10th U.S. National Congress on Computational Mechanics

Extraction of Cohesive Properties of Elasto-Plastic material using Inverse Analysis

Arun Lal Gain, Jay Carroll, Glaucio H. Paulino, John Lambros

University of Illinois at Urbana Champaign

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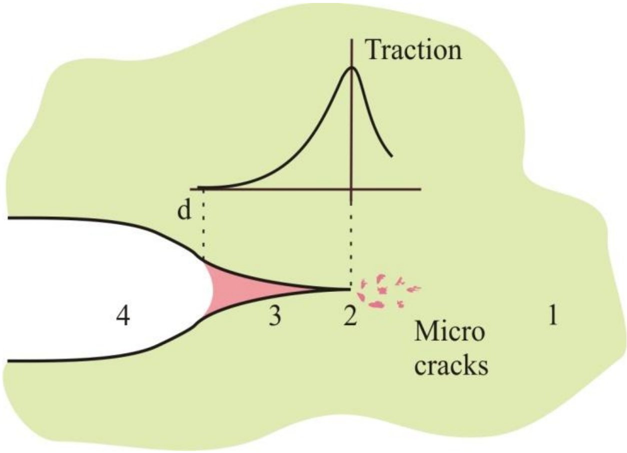
Introduction: Cohesive Zone Model

- **Cohesive Zone Modeling** - fracture seen as phenomenon of gradual separation taking place across cohesive zone (path of crack) and resisted by cohesive tractions
- Four staged failure
  - Stage 1: Material homogeneous
  - Stage 2: Crack Initiation
    - Criterion: Stress reaching tensile strength (simplified)
  - Stage 3: Crack propagation based on traction v/s separation curve
  - Stage 4: Complete failure
    - Criterion: e.g. Crack width reaches certain predefined value



Introduction: Cohesive Zone Model

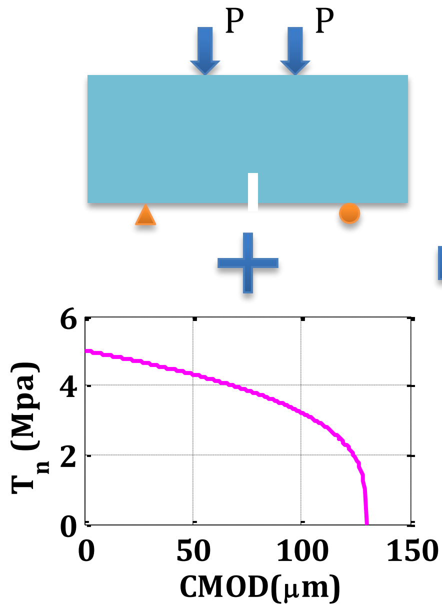
- Various approaches to obtain cohesive zone model are available in literature
  - Obtain through experiments
    - Direct tension test
      - van Mier, van Vliet, *Uniaxial tension test for determination of fracture parameters of concrete*, Fracture of Concrete & Rock, 2002
  - Assume the shape
    - CZM shape significantly affects fracture analysis results – should be chosen carefully
      - Song S.H, Paulino G.H, Buttlar G.H, *Influence of cohesive zone model shape parameter on asphalt concrete fracture behavior*, American Institute of Physics, 2008

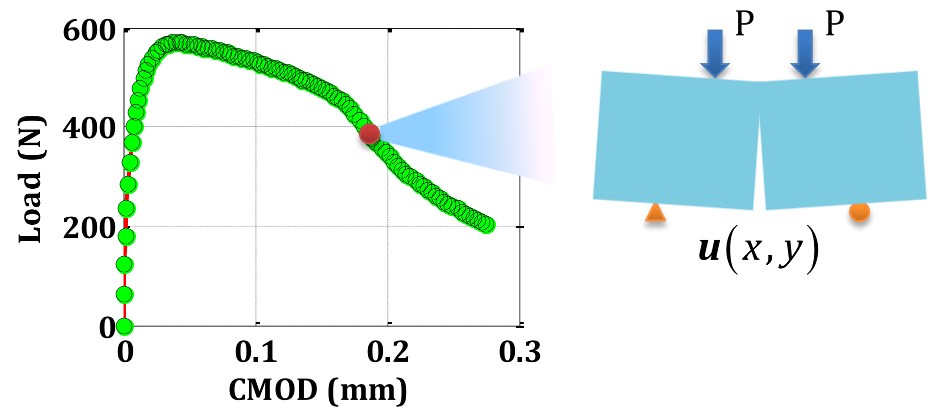
Introduction: Cohesive Zone Model

- Indirect method: Inverse analysis
  - Development in experimental stress analysis techniques like photo-elasticity, DIC have made Inverse Analysis attractive
    - van Mier, *Fracture processes of concrete : assessment of material parameters for fracture models*, CRC Press, 1997
    - Hanson J. H. , *An experimental - computational evaluation of the accuracy of the fracture toughness tests on concrete*, PhD Thesis, Cornell university, 2000
    - Hanson J.H., Ingraffea A.R. *Using numerical simulations to compare the fracture toughness values for concrete from the size-effect, two-parameter and fictitious crack models*, Engineering Fracture Mechanics, 2003

Introduction: Forward v/s Inverse Problem

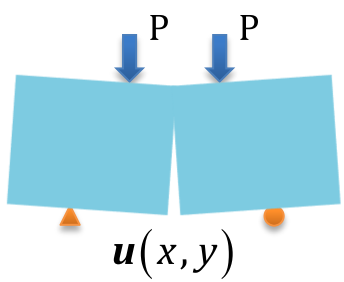
Forward Problem





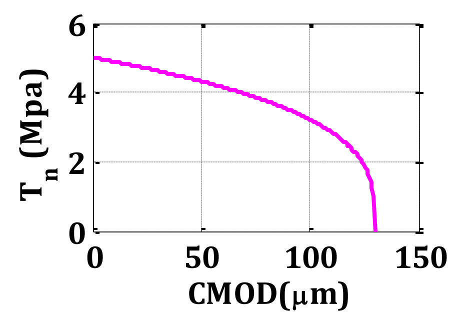
Global Response

Inverse Problem



Optimization

Nelder-Mead Scheme

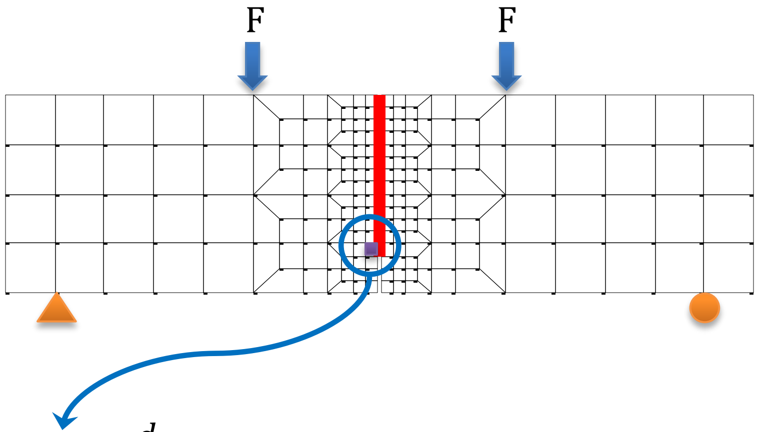


DIC / Synthetic Data from forward problem

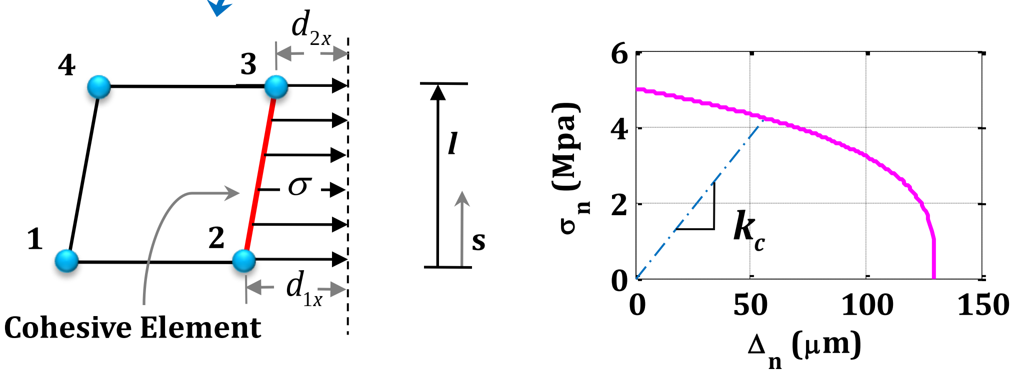
Constitutive Response

Elasto-Plastic Forward Problem

$$\left(\mathbf{K}_b\left(\mathbf{u}_i^j\right)+\mathbf{K}_{\text{coh}}\left(\mathbf{u}_i^j,\lambda_{\text{coh}}\right)\right)\cdot\mathbf{u}_i^{j+1}=\mathbf{F}_i^{ext}$$



$$\mathbf{K}_b \rightarrow \text{Plane Stress J2 Plasticity}$$



$$\mathbf{K}_{\text{coh}}^{\text{element}}=\int_{-1}^{1}k_c\left(\Delta_n\right)\mathbf{N}\mathbf{N}^T\,t\,l\,d\eta$$

$$\mathbf{d}_x=\left\{\mathbf{d}_{1x},\mathbf{d}_{2x}\right\}^T$$

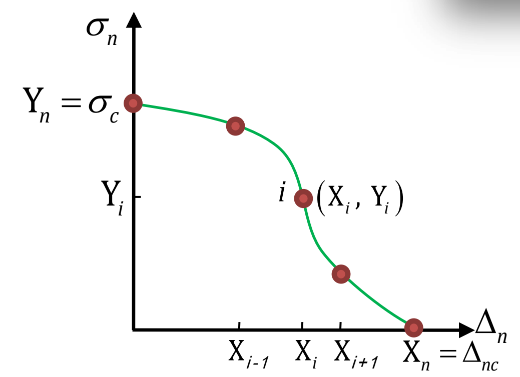
$$f_{\text{coh}}=\mathrm{K}_{\text{coh}}^{\text{element}}\,\mathrm{u}^{\text{element}}=\frac{1}{2}\int_{-1}^{1}\sigma_n\left(-2\,\mathbf{d}_x^T\,\mathbf{N}\right)\mathbf{N}\,t\,l\,d\eta$$

Shen, B., 2009, *"Functionally Graded Fiber – Reinforced Cementitious Composites : Manufacturing and Extraction of Cohesive Properties using Finite Elements and Digital Image Correlation"*, PhD Thesis, UIUC

Modeling Approaches: Shape Regularization

- Elasto-Plastic Inverse Problem

$$\left(\mathbf{K}_b(\mathbf{u})+\mathbf{K}_{\text{coh}}(\mathbf{u},\boldsymbol{\lambda}_{\text{coh}})\right)\cdot\mathbf{u}=\mathbf{F}^{ext}$$



Nelder-Mead Optimization

$$\min_{\lambda}\ \Phi(\boldsymbol{\lambda}) = w_1\left\|\mathbf{F}^{\text{ext}}-\mathbf{F}^{\text{int}}\right\|+w_{f_1}f_1(\mathbf{Y})+w_{f_2}f_2(\mathbf{X}),\quad \Phi:\mathrm{R}^{\mathrm{M}}\rightarrow\mathrm{R}$$

$$f_1(\mathbf{Y})=\sum_i 10^{\psi_1(\gamma_1-Y_i)},\qquad f_2(\mathbf{X})=\sum_i 10^{\psi_2(\zeta_i-\gamma_2)}$$

$\boldsymbol{\lambda}_{coh}$ = Cohesive Parameters

$$=\{X_1,X_2,..,X_n,Y_1,Y_2,..,Y_n\}$$

where, $$\zeta_i=\left|\frac{X_i-(X_{i+1}+X_{i-1})/2}{(X_{i+1}-X_{i-1})/2}\right|$$

$$\gamma_i<<1,\qquad \psi_i>>1$$

Constraints

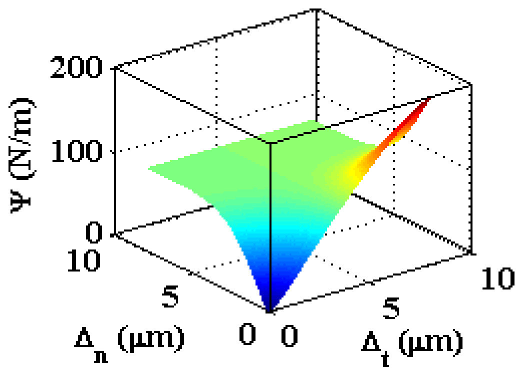
$$\sigma>0$$

$$0<X_1<X_2<..<X_n$$

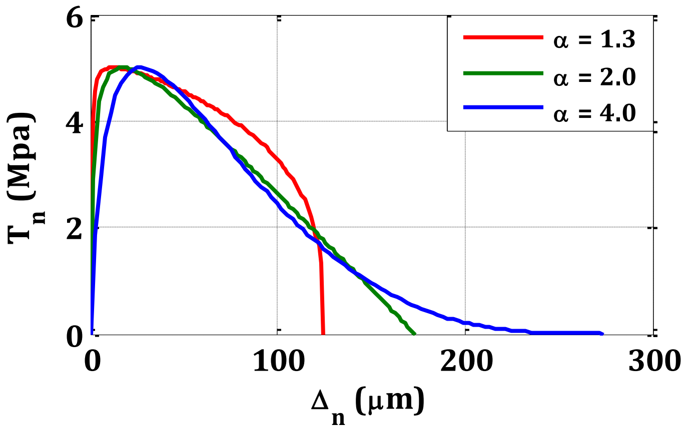
Modeling Approaches: PPR model

- Unified potential based model: PPR (Park-Paulino-Roesler)

$$\psi(\Delta_n,\Delta_t)=\min(\phi_n,\phi_t)+\left[\Gamma_n\left(1-\frac{\Delta_n}{\delta_n}\right)^{\alpha}\left(\frac{m}{\alpha}+\frac{\Delta_n}{\delta_n}\right)^{m}+\langle\phi_n-\phi_t\rangle\right]\left[\Gamma_t\left(1-\frac{|\Delta_t|}{\delta_t}\right)^{\beta}\left(\frac{n}{\beta}+\frac{|\Delta_t|}{\delta_t}\right)^{n}+\langle\phi_t-\phi_n\rangle\right]$$



$$T_n(\Delta_n,\Delta_t)=\frac{\partial\psi}{\partial\Delta_n}\qquad T_t(\Delta_n,\Delta_t)=\frac{\partial\psi}{\partial\Delta_t}$$

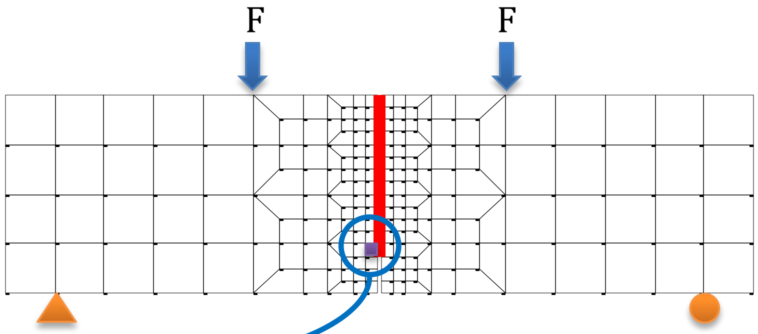


Park K., Paulino G.H., Roesler J.R., 2009, *A unified potential-based cohesive model of mixed-mode fracture,* Journal of the Mechanics and Physics of Solids, 57, 891

Modeling Approaches: PPR model

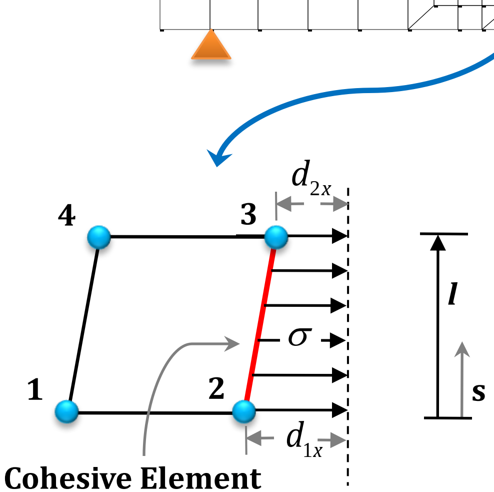
- Elasto-Plastic Forward Problem

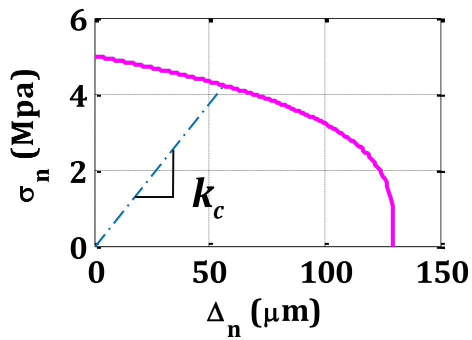
$$\left(\mathbf{K}_b\left(\mathbf{u}_i^j\right)+\mathbf{K}_{\mathrm{coh}}\left(\mathbf{u}_i^j,\boldsymbol{\lambda}_{\mathrm{coh}}\right)\right)\cdot\mathbf{u}_i^{j+1}=\mathbf{F}_i^{ext}$$



$$\boldsymbol{\lambda}_{\mathrm{coh}} = \text{Cohesive Parameters}$$

$$=\{\phi_n,\ \sigma_{\max},\ \alpha\}$$





$$\mathbf{K}_{\mathrm{coh}}^{\mathrm{element}}=\int_{-1}^{1}k_c\left(\Delta_n\right)\mathbf{N}\mathbf{N}^T\ t\,l\,d\eta$$

$$\mathbf{d}_x=\left\{\mathbf{d}_{1x},\mathbf{d}_{2x}\right\}^T$$

$$f_{\mathrm{coh}}=\mathrm{K}_{\mathrm{coh}}^{element}\ \mathrm{u}^{element}=\frac{1}{2}\int_{-1}^{1}\sigma\left(-2\,\mathbf{d}_x^T\,\mathbf{N}\right)\mathbf{N}\,t\,l\,d\eta$$

Modeling Approaches: PPR model

- Elasto-Plastic Inverse Problem

$$\left(\mathbf{K}_b\left(\mathbf{u}\right)+\mathbf{K}_{\mathrm{coh}}\left(\mathbf{u},\boldsymbol{\lambda}_{\mathrm{coh}}\right)\right)\cdot\mathbf{u}=\mathbf{F}^{ext}$$

Nelder-Mead Optimization

$$\min_{\lambda}\ \Phi(\boldsymbol{\lambda}) = \mathrm{w}_1\left\|\mathbf{F}^{\mathrm{ext}}-\mathbf{F}^{\mathrm{int}}\right\|+w_{f_1}f_1(\alpha), \quad \Phi:\mathrm{R}^3\rightarrow\mathrm{R}$$

$\boldsymbol{\lambda} =$ Cohesive Parameters

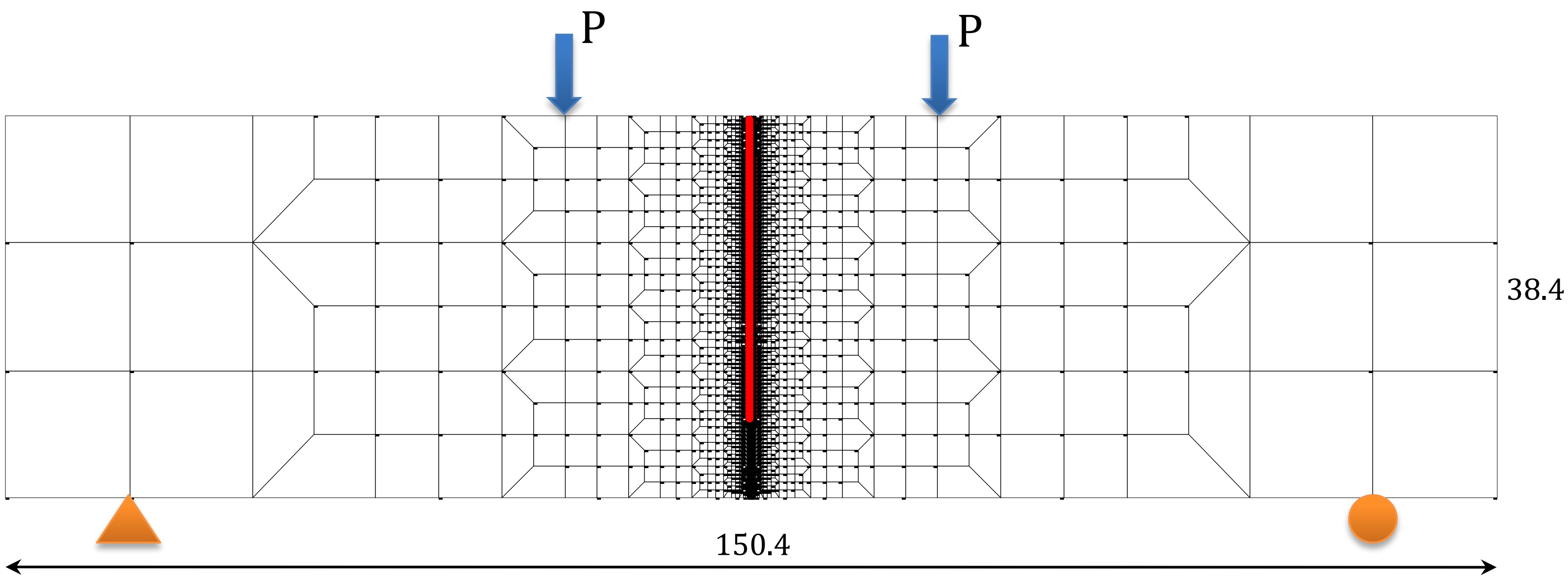
$= \{\phi_n,\ \sigma_{\max},\ \alpha\}$

Constraints: $\alpha > 1$

Barrier Function: $f_1(\alpha)=\sum 10^{\psi(1-\alpha)}$, $\psi >> 1$

Numerical Simulations

- Problem Details



$$E = 70\,\text{GPa}, \quad \nu = 0.25,$$

$$D_{app} = -0.14\,mm$$

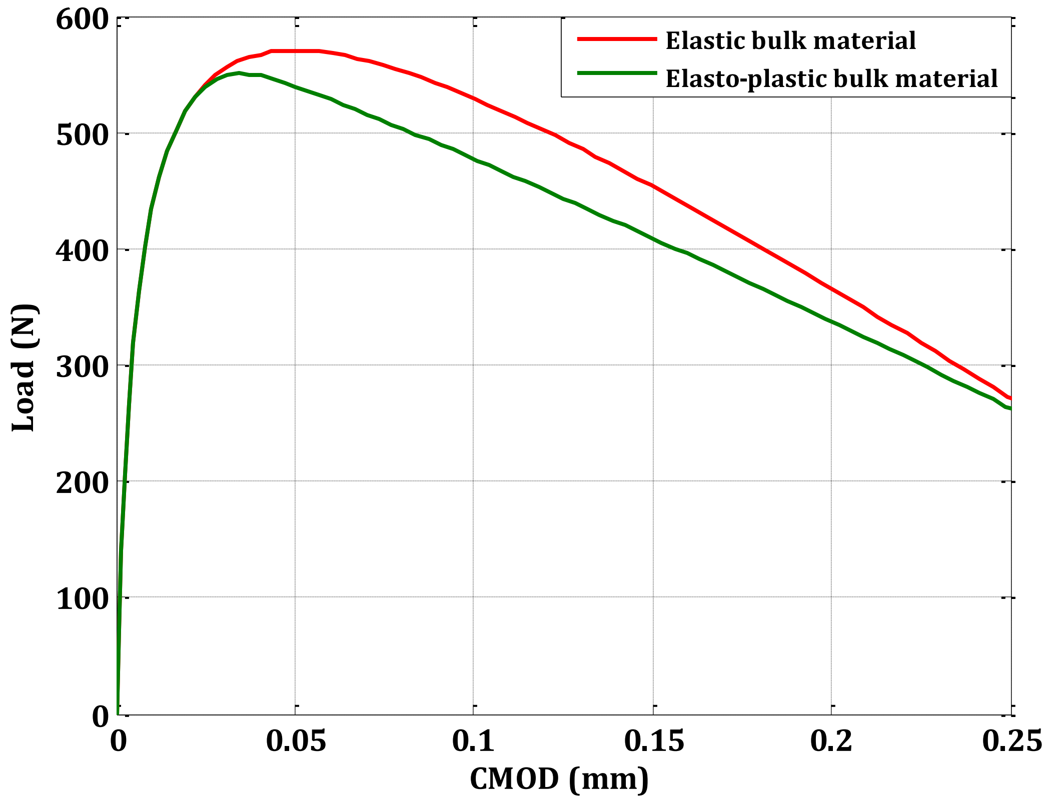
Isotroplic Hardening

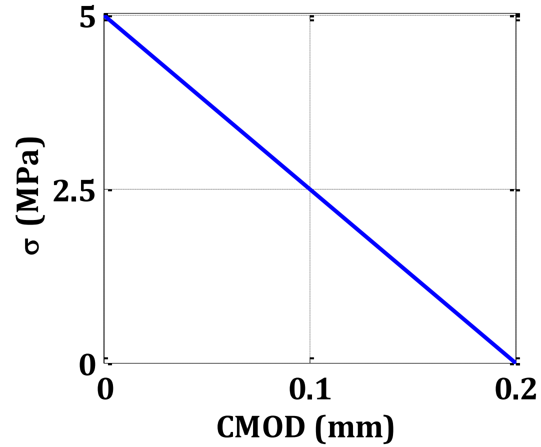
$$\rightarrow \ \sigma_y = 20\ \text{MPa}, \quad K = 100\ \text{MPa}$$

- 5782 Nodes
- 5346 Q4 Elements
- 304 Cohesive Elements
- Displacement Ctrl: 100 Steps

Numerical Simulations: Shape Regularization

- Forward Problem

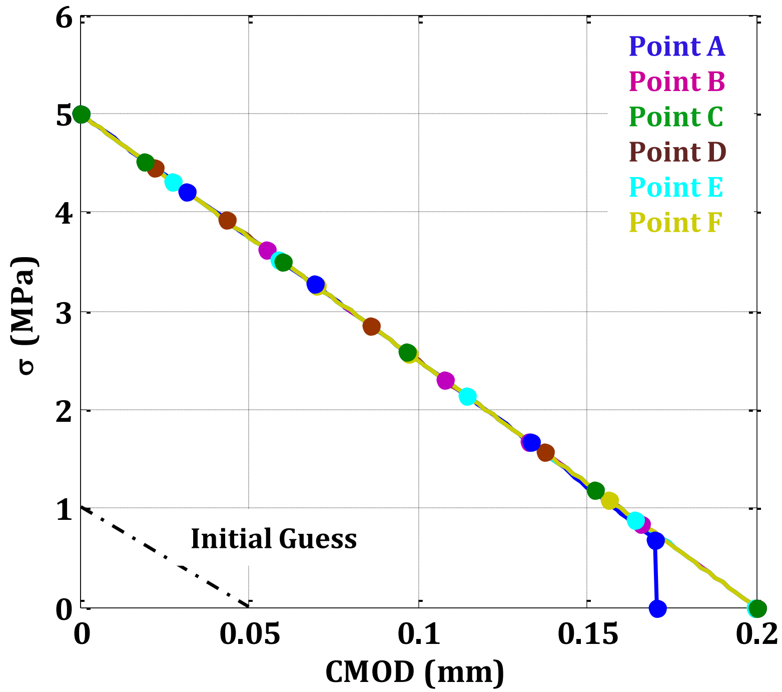


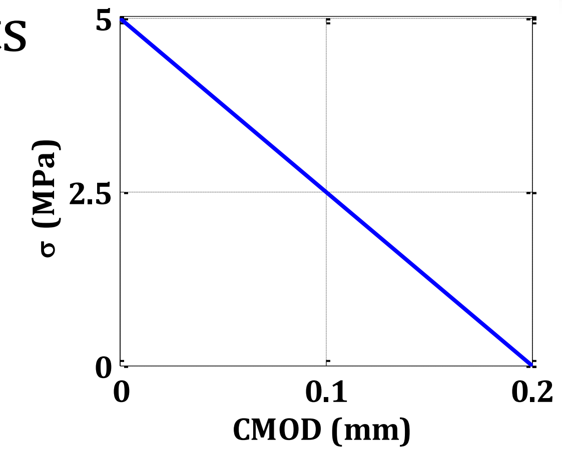


Linear softening CZM

Numerical Simulations: Shape Regularization

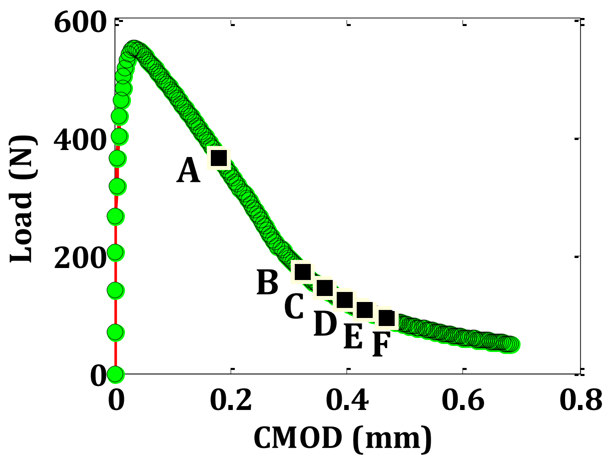
- Inverse Problem: Different Loading Points





Linear softening CZM

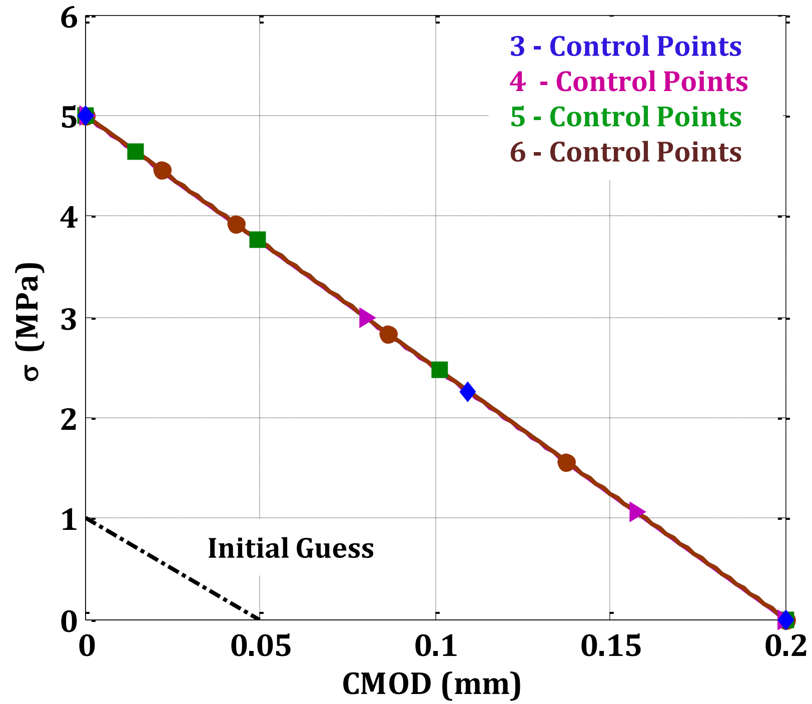
- 6 Control Points
- Piecewise Cubic Hermite interpolation
- Synthetic data without any noise

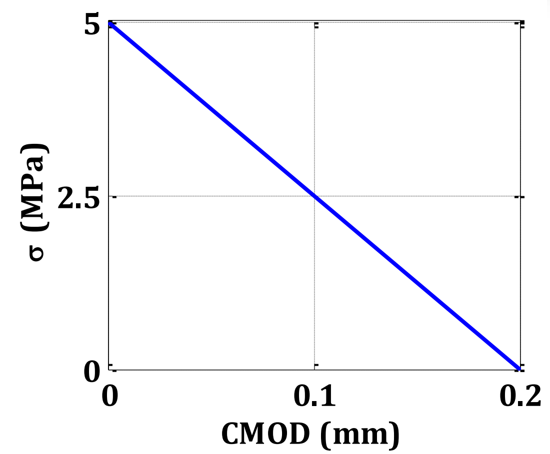


Forward Problem Plot

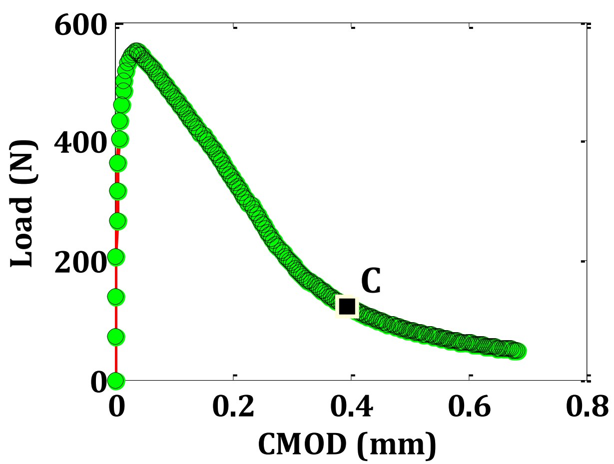
Numerical Simulations: Shape Regularization

- Inverse Problem: Various Control Points





Linear softening CZM

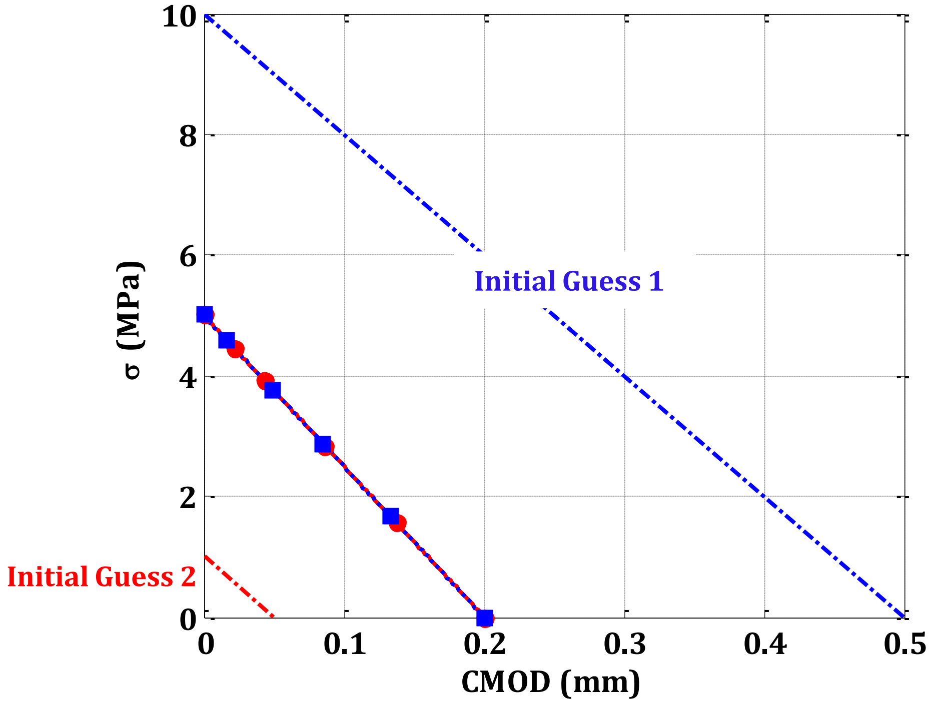


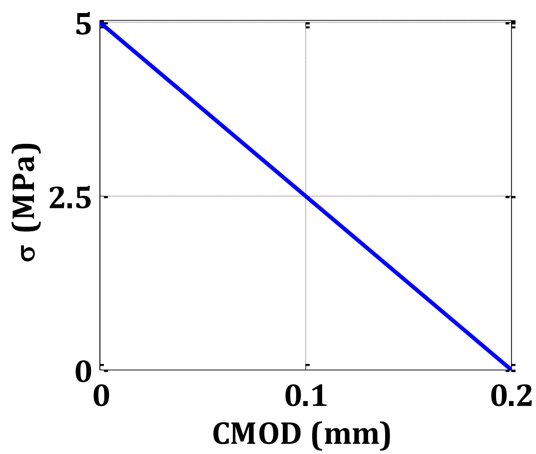
Forward Problem Plot

- Displacement field taken from loading point C = 122.1 N
- Piecewise Cubic Hermite interpolation
- Synthetic data without any noise

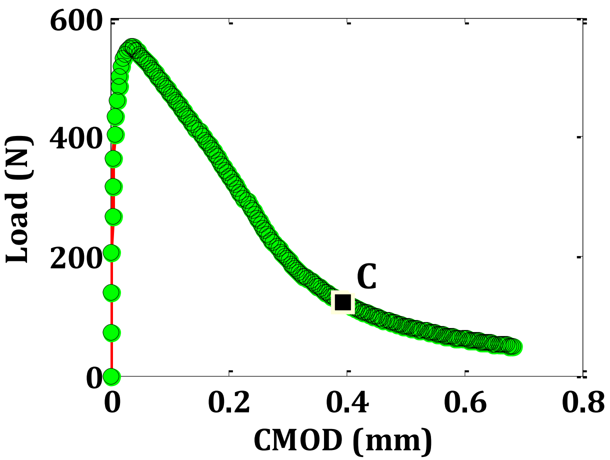
Numerical Simulations: Shape Regularization

- Inverse Problem: Different Initial Guess





Linear softening CZM

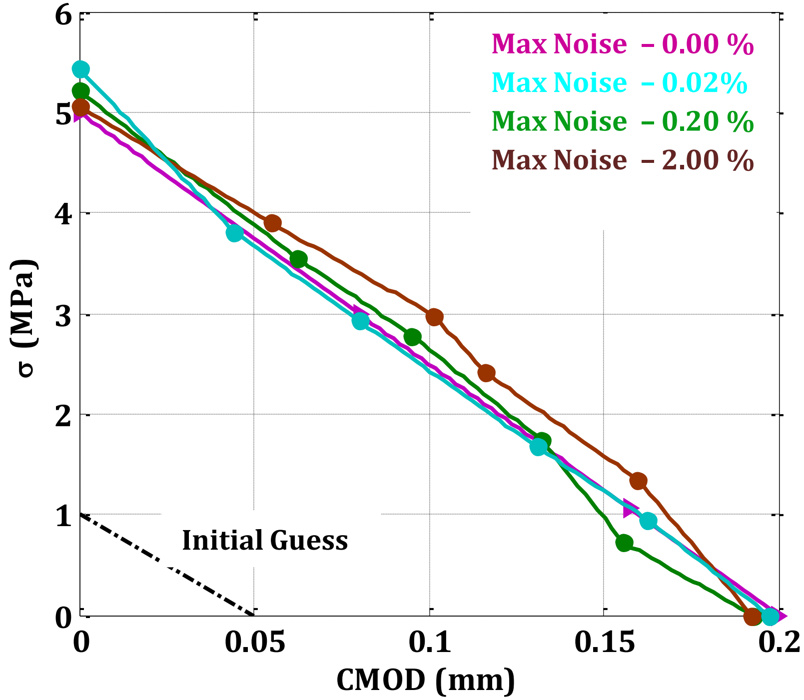


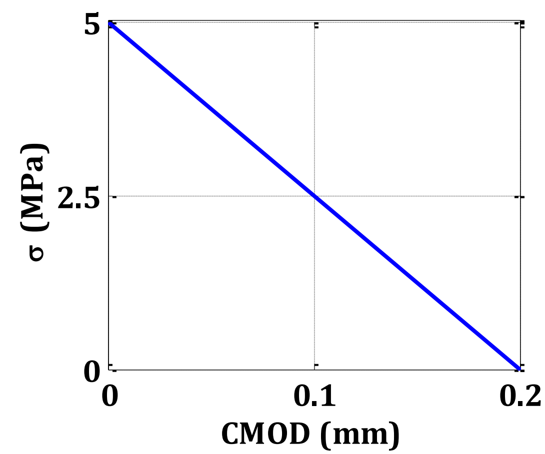
Forward Problem Plot

- Displacement field taken from loading point C = 122.1 N
- 6 Control Points
- Piecewise Cubic Hermite interpolation
- Synthetic data without any noise

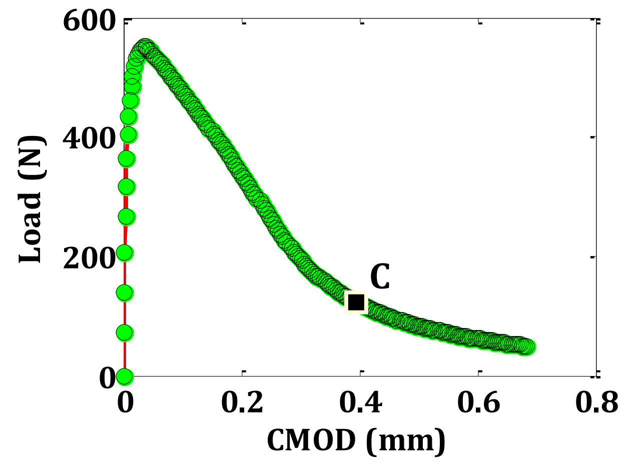
Numerical Simulations: Shape Regularization

- Inverse Problem: Noise in Synthetic Data





Linear softening CZM

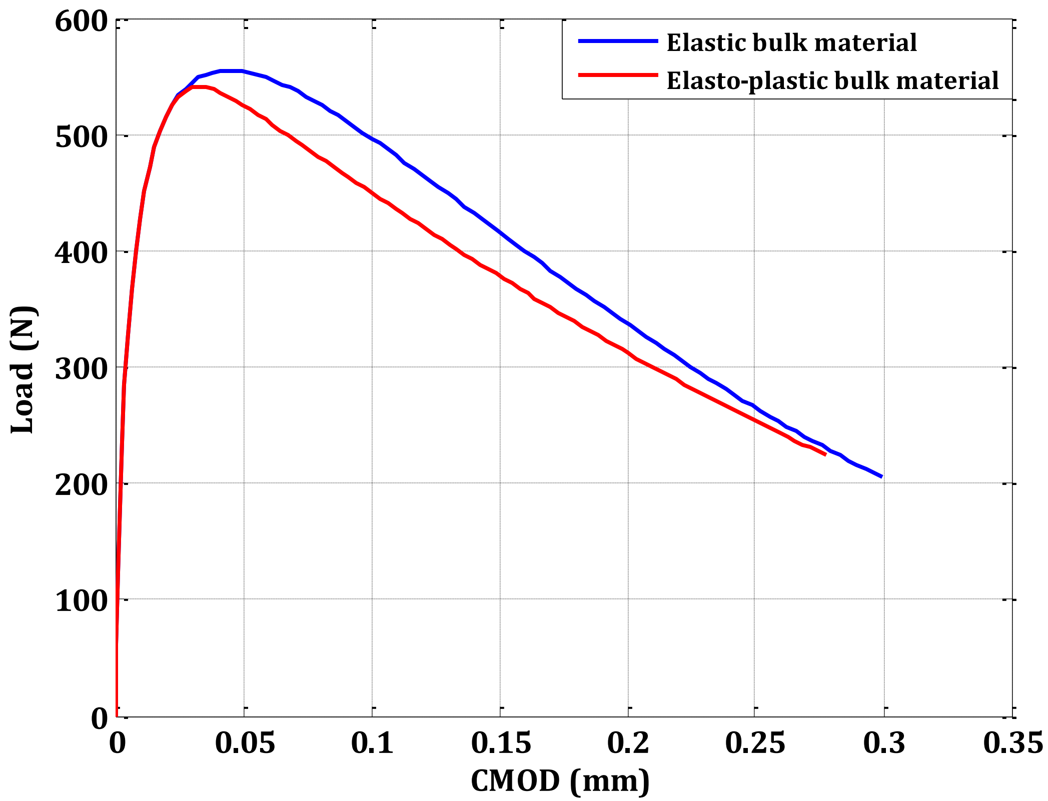


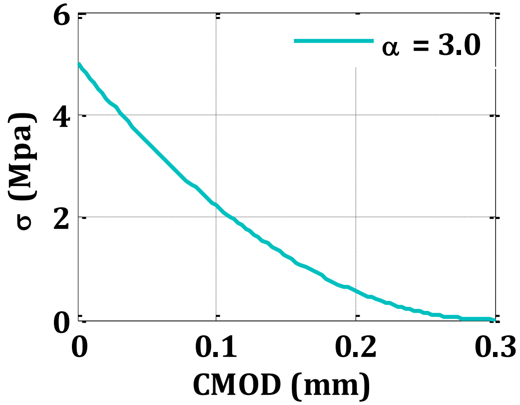
Forward Problem Plot

- Displacement field taken from loading point C = 122.1 N
- 6 Control Points
- Piecewise Cubic Hermite interpolation
- Synthetic data with varying amount of noise

Numerical Simulations: PPR model

- Forward Problem





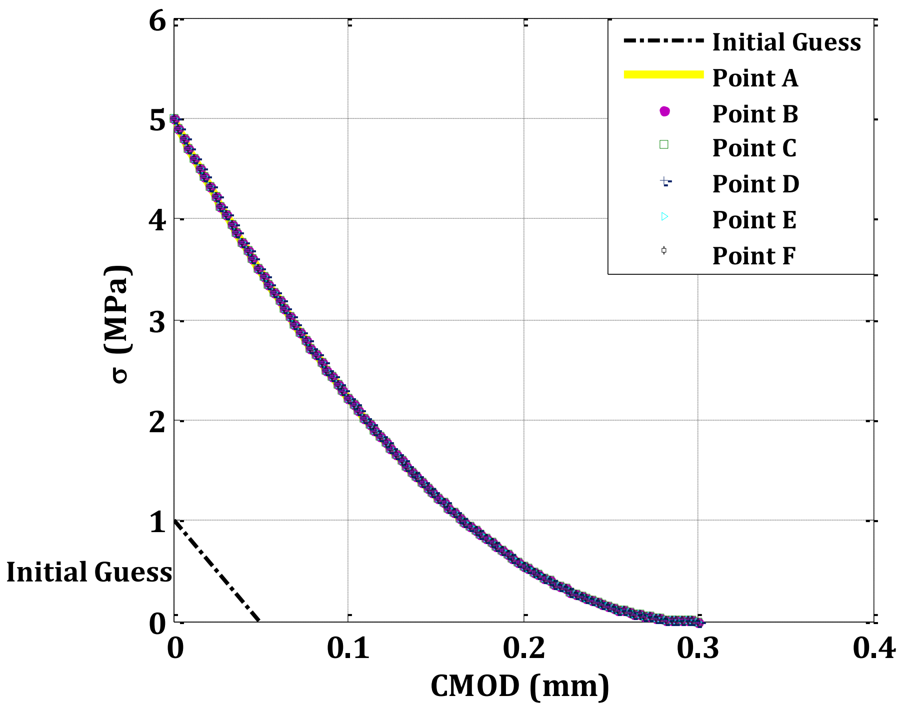
$\phi_n = 500 \text{ N / m}$

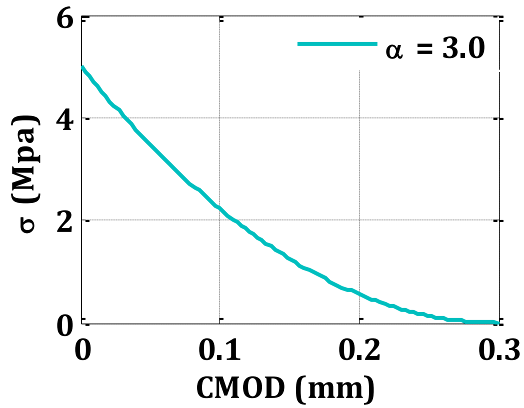
$\sigma_{\max} = 5 \text{ MPa}$

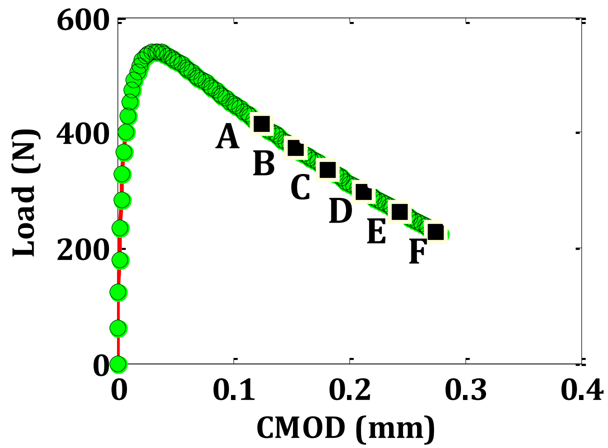
$\alpha = 3$

Numerical Simulations: PPR model

- Inverse Problem: Different Loading Points





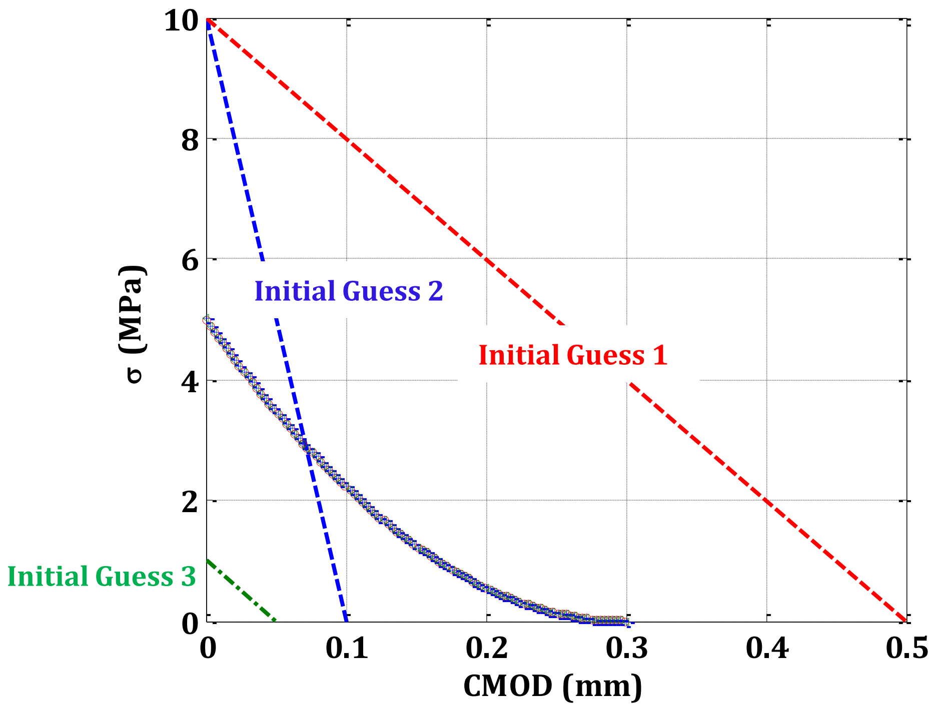


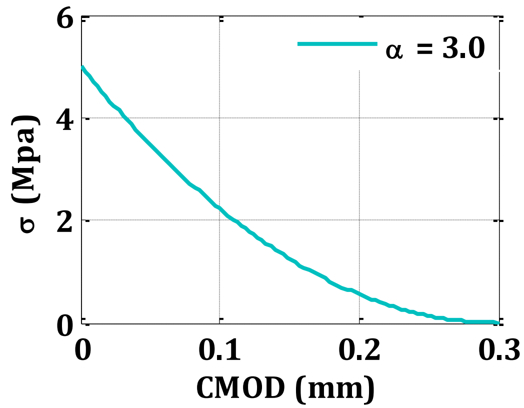
Forward Problem Plot

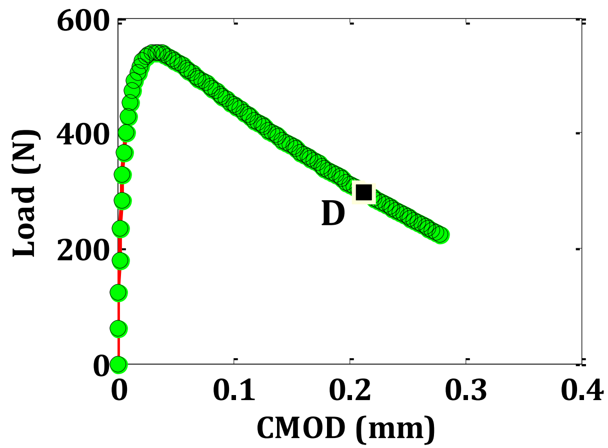
- Synthetic data without any noise

Numerical Simulations: PPR model

- Inverse Problem: Various Initial Guesses





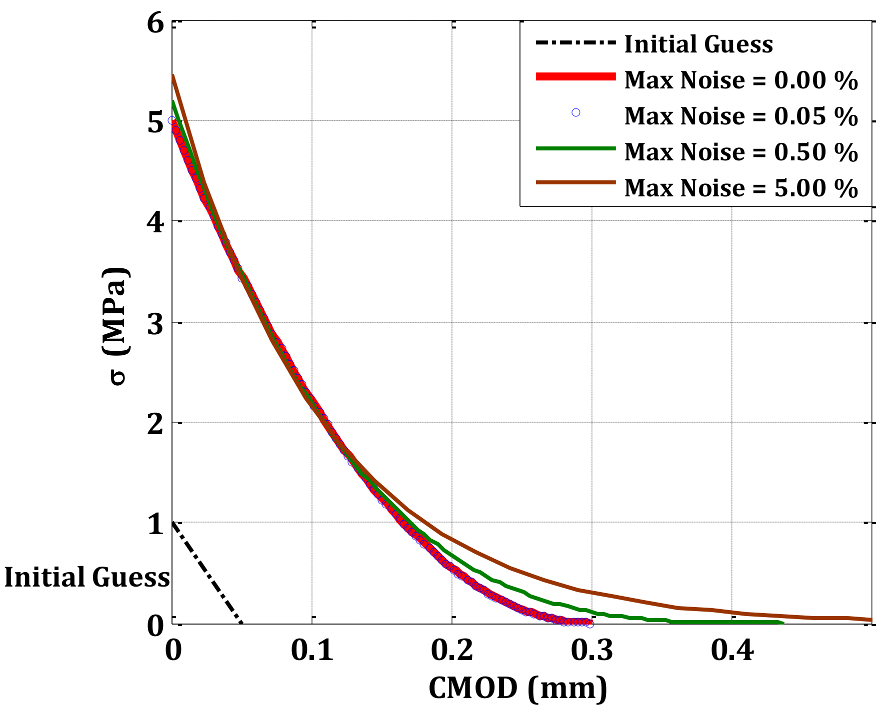


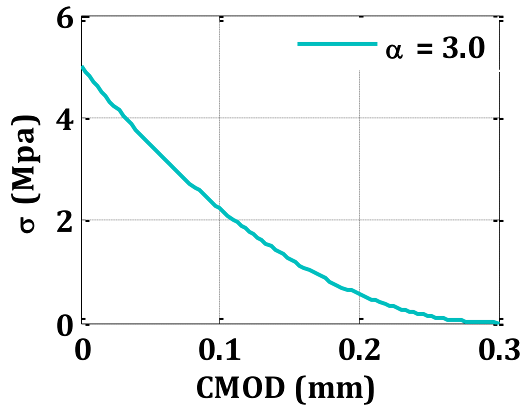
Forward Problem Plot

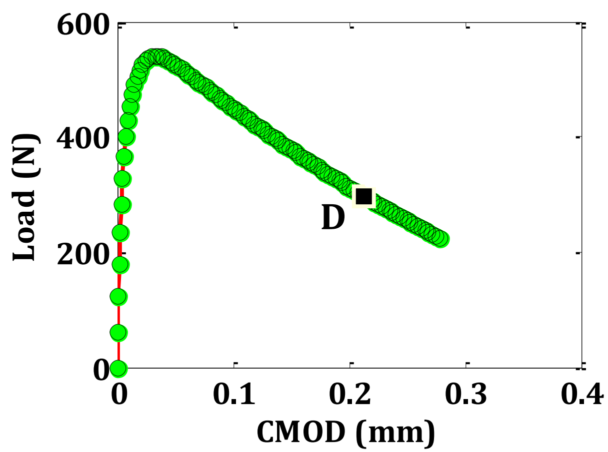
- Displacement field taken from loading point D = 296.4 N
- Synthetic data without any noise

Numerical Simulations: PPR model

- Inverse Problem: Noise in Synthetic Data





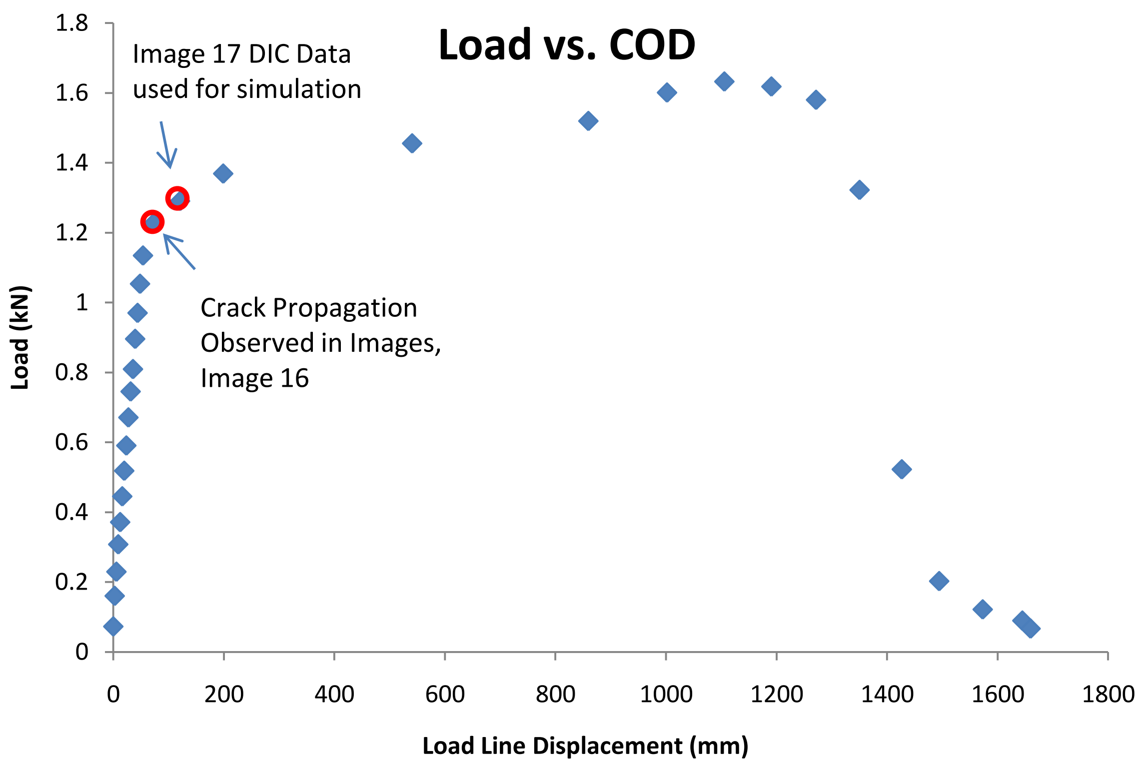


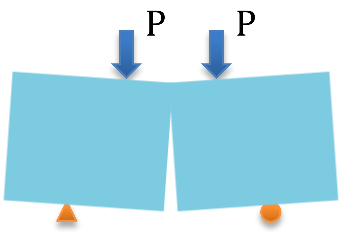
Forward Problem Plot

- Displacement field taken from loading point D = 296.4 N
- Synthetic data with varying amount of noise

Inverse Analysis using DIC

- Load v/s CMOD from experiment

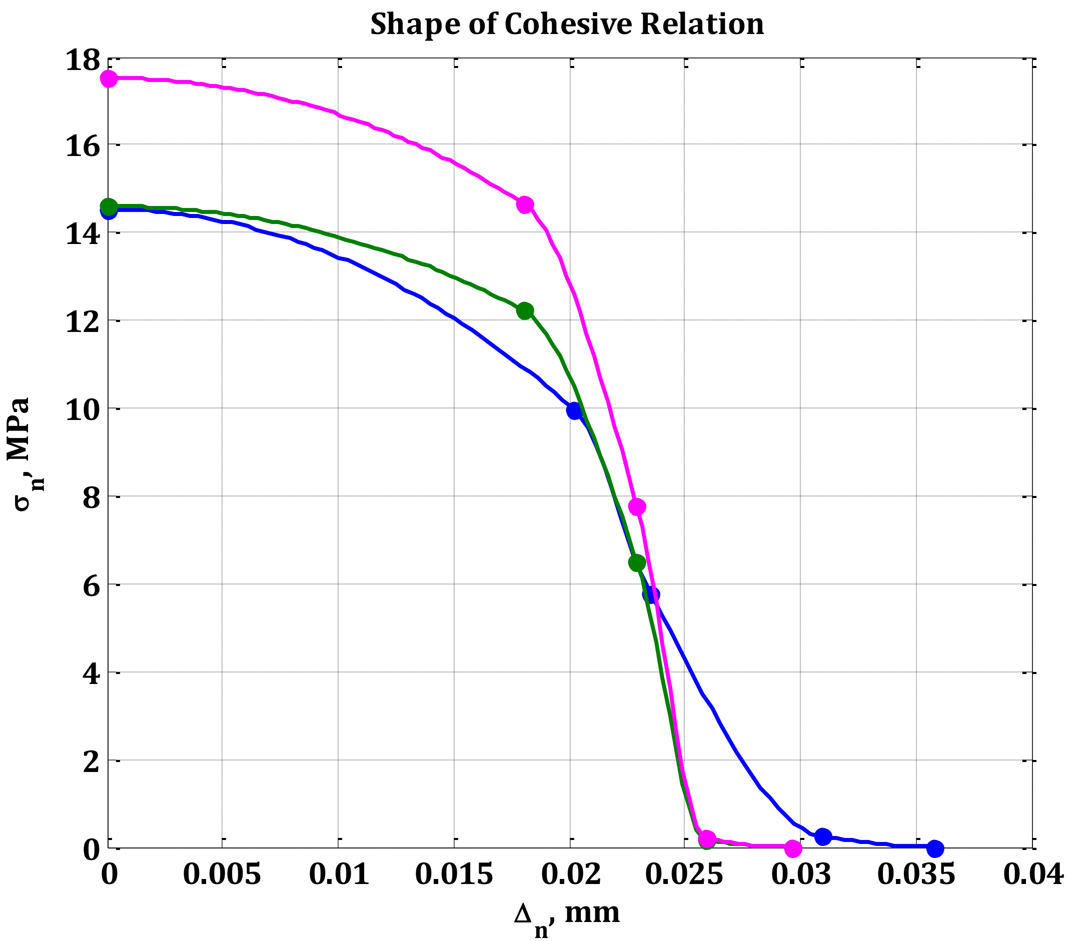




Preliminary results using PMMA

Inverse Analysis using DIC

- Extracted Cohesive Relation using Inverse Analysis



- Results from different simulation runs
- Shape similar to the one used in reference below

Preliminary results using PMMA

Elices M., Guinea G.V., Gomez J. & Planas J., 2002, *The Cohesive Zone Model: Advantages, Limitations and Challenges*, EFM, 69, 137

Inverse Analysis using DIC

- Traction Separation relation for PMMA used by Elices et al.

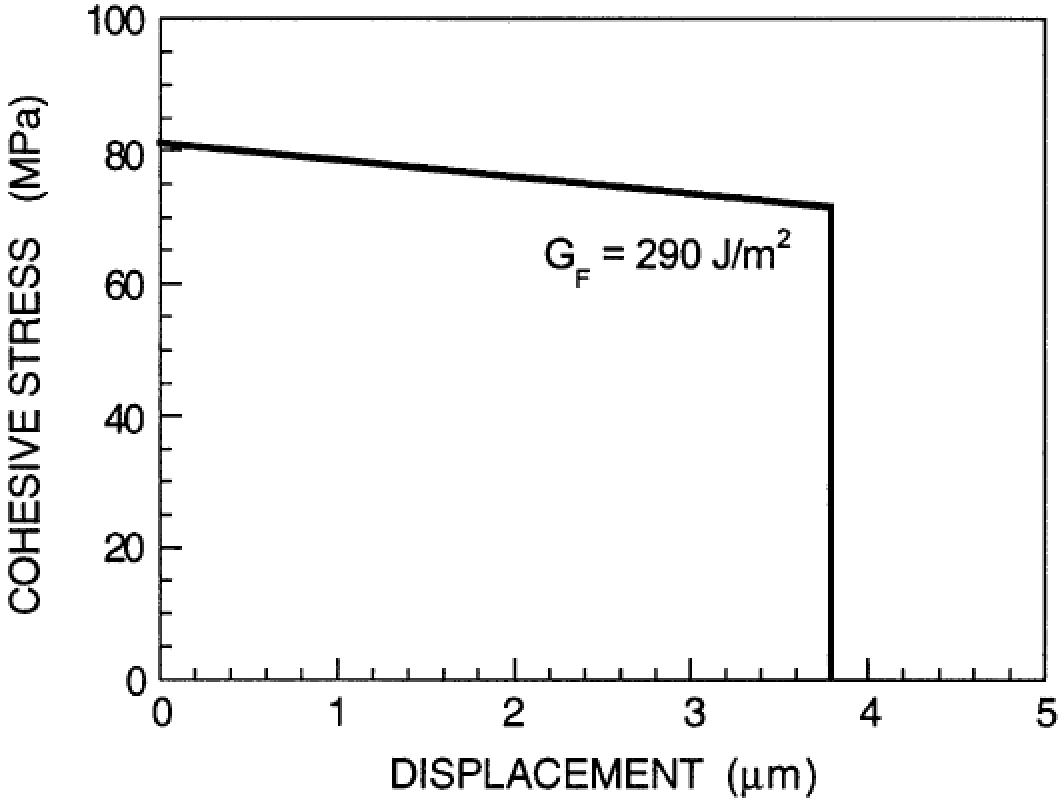


Fig. 19. Softening function for PMMA.

Elices M., Guinea G.V., Gomez J. & Planas J., 2002, *The Cohesive Zone Model: Advantages, Limitations and Challenges,* EFM, 69, 137

Summary & Conclusions

- Developed inverse analysis techniques to extract cohesive fracture properties of elasto-plastic materials
  - Shape regularization
  - PPR model
- Verified implementation for various conditions
- Ongoing collaborative work: Hybrid technique (Experimental DIC + Inverse analysis) for polymers and metal/metal composites such as Ti/Ti composites

Future Work

- Inverse analysis for fatigue loading
- Validation of elasto-plastic inverse analysis using DIC experiments
- Extension of elasto-plastic inverse analysis to plates and shells

Thank You !

Acknowledgements:: Bin Shen, Jason Patric