Design of Structural Braced Frames Using Group Optimization

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ABSTRACT

Though topology optimization has been applied to many fields, ranging from mechanical to aerospace engineering, more work must be done to tailor it to the needs of the structural engineer, especially in regards to the design of high-rise buildings. Thus, this work aims to improve its application to structural engineering by describing an integrated topology optimization approach involving continuum and discrete finite elements to design the lateral systems in structural braced frames for high-rise buildings. The approach is implemented using concurrent continuum finite elements and discrete beam/truss elements to simplify and improve the overall design process by creating optimal geometries for a given volume of material. For example, after an engineer develops a structural frame consisting of beams and columns sized for gravity loads, topology optimization on the continuum (e.g. quadrilateral) elements is used to create a conceptual design for the braces of the lateral system resulting in highly efficient structures. Several practical examples are demonstrated to show the importance and relevance of this work to the structural design industry.

INTRODUCTION

In the construction industry, many works of modern architecture are lacking balance between engineering principals and architectural aesthetics. Often, the engineering can be seen as a means to make the architecture stand, where in this case the architecture is similar to a work of art. Often these structures may use much more material than required or cause difficulties in constructability and realization of the design. Other cases include buildings which are built solely to serve a specific function at low cost, with little or no regard for aesthetics. Examples of such structures include big box stores, warehouses, convention centers, and strip malls. This lack of balance provides motivation to introduce topology optimization to connect engineering and architecture in the design of structures. Topology optimization provides unique optimal solutions for a given design space and set of applied forces and boundary conditions. Such optimal solutions often provide aesthetic value in the realization of patterns in the final design as well.

Though topology optimization is typically know for its use in mechanical and aeronautical engineering, and it is becoming more common for civil engineering applications in recent years. Examples of such applications include the multi-story building design or long span bridge design applications presented in Stromberg et al. (2011a), Allahdadian and Boroomand (2010), Neves et al. (1995), or Huang and Xie (2008). Though the applications within the civil engineering field encompass a wide range of topics, the focus of this work is towards the high-rise building industry to provide engineers with a tool that can be used to identify the optimal topology of the lateral bracing systems in addition to minimize material usage and corresponding cost. Thus, this work aims to develop a methodology that enables engineers to design the lateral system from the conceptual optimal bracing angles to the final sizing of the members.

By using topology optimization to express the engineering together with the architecture, buildings can be developed with unique bracing systems. Such braced frame and moment frame structural systems are commonly deployed in the lateral design of high-rise buildings, such as the John Hancock Center (Chicago, IL), Broadgate Tower (London, UK) and Bank of China Tower (Hong Kong). However, the design of such systems is traditionally based on diagonal braces arranged according to 45 through 60 degree angles, though there have been few engineering studies in the past to identify the optimal bracing angle and the parameters affecting such angles (Huang et al., 2010). This work, on the other hand, aims to optimize the geometry of the frame members in terms of structural performance (maximum stiffness) while minimizing the material usage. Other measures of structural performance might also include tip deflection, frequency, critical buckling load, etc.

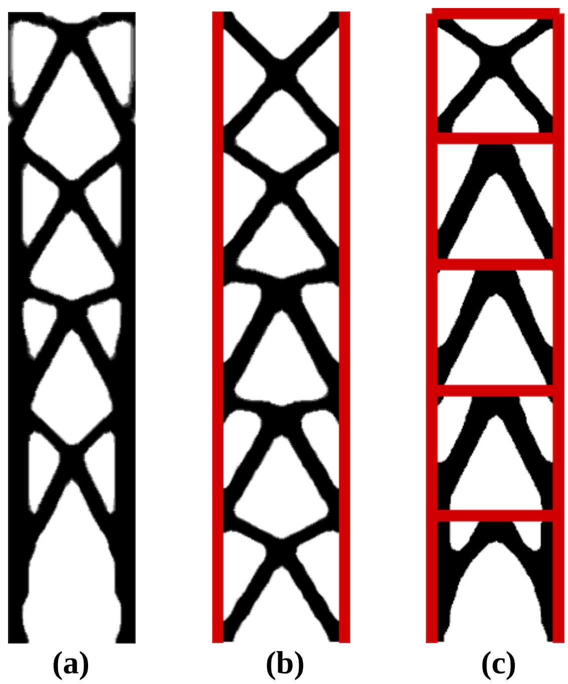


Figure 1. Comparison of existing topology optimization techniques with new methods proposed in this work: (a) continuum topology optimization with no frame elements, (b) with column elements (in red), (c) with beam and column frame elements (in red)

In Stromberg et. al (2011a), an early attempt was made to identify optimal bracing angles using continuum elements. However, some limitations were evident in the optimization results as seen in Figure 1 (left). For example, the results show high concentrations of material towards the extreme edges of the domain, as expected from the web-flange behavior discussed in Stromberg et. al (2011a). These dense regions of material make it difficult to locate the bracing work points (i.e. the locations where the diagonals intersect the columns). Furthermore, such high concentrations may lead to incorrect flexural stiffness in the analysis of the structure. Moreover, with a constraint imposed on the fraction of material available for the design, if the majority of the material is "optimal" at the extreme edges of the domain, relatively small amounts of material are leftover to form the diagonal members and the structure has an incomplete diagonalization, which is not practical for realistic building design. An additional constraint on the material distribution between the columns and diagonals might be imposed to circumvent the issue, but the addition of beam-column elements to the design domain eliminates the problem altogether.

For instance, with the additional column elements to the design problem as shown in Figure 1 (center), the issues mentioned are no longer of concern for structural design. The discrete column elements, much narrower in width, now give practical bending stiffness to the structure. Furthermore, the diagonalization is complete along the structure's height and the bracing members are clearly identified. We highlight next how this methodology might be incorporated into the structural design process.

GROUP OPTIMIZATION CONCEPTS

In this work, *group optimization* refers to the combination of different groups of elements (e.g. Q4, bar, beam, etc.) in designing an optimal structure. Using *group optimization*, a clear, complete diagonalization results for maximum stiffness design with minimal volume. This integration of beam and quadrilateral elements can be modeled using one of the two connections types described next, which can be incorporated into the classical topology optimization formulation by introducing a few modifications as described below.

Combining Q4 and Beam Elements. The beams and quadrilateral elements are connected in this work either at the extreme ends of the beam only, or continuously along the beam line, as illustrated by Figure 2. In the first case, the degrees of freedom shared between elements (shown in red) are only the two translations at the corners of the quadrilateral and edges of the beam mesh. For the latter, the two translations at coincident nodes are shared (in red) along both the beam and quadrilateral meshes.

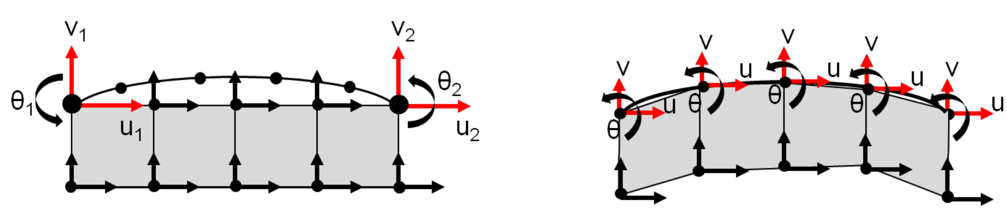


Figure 2. Connection types for combining continuum and discrete elements: (left) attached at extreme ends only, (right) attached continuously along the beam line

Problem Statement. In this work, the objective of the optimization problem is to maximize the overall stiffness of a building, or minimize the compliance. The outer skin or shell of the building is taken as the design domain (see Figure 3) so that the structural system would be expressed in the exterior together with the architecture. Thus, the optimal layout problem in terms of minimum compliance can be stated using the density, ρ, and the displacements, $\boldsymbol{u}$, as follows:

$$
\begin{aligned}
\min_{\rho,\boldsymbol{u}} \quad & c(\rho,\boldsymbol{u}) \\
s.t. \quad & \boldsymbol{K}(\rho)\boldsymbol{u} = \boldsymbol{f} \\
& \int_{\Omega} \rho \, dV < V_s \\
& \rho(\boldsymbol{x}) \in [0,1] \;\; \forall \boldsymbol{x} \in \Omega
\end{aligned}
$$

The overall compliance of the structure is denoted by c in the equations above, while $\boldsymbol{K}(\rho)$ represents the global stiffness matrix which depends on the material densities, $\boldsymbol{u}$ and $\boldsymbol{f}$ are the vectors of nodal displacements and forces, respectively, V_s is the volume fraction constraint which represents the maximum volume of material permitted for the design of the structure, and ρ is the material density for each design variable. A void is signified by a null material density, $\rho \rightarrow 0$, and $\rho \rightarrow 1$ represents solid material. For regions of gray material or intermediate densities, the commonly used Solid Isotropic Material with Penalization (SIMP) model is employed (Zhou and Rozvany (1991), Rozvany et al. (1992), Bendsoe (1989), Bendsoe and Sigmund (1999)):

$$
E(\boldsymbol{x}) = \rho(\boldsymbol{x})^p E_0
$$

This power-law relationship between stiffness and element density uses the Young's Modulus of solid material E_0 and penalization power $p \geq 1$ to force the material to tend towards 0 or 1 (void or solid respectively) where the element density ρ assumes a value somewhere in this range. The optimization process presented in this work for braced frames also includes continuation on the penalization power from 1 to 4 in steps of 0.5 until convergence.

Figure 3. Design domain (outer skin or shell) for topology optimization of a building

Design Process. The optimization techniques described in Baker (1992) help streamline the design decisions at various stages of a project from the conceptual characterization of a braced frame layout to the final sizing of the members. Combining the Principle of Virtual Work with the Lagrange Multiplier Method, the deflection, Δ, is expressed as

$$\Delta = \sum_i \frac{F_i f_i L_i}{E_i A_i} + \lambda \left(\sum_j A_j L_j - V \right)$$

where F_i and f_i are the internal forces due to the applied and unit loads respectively, L_i is the length, E_i is the Young's Modulus, and A_i is the cross-sectional area for the i^{th} member, and V is the total volume of the structure. Solving, the final cross-sectional areas required to achieve a target deflection, Δ_{req}, are given as

$$(A_i)_{req} = \frac{1}{\Delta_{req} E} (F_i f_i)^{0.5} \left[\sum_j L_j \left(F_j f_j \right)^{0.5} \right]$$

Once the overall shape or outer skin of the building is known, the optimal bracing layout could be established assuming that frame columns are arranged around the outer perimeter at a regular spacing to ensure that the tributary areas for the columns are similar. At each floor level, a horizontal beam (spandrel) would span between two subsequent columns. Beams and columns would be modeled using beam elements while the space bounded by two columns and two beams would be meshed using quadrilateral elements. After the finite element mesh is completed the following steps can be applied in sequence in the design flow process:

1. size vertical line elements (columns) according to gravity load combinations (accounting for dead, superimposed dead and live loads) according to Baker (1992)
2. run topology optimization on the continuum elements for lateral load combinations (accounting for wind and seismic loads)
3. identify the optimal bracing layout based on results and create frame model
4. optimize the member sizes using the virtual work methodology

These steps describe a complete process from the conceptual design and geometry up to the final sizing of the frame members. Throughout the design phase, these steps may be implemented in isolation or the entire process may be repeated to resize members.

OPTIMAL BRACED FRAMES

In this section, we explore several important analytical aspects of optimal braced frames in regards to the frame geometry.

Fully Stressed Design. The energy-based design method presented in Baker (1992) and described in the previous section implies that any frame with optimal cross-sectional members subject to a point load at the top is under a state of constant stress (Fully Stressed Design) as demonstrated next.

By taking the derivative of the equation for deflection with respect to the areas A_i and solving for the Lagrangian multiplier, we obtain:

$$\lambda = \frac{F_i f_i}{E A_i^2}$$

When the structure is loaded with a single point load at the top of the frame, the force can be expressed by $f_i = k \cdot F_i$ where k is a proportionality constant, thus the following equation holds:

$$\lambda = \left(\frac{F_i}{A_i}\right)^2 \frac{k}{E} = const$$

In the above expression the Lagrangian multiplier is a constant, therefore the stress in the i^{th} member, $\sigma = F_i/A_i$, is also a constant. This conclusion applies to each of the i^{th} members of the frame, therefore the stress level is constant in every member. It follows that the stress is then constant throughout the frame.

For structures where multiple loads are applied, the compliance is given using the external and internal work, W_{ext} and W_{int} and the applied forces and displacements P_i and u_i at the i^{th} node as follows:

$$W_{ext} = \sum_i P_i u_i = \sum_j \frac{F_j^2 L_j}{E A_j} = W_{int}$$

Using the Lagrange multiplier method as was done previously,

$$W_{ext} = \sum_j \frac{F_j^2 L_j}{E A_j} + \lambda \left(\sum_j A_j L_j - V \right)$$

Minimizing the compliance and solving for the Lagrange multiplier, we obtain the following expression:

$$\lambda = \left(\frac{f_i}{A_i}\right)^2 \frac{1}{E} = const$$

This expression shows that the minimization of compliance leads to a state of constant stress. We note that for the compliance minimization problem, a state of *constant strain energy density* represents the *condition of optimality* (Bendsoe and Sigmund, 2002). Since the Von Mises stresses and the strain energy density are proportional (Hill, 1950; Lubliner, 1990), the effective stresses in optimal structures are constant.

The constant stress condition is verified in the continuum approach for the structure in Figure 4 (bottom right) which was derived using a Q4 element mesh. As shown here, the Von-Mises stresses are nearly constant within each optimized member.

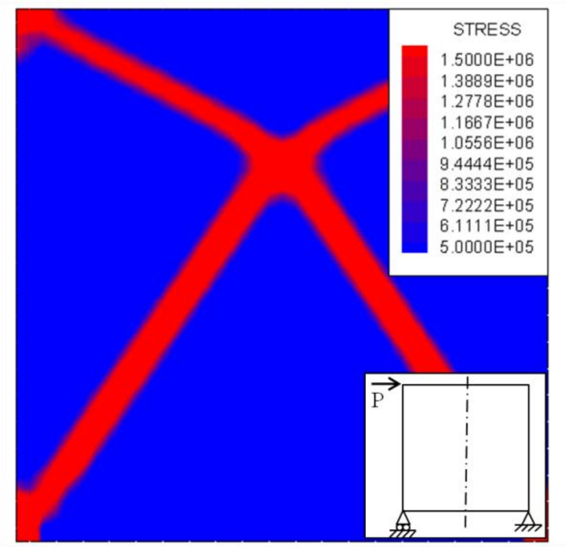


Figure 4. Plot of Von Mises Stresses for topology optimization problem of symmetric cantilever problem

Optimal Frame Geometry. The optimal geometry of braced frames can be described by considering the geometry shown in Figure 5, with the overall width of the structure given as $2B$, the total height as H and the unknown location of the base to the optimal bracing point as z.

By applying a unit load and taking advantage of the symmetry of the problem, the internal forces of each member are given as

$$f_1 = \frac{H - z}{B}$$

$$f_2 = \frac{\sqrt{B^2 + z^2}}{B}$$

$$f_3 = \frac{-\sqrt{B^2 + (H - z)^2}}{B}$$

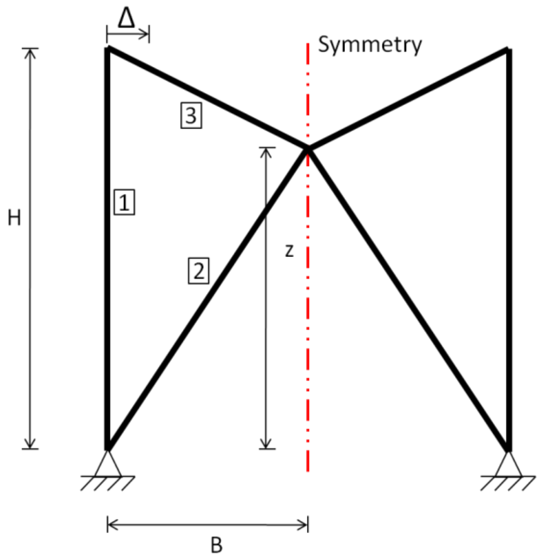


Figure 5. Single module frame geometry for discrete optimization problem with unknown bracing height, *z*

Assuming each member to have a constant stress, σ, (as previously demonstrated), the tip displacement can be written as

$$\Delta = \frac{F_i}{EA_i}\sum_i f_i L_i = \frac{\sigma B}{E}\sum_i \frac{f_i L_i}{B}$$

Thus, the tip displacement, or compliance in this problem, is minimal when

$$\frac{\partial \Delta}{\partial z} = \frac{\sigma B}{E}\frac{\partial}{\partial z}\left(\sum_i \frac{f_i L_i}{B}\right)$$

$$= \frac{\sigma B}{E}\frac{\partial}{\partial z}\left(H\left(\frac{H-z}{B^2}\right) + \frac{B^2+z^2}{B^2} + \frac{B^2+(H-z)^2}{B^2}\right)$$

$$= \frac{\sigma B}{E}\left(-\frac{H}{B} + \frac{2z}{B} - \frac{2(H-z)}{B}\right) = 0$$

So finally, the optimal brace work point height gives the minimum deflection when

$$z = \frac{3}{4}H$$

The rationale behind this solution can be explained by considering the topology optimization of a simple structural frame with a point load applied to the top left corner and symmetry constraints (see Figure 6). This problem (a simplification of a cantilever beam representing a "high-rise" problem) run with topology optimization does not lead to the 45 degree diagonalization, as one might expect, but rather to the "high-waisted" cross-brace with a working point at 75% of the structure's height. The cantilever ("high-rise") problem must also account for the overturning moment, PH, which is not present in the pure shear problem, causing a vertical shift in the intersection of the braces. (Note: the results presented here are dependent on the assumption of constant stresses in the members.)

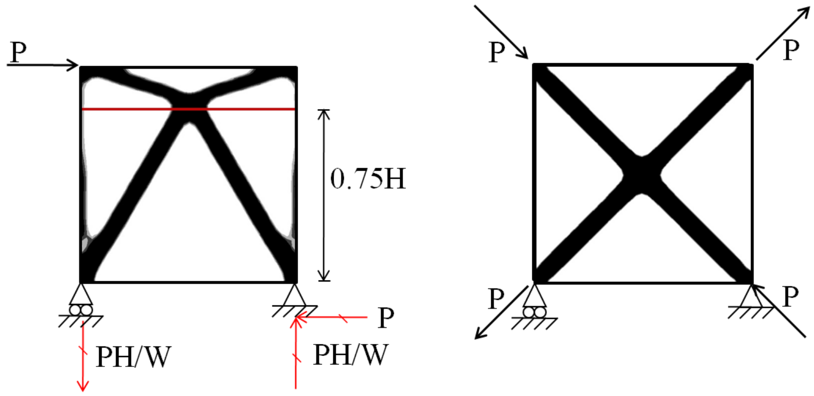


Figure 6. Comparison of optimal geometries for a cantilever problem (left) and a pure shear problem (right)

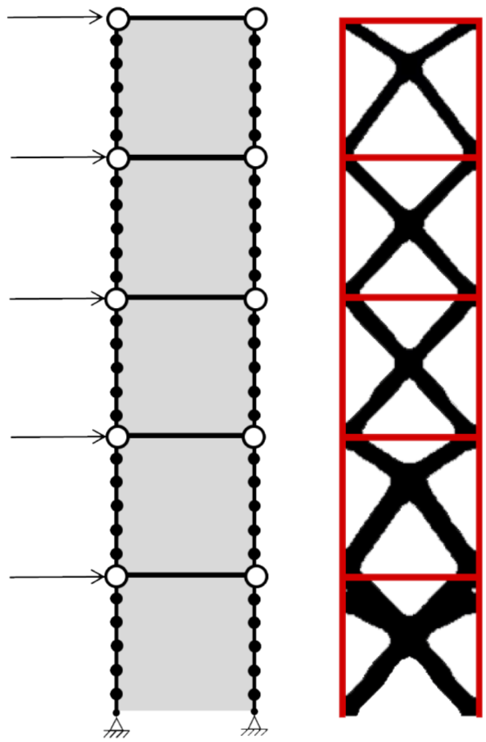
Similarly, the optimal geometry analysis can be extended for the case of multiple modules as is shown in Stromberg et. al (2011b), where the optimal bracing work point location of each module still remains at 75% of the module height.

NUMERICAL RESULTS

Numerical results are given here based on a code written by the authors in C++ using the methodology described in this work for a realistic building system. The structural frame is modeled in Figure 7 by assuming the beam elements are not engaged vertically by the quadrilateral mesh, while the column elements are attached continuously along their length. Furthermore, the connections from the beams (used to represent the floor levels) to the columns are pinned for the prototypical high-rise building. This structure is loaded with uniform point loads at each module. The volume of material for the topology optimization problem is 30% (i.e. 30% of the design domain will be filled with solid material).

The resulting braced frame geometry shows working points near 75% of each modules height, as was derived analytically in the previous section. The discrepancy between the numerical results shown here and the analytical derivation is due to the nonuniformity of stress in the optimized solution. This behavior was explained by Bendsoe and Sigmund (2002) by noting that the strain energy density is constant for the intermediate densities but not for all possible values (e.g. the extreme regions of solid and void, or 1 and 0). In Figure 4, this behavior is shown by observing the Von Mises stresses are constant for the intermediate densities.

Another interesting feature in the optimized structure of Figure 7 is that the size of the bracing members increases from the top to the base of the building as expected, due to the increase in shear forces. Note that the discrete columns are sized *a priori* to increase in size along the height, in accordance with gravity loading.



(a) **(b)**

Figure 7. Braced frame design using topology optimization: (a) problem statement, (b) optimization results using continuum (black) and discrete elements sized *a priori* (red)

CONCLUSIONS

The methodology described in this work provides structural engineers with an effective and efficient means to design the lateral bracing systems for high-rise buildings by using topology optimization with a combination of continuum (Q4) and discrete (beam) elements. The main contributions of this work are as follow:

- A method to describe optimal braced frames was proposed.
- The constant state of stress in the members of an optimized frame was verified.
- The relevance of this work has been demonstrated in the structural engineering industry.
- The optimal geometry of these frames was described analytically and numerically.

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