Asphalt Pavement Aging and Temperature Dependent Properties through a Functionally Graded Viscoelastic Model –I: Development, Implementation and Verification

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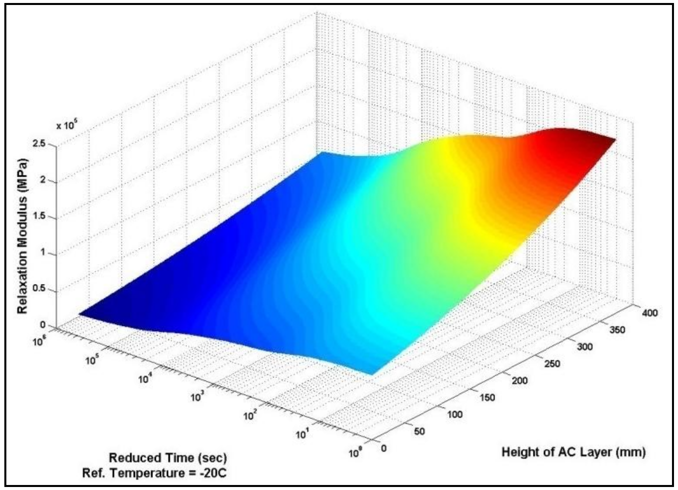
Outline

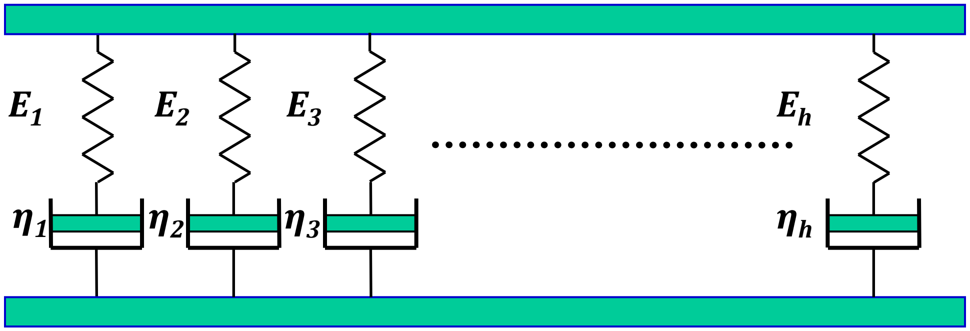
- Part – I
  - Graded Finite Elements
  - Viscoelasticity and FGMs
  - Finite Element Formulations
  - Verification
  - Concluding Remarks



- Part – II (Companion presentation)
  - Asphalt Pavements
  - Effect of Aging
  - Simulations
  - Concluding Remarks

Objectives

- Develop efficient and accurate simulation scheme for viscoelastic functionally graded materials (VFGMs)
- Correspondence Principle based formulation
- Application: Asphalt concrete pavements (Part II)



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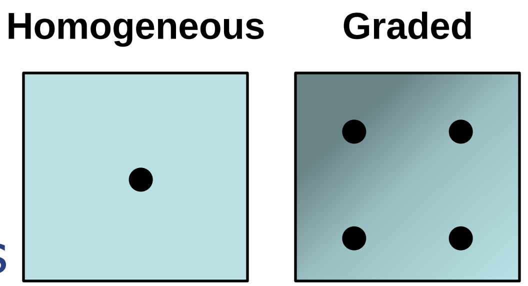
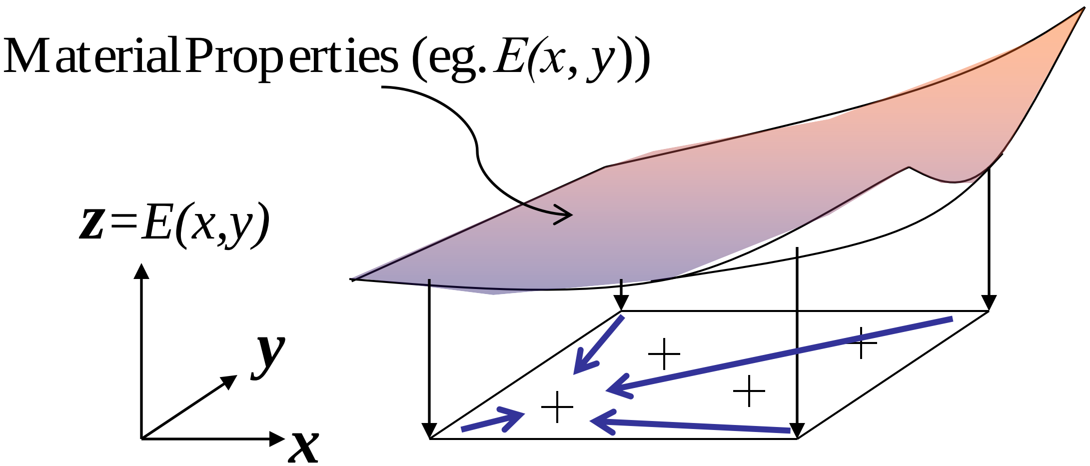
Graded Finite Elements



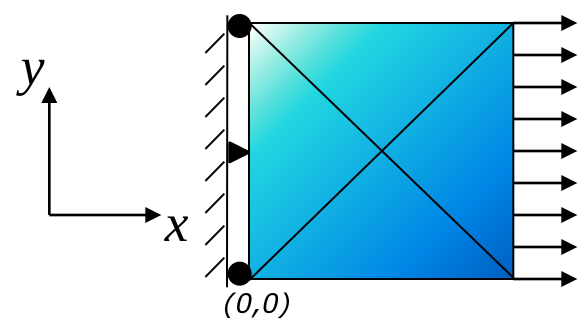
- Graded Elements: Account for material non-homogeneity within elements unlike conventional (homogeneous) elements
- Lee and Erdogan (1995) and Santare and Lambros (2000)
  - Direct Gaussian integration (properties sampled at integration points)
- Kim and Paulino (2002)
  - Generalized isoparametric formulation (GIF)
- Paulino and Kim (2007) and Paulino et al. (2007) further explored GIF graded elements
  - Proposed patch tests
  - GIF elements should be preferred for multiphysics applications
- Buttlar et al. (2006) demonstrated need of graded FE for asphalt pavements (elastic analysis)

Generalized Isoparametric Formulation (GIF)

- Material properties are sampled at the element nodes
- Iso-parametric mapping provides material properties at integration points
- Natural extension of the conventional isoparametric formulation



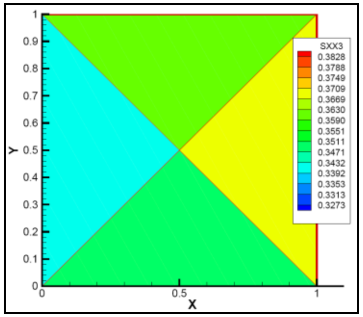
$$E(x,y) = E_0 Exp[3x - 2y]$$



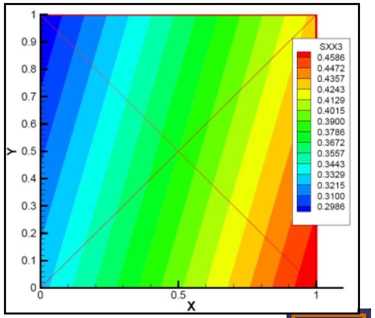
$$E = \sum_{i=1}^{m} N_i E_i$$

N_i = Shape function corresponding to node, i

m = Number of nodes per element



Conventional Homogeneous



GIF

Viscoelasticity: Basics

- Constitutive Relationship for linear viscoelastic body:

$$\sigma_{ij}^{\;d}(x,t) = 2\int_{t'=-\infty}^{t'=t} G_{ijkl}\left(x,\xi(t)-\xi(t')\right)\varepsilon_{kl}^{\;d}(x,t')\,dt'$$

$$\sigma_{kk}(x,t) = 3\int_{t'=-\infty}^{t'=t} K_{kkll}\left(x,\xi(t)-\xi(t')\right)\varepsilon_{ll}(x,t')\,dt'$$

- σ_{ij} are stresses, ε_{ij} are strains
- Superscript d represents deviatoric components
- G_{ijkl} and K_{ijkl} : shear and bulk moduli (space and time dependent)
- Assumptions: no body forces, small deformations
- Equilibrium: $\sigma_{ij,j} = 0$
- Strain-Displacement: $\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$
- u_i : displacements

Viscoelasticity: Correspondence Principle

- Correspondence Principle (Elastic-Viscoelastic Analogy): "Equivalency between transformed (Laplace, Fourier etc.) **viscoelastic** and **elasticity** equations"

Elastic

$$\sigma^d = 2G\varepsilon^d$$
$$\sigma = 3K\varepsilon$$

Transformed Viscoelastic

$$\tilde{\sigma}^d = 2\tilde{G}\tilde{\varepsilon}^d$$
$$\tilde{\sigma} = 3\tilde{K}\tilde{\varepsilon}$$

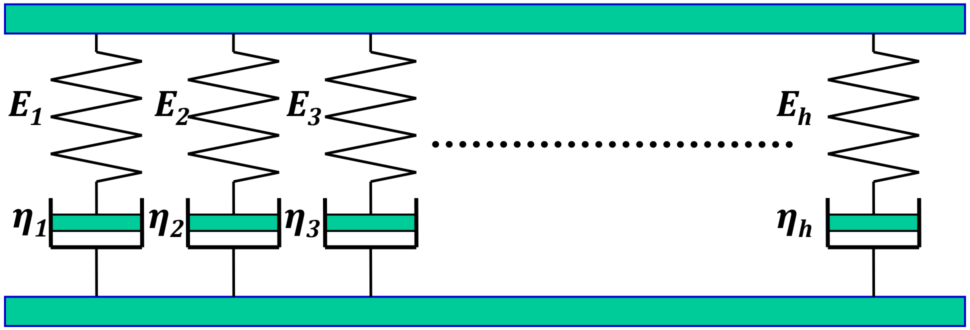
- Extensively utilized to solve variety of nonhomogeneous viscoelastic problems:
  - Hilton and Piechocki (1962): *Shear center of non-homogeneous viscoelastic beams*
  - Chang et al. (2007): *Thermal stresses in graded viscoelastic films*

Viscoelastic Model

- Prony series form: <u>*Generalized Maxwell Model*</u>
  - Equivalency between compliance and relaxation forms
  - Flexibility in fitting experimental data
  - Transformations are well established
  - Readily applicable to asphaltic and other viscoelastic materials (polymers, etc)

$$E(t) = \sum_{i=1}^{h} E_i \, Exp\left[-t/\tau_i\right]$$

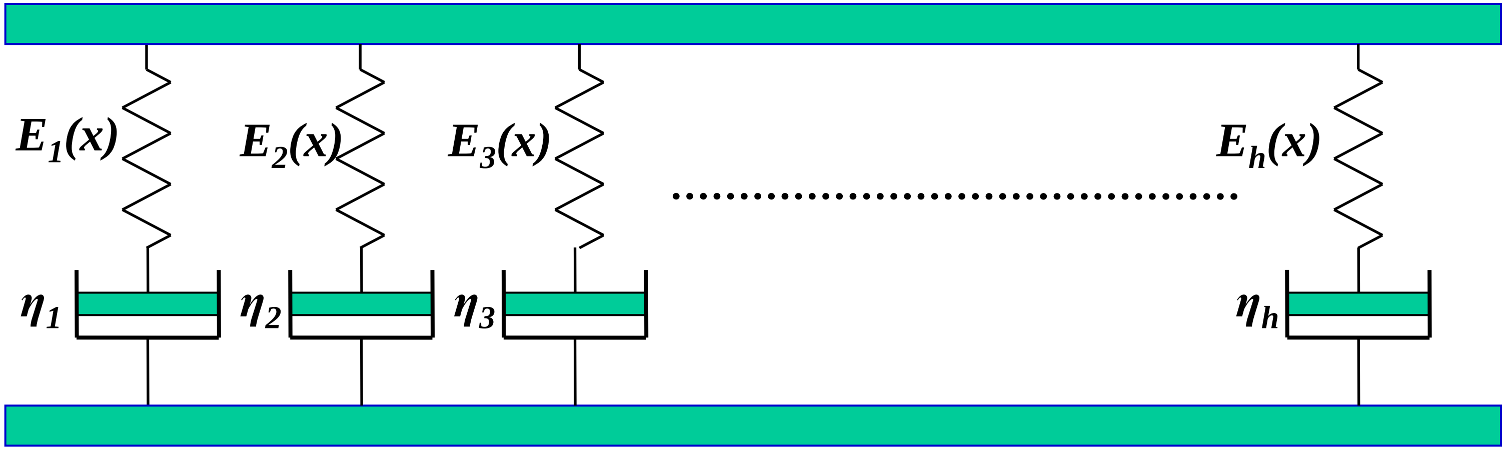
$$\tau_i = \frac{\eta_i}{E_i}$$



Viscoelastic FGMs

- Paulino and Jin (2001); Mukherjee and Paulino (2003)
  - Material with "Separable Form"

$$\mathbf{E}(\mathbf{x},t) = \mathbf{E}(\mathbf{x})\mathbf{f}(t)$$



General FE Implementation

- Correspondence principle based implementation using Laplace transform (Yi and Hilton, 1998)
- Variational Principle (Potential): (Taylor et al., 1970)

$$\boldsymbol{\Pi} = \int_{\Omega} \int_{t''=-\infty}^{t''=t} \int_{t'=-\infty}^{t'=t-t''} \frac{1}{2} \boldsymbol{C}_{ijkl} \left[\boldsymbol{x}, \boldsymbol{\xi}_{ijkl}(t-t'') - \boldsymbol{\xi}'_{ijkl}(t') \right] \frac{\partial \boldsymbol{\varepsilon}_{ij}(\boldsymbol{x},t')}{\partial t'} \frac{\partial \boldsymbol{\varepsilon}_{kl}(\boldsymbol{x},t'')}{\partial t''} dt' dt'' d\Omega$$

$$- \int_{\Omega} \int_{t''=-\infty}^{t''=t} \int_{t'=-\infty}^{t'=t-t''} \boldsymbol{C}_{ijkl} \left[\boldsymbol{x}, \boldsymbol{\xi}_{ijkl}(t-t'') - \boldsymbol{\xi}'_{ijkl}(t') \right] \frac{\partial \boldsymbol{\varepsilon}^{*}_{ij}(\boldsymbol{x},t')}{\partial t'} \frac{\partial \boldsymbol{\varepsilon}^{*}_{kl}(\boldsymbol{x},t'')}{\partial t''} dt' dt'' d\Omega$$

$$- \int_{S} \int_{t''=-\infty}^{t''=t} \boldsymbol{P}_{i}(\boldsymbol{x}, t-t'') \frac{\partial \boldsymbol{u}_{i}(\boldsymbol{x},t'')}{\partial t''} dt'' dS$$

FE Implementation: Basis

- Stationarity:

$$\delta\Pi = \int_{\Omega_u} \int_{t'=-\infty}^{t'=t} \int_{t''=-\infty}^{t''=t-t'} \left\{ C_{ijkl}\left[x, \xi_{ijkl}\left(t-t''\right) - \xi'_{ijkl}\left(t'\right)\right] \frac{\partial}{\partial t'}\left(\varepsilon_{ij}\left(x,t'\right) - \varepsilon^{*}_{ij}\left(x,t'\right)\right) \frac{\partial \delta\varepsilon_{kl}\left(x,t''\right)}{\partial t''} \right\} dt'\, dt''\, d\Omega_u$$

$$-\int_{\Omega_\sigma} \int_{t''=-\infty}^{t''=t} P_i\left(x, t-t''\right) \frac{\partial \delta u_i\left(x,t''\right)}{\partial t''} dt''\, d\Omega_\sigma = 0.$$

Ω : volume, S surface with traction Pi

C_{ijkl}: constitutive properties

ε_{ij}: mechanical strains, ε_{ij}^{*} : thermal strains, u_i: displacements,

ξ: reduced time related to real time through time-temperature superposition principle given by:

$$\xi(t) = \int_0^t a\left(T\left(t'\right)\right) dt'$$

a is time-temperature shift factor, and T is temperature

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FEM

- Element stiffness matrix:

$$k_{ij}\left(x,t\right)=\int\limits_{\Omega_u} B_{ik}^{T}\left(x\right)C_{kl}\left(x,\xi\left(t\right)\right)B_{lj}\left(x\right)d\Omega_u$$

- Force vectors:

Mechanical: $$f_i\left(x,t\right)=\int\limits_{\Omega_\sigma} N_{ij}\left(x\right)P_j\left(x,t\right)d\Omega_\sigma$$

Thermal: $$f_i^{th}\left(x,t\right)=\int\limits_{\Omega_u}\int\limits_{-\infty}^{t} B_{ik}\left(x\right)C_{kl}\left(x,\xi(t)-\xi\left(t'\right)\right)\frac{\partial \varepsilon^{*}{}_{l}\left(x,t'\right)}{\partial t'}dt'\,d\Omega_u$$

k_{ij} : element stiffness matrix,
f_i : element force (load) vector
u_i : displacement vector
ε_i : strains related to nodal degrees of freedom q_j through isoparametric shape functions N_{ij} and their derivatives B_{ij}

$$u_i\left(x,t\right)=N_{ij}\left(x\right)q_j\left(t\right)$$

$$\varepsilon_i\left(x,t\right)=B_{ij}\left(x\right)q_j\left(t\right)$$

FEM: Assembly and Solution

- Assembling provides global stiffness matrix, K_{ij} and force vectors, F_i
- Equilibrium:

$$K_{ij}\left(x,\xi(t)\right)U_j(0)+\int_{0^+}^{t}K_{ij}\left(x,\xi(t)-\xi\left(t^{'}\right)\right)\frac{\partial U_j\left(t^{'}\right)}{\partial t^{'}}dt^{'}=F_i\left(x,t\right)+F_i^{th}\left(x,t\right)$$

- Correspondence principle:

$$\tilde{K}_{ij}\left(x,s\right)\tilde{U}_j\left(s\right)=\tilde{F}_i\left(x,s\right)+\tilde{F}_i^{th}\left(x,s\right)$$

$\tilde{a}(s)$ is Laplace transform of $a(t)$, s is transformation variable

$$\tilde{a}(s)=\int_0^\infty a(t)\,Exp[-st]dt$$

FEM: Implementation

Define problem in time-domain (evaluate load vector, $F(x, t)$ and stiffness matrix components $K(x)$ and $\Lambda(t)$)

↓

Perform Laplace transform to evaluate $\tilde{F}(x, s)$ and $\tilde{\Lambda}(s)$

↓

Solve linear system of equations to evaluate nodal displacement, $\tilde{U}(x,s)$

↓

Perform inverse Laplace transforms to get the solution, $U(x,t)$

↓

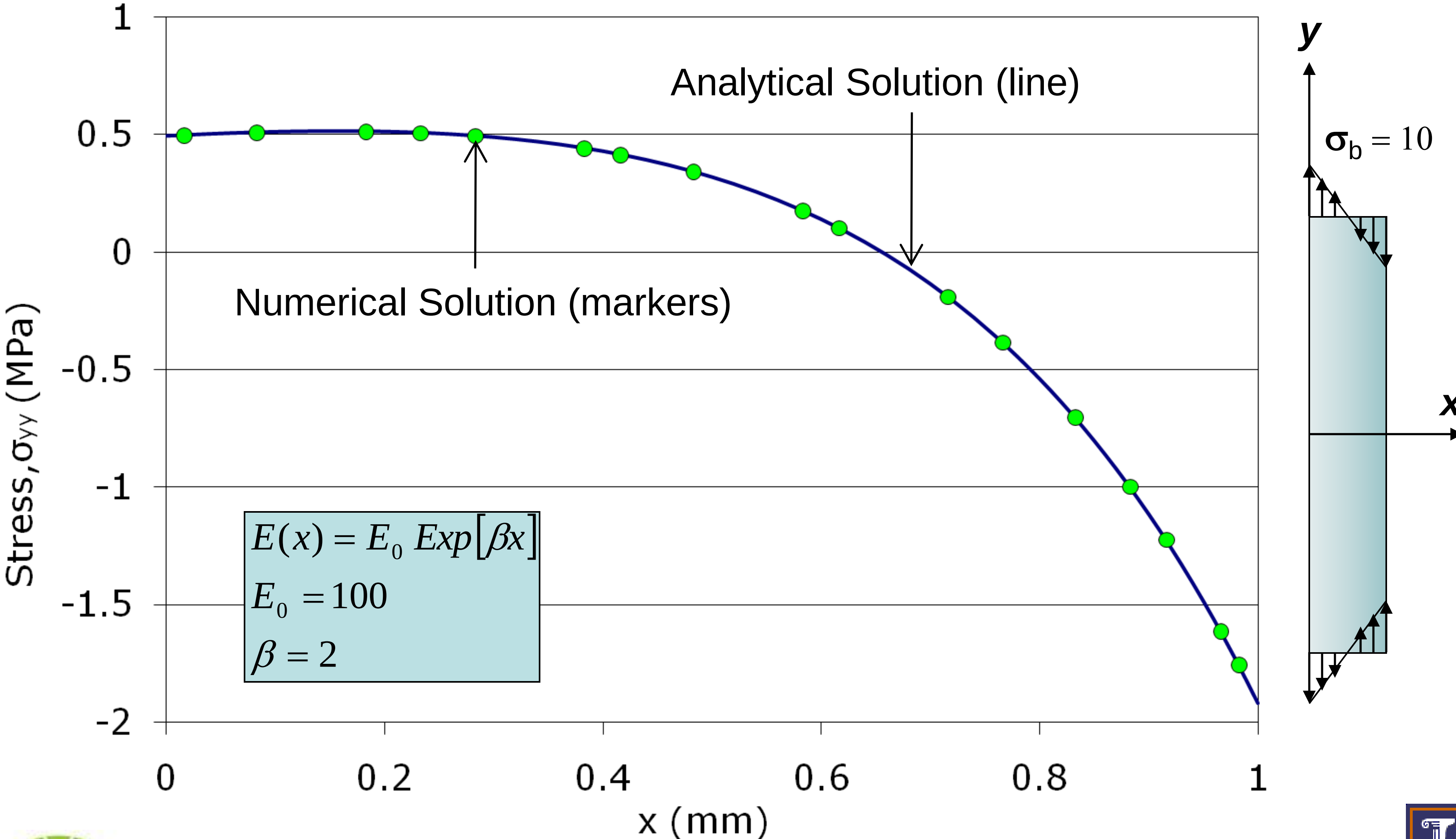
Post-process to evaluate field quantities of interest

FEM: Verification

- MATLAB® code using GIF and correspondence principle
- GIF
  - Compare analytical and numerical solutions for graded boundary value problems
- Viscoelasticity
  - Compare analytical and numerical solutions for viscoelastic bar imposed with creep loading
- Comparison with Commercial Code *ABAQUS*® (Layered Approach)

Graded Finite Element Performance

Bending example



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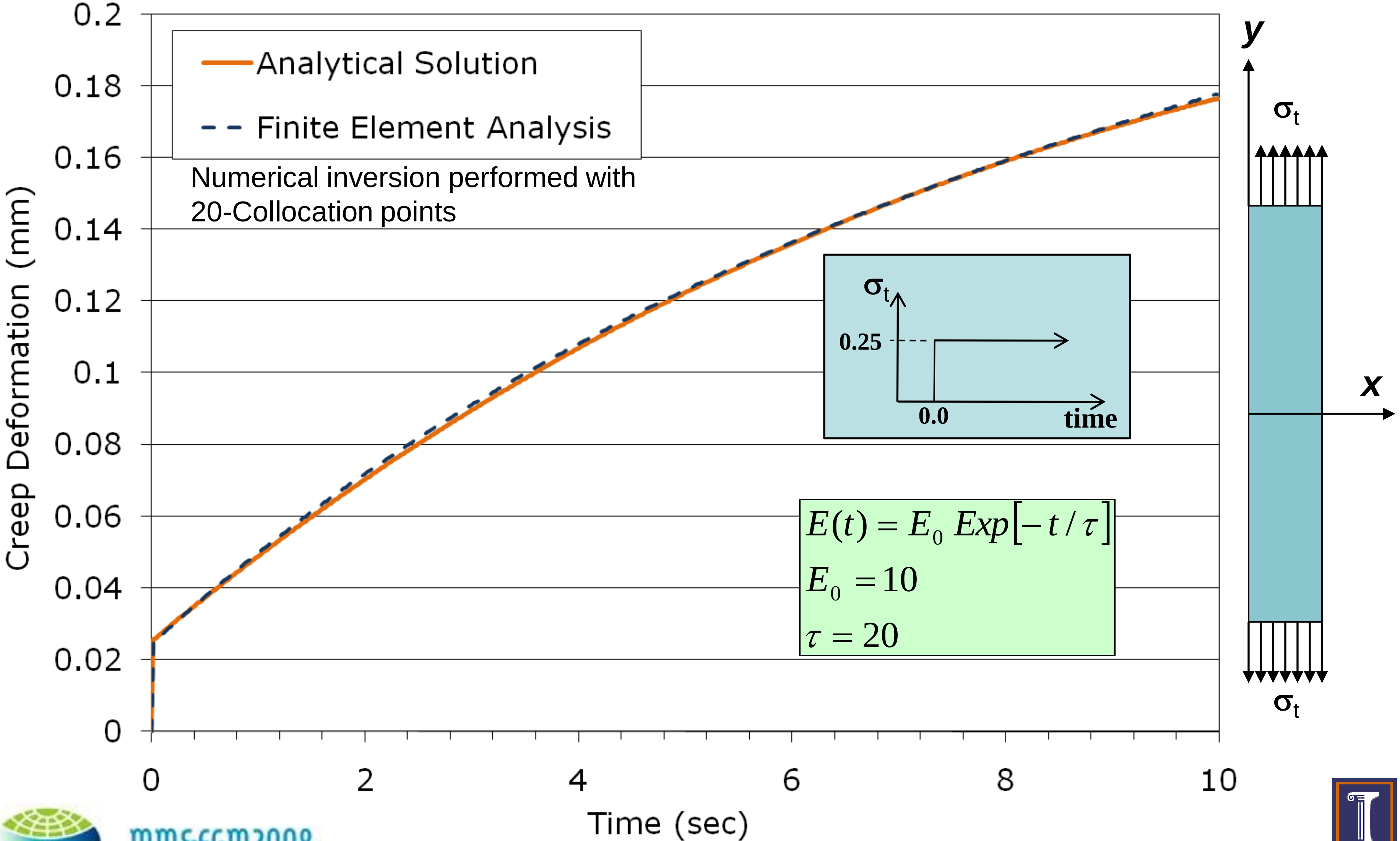
Homogeneous Viscoelastic Verification

Creep example shown here



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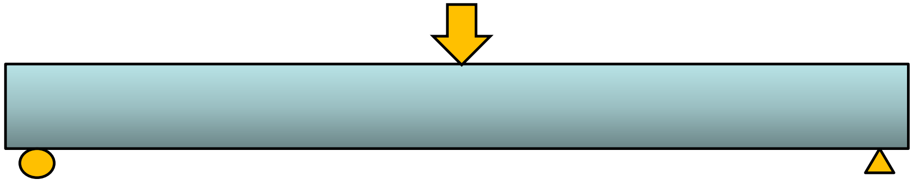
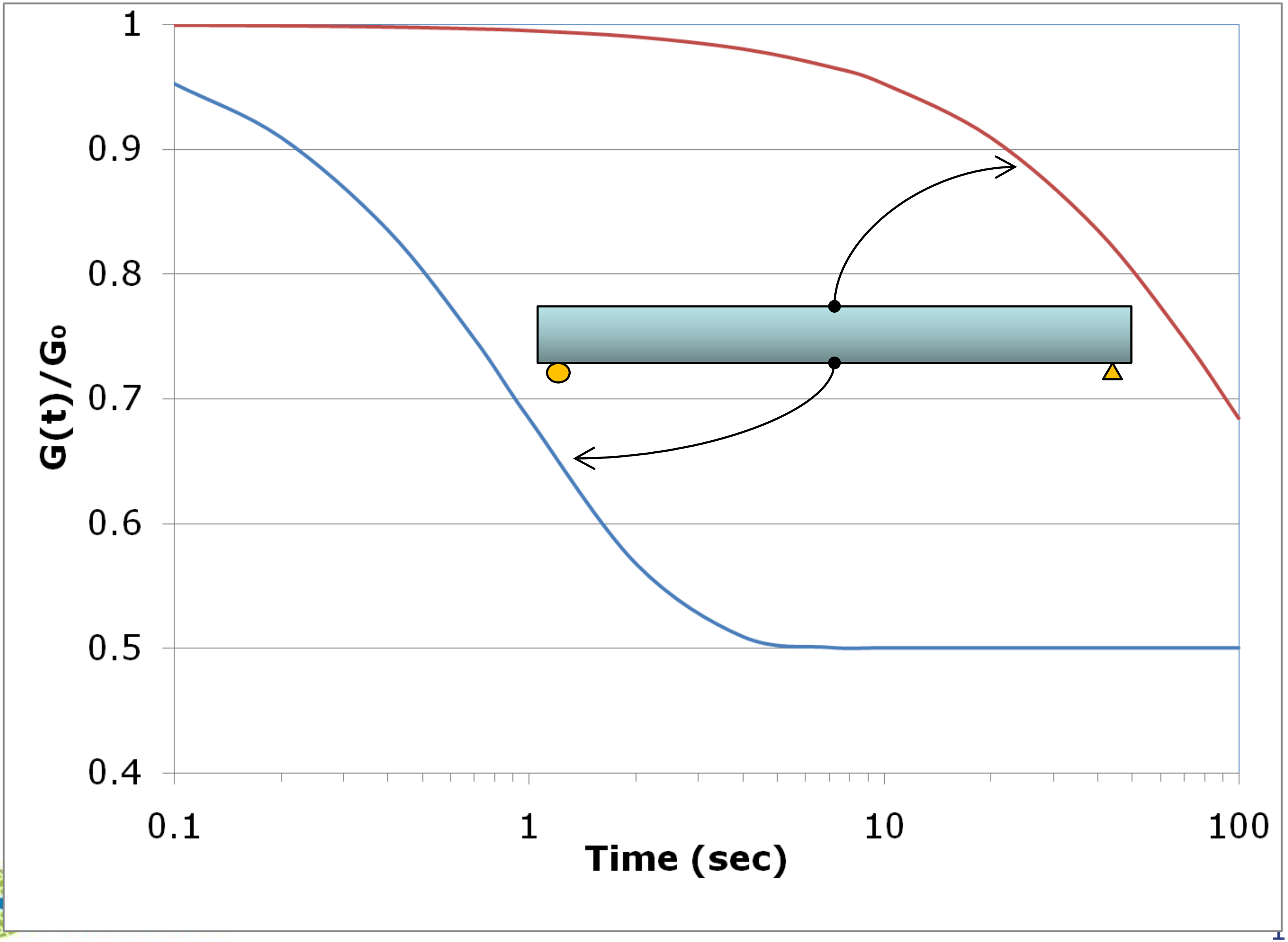
FGM Verification with ABAQUS

- Simply supported beam in 3-point bending
  - 100-second creep loading
- Graded viscoelastic material properties



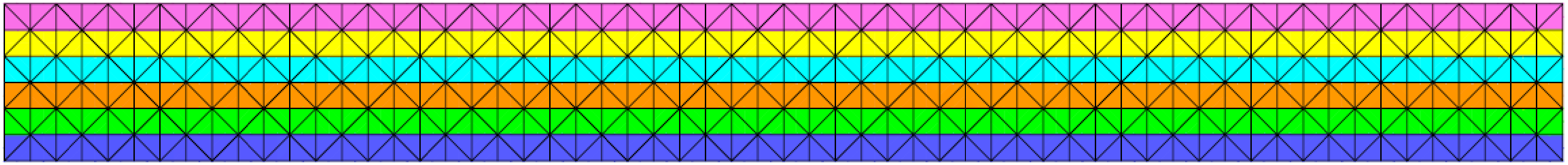
- FE simulation:
  - Homogeneous: Averaged properties
  - Layered (ABAQUS):
    - *6-Layers*
    - *12-Layers*
  - Graded:
    - *Same mesh structure as 6-Layers*

Reference Material Properties

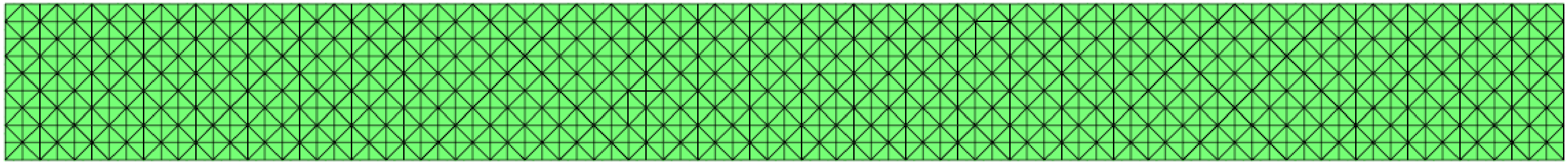


FEM Meshes

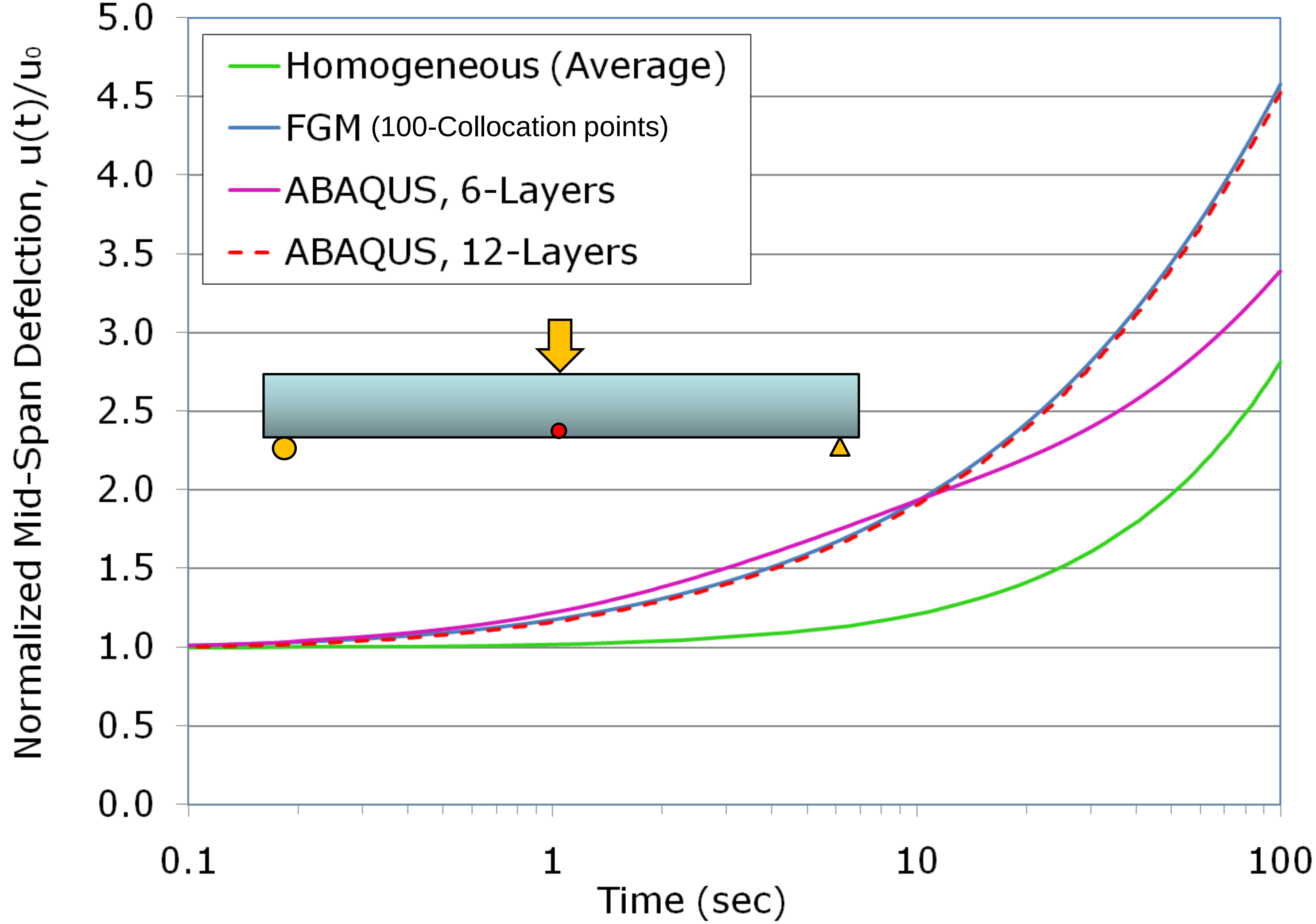
- 6-Layers / FGM / Homogeneous
  - 3146 DOFs
  - 6-node triangle elements



- 12-Layers
  - 6878 DOFs
  - 6-node triangle elements



Numerical Results



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Concluding Remarks

- Main Contribution:
  development of graded viscoelastic elements
- Extension of the Generalized Isoparametric Formulation (Elastic) to rate-dependent materials (viscoelastic)
- Correspondence Principle based formulation: separable material properties
- Companion presentation (paper) demonstrates application of this work to field of asphalt pavements

- Extension: Graded Viscoelastic formulation in time domain

Thank you for your attention!!