Tailoring the Phase Field Method for Structural Topology Optimization with Polygonal/Polyhedral Finite Elements

Arun L. Gain, Glaucio H. Paulino

Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, U.S.A.

Motivation

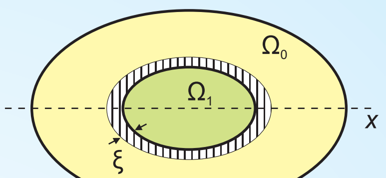
- Implicit function methods such as level-set method, although attractive, require periodic reinitializations to maintain signed distance characteristics for numerical convergence
- Polygonal/polyhedral elements circumvent the mesh bias caused by the intrinsic simplex geometry of standard finite elements (triangles/tetrahedra or quads/bricks)
- Explore general and curved domains rather than the traditional Cartesian domains (box-type) that have been extensively used for topology optimization

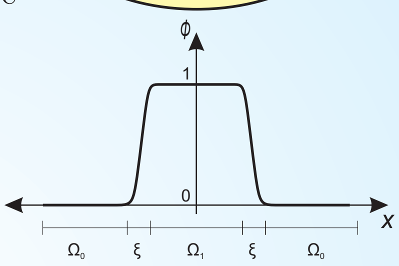
Phase Field Method

$$\phi = 1, \qquad \boldsymbol{x} \in \Omega_1,$$

$$0 < \phi < 1, \qquad \boldsymbol{x} \in \xi, \qquad \text{Diffuse interface}$$

$$\phi = 0, \qquad \boldsymbol{x} \in \Omega_0,$$





Allen-Cahn Equation

$$\frac{\partial \phi}{\partial t} = \kappa \nabla^2 \phi - f'(\phi)$$

$$\frac{\partial \phi}{\partial t} = \kappa \nabla^2 \phi + \phi(1-\phi)\left[\phi - \frac{1}{2} - 30\eta \frac{\bar{J}'(\phi_{t1})}{\|\bar{J}'(\phi_{t1})\|}\phi(1-\phi)\right]$$

Finite Volume Method

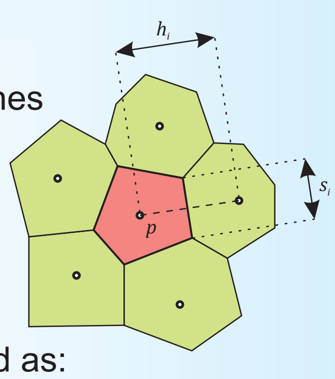
$$\int_{t,\Omega} \frac{\partial \phi}{\partial t} dt d\Omega = \int_{t,\Gamma} \kappa \boldsymbol{\nabla}\phi \cdot \boldsymbol{n} dt d\Gamma - \int_{t,\Omega} f'(\phi)\, dt d\Omega$$

$$\phi_{i,j}^{n+1} = \begin{cases} \dfrac{\Omega_p \phi_{i,j}^n + P_3}{\Omega_p\left(1-\left(1-\phi_{i,j}^n\right) r\left(\phi_{i,j}^n\right)\Delta t\right)} & \text{for } r\left(\phi_{i,j}^n\right) \le 0 \\ \dfrac{\Omega_p \phi_{i,j}^n \left(1+r\left(\phi_{i,j}^n\right)\Delta t\right)+P_3}{\Omega_p\left(1+\phi_{i,j}^n r\left(\phi_{i,j}^n\right)\Delta t\right)} & \text{for } r\left(\phi_{i,j}^n\right) > 0 \end{cases}$$

Polygonal Finite Elements

Simple approach to discretize complex geometries using polygonal/polyhedral meshes

Natural neighbor scheme based Laplace interpolants to construct a finite element space of polygonal elements



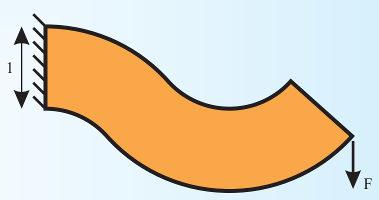
Laplace shape function for node p_i is defined as:

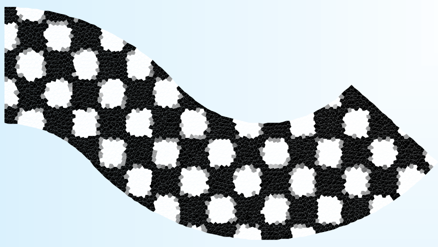
$$N_i(\boldsymbol{x}) = \frac{\alpha_i(\boldsymbol{x})}{\sum_{\mathcal{P}} \alpha_j(\boldsymbol{x})}, \qquad \alpha_i(\boldsymbol{x}) = \frac{s_i(\boldsymbol{x})}{h_i(\boldsymbol{x})}, \qquad \boldsymbol{x} \in \mathbf{R}^2$$

$$\mathcal{P} = \{p_1, p_2, ..., p_n\}$$

Minimum Compliance Design

Difficult to accurately discretize such a domain using Cartesian grids



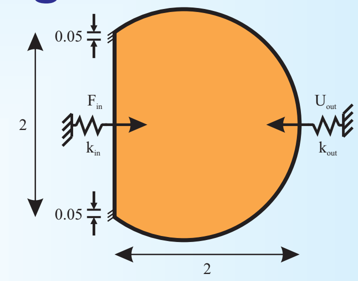


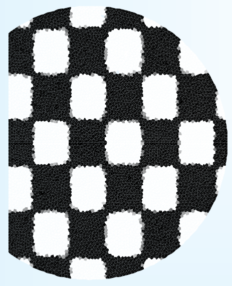
Initial Topology

Converged Topology

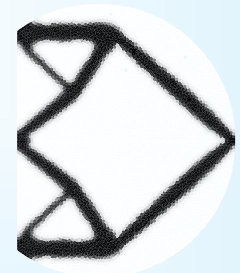
Compliant Mechanism Design

No single node connections observed in the converged topology





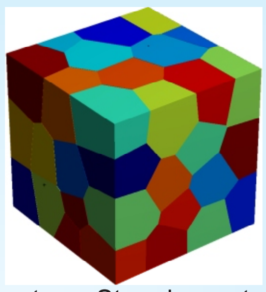
Initial Topology



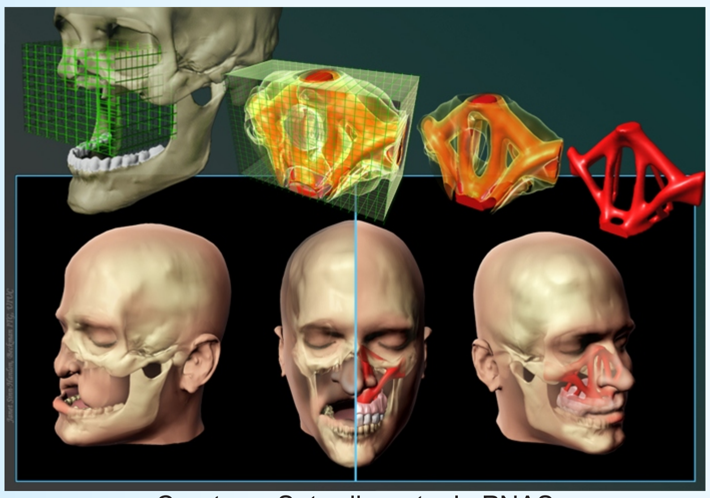
Converged Topology

Extensions

Natural extension to 3D using polyhedral meshes



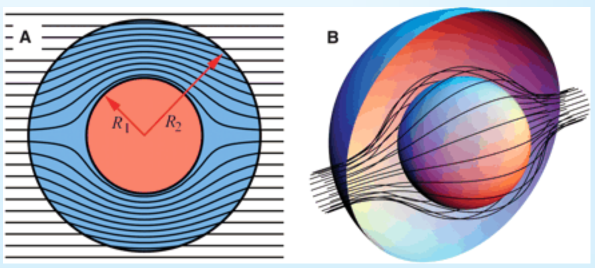
Courtesy: Stromberg et. al.



Courtesy: Sutradhar et. al., PNAS

Phase field method using polyhedral meshes paves the way for medical engineering applications including craniofacial segmental bone replacement

Phase field method with sharpness control of diffuse interfaces offers an attractive framework for phononic metamaterial cloaking design



Courtesy: Pendry et. al., Science 312

Conclusions

- Implicit function-based phase field method offers a general framework for topology optimization on arbitrary domains
- Meshes based on simplex geometry such as quads/bricks or triangles/tetrahedra introduce intrinsic bias in standard FEM, but polygonal/polyhedral meshes do not.
- Polygonal/polyhedral meshes based on Voronoi tessellation not only remove numerical anomalies such as one-node connections and checker-board pattern but also provide greater flexibility in discretizing non-Cartesian design domains.

References

- Gain AL, Paulino GH (2011) Topology optimization using phase field method and polygonal finite elements. *Submitted for review.*
- Sutradhar A, Paulino GH, Miller MJ, Nguyen TH (2010) Topology optimization for designing patient-specific large craniofacial segmental bone replacements. *Proceedings of the National Academy of Sciences* 107(30): 13222-13227
- Talischi C, Paulino GH, Pereira A, Menezes IFM (2010) Polygonal finite elements for topology optimization: A unifying paradigm. *International Journal for Numerical Methods in Engineering* 82: 671-698