



Deployable Folded-core Sandwich Panels Guided by a Generating Surface

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Abstract

We propose a composite panel that can be assembled in a planar state and can be transformed with a one-DOF kinematic mechanism after the assembly. The proposed structure is comprised of generalized rigid-foldable tubes. The tubes are assembled such that they are non-trivially compatible with one another, but still share a desired single-curved surface. Because of the nontrivial assembly, the structure is expected to be flexible only in the desired one-DOF motion, deploying from a flat state to a 3D state, while it is significantly stiffer against other motions. The geometric construction follows the following procedure; (1) obtain an equivalent origami structure from a generating surface, (2) attach compatible tubular assemblies on both sides of the surface. This method produces a wide range of rigidly-foldable composite structures including corrugated surfaces with a flat-foldable compact state.

Keywords: origami, deployable structure, transformable structure, folded-core sandwich

1. Introduction

Origami is a technology that can be used to make lightweight stiff structures or deployable systems. Folded-core and honeycomb-core sandwich panels are examples of the former applications. Folded-core panels are lightweight stiff composite structures that can be fabricated by folding and assembling thin sheets [1]. These core structures can also be fitted to a single curved surface by changing the folding or cutting patterns of the core [2]. These structures are assembled in a folded 3D state, and once assembled, they become rigid and non-deformable. The latter type of application for deployable mechanisms can be realized through a geometric system of rigid origami, where rigid panels are connected by rotational hinges [3]. Rigid foldable origami tubes and cellular materials have been investigated previously [4,5,6]. Cellular rigid origami are expected to have meta-materialistic properties where the structure is flexible in one-mode and stiffer in other modes [7].

The authors have introduced and explored coupling methods for rigid-foldable tubes [8], that can be used to create flexibly transformable but stiff, lightweight structures. In this paper, we generalize ideas of tube coupling to obtain a variety of designs for sandwich panels that can be deformed only in one prescribed folding mode. First, we review the basic ideas of tube coupling and folded cores in Section 2. Then, we propose generalized construction process for rigid-foldable tubular assemblies in Sections 3 and 4.

2. Basic Structure

2.1 Corrugated Sandwich Systems

Figure 1 shows a rigid foldable Miura-ori tube, its aligned assembly [4] and the new assembly introduced in [8]. The stiffness and flexibility of such origami structures can be examined through eigenvalue analysis [9]. In [8], we show a detailed investigation of the different assemblies, and we show that a new coupling technique can increase the stiffness of the structure while still permitting for flexible folding and unfolding. The different coupling methods can be repeatedly translated and connected to form a corrugated sandwich surface (Figure 2). This structure can flat fold in both directions and is expected to have improved structural properties when in a coupled form. The sandwich surface consists of a corrugated surface in the middle, which we call a *generating surface*. This surface is offset in both directions, connected by corrugated walls, and thus forms individual tubes. The extrusion vector defining the corrugated walls are different on the top and bottom sides. The prime objective of this study is to obtain varieties of forms by changing the geometric parameters of the tubes.

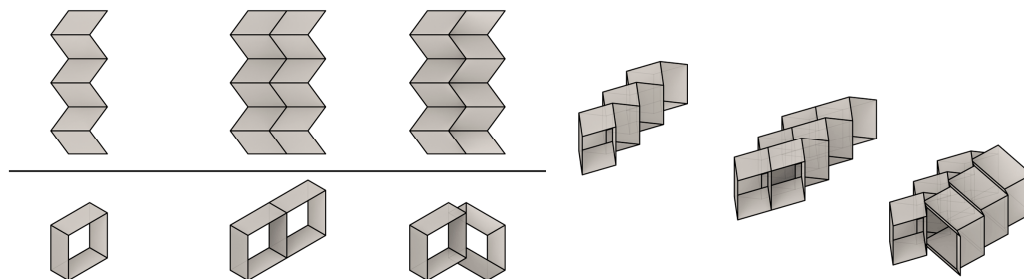


Figure 1. Rigid foldable Miura-ori tube (Left) and its assemblies: aligned (Middle) and new (Right) [8].

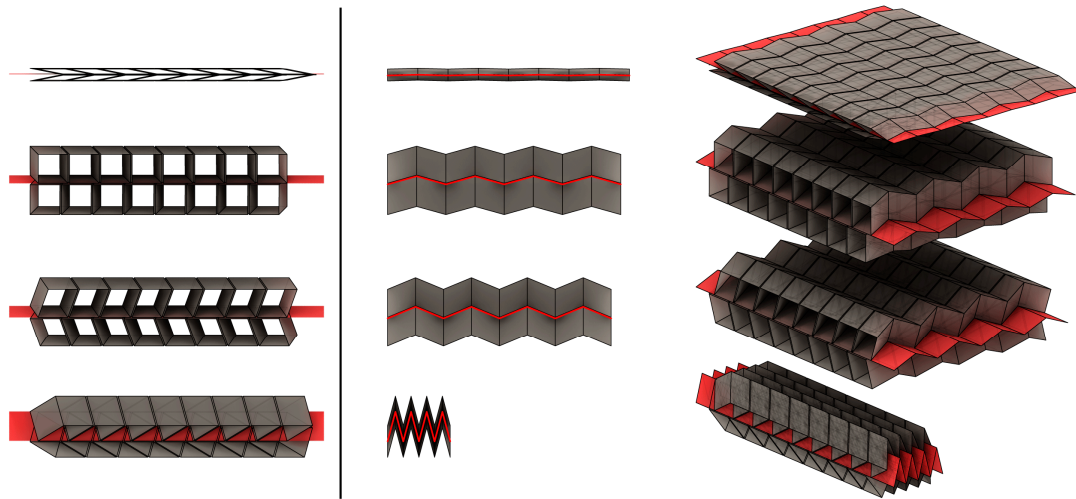


Figure 2. A transformable corrugated sandwich surface. Red indicates the generating surface. In the design phase, we construct the top and bottom offset structures based on the given middle surface.

2.2 Problem Description

In this paper, we consider the following inverse problem: given a generating surface as a general singly-curved polyhedral surface, construct a rigidly foldable sandwich assembly with different extrusion directions on the top and bottom sides (Figure 3). So, we decide to explore the design variations using the generating surface as the guiding geometry. Here, the generating surface is given by a quadrilateral strip, which is not necessarily constrained to cylinders. From fine subdivision, we can naturally extend the geometric construction to smooth developable surfaces including general cylinders, cones, and tangent surfaces.

Constructing the structure only on one side is relatively straightforward. The structure is an assembly of aligned tubes, which is obtained by sweeping extruding the parallelogram cross-section grid along an arbitrary guiding polyline on the generating surface (Figure 2). The sweeping is sequential: first, a section grid is extruded along a segment of the guiding polyline by sufficient length; then the extruded surface is trimmed by a plane passing through the fold line of the generating surface; the new section forms another parallelogram grid, so we continue the extrusion along the next segment of the guiding polyline. Since the cross-section remains a parallelogram grid when folded, these aligned tubular structures possess a one-DOF mechanism.

However, combining top and bottom structures is a non-trivial problem. If we separately obtain top and bottom structures, we will end up with two separate one-DOF structures that cannot synchronize once they start to fold. This is because the folding motions of the shared generating surfaces can be different in general. We need to explore geometric constraints that make the top and bottom mechanisms compatible with each other. One obvious example of compatible mechanisms are ones sharing the same extrusion vectors on the top and bottom, and this would create an aligned type assembly in both directions. However, we would like to use the new type of coupling, because of the improved structural

properties discussed in [8]. We need to explore non-trivial intrinsic symmetry in order to explore these special cases.

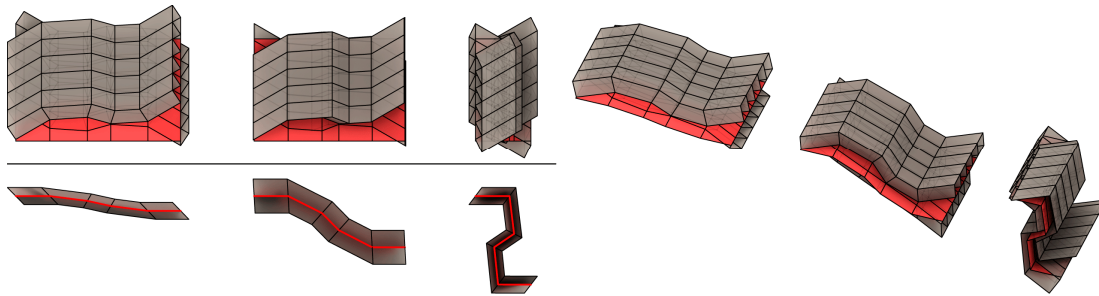


Figure 3. Folding of a transformable structure obtained from a given generating surface.

2.3. Equivalent Origami Structures

In order to simplify the geometric constraints, we extract the fundamental elements from the mechanism comprising of the generating surface and two distinct wall surfaces (Figure 4). By connecting portions of these surfaces, we construct an *equivalent origami* mechanism composed of $3 \times n$ quadrilateral panels, mechanically equivalent to the original structure. Because the equivalent origami gives different extrusion vectors for the top and bottom, we can easily construct the whole cellular structure from it. Therefore, the design problem of rigid foldable sandwich structures is reduced to obtaining a rigid foldable equivalent origami structure with $3 \times n$ quadrilateral panels. Here, note that the equivalent origami structure does not necessarily have to be developable in order to form a mechanism. If it is developable, the sandwich structure has a completely flat state; fabrication is especially easy in such a state because we can use flat sheets and assemble in a flat state.

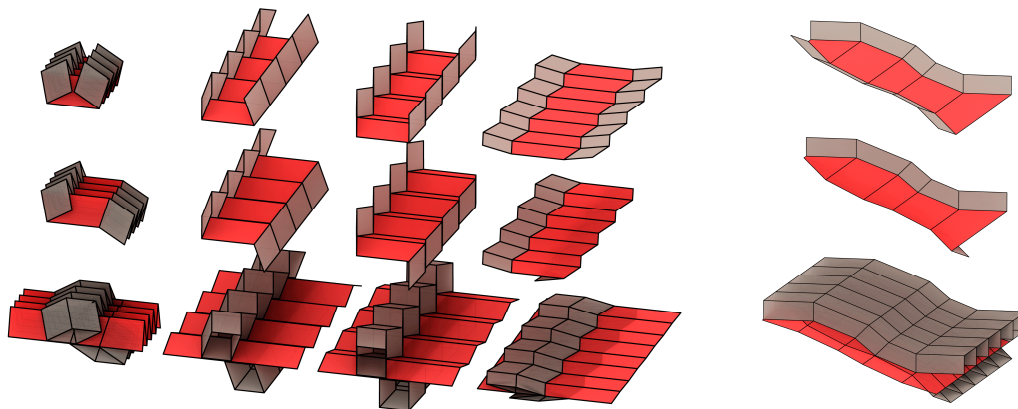


Figure 4. Left four columns: Extraction of equivalent origami from structures in Figure 2. Right column: Equivalent origami of model in Figure 3. Top row: equivalent surface composed of $3 \times n$ quadrilateral panels. Middle row: extracted surfaces. Bottom row: generating surface and top and bottom tubes.

3. Cylindrical Generating Surface with Mirror Symmetric Walls

Equivalent origami structures can be constructed when the generating surface is cylindrical, i.e., the fold lines are parallel. In this case, we can construct a mirror plane perpendicular to the surface. Arbitrarily constructed wall surface of one side can be mirror reflected to construct the wall on the other side. Figure 5 shows an example design of mirror-reflected construction and generated sandwich structures.

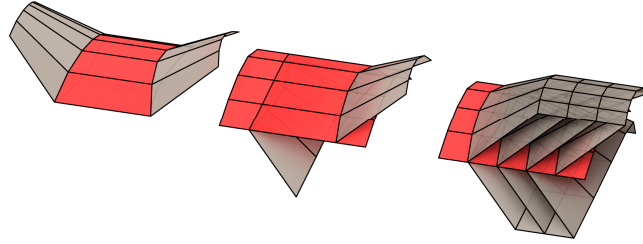


Figure 5. Cylindrical surface with mirror symmetric wall surfaces.
In this example, the structure does not have a flat state.

4. General Generating Surface with Bi-Directionally Flat-Foldable Structures

A family of equivalent origami structures can be obtained using bi-directionally flat-foldable quadrivalent mesh structure, i.e., a polyhedral surface where each vertex comprises of four corners with angles $\alpha, \beta, \alpha, \beta$ or $\alpha, \beta, \pi - \alpha, \pi - \beta$, in this order. If this type of structure has a non-flat configuration, it has a mechanism where the tangent of half fold angles (supplementary angle of dihedral angle) of all fold lines are always proportional to each other [10]. The motion of the generating surface is predefined by these constraints, and independent of other parameters of the attached structures. Top and bottom wall structures fold in a compatible motion if they share a common generating surface. Because of bidirectional flat foldability, the structure can fold into two extreme flat states. It can fold to a developed state, and to a completely flat-folded state if we ignore collision.

We start from the generating surface and obtain a wall structure. Figure 6 shows the process of construction. First, the generated surface is given as a sequence of planes g_1, g_2, \dots, g_n . Then, a wall surface w_1, w_2, \dots, w_n can be sequentially constructed such that each vertex is bi-directionally flat-foldable. The construction is as follows. Consider a vertex v_i surrounded by $g_i, w_i, w_{i+1}, g_{i+1}$ comprising corner angles $\alpha_i, \beta_i, \pi - \alpha_i, \pi - \beta_i$, respectively, in counterclockwise order. According to [10], if we denote the tangent of the half fold angle between g_i and w_i by $\tan \frac{\rho_i}{2}$ and that between g_i and g_{i+1} by $\tan \frac{\phi_i}{2}$, the fold angles around the vertex in counterclockwise order is

$$\left(\tan \frac{\rho_i}{2}, -\tan \frac{\phi_i}{2}, \tan \frac{\rho_{i+1}}{2}, \tan \frac{\phi_i}{2} \right) = (t, -k_i t, t, k_i t), \quad (1)$$

where $t = \tan \frac{\rho}{2} = \tan \frac{\rho_1}{2} = \dots = \tan \frac{\rho_n}{2}$, and k_i is the *fold coefficient* of a vertex defined by

$$k_i = \frac{\tan \frac{\alpha_i}{2} - \tan \frac{\beta_i}{2}}{\tan \frac{\alpha_i}{2} + \tan \frac{\beta_i}{2}} \quad (2)$$

If we assume that the vertex structure at v_{i-1} is solved, then, parameters α_i , t and $\tan \frac{\phi_i}{2} = k_i t$ are known. By solving equation (2), we obtain β_i as

$$\tan \frac{\beta_i}{2} = \frac{1 - k}{1 + k} \tan \frac{\alpha_i}{2}. \quad (3)$$

This determines the configuration of v_i with parameter α_{i+1} . Therefore, after choosing initial angles α_1 and $t = \tan \frac{\rho}{2}$, the rest of the wall structure is obtained sequentially. In the same manner, we can construct the structure on the opposite side with a different parameter of α_1 and with different or the same parameter t . The resulting sandwich structure is shown in Figure 7.

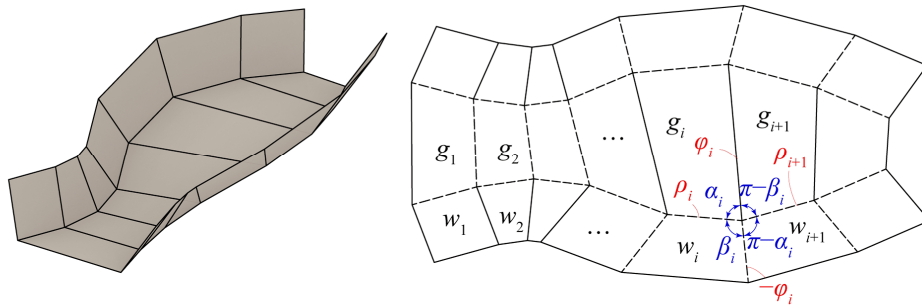


Figure 6. Sequential construction of a structure from a generating surface. Solid and dashed lines indicate mountain and valley folds with negative and positive fold angles, respectively.

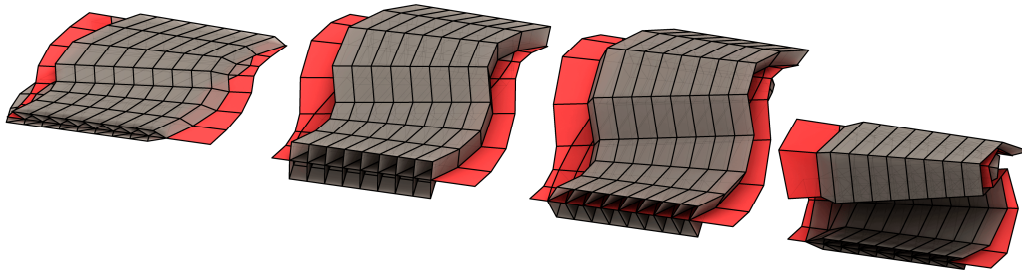


Figure 7. Generated sandwich structure.

5. Conclusion

We have shown a method for obtaining rigidly foldable sandwich structures that follow given singly curved surfaces. The proposed structures are composed of the assembly of aligned rigid foldable tubes on top and bottom, where the compatibility of the top and bottom mechanisms are satisfied by making an equivalent origami structure rigidly foldable. We have shown a family of structures that follow

general cylindrical surfaces, and a family of bi-directionally flat-foldable structures that follow generic single curved surfaces, i.e., general planar quad strips or a developable surfaces. The latter family allows for high degree of freedom of design, while it has potentially useful property that the structures can be fabricated in a flat state. The thin sheet becomes a thick stiff structure as the surface bends into a desired single-curved surface. Potential applications of the proposed structures include deployable vault structures (Figure 8) and deployable sandwich surface patches that can cover doubly curved surfaces.



Figure 8. Model of deployable vault structure.

Acknowledgement

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