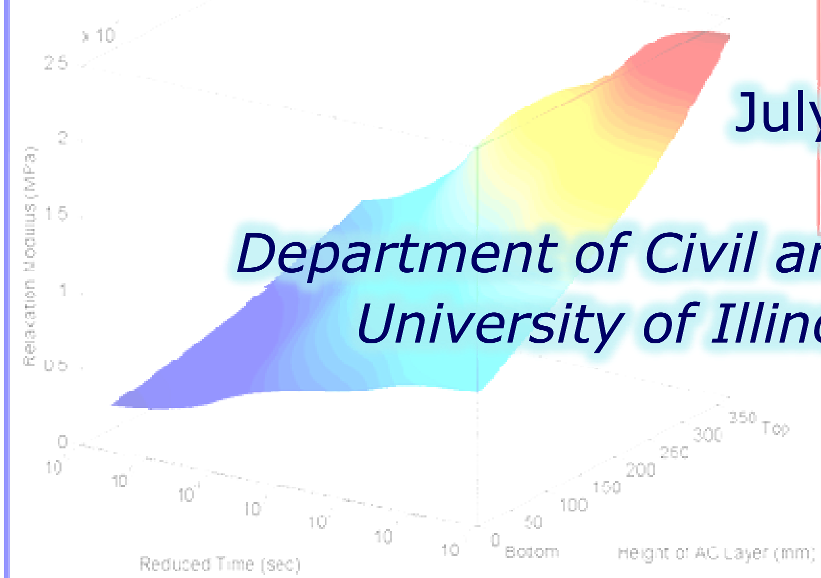
10th US National Congress on Computational Mechanics
Columbus, OH

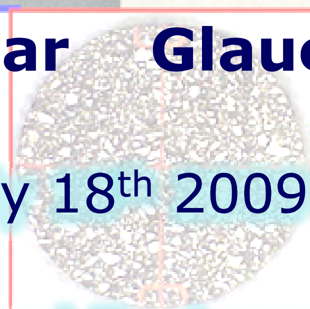
Framework for Consideration of Aging and Thermal Gradients in Asphalt Concrete Pavement Simulations

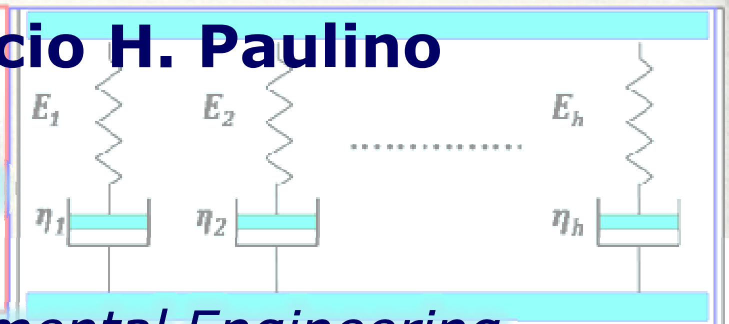
Eshan V. Dave

William G. Buttlar   Glaucio H. Paulino

July 18th 2009







Department of Civil and Environmental Engineering
University of Illinois at Urbana-Champaign

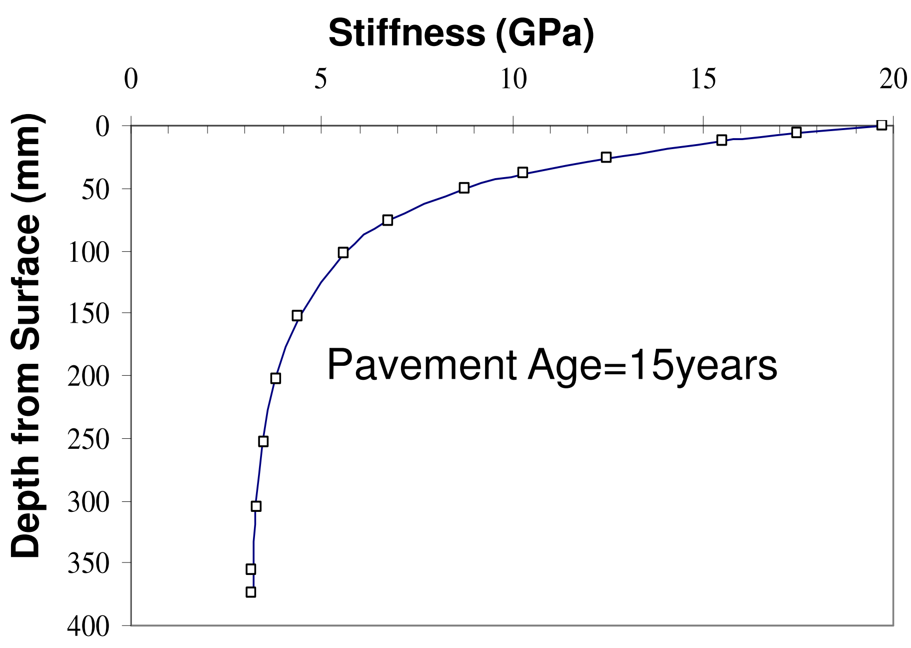
ILLINOIS
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

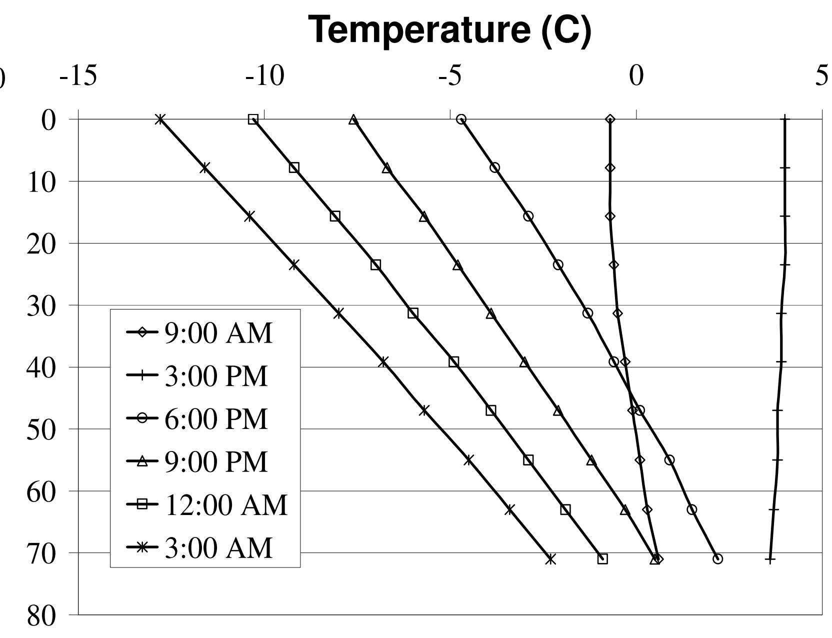
Functionally Graded Viscoelastic Model for Asphalt Concrete

- Motivation and Introduction
- Viscoelastic FGM Finite Elements
- Time Integration Analysis
- Application Examples:
  - Asphalt Pavement
  - Boundary Layer Model for Fracture
- Summary and Conclusions

Asphalt Pavements are Non-Homogeneous Structures

- Sources of Non-Homogeneity:
  1. *Oxidative aging*
  2. *Temperature dependence of material properties*
  3. *Other sources (construction, additives etc.)*





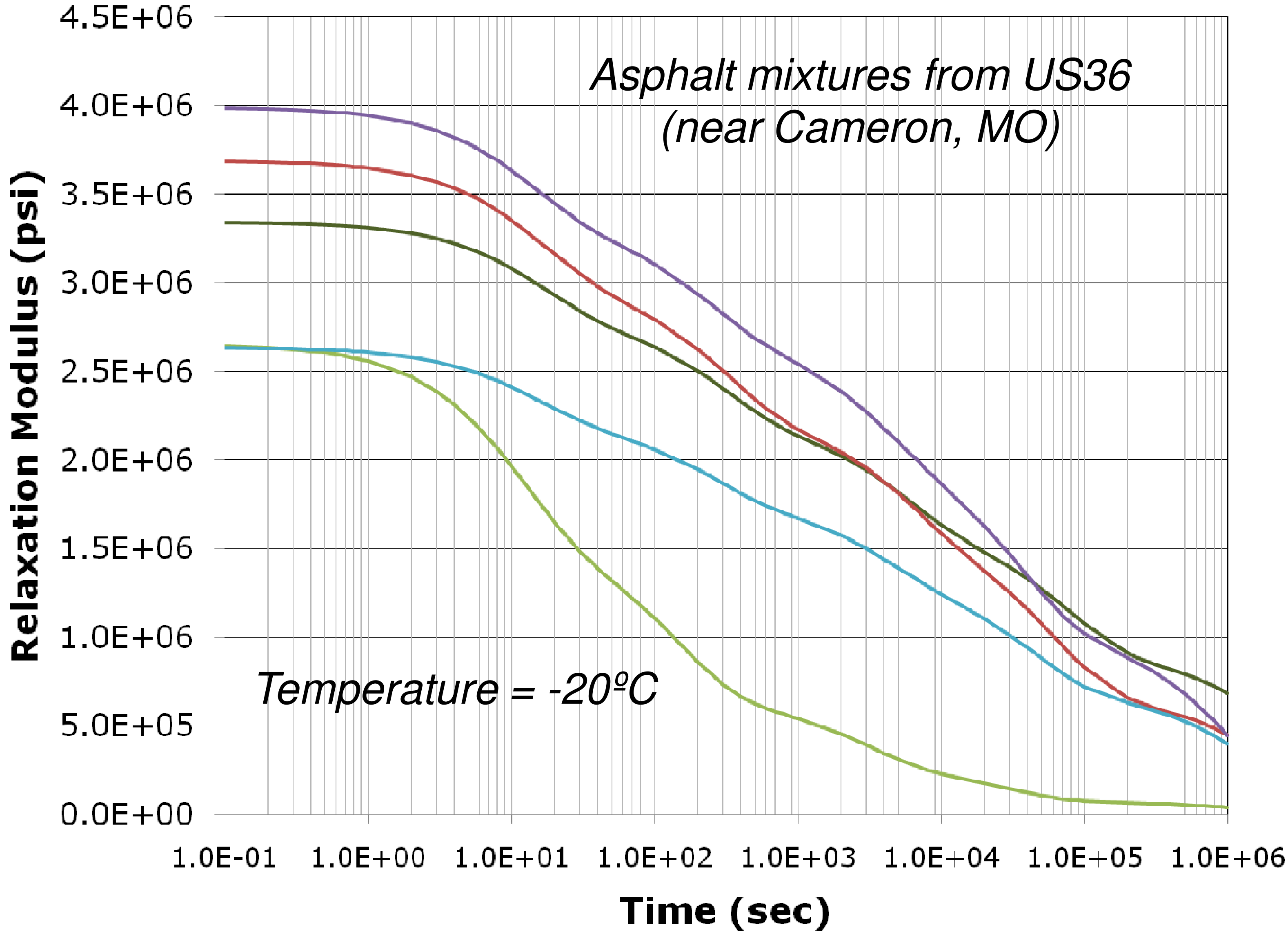
Aging gradient generated using "Global aging model" by Mirza and Witczak (1996)

Temperature profiles generated using "EICM" from AASHTO MEPDG (2002)

M.W. Mirza, and M.W. Witczak, (1996) "Development of a global aging system for short and long term aging of asphalt cements," Journal of Asphalt Paving Technologists, Vol. 64 393-430.

Guide for Mechanistic-Empirical Design of New and Rehabilitated Pavement Structures, NCHRP Project 1-37A Final Report, 2002.

Asphalt Concrete is Viscoelastic



Objectives

- Develop efficient and accurate simulation scheme for asphalt concrete pavements
- Account for:
  - Aging gradients
  - Temperature dependent property gradients
- Viscoelastic analysis
  - a) Correspondence Principle (Dave et al., 2009)
  - b) Time Integration Scheme (current presentation)
- Applications: (selected examples discussed here)
  - Asphalt concrete pavements and overlay systems
  - Boundary layer fracture analysis

E.V. Dave, G.H. Paulino, and W.G. Buttlar, (2009) " Viscoelastic functionally graded finite element method using correspondence principle," Journal of Materials in Civil Engineering, 2009 (in-review)

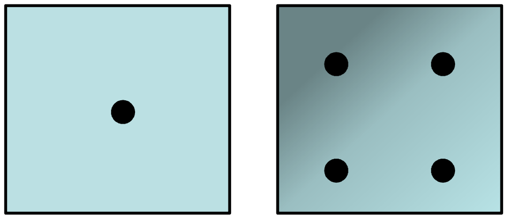
Viscoelasticity: Basics

- Constitutive Relationship:

$$\sigma(x,t)=\int_{t'=-\infty}^{t'=t} C\left(x,\xi-\xi'\right)\frac{\partial\varepsilon\left(x,t'\right)}{\partial t'}dt'$$

σ : Stresses, $C(x,\xi)$: Relaxation Modulus, ε : Strains

Graded Finite Elements



- Graded Elements: Account for material non-homogeneity within elements unlike conventional (homogeneous) elements
- Lee and Erdogan (1995) and Santare and Lambros (2000)
  - Direct Gaussian integration (properties sampled at integration points)
- Kim and Paulino (2002)
  - Generalized isoparametric formulation (GIF)
- Paulino and Kim (2007) and Silva et al. (2007) further explored GIF graded elements
  - Proposed patch tests
  - GIF elements should be preferred for multiphysics applications
- Buttlar et al. (2006) demonstrated need of graded FE for asphalt pavements (elastic analysis)

J.H. Kim, and G.H. Paulino, (2002) "Isoparametric graded finite elements for nonhomogeneous isotropic and orthotropic materials," Journal of Applied Mechanics, 69:502-14.

G.H. Paulino, and J.H. Kim, (2007) "The weak patch test for nonhomogeneous materials modeled with graded finite elements," Journal of the Brazilian Society of Mechanical Sciences and Engineering, 29:63-81.

Y.D. Lee, and F. Erdogan, (1995) "Residual/thermal stresses in FGM and laminated thermal barrier coatings," International Journal of Fracture, 69:145-65.

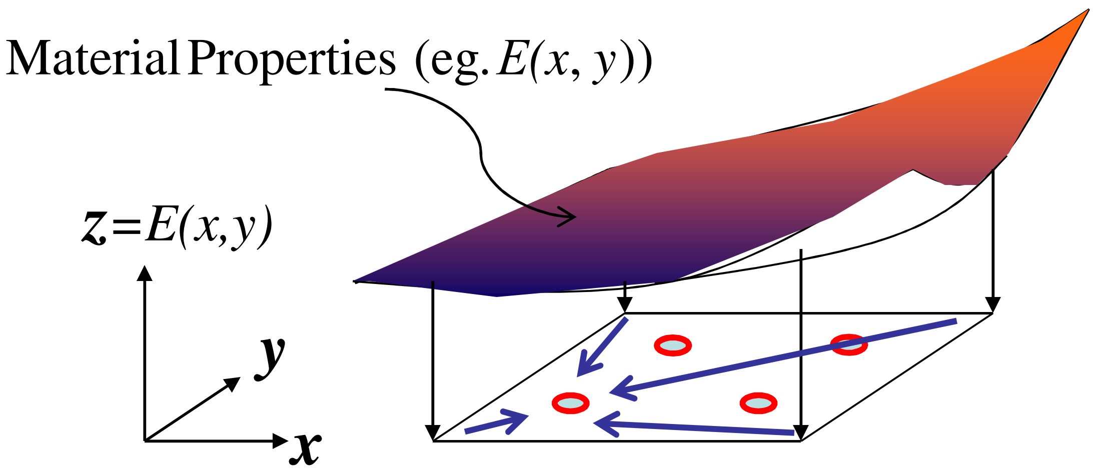
W.G. Buttlar, G.H. Paulino, and S.H. Song, (2006) "Application of graded finite elements for asphalt pavements," Journal of Engineering Mechanics, 132:240-249.

E.C.N. Silva, R.C. Carbonari, and G.H. Paulino, (2007) "On graded elements for multiphysics applications," Smart Materials and Structures, 16:2408-2428.

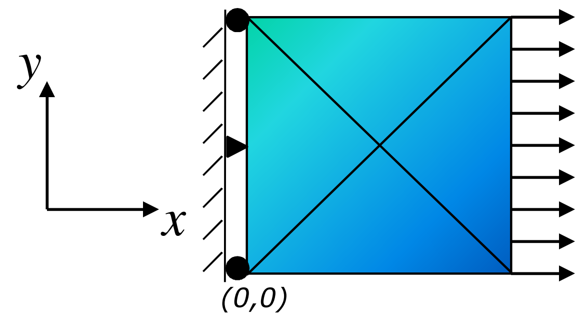
M.H. Santare and J. Lambros, (2000) "Use of graded finite elements to model the behavior of nonhomogeneous materials," Journal of Applied Mechanics, 67:819-22.

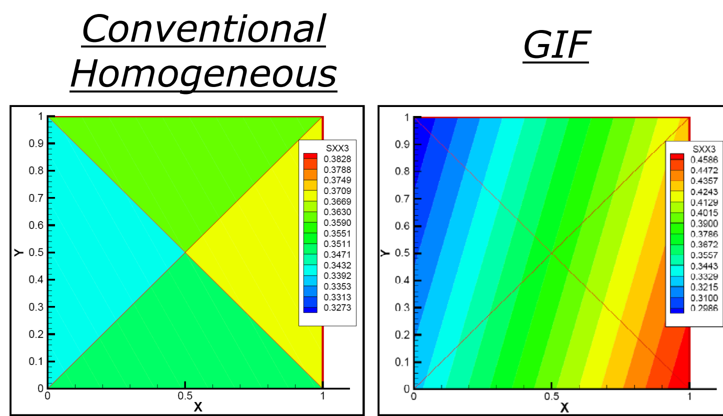
Generalized Isoparametric Formulation (GIF)

- Material properties are sampled at the element nodes
- Iso-parametric mapping provides material properties at integration points
- Natural extension of the conventional isoparametric formulation



$$E(x,y)=E_0 Exp[3x-2y]$$





$$E(t)=\sum_{i=1}^{m} N_i\left[E(t)\right]_i$$

N_i = Shape function corresponding to node, i

m = Number of nodes per element

General FE Implementation

- Variational Principle (Potential):

$$\pi = \int_{\Omega_u} \int_{t'=-\infty}^{t'=t} \int_{t''=-\infty}^{t''=t-t'} \frac{1}{2} C\left[x, \xi\left(t-t''\right) - \xi'\left(t'\right)\right] \frac{\partial \varepsilon\left(t'\right)}{\partial t'} \frac{\partial \varepsilon\left(t''\right)}{\partial t''} dt'\, dt''\, d\Omega_u$$

$$- \int_{\Omega_\sigma} \int_{t''=-\infty}^{t''=t} P\left(x, t-t''\right) \frac{\partial u\left(t''\right)}{\partial t''} dt''\, d\Omega_\sigma$$

Where, π is Potential, ε are strains for body of volume Ω_u,

P is the prescribed traction on surface Ω_σ and u is the corresponding displacement

- Stationarity forms the basis for problem description:

$$\delta\pi = \int_{\Omega_u} \int_{t'=-\infty}^{t'=t} \int_{t''=-\infty}^{t''=t-t'} \left\{ C\left[x, \xi\left(t-t''\right) - \xi'\left(t'\right)\right] \frac{\partial \varepsilon\left(t'\right)}{\partial t'} \frac{\partial \delta\varepsilon\left(t''\right)}{\partial t''} \right\} dt'\, dt''\, d\Omega_u$$

$$- \int_{\Omega_\sigma} \int_{t''=-\infty}^{t''=t} P\left(x, t-t''\right) \frac{\partial \delta u\left(t''\right)}{\partial t''} dt''\, d\Omega_\sigma = 0$$

Non-Homogeneous Viscoelastic FEM

- Equilibrium:

$$K_{ij}\left(x,\xi(t)\right)u_j(0)+\int_{0^+}^{t}K_{ij}\left(x,\xi(t)-\xi\left(t^{'}\right)\right)\frac{\partial u_j\left(t^{'}\right)}{\partial t^{'}}dt^{'}=F_i(x,t)$$

- Solution approaches:

1. Correspondence Principle (CP)

$$\left[K^0(x)\,s\tilde{K}^t(s)\right]_{ij}\tilde{u}_j(s)=\tilde{F}_i(x,s)$$

$\tilde{a}(s)$ is Laplace transform of $a(t)$; s is transformation variable

$$\tilde{a}(s)=\int_0^{\infty}a(t)\,Exp[-st]dt$$

2. Time-Integration Schemes

- *Recursive Formulation*

Non-Homogeneous Viscoelastic FEM

1. Correspondence Principle (CP)

- Benefits:
  - Solution does not require evaluation of hereditary integral
  - Direct extension of elastic formulations
- Limitations:
  - Inverse transformations are computationally expensive
  - Transform/Convolution should exist for material model and boundary conditions

2. Time-Integration Schemes (Recursive formulation)

- Benefits:
  - Fewer limitations on material model and boundary conditions
- Limitations:
  - Convergence studies are required to determine time step size
  - Elaborate formulation and implementation

Time Integration Approach

$$K_{ij}(x,\xi)u_j(0)+\int_{0^+}^{t}K_{ij}(x,\xi-\xi')\frac{\partial u_j(t')}{\partial t'}dt'=F_i(x,t)$$

- Above could be solved sequentially using Newton-Cotes expansion (material history effect needs to be considered)

$$u_j(t_n)=\left[K_{ij}(x,0)+K_{ij}(\xi_n-\xi_{n-1})\right]^{-1}\left\{\begin{array}{l}2F_i(t_n)-\left[K_{ij}(\xi_n)-K_{ij}(\xi_n-\xi_1)\right]u_j(0)\\ -\sum_{m=1}^{n-1}\left[K_{ij}(\xi_n-\xi_{m-1})-K_{ij}(\xi_n-\xi_{m+1})\right]\boxed{u_j(t_m)}\end{array}\right\}$$

- Alternatively, recursive formulation could be developed that requires only few previous solutions

Time-Integration Analysis

Recursive Formulation (extension from Yi and Hilton, 1994):

$$\left[\sum_{h=1}^{m}\left(K_{ij}^{e}(x)\right)_h\cdot\left[\left(v_{ij}^{1}(x,t_n)\right)_h\Delta t-\left(v_{ij}^{2}(x,t_n)\right)_h\right]\frac{2}{\Delta t^{2}}\right]u_j(t_n)=F_i(t_n)$$

$$+\sum_{h=1}^{m}\left[\left[\left(K_{ij}^{e}(x)\right)_h\cdot Exp\left[-\frac{\xi(t_n)}{\left(\tau_{ij}(x)\right)_h}\right]\right]\right]\left\{\left(v_{ij}^{1}(x,t_{n-1})\right)_h\left[u_j(t_{n-1})\frac{2}{\Delta t}+\dot{u}_j(t_{n-1})\right]\right.$$

$$\left.-\frac{2}{\Delta t^{2}}\left(v_{ij}^{2}(x,t_{n-1})\right)_h\left[u_j(t_{n-1})+\dot{u}_j(t_{n-1})\Delta t\right]-u_i(t_0)+\left(v_{ij}^{1}(x,t_0)\right)\dot{u}_j(t_0)\right\}+\left(R_i(t_n)\right)_h\Bigg]$$

Where,

$$\left(v_{ij}^{1}(x,t_n)\right)_h=\int_{0}^{t_n}Exp\left[-\xi(t^{'})/\left(\tau_{ij}(x)\right)_h\right]dt^{'};\ \left(v_{ij}^{2}(x,t_n)\right)_h=\int_{t_{n-1}}^{t_n}\left(v_{ij}^{1}(x,t^{'})\right)_h dt^{'}$$

$$\left(R_i(t_n)\right)_h=K_{ij}^{e}\cdot Exp\left[-\xi(t^{'})/\left(\tau_{ij}(x)\right)_h\right]\cdot\left(v_{ij}^{2}(x,t_n)\right)_h\ddot{u}_j(t_{n-1})$$

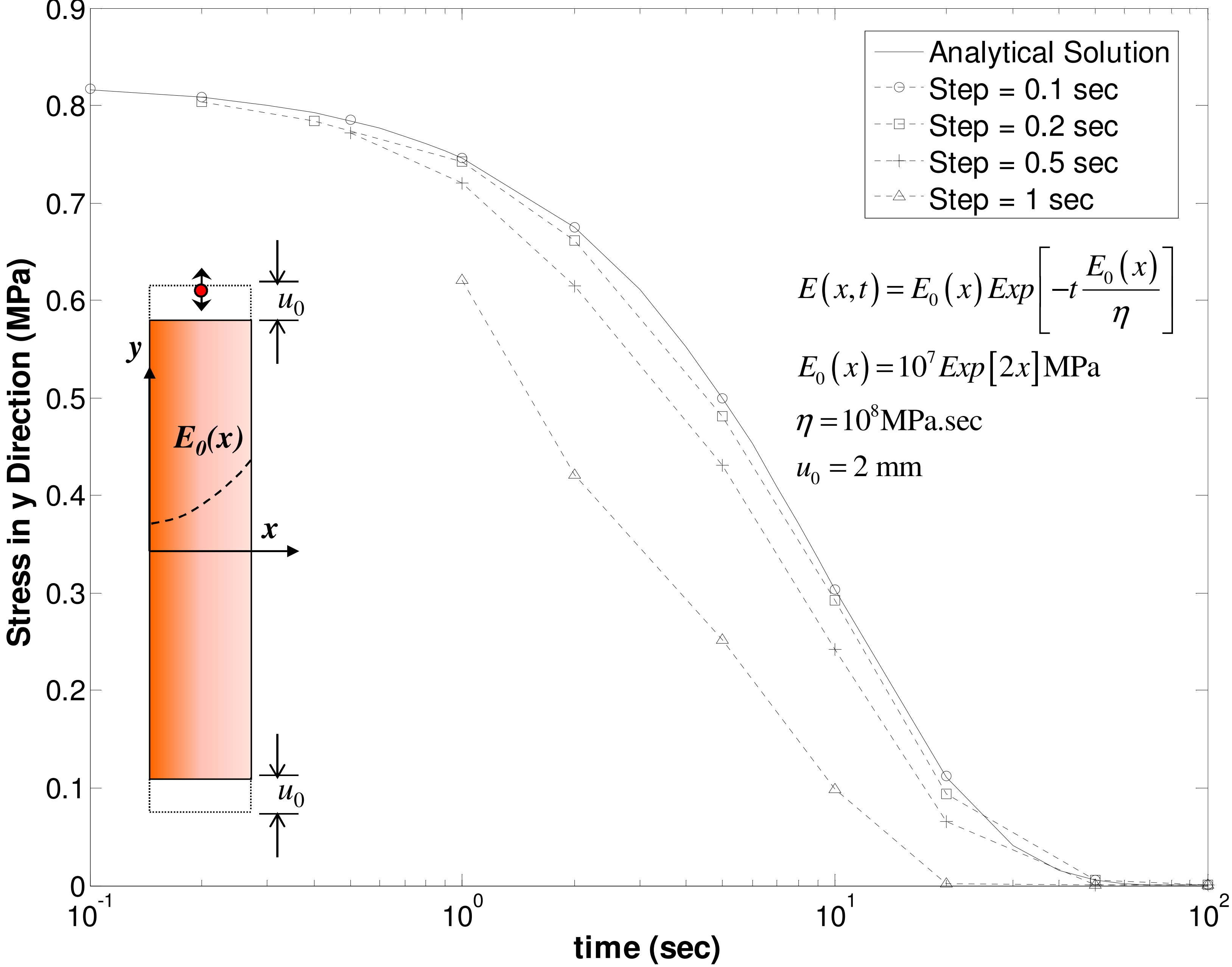
$$+Exp\left[-\xi(t^{'})/\left(\tau_{ij}(x)\right)_h\right]\left(R_j(t_{n-1})\right)_h$$

Verification examples:

S. Yi, and H.H. Hilton, (1994) "Dynamic finite element analysis of viscoelastic composite plates in the time domain," International Journal for Numerical Methods in Engineering, 37:4081-96.

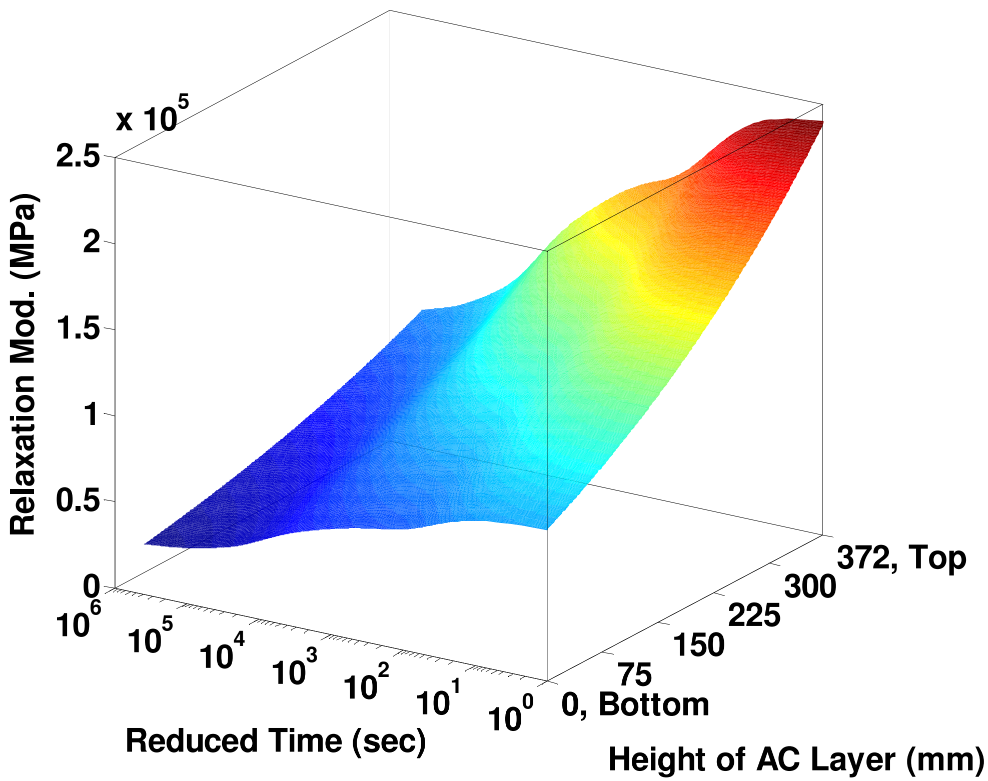
1. Analytical solution (Stress relaxation shown here)
2. Comparison with commercial software

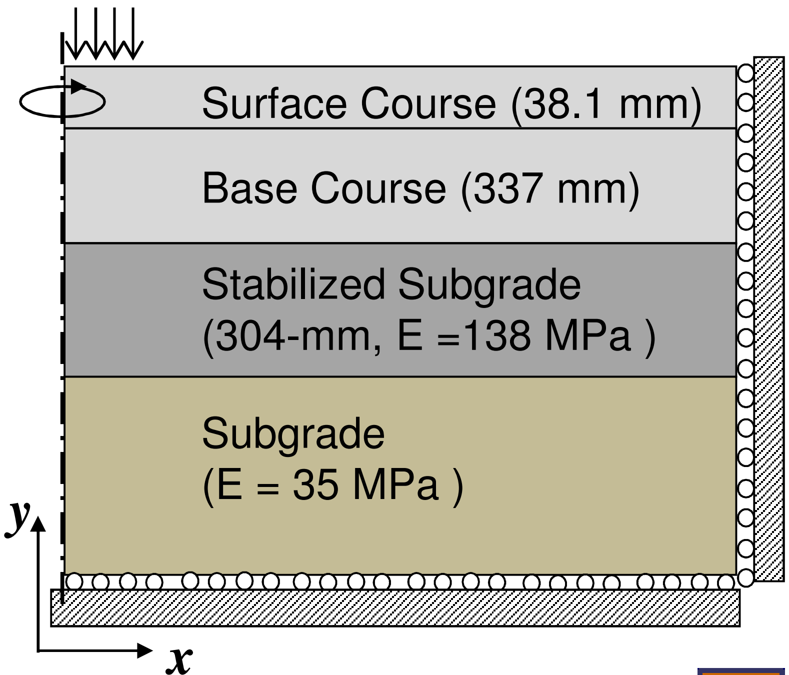
Verification: Analytical Solution



Example-1: Full Depth AC Pavement

- Based on I-155, Lincoln IL
- Single Tire load simulated (up to 1000 sec loading time)
- Aged material properties (Apeagyei et al., 2008)
  - Surface of AC: Long term aged
  - Bottom of AC: Short term aged



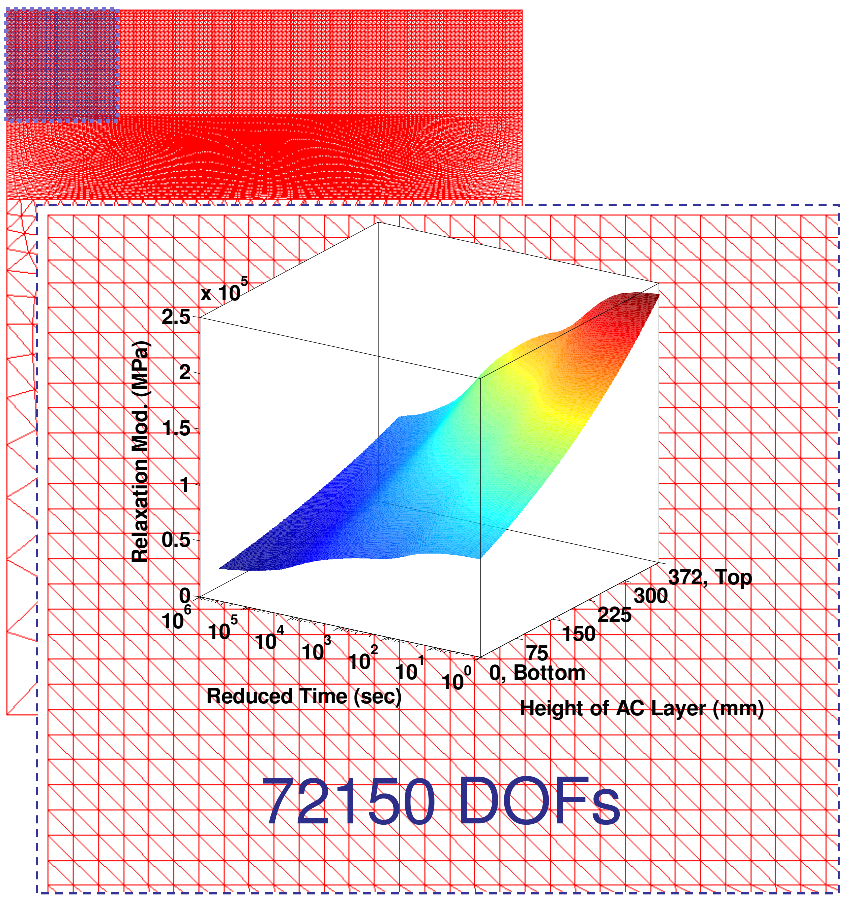


A.K. Apeagyei, W.G. Buttlar, and B.J. Dempsey, (2008) "Investigation of Cracking Behavior of Antioxidant-Modified Asphalt Mixtures," Journal of Asphalt Paving Technologists, Vol. 77.

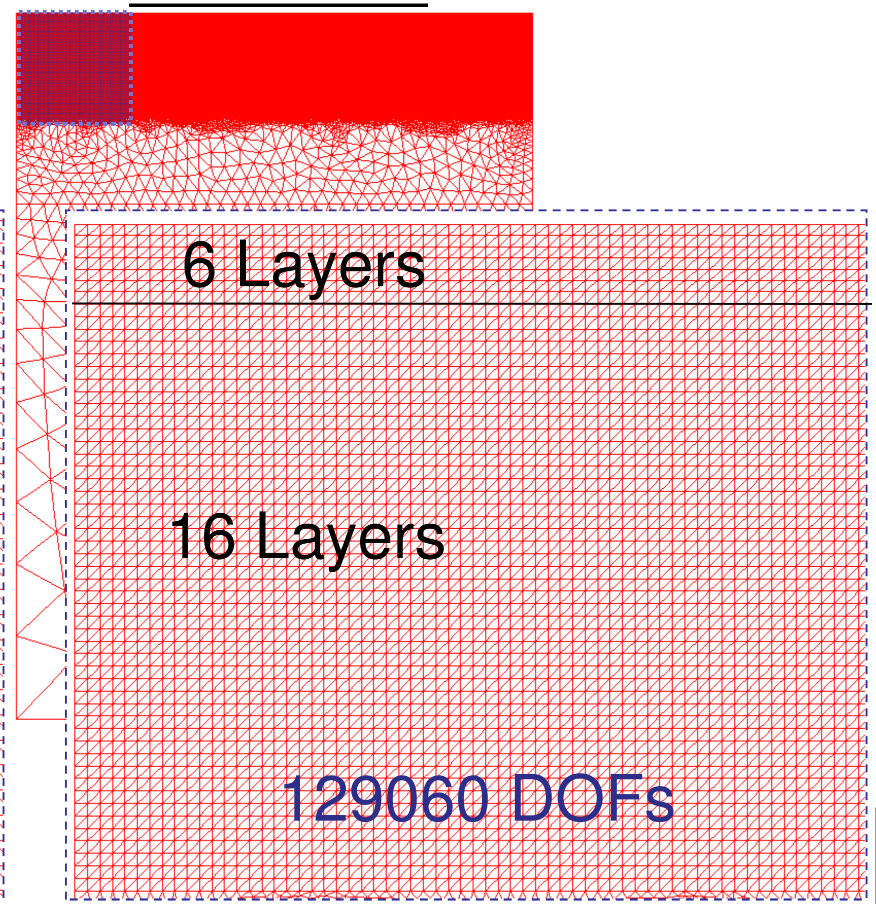
Example-1: FEM Discretization

- Two mesh refinement levels: (material distributions)
  - Coarse mesh: Graded and Homogeneous simulations
  - Fine Mesh: Layered simulations

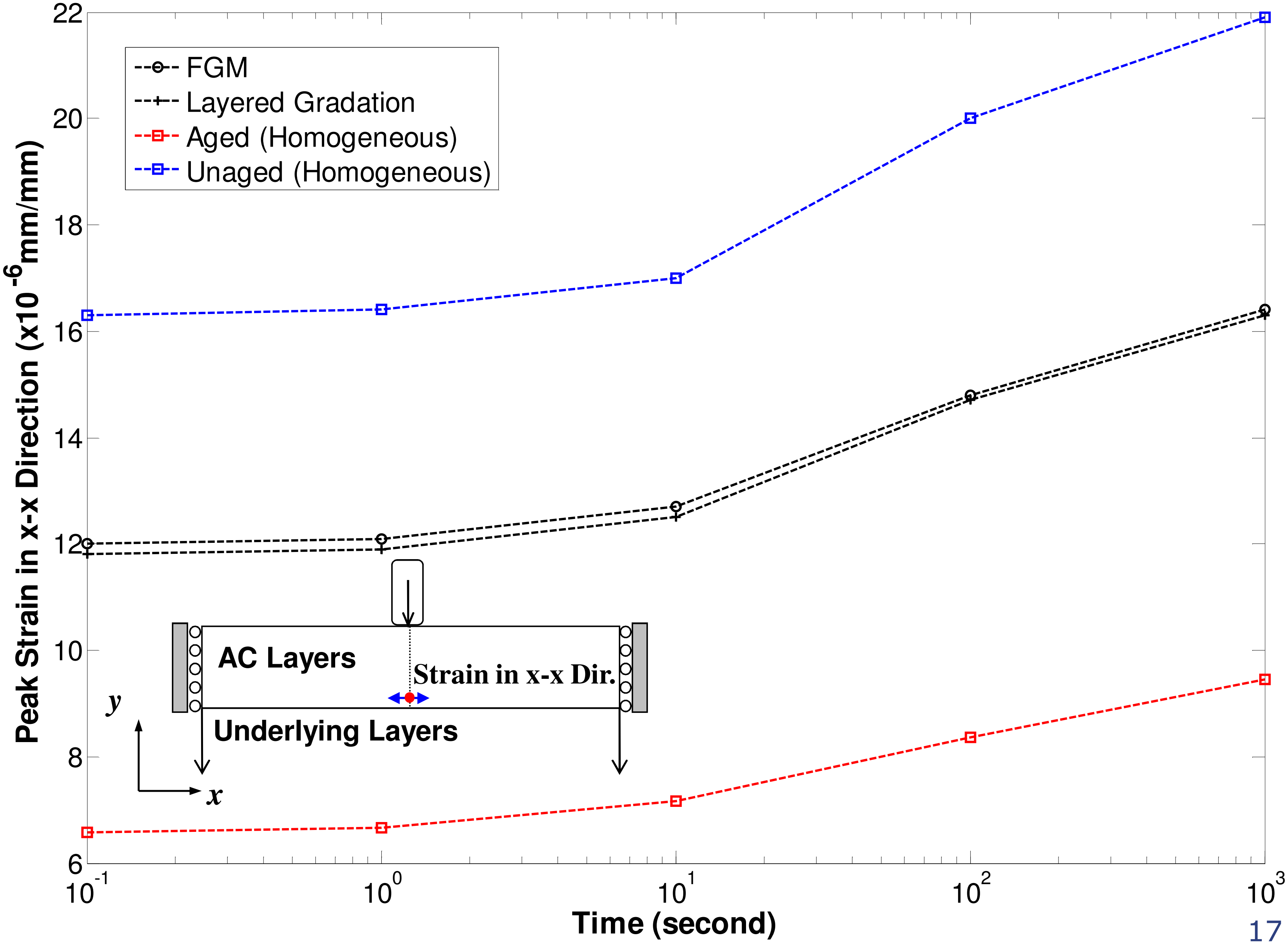
Coarse Mesh



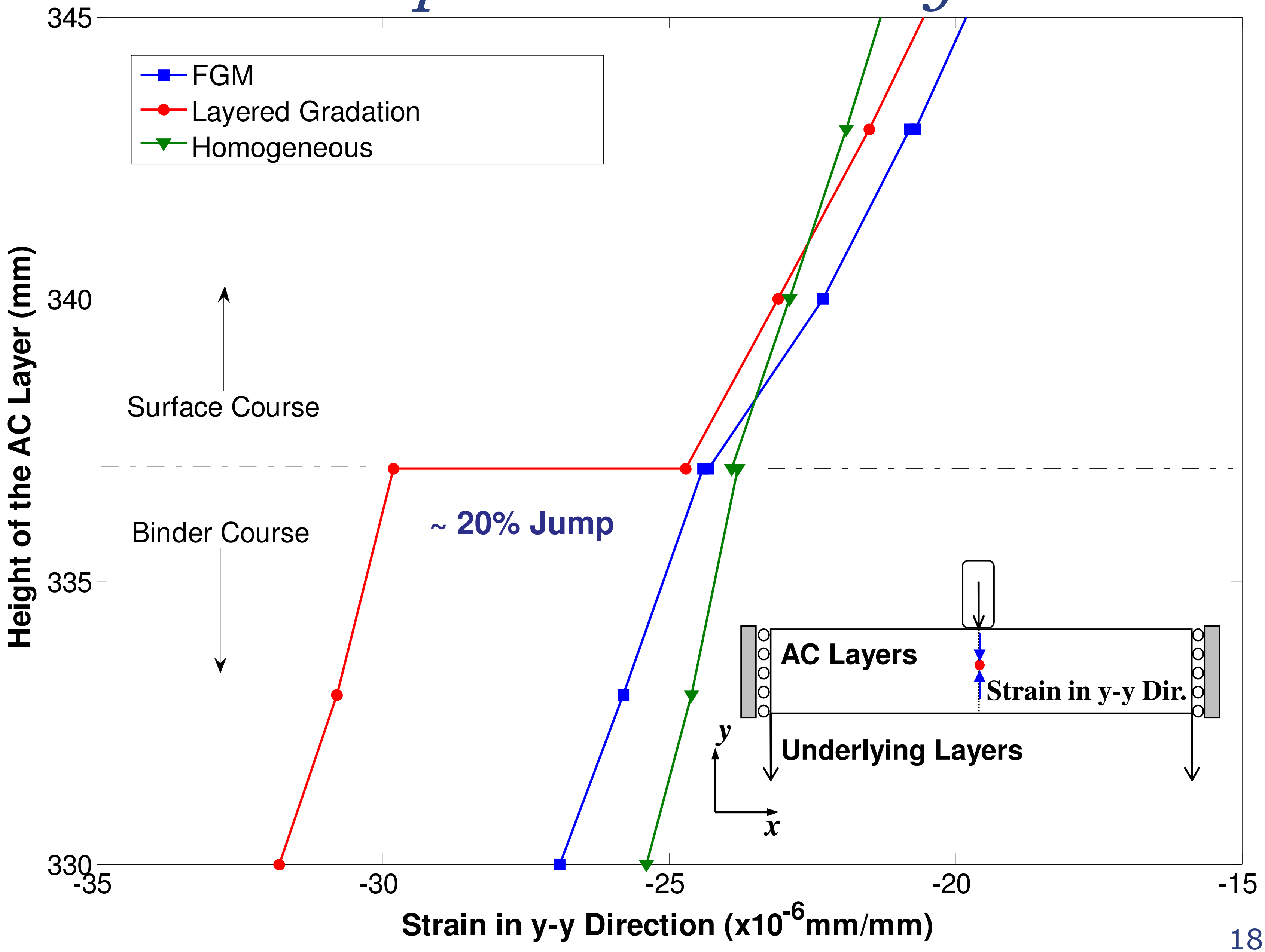
Fine Mesh



Example-1: Strain at Bottom of AC



Example-1: FGM vs. Layered



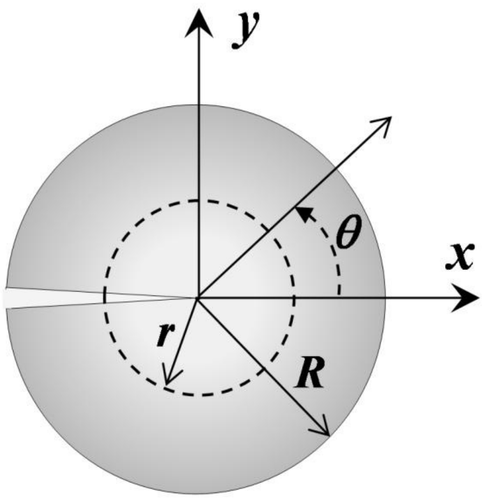
Example 2: Boundary Layer Model

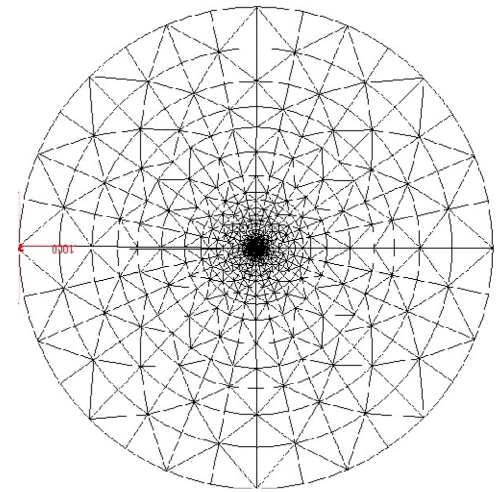
- Displacement loading for Mode-I: (Kim, 2003)

$$u_i = \frac{K_I}{G_{tip}} \sqrt{\frac{r}{2\pi}} g_i^I(\theta);$$

where, $g_1^I(\theta) = \cos\frac{\theta}{2}\left[\frac{1}{2}(\kappa - 1) + \sin^2\frac{\theta}{2}\right]$ and

$$g_2^I(\theta) = \sin\frac{\theta}{2}\left[\frac{1}{2}(\kappa + 1) - \cos^2\frac{\theta}{2}\right]$$

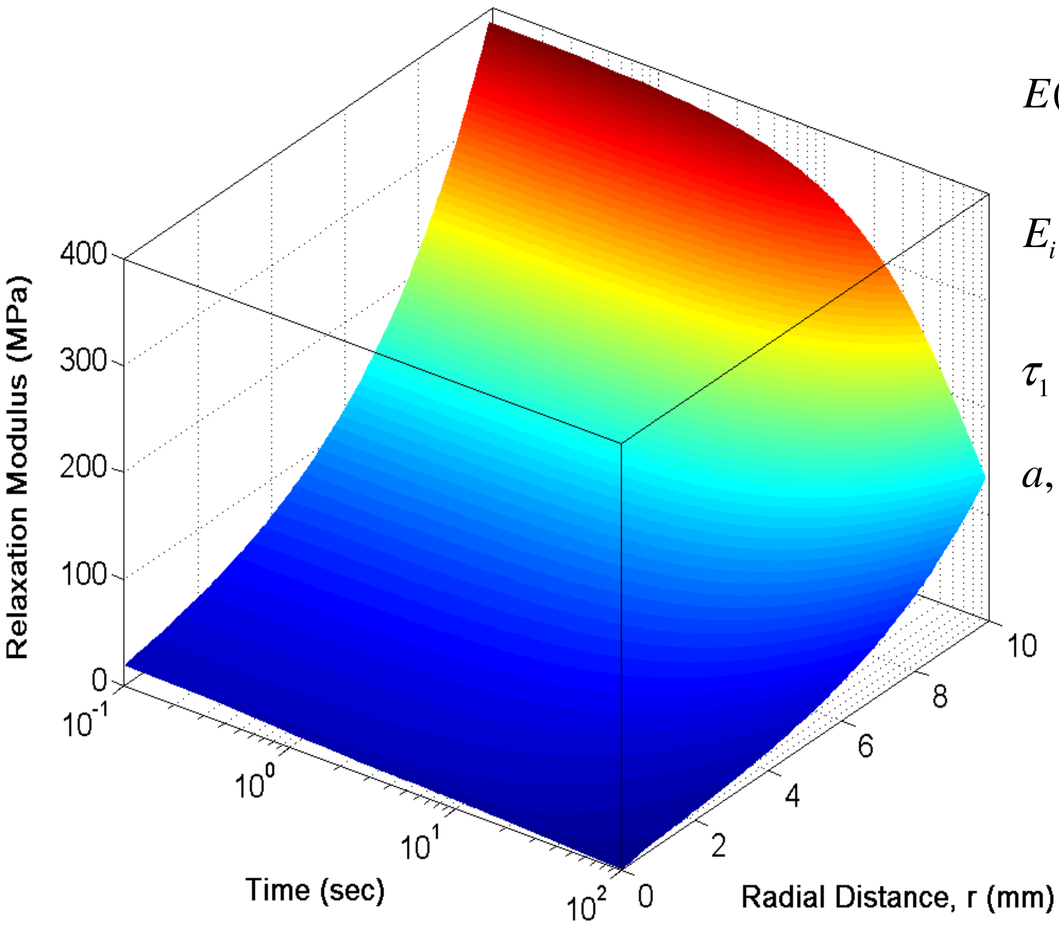




J.H. Kim,"Mixed-Mode Crack Propagation in Functionally Graded Materials," Doctorate Thesis, University of Illinois at Urbana-Champaign, Urbana, IL, 2003.

Material Properties

- Radial Gradation:



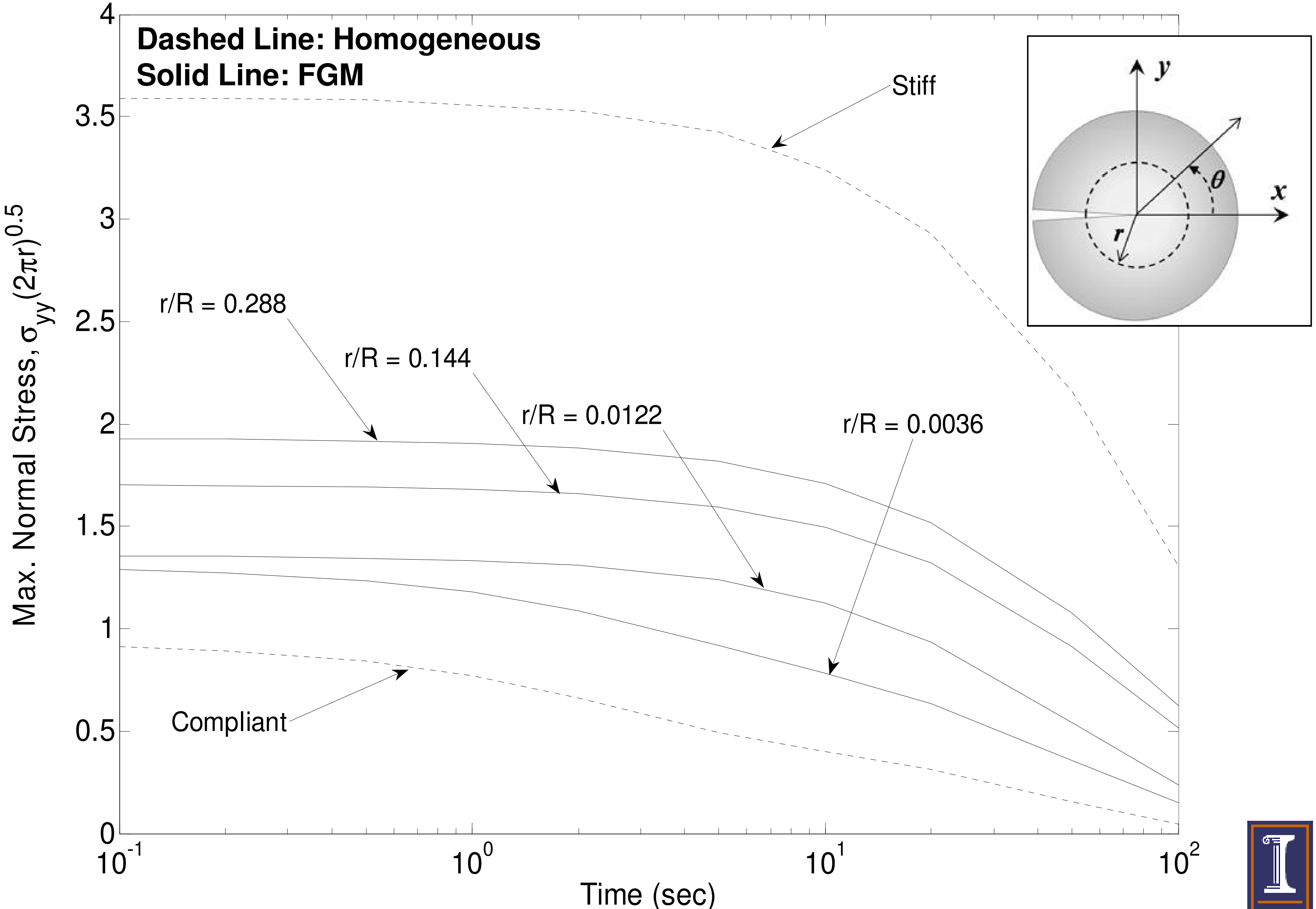
$$E(r,t)=\sum_{i=1}^{2} E_i(r)Exp\left(-\frac{t}{\tau_i(r)}\right);$$

$$E_i(r)=E_0 Exp\left(a\frac{r}{R}\right);$$

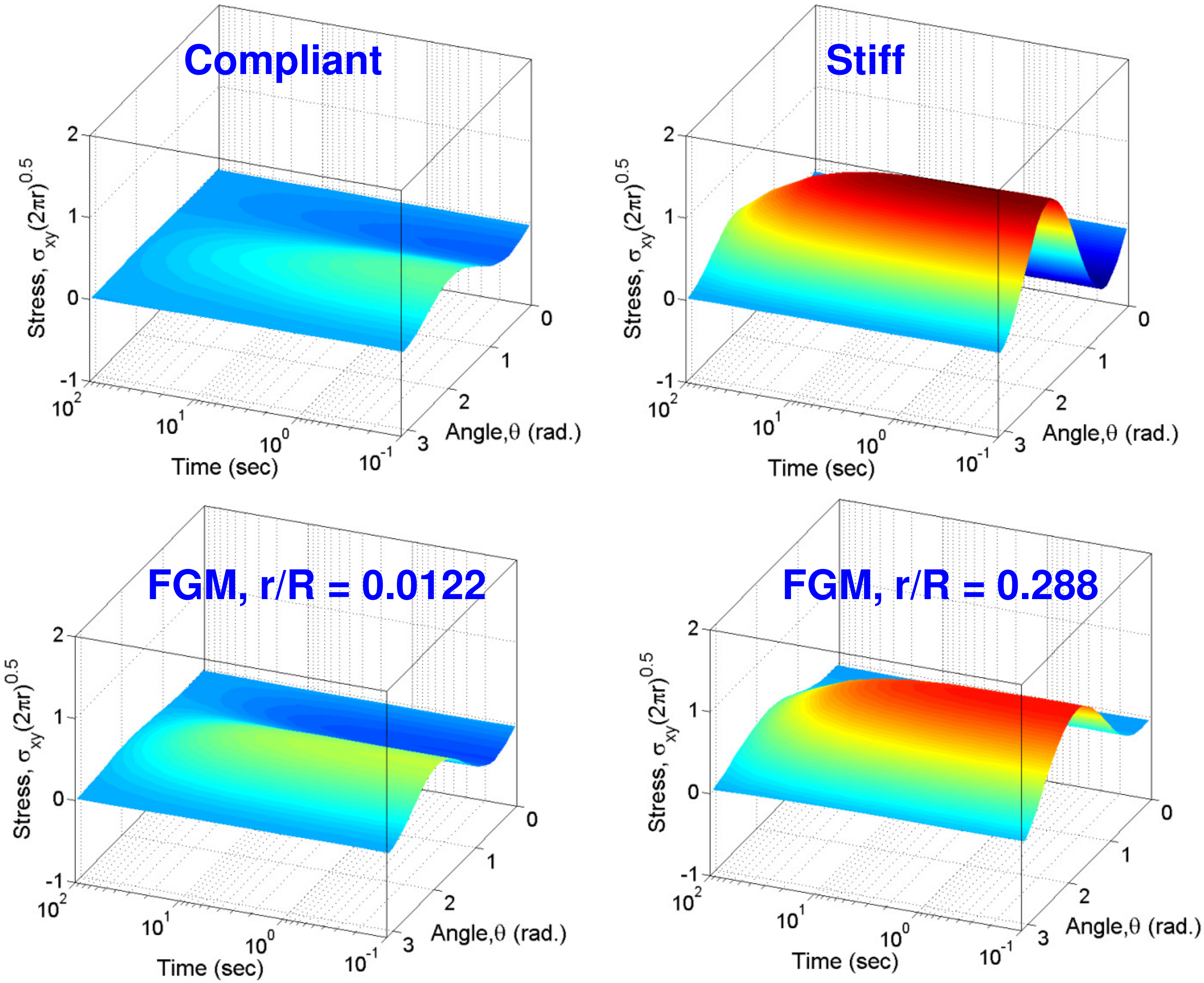
$$\tau_1(r)=b\left(1+\frac{r}{R}\right)^c;\ \tau_2(r)=d\left(1-Exp\left(e\frac{r}{R}\right)\right)$$

$a,b,c,d,$ and e are all scalar material constants

Viscoelastic Results, Peak Normal Stresses



Viscoelastic Results, Shear Stresses



Summary

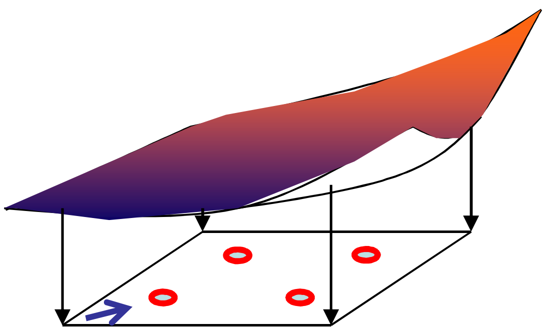
- Viscoelastic graded finite elements using GIF are proposed
- Recursive formulation is utilized for time-integration analysis
- Verifications are performed by comparison of present approaches with:
  - Analytical solutions
  - Commercial software (*ABAQUS*)
- Application Examples:
  - Aged Asphalt Pavement
  - Boundary layer model for fracture analysis of viscoelastic FGM

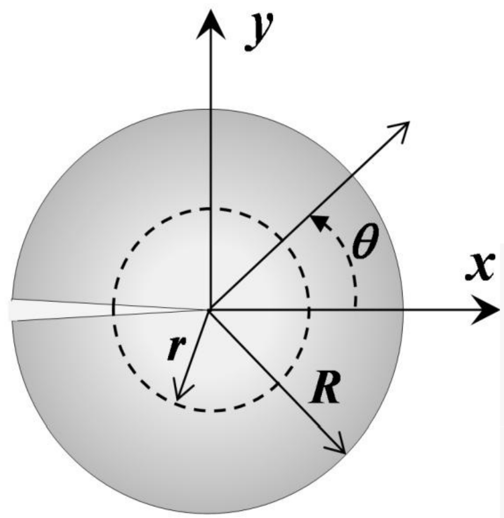
Conclusions

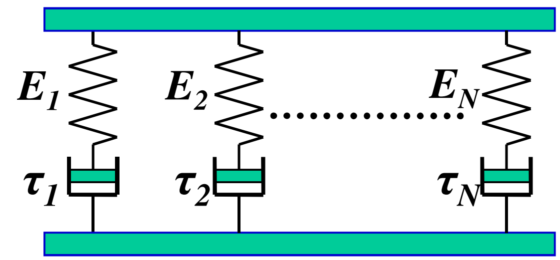
- Aging and temperature dependent property gradients should be considered in simulation of asphalt pavements
- Non-homogeneous viscoelastic analyses procedures presented here are suitable and preferred for simulation of asphalt pavement systems
- Proposed procedures yield greater accuracy and efficiency over conventional approaches
- Layered gradation approach can yield significant errors
  - Most pronounced errors are at layer interfaces in the stress and strain quantities.

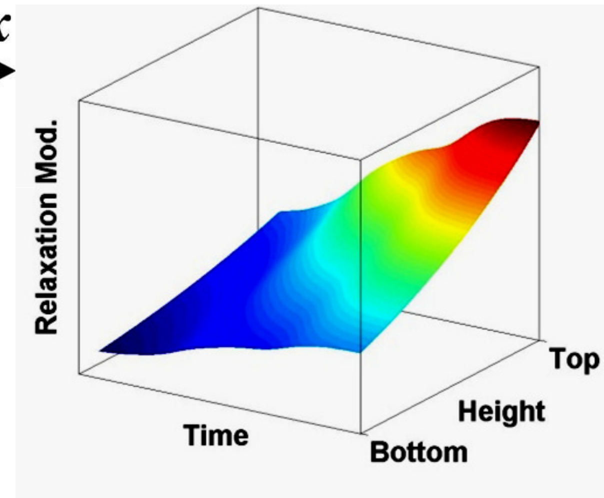
Thank you for your attention!!

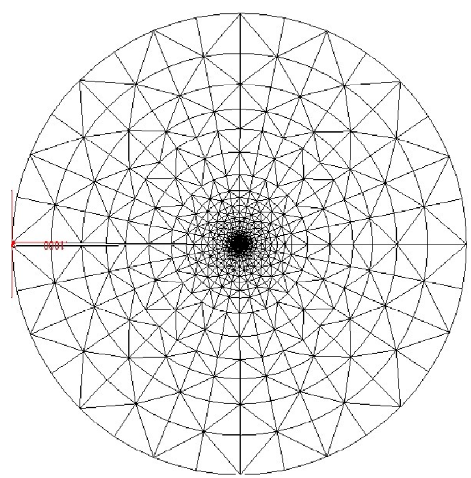
Acknowledgement: USDOT Nextrans Research Center









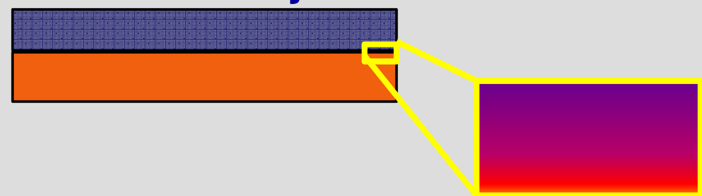


Other Applications and Future Extensions

Applications and Extensions

Micromechanical Predictions

Transition Layers Characteristics



Graded Viscoelastic + Stress Dependent Model

Other material models:
- 2S2P1D
- Power law etc.

Analysis of graded viscoelastic materials:
- Metals at high temperature
- Polymers
- Geotechnical materials
- PCC/FRC
- Biomaterials
- Food industry

Design of Viscoelastic FGMs

$E(x,t)$

ABAQUS UMAT Subroutine