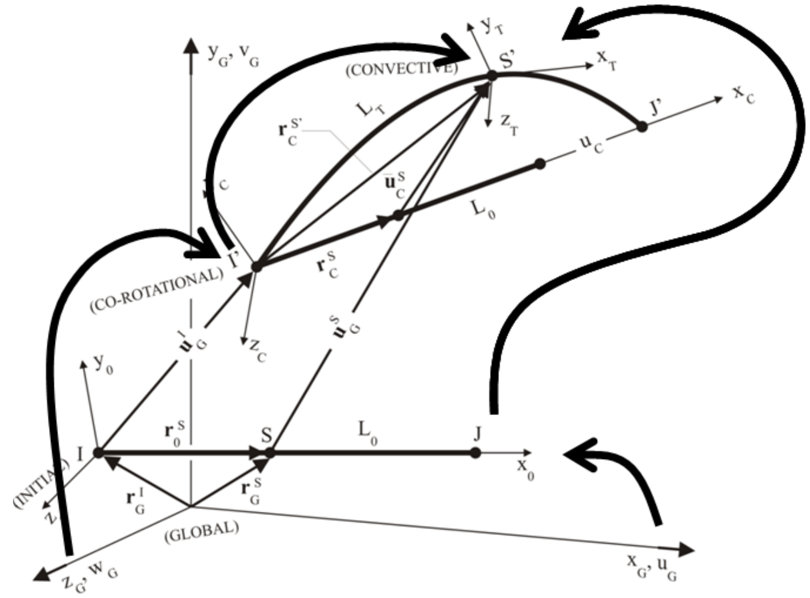
I relative to element node I
S relative to a general position along the element's length
C relative to corotational Coordinates
T relative to convective coordinates
0 relative to undeformed (initial) coordinates

INTRODUCTION

The large demand for oil has brought its exploitation to more difficult and risky proven reserve areas, what has driven engineering to a significant increase for research and technological solutions - as in deep seashore waters -, requiring the development of new types of marine structures. The riser is one of the most keen and sensitive of such structures. It is basically a very long and slender pipe designed to convey oil products from deep sea to a tank placed in a floating platform. Due to adverse working conditions the riser structure should sustain, in good balance, the thermo-mechanical strength needed to accommodate thermal induced loadings, both along its length as per through the cross-section thickness, as well as general mechanical forces due to self weight – which includes the inside fluid weight -, bouyance, sea currents and waves, concentrated and distributed buoys, sea bottom contact and platform motions. Moreover, working conditions required at inner and outer surfaces of a riser are most of the time quite distinct, demanding for a material that should combine the best properties of ceramics and metals such as low density and high strength, good temperature and corrosion resistance, high toughness and good machinability, just to mention a few. To try to accomplish such demand, new materials have been formed which possess such properties, in a desired property gradation, in spatial directions. These continuous materials are able to reduce thermal stresses, residual stresses and stress concentration in the transition region, as occurring in multi-layered type risers.

THE FINITE ELEMENT FORMULATION

In this work a numerical framework for the dynamic behavior representation of such structures, considering the use of functionally graded materials (FGM), is presented. The riser is numerically represented by a corotational finite element beam formulation [1,3], including the effects of geometric nonlinearities as the structure undergoes to very large displacements and rotations. Thus, a two node finite element beam formulation that represents the important straining due to stretch, torsional and bending kinematics - all referred to the reference frame attached to the element initial node coordinates, in a Total Lagrangian formulation scheme - was employed. In the kinematics considered, as shown in Fig. 1, the element centre line moves from an initial configuration I-J to its current (final) spatial position I'-J', with the corotational coordinate system defined from the coordinates of the element nodes in the final configuration.



$$(x_G , y_G , z_G) \; - \; \mathbf{R}_G^C \; \rightarrow \; (x_C , y_C , z_C) \; - \; \mathbf{R}_C^T \; \rightarrow \; (x_T , y_T , z_T)$$

$$(x_G , y_G , z_G) \; - \; \mathbf{R}_G^0 \; \rightarrow \; (x_0 , y_0 , z_0) \; - \; \mathbf{R}_0^T \; \rightarrow \; (x_T, y_T , z_T)$$

Figure 1 – Two node beam element coordinates and frame transformations

$\mathbf{R}_G^C$, $\mathbf{R}_C^T$, $\mathbf{R}_G^0$ and $\mathbf{R}_0^T$ are frame transformations between element space coordinates from global to corotational, from corotational to convective, from global to initial and from initial to convective, respectively. Considering a typical point S at the element center line and longitudinal initial position $\mathbf{r}_0^S$, the associated effective displacement vector – displacement quantity measured from the corotational frame - is given , in global coordinates, by the following vector sum,

$$\bar{\mathbf{u}}_{\mathbf{C}}^{S} = \mathbf{R}_{\mathbf{G}}^{\mathbf{C}} (\mathbf{r}_{\mathbf{G}}^{S} + \mathbf{u}_{\mathbf{G}}^{S} - \mathbf{r}_{\mathbf{G}}^{I} - \mathbf{u}_{\mathbf{G}}^{I}) - \mathbf{r}_{\mathbf{0}}^{S} \tag{1}$$

with all vector quantities pictorially defined in Fig. 1. Out of the element center line the displacements are evaluated under the assumption that cross sections initially plane must remain plane after beam deformations, requiring that material global rotations - at any position point S - must be defined. This is accomplished by using the identity transformation condition,

$$\mathbf{R}_{\mathbf{G}}^{\mathbf{C}} \, \mathbf{R}_{\mathbf{C}}^{\mathbf{T}} = \mathbf{R}_{\mathbf{G}}^{\mathbf{0}} \, \mathbf{R}_{\mathbf{0}}^{\mathbf{T}} \tag{2}$$

which results in

$$\mathbf{R}_{\mathbf{C}}^{\mathbf{T}} = \left(\mathbf{R}_{\mathbf{G}}^{\mathbf{C}} \right)^{T} \mathbf{R}_{\mathbf{G}}^{\mathbf{0}} \, \mathbf{R}_{\mathbf{0}}^{\mathbf{T}} \tag{3}$$

with local – effective – cross-section rotation vector at S obtained from the following skew-symmetric matrix evaluation [2],

$$\overline{\vartheta}^{S}_{C} = \begin{bmatrix} \overline{\vartheta}^{S}_{x\,C} & \overline{\vartheta}^{S}_{y\,C} & \overline{\vartheta}^{S}_{z\,C} \end{bmatrix} = \log_e \mathbf{R}^{\mathbf{T}}_{\mathbf{C}} \tag{4}$$

Equations (1) and (4) furnish a set of local corotational displacements and rotations, at any section-S, all referred to the global coordinate system considered and free from rigid body motions. Element I-J strain measures are then set from these displacements that can furnish the strain energy due to beam modes of deformation for a homogeneous material model, as in the form

$$U = \frac{1}{2}\int_0^{L_0} (EA\varepsilon^2 + GA\gamma^2 + GJ\,\kappa_x^2 + EI_y\,\kappa_y^2 + EI_z\,\kappa_z^2)\,dx_0 \tag{5}$$

where ε, γ and $\kappa_{y,z}$ are the center line 'local' linear and shear strain components and, curvature measures, respectively, κ_x is torsional rotation, A, $I_{y,z}$ and J are cross section geometric parameters and E and G are the material constants.

Thus, element internal forces and tangent stiffness matrix are obtained through successive differentiations of U with respect to displacements referred to co-rotational reference frame, as in the following equations (with i,j=1,2,3)

$$\mathbf{F}^T = \begin{bmatrix} \dfrac{\partial U}{\partial \overline{u}^{S}_{i\,C}} & \dfrac{\partial U}{\partial \overline{\vartheta}^{S}_{i\,C}} \end{bmatrix} \qquad k_{ij} = \begin{bmatrix} \dfrac{\partial^2 U}{\partial \overline{u}^{S}_{i\,C}\,\partial \overline{u}^{S}_{j\,C}} & \dfrac{\partial^2 U}{\partial \overline{u}^{S}_{i\,C}\,\partial \overline{\vartheta}^{S}_{j\,C}} \\ \text{symm.} & \dfrac{\partial^2 U}{\partial \overline{\vartheta}^{S}_{i\,C}\,\partial \overline{\vartheta}^{S}_{j\,C}} \end{bmatrix} \tag{6}$$

where the following notation applies

$$\begin{matrix} \overline{u}^{S}_{1\,C} \equiv \overline{u}^{S}_{x\,C} & \overline{u}^{S}_{2\,C} \equiv \overline{u}^{S}_{y\,C} & \overline{u}^{S}_{3\,C} \equiv \overline{u}^{S}_{z\,C} \\ \overline{\vartheta}^{S}_{1\,C} \equiv \overline{\vartheta}^{S}_{x\,C} & \overline{\vartheta}^{S}_{2\,C} \equiv \overline{\vartheta}^{S}_{y\,C} & \overline{\vartheta}^{S}_{3\,C} \equiv \overline{\vartheta}^{S}_{z\,C} \end{matrix} \tag{7}$$

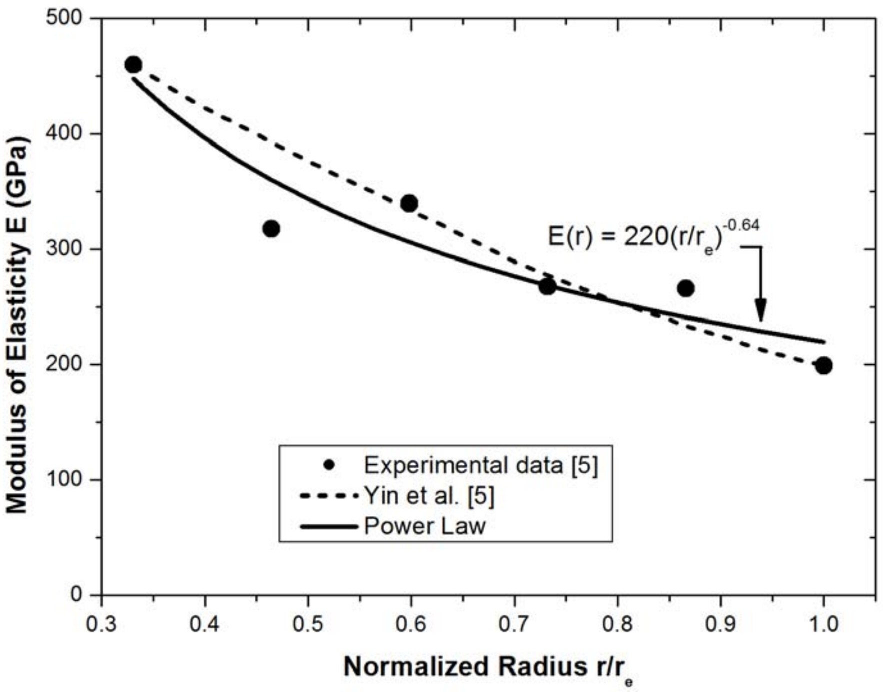


Figure 2 – Power Law Approximation of FGM Young's Modulus Variation Along Riser Thickness

In the element implementation Hermitian functions were employed in displacement representations along the length of the element. Therefore, linear functions were used for the axial displacements and torsional rotations, quadratic functions for the bending rotations and cubic functions for the element transverse displacements.

In considering the FGM non-homogeneous isotropic material model of pipe cross section risers, in its linear elastic range, a continuous variation of the elastic constants through the wall thickness were obtained from previously published experimental results, provided by Paulino, Yin et al. in [4,5], for TiC-Ni_3Al alloy. As shown in Fig. 2 published experiments were approximated by a continuous function to provide the "material law" for the Young's Modulus r-variation through the pipe thickness, using a least square numerical approximation. In this particular, a power law equation was established and equivalent values for EA, GJ, EI_y, EI_z constants, in Eq (5), have been derived in closed form. This approach was used in the examples considered in the next section.

SAMPLE ANALYSES

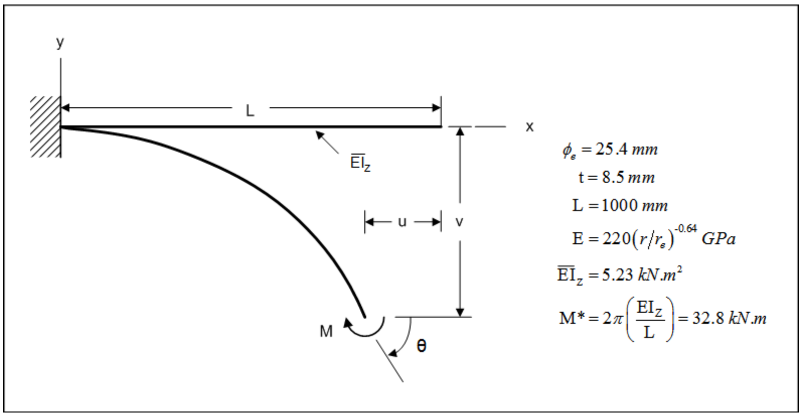


Figure 3 – The cantilever pipe beam considered in the analysis

In this first study a ten equally spaced element model of a cantilever pipe, submitted to constant bending moment is considered. Cross section geometry parameters and the material law equation used are shown in Fig. 3. M* is the applied moment required for the beam to undergo to considerable large displacements and rotations as shown, qualitatively, in Fig. 4.

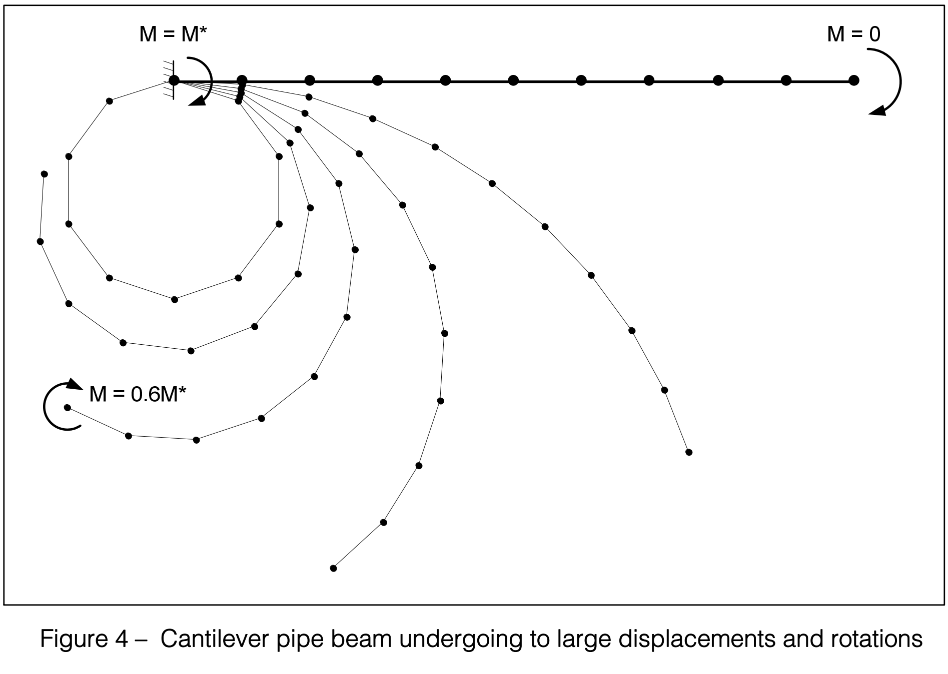


Figure 4 – Cantilever pipe beam undergoing to large displacements and rotations

Figure 5 shows comparisons of the obtained numerical results for displacements u and v and rotation θ at the tip of the beam to solutions presented in [6], for 0≤M≤M*. The numerical results were obtained using 100 equal load steps with displacement incrementally iterative procedure, as described in [1]. A good agreement in the results can be observed. Figure 6 displays two cuts of the normal stress radial distribution at the pipe cross section, for M= 0.6M*. Notice that for the FGM considered, the results are quite apart from the typical linear distribution obtained with homogeneous material beams. However, for solid section beams the two patterns cross the beam center line axis – or the neutral axis - .

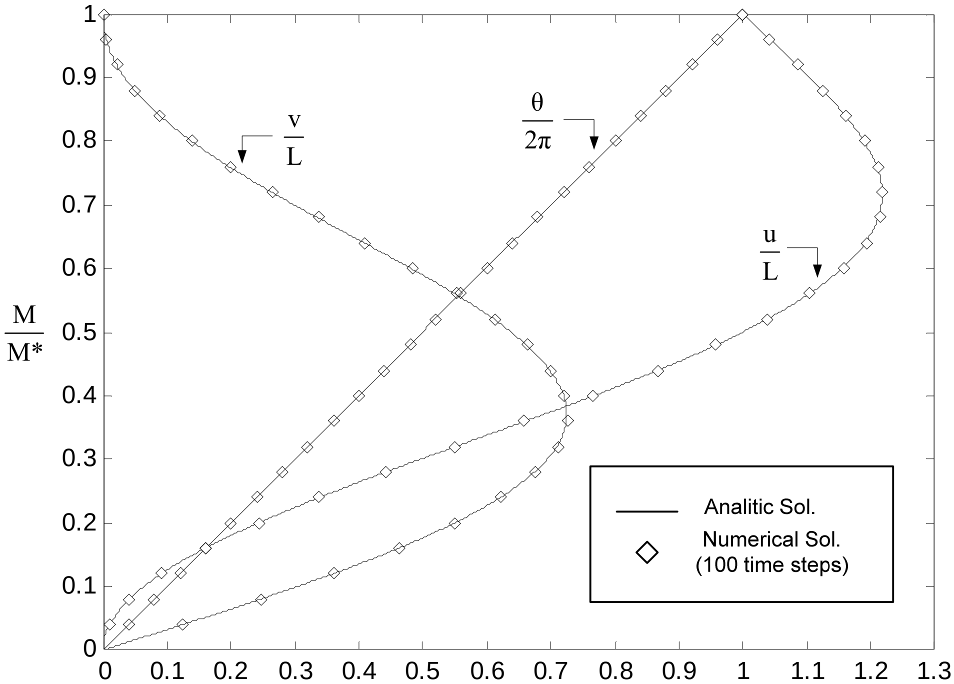


Figure 5 – Tip displacements and rotations of cantilever beam undergoing large displacements

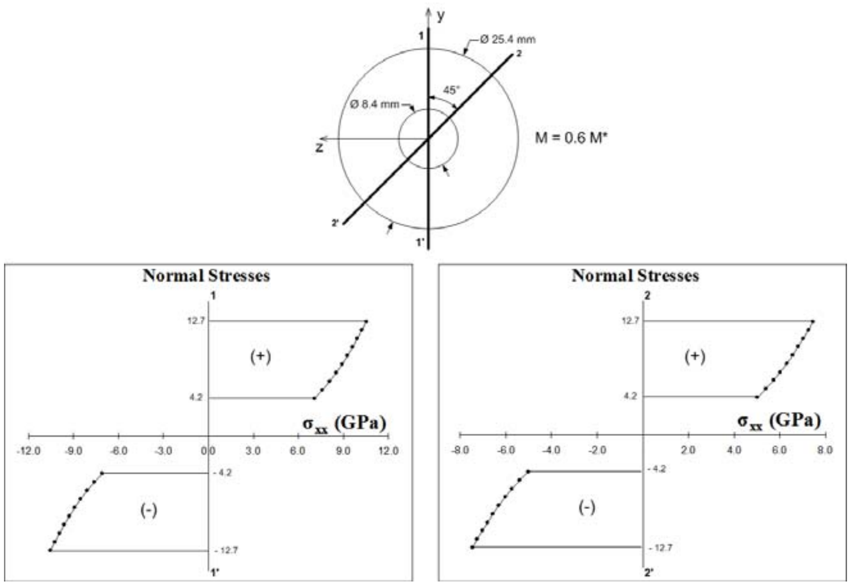


Figure 6 – Normal stresses at the cantilever pipe beam cross-section, for M=0.6 M*.

In a second application we use the bi-material (alumina-titanium alloy) pipe beam model where the transition is made smooth by inserting a FGM layer which thickness t_{FGM} can vary from 0% to 100% of the pipe thickness h. In all situations the FGM layer is placed at the mid-depth and the thicknesses of the alumina and the titanium are always kept equal. The objective is to evaluate their relative effects on free vibration. The FGM elastic modulus and mass density is varied according to power law, as shown in Fig. 7. Equivalent values for the beam rigidity modulus EI and line mass distribution ρA are evaluated for various cross-section material configurations and the numerical values for the natural frequencies associated to bending are obtained, considering the beam under two boundary conditions: a) the cantilevered and b) the cantilever-simply supported. These frequencies values are compared to analytical results furnished in Ref. [7]. A good agreement in the results was observed when ten or more equally spaced elements are used in the model, as in the plots shown in Fig. 8. From these comparisons, it is observed quite small differences in the first two frequencies, as the FGM layer thickness is increased. However these differences are magnified when higher mode frequencies are extracted. From these plots, the amplification factors are sequentially equal to 9.0-2.77-1.96-1.65 and 3.24-2.08-1.71-1.53 for cases a) and b), respectively.

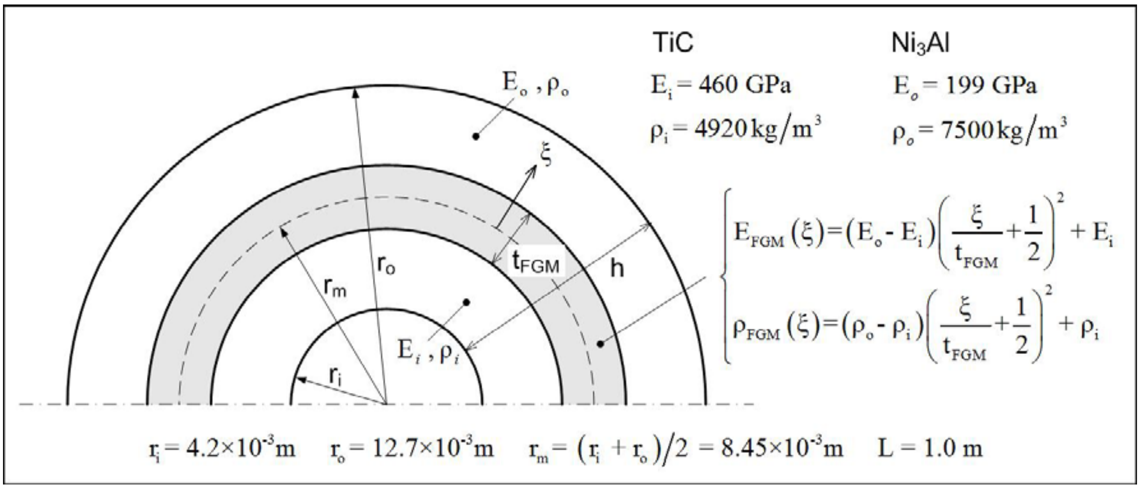


Figure 7 – Composed cross-section details as considered in the numerical analysis.

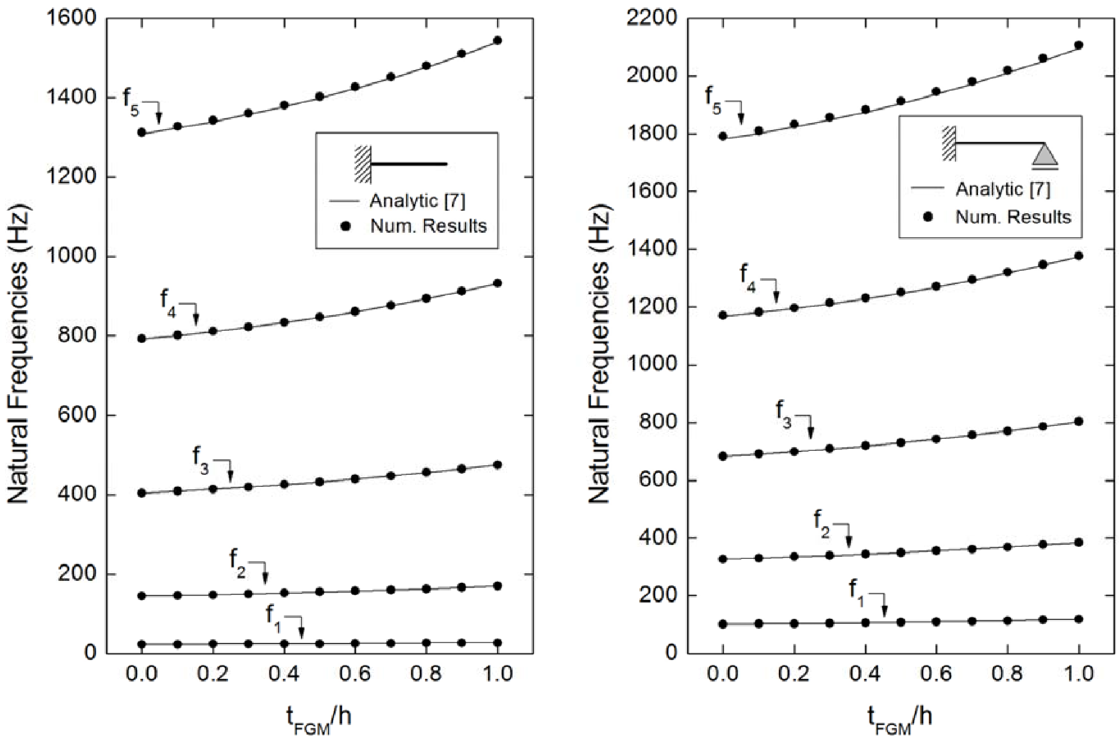


Figure 8 – First five flexure natural frequencies obtained for the composed beams considered .

CONCLUSIONS

A shear deformable two node finite element beam is presented, based on the corotational formulation referred to the spatial coordinates defined by element node. The element is based on the Timoshenko's constant transverse shear deformation theory accounting for large displacement kinematics, but in the small strain theory range. In this work, the element is used to study static and free vibration in FGM and composed risers sections, by adjusting the beam theory rigidity parameters to the equivalent ones considering cross-section evaluations in closed form. The Hermitian's shape functions used grant the element formulation exact results only for loadings that can be represented by linear bending moments and constant shear forces. Otherwise, FEM refinement procedures for element spatial displacement convergence results must be used.

It has been found that significant difference occurs in FGM riser stress results when they are compared to its parental homogeneous material beam mainly due to rigidity parameters evaluations and, most of all, to its variation along the pipe thickness. In the analyses considered a power law was obtained from previously published experiments. It can be re-evaluated using the numerical tool employed in the analyses. Also, as it does happen in risers the use of FGM layers in the cross section can be regarded as an effective way to smoothen stress jumps in multi-metal beams,.

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