

# Potential-Based Dynamic Fracture Simulation with Adaptive Topological Operators

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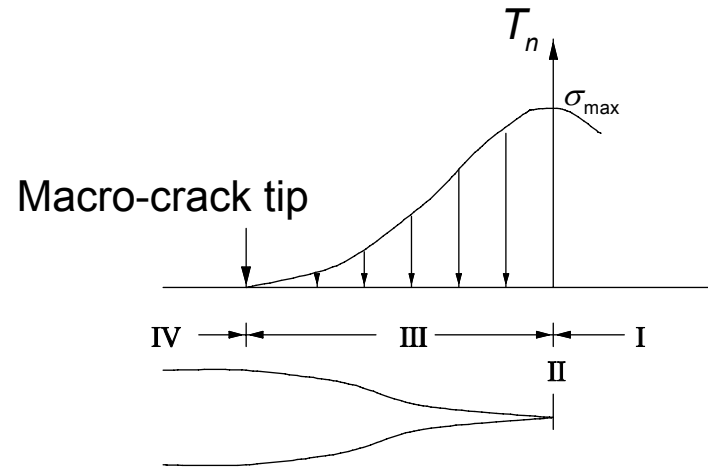
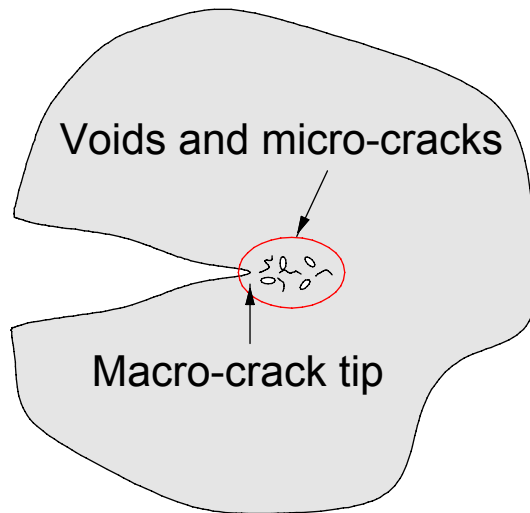
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Pontifical Catholic University of Rio de Janeiro (PUC-Rio)

# Outline

- **Cohesive Zone Modeling**
- **Potential-Based Cohesive Model**
- **Adaptive Topological Operators**
- **Computational Examples**
  - Mode I pre-defined crack path
  - Mixed-mode fracture problem
  - Micro-Branching
- **Summary**



# Cohesive Zone Modeling



## □ Constitutive Relationship of Cohesive Fracture

- Non-potential based-model vs. Potential based-model

## □ Computational Methods

- Cohesive surface elements, enrichment functions, embedded discontinuities

# Previous Potentials for Fracture

## □ Needleman A. (1987)

- Polynomial potential / linear shear interaction

## □ Needleman A. (1990)

- Exponential potential / periodic dependence

## □ Beltz G.E. and Rice J.R. (1991)

- Generalized the potential (Exponential + Sinusoid)

## □ Xu X.P. and Needleman A. (1993)

- Exponential potential (Exponential + Exponential)

■ Needleman A. 1987, A continuum model for void nucleation by inclusion debonding, *Journal of Applied Mechanics*, 54, 525-531

■ Needleman A. 1990, An analysis of tensile decohesion along an interface, *Journal of the Mechanics and Physics of Solid*, 3, 289-324

■ Beltz GE and Rice JR, 1991, Dislocation nucleation versus cleavage decohesion at crack tip, *Modeling the Deformation of Crystalline Solids*, 457-480.

■ Xu XP and Needleman, 1993, Void nucleation by inclusion debonding in a crystal matrix, *Modeling Simulation Material Science Engineering*, 1, 111-132.



# PPR: Unified Mixed Mode Potential

$$\Psi(\Delta_n, \Delta_t) = \min(\phi_n, \phi_t) + \left[ \Gamma_n \left(1 - \frac{\Delta_n}{\delta_n}\right)^\alpha \left(\frac{m}{\alpha} + \frac{\Delta_n}{\delta_n}\right)^m + \langle \phi_n - \phi_t \rangle \right] \\ \left[ \Gamma_t \left(1 - \frac{|\Delta_t|}{\delta_t}\right)^\beta \left(\frac{n}{\beta} + \frac{|\Delta_t|}{\delta_t}\right)^n + \langle \phi_t - \phi_n \rangle \right]$$

- Energy Constants:  $\Gamma_n$  and  $\Gamma_t$
- Exponents:  $m$  and  $n$
- Characteristic length scales:  $\delta_n$  and  $\delta_t$
- Shape parameters :  $\alpha$  and  $\beta$

- Fracture energy
- Cohesive strength
- Cohesive interaction shape
- Initial slope

**USNCCM10: Sunday Technical Session 10, 9:30AM, Rm D144**

K. Park, GH. Paulino, JR. Roesler, 2008, A unified potential-based cohesive model of mixed-mode fracture, *Journal of the Mechanics and Physics of Solids* 57, 891-908.



# Extension for the **EXTRINSIC** Model

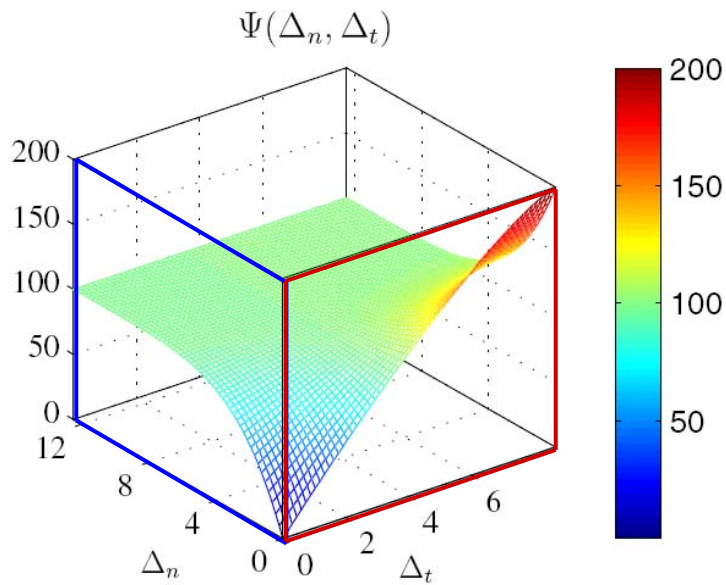
## □ Correct Limit Procedure

- Limit of initial slope indicators in the potential

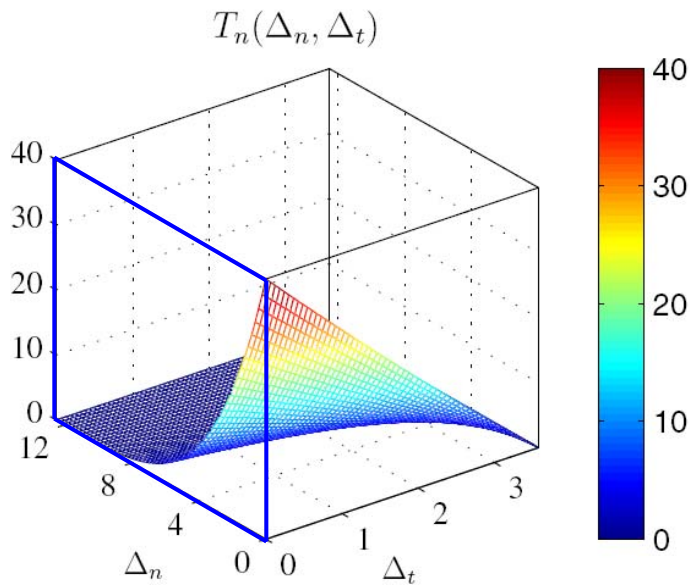
$$\Psi(\Delta_n, \Delta_t) = \min(\phi_n, \phi_t) + \left[ \Gamma_n \left( 1 - \frac{\Delta_n}{\delta_n} \right)^\alpha + \langle \phi_n - \phi_t \rangle \right] \left[ \Gamma_t \left( 1 - \frac{|\Delta_t|}{\delta_t} \right)^\beta + \langle \phi_t - \phi_n \rangle \right]$$

- Energy constants:  $\sqrt{\Gamma_n}$  and  $\sqrt{\Gamma_t}$
  - Characteristic length scales:  $\delta_n$  and  $\delta_t$
  - Shape parameters:  $\alpha$  and  $\beta$
- Exclude elastic behavior → **Extrinsic model**
- Consider different fracture energy:  $\phi_n, \phi_t$
- Describe different cohesive strength:  $\sigma_{\max}, \tau_{\max}$
- Represent various cohesive shape:  $\alpha, \beta$

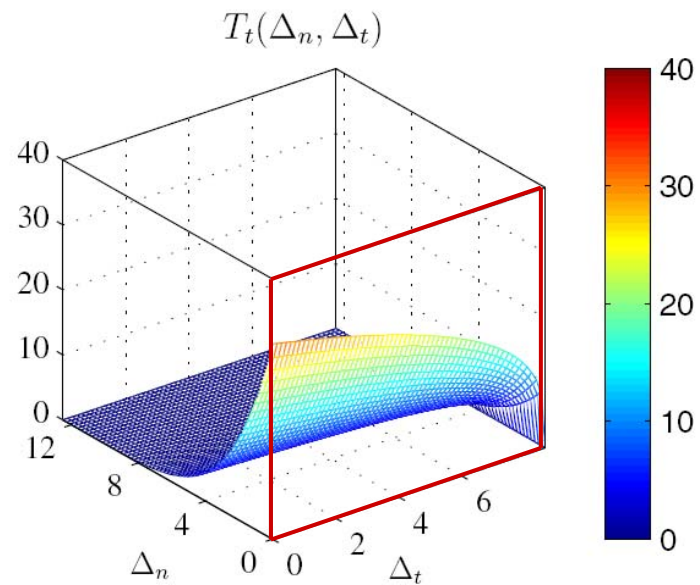




$\phi_n = 100 \text{ N/m}$	$\phi_t = 200 \text{ N/m}$
$\sigma_{\max} = 40 \text{ MPa}$	$\tau_{\max} = 30 \text{ MPa}$
$\alpha = 5$	$\beta = 1.3$



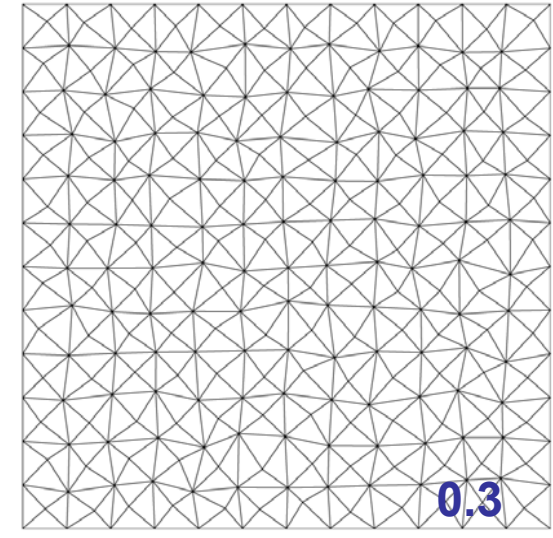
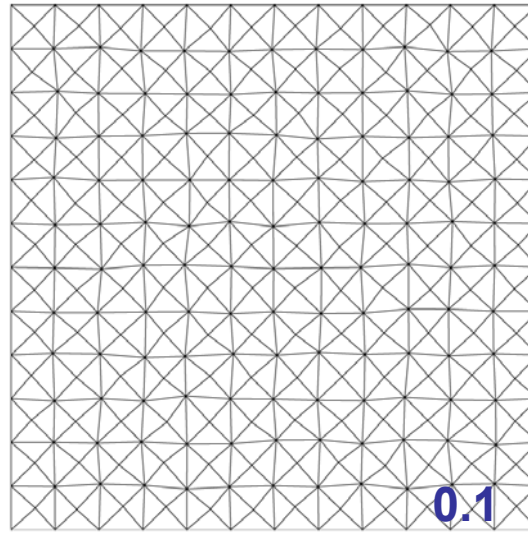
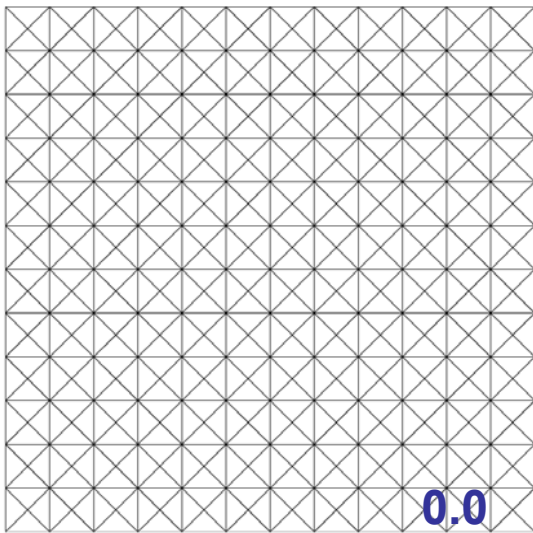
**Mode I**



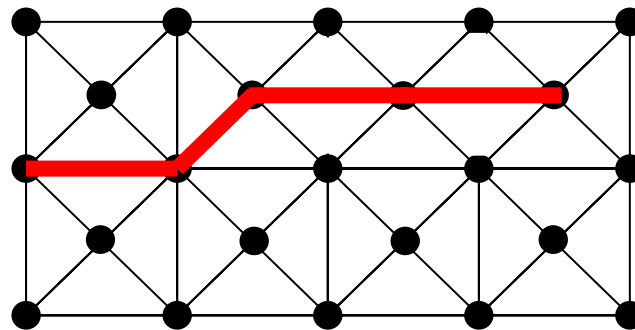
**Mode II**

# Topological Operators

## □ Nodal Perturbation



## □ Edge-Swap

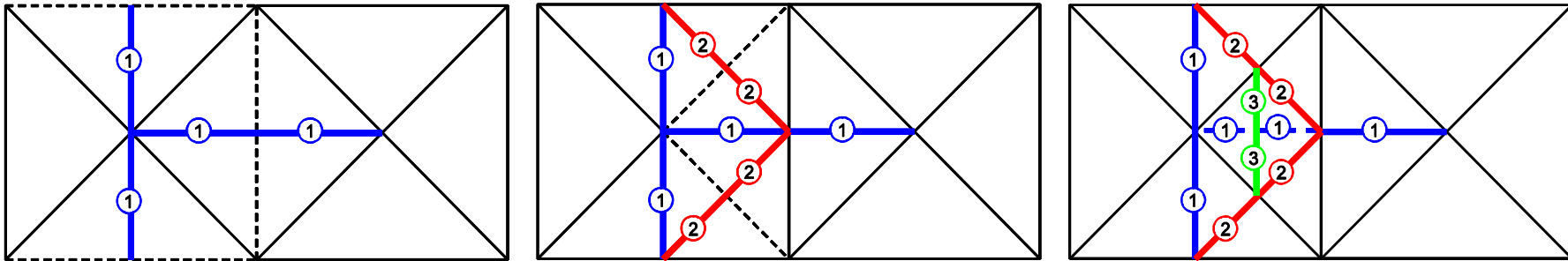




# Topological Operators

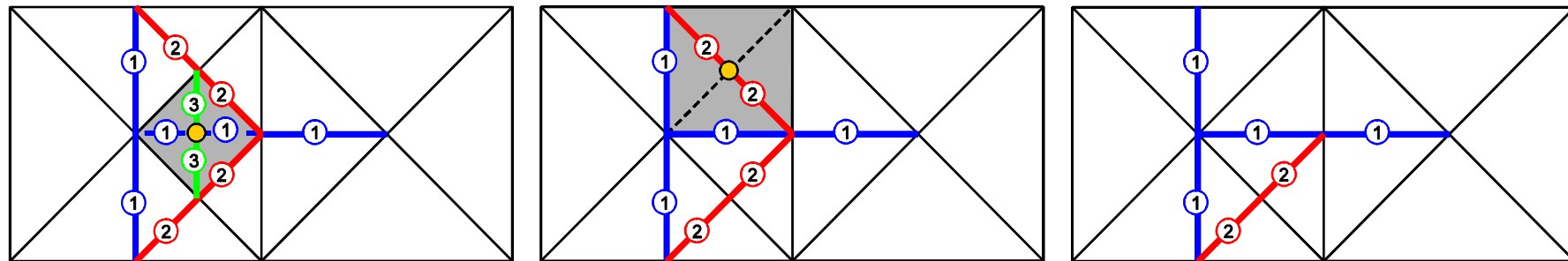
## □ Edge-Split

- Adaptive mesh refinement based on *a priori* knowledge

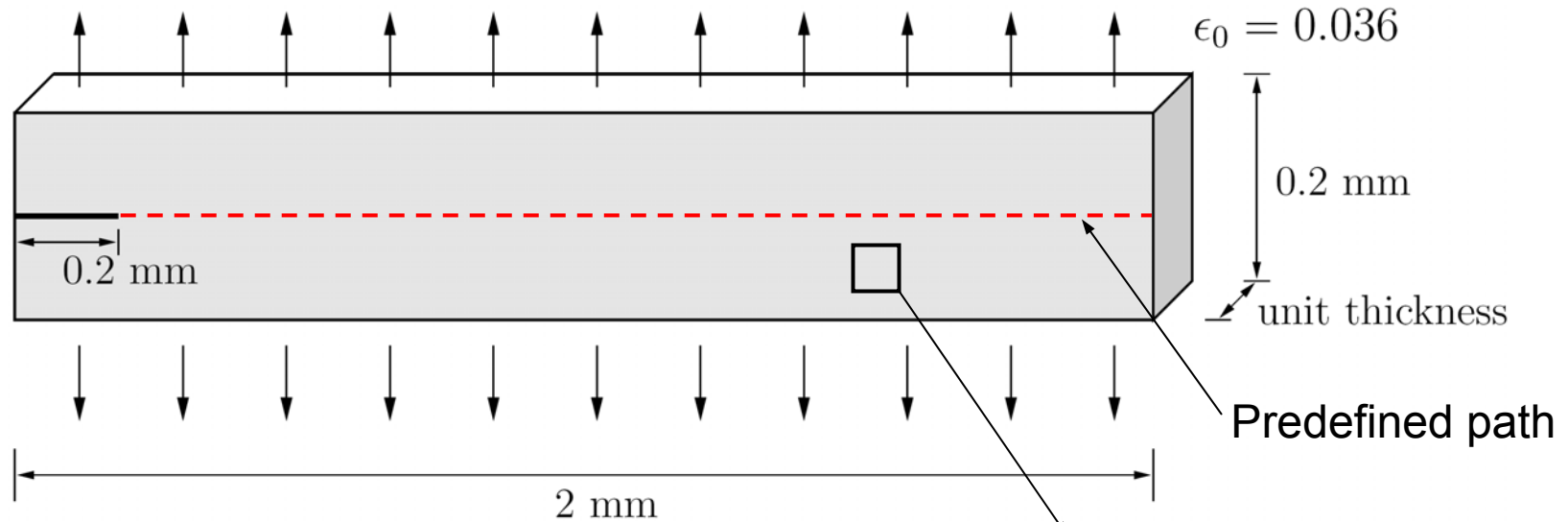


## □ Vertex-Removal (or Edge-Collapse)

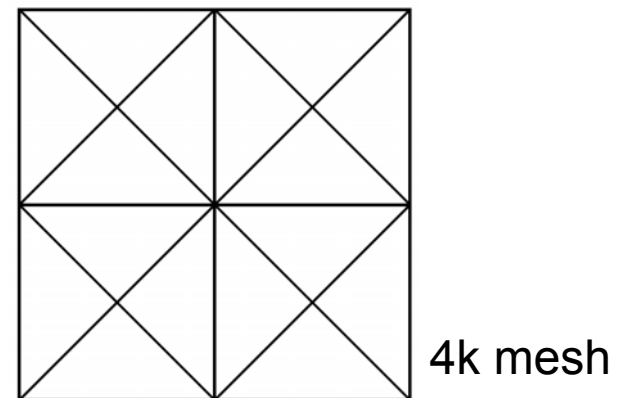
- Adaptive mesh coarsening based on *a posteriori* error estimation, i.e. root mean square of strain error



# Mode I Pre-defined Crack Propagation



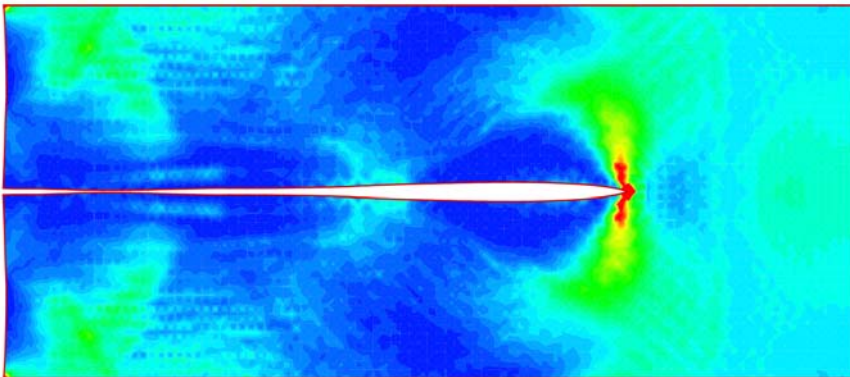
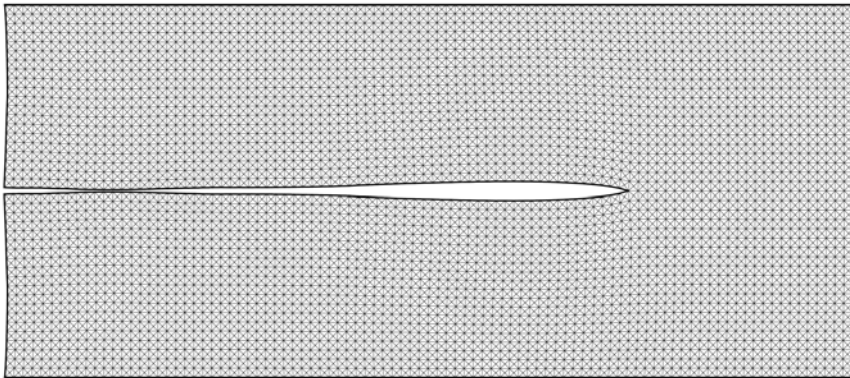
$E$	$\nu$	$\rho$	$G_I$	$T_{\max}$
3.24 GPa	0.35	1190 kg/m <sup>3</sup>	352 N/m	324 MPa



# Computational Results

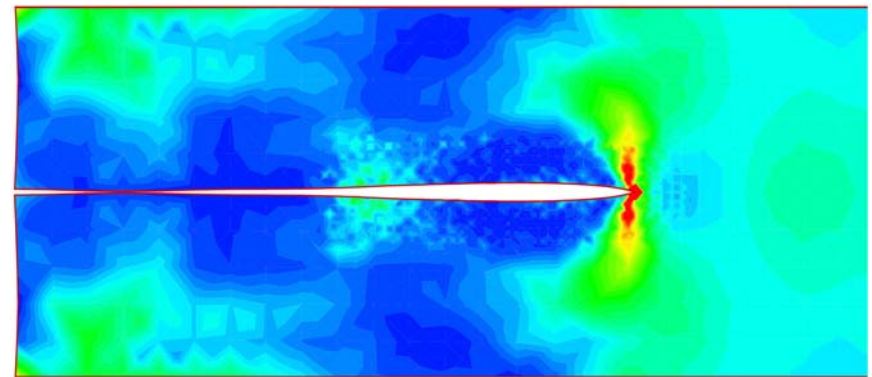
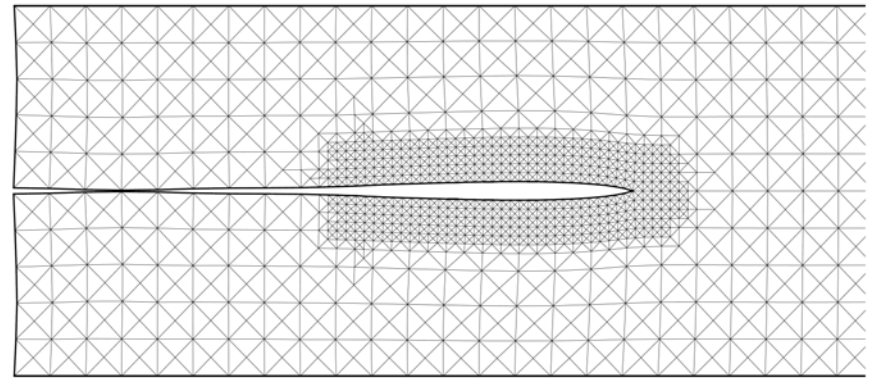
## □ Uniform Mesh Refinement

- 400x40 mesh grid
- Element size:  $5\mu\text{m}$
- 64000 elements, 128881 nodes

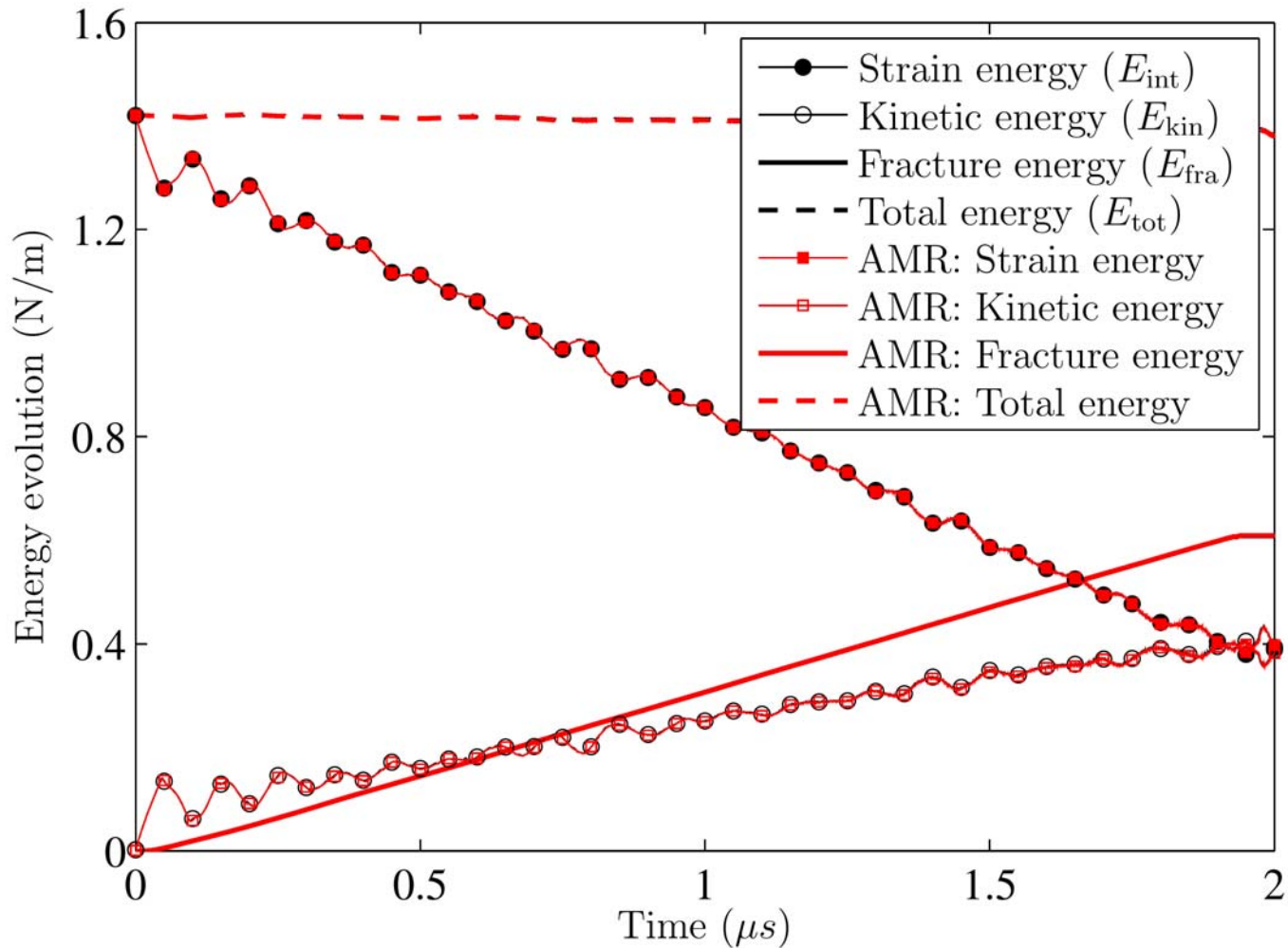


## □ Adaptive Mesh Refinement

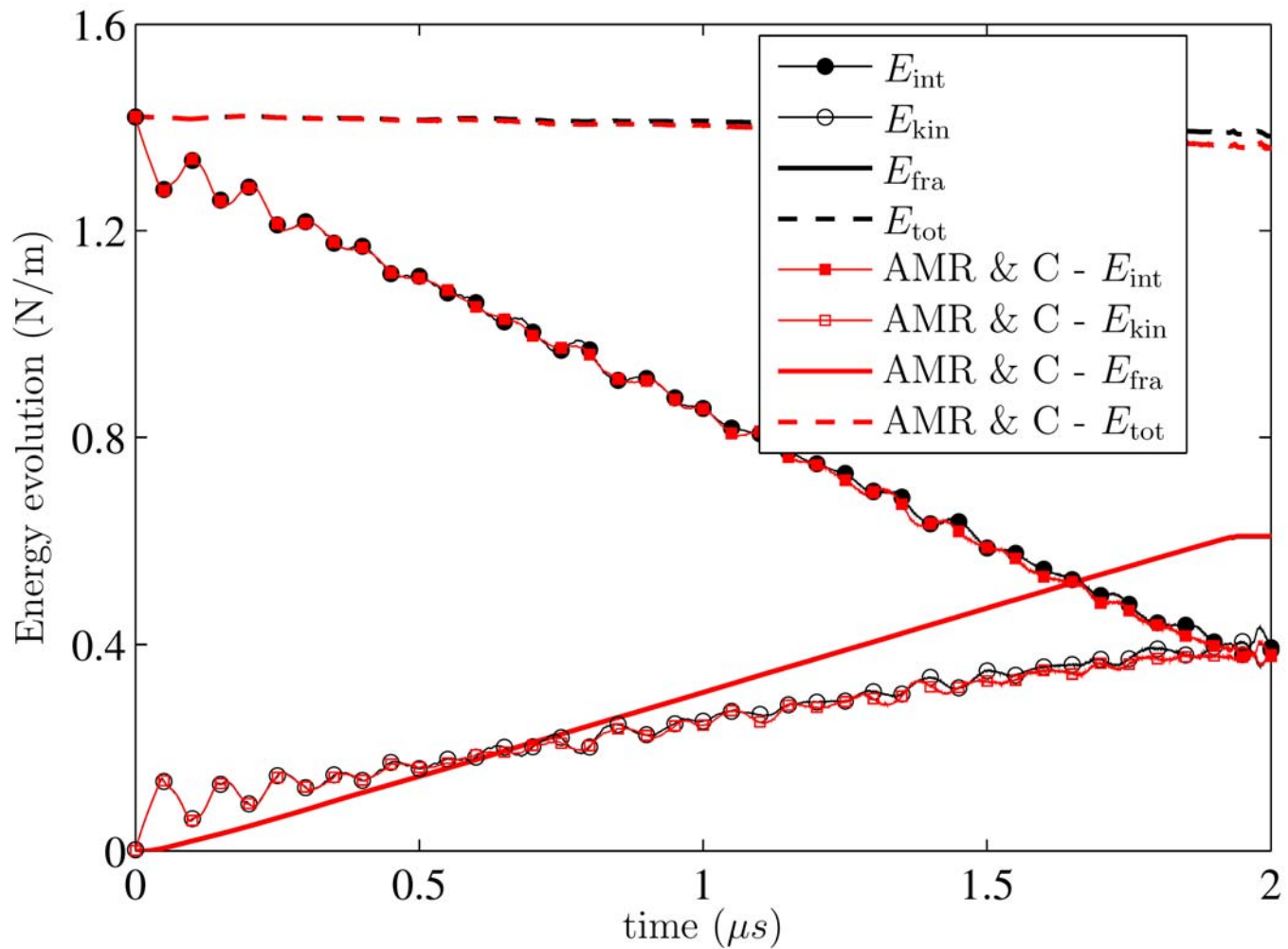
- 100x10 mesh grid
- Element size:  $20\sim 5\mu\text{m}$
- 4448 elements, 9147 nodes



# Computational Results (AMR)

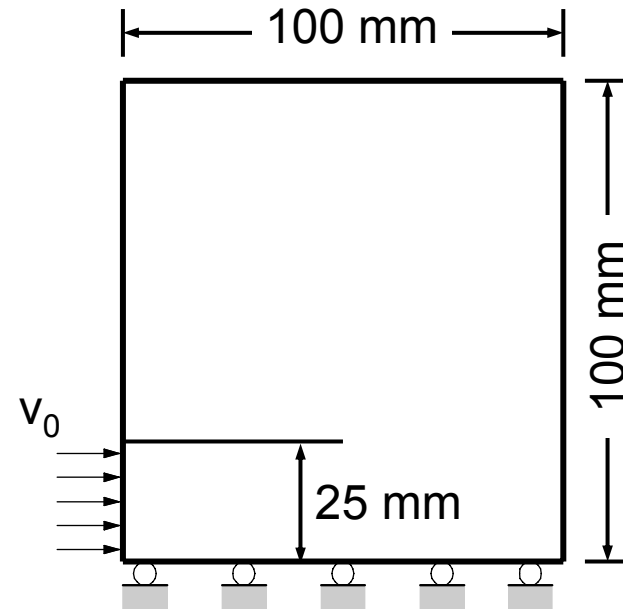
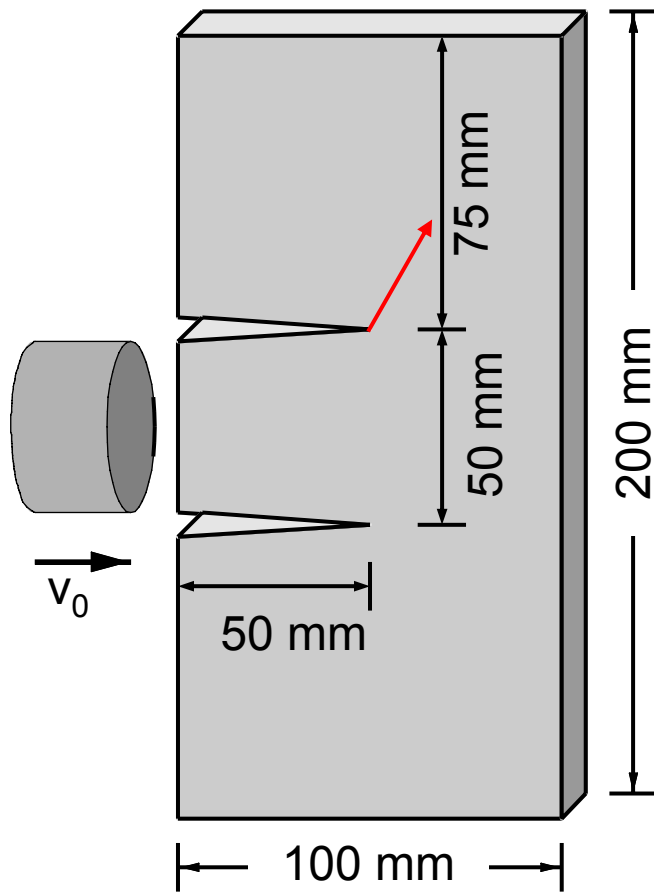


# Computational Results (AMR+C)



# Mixed-Mode Crack Propagation

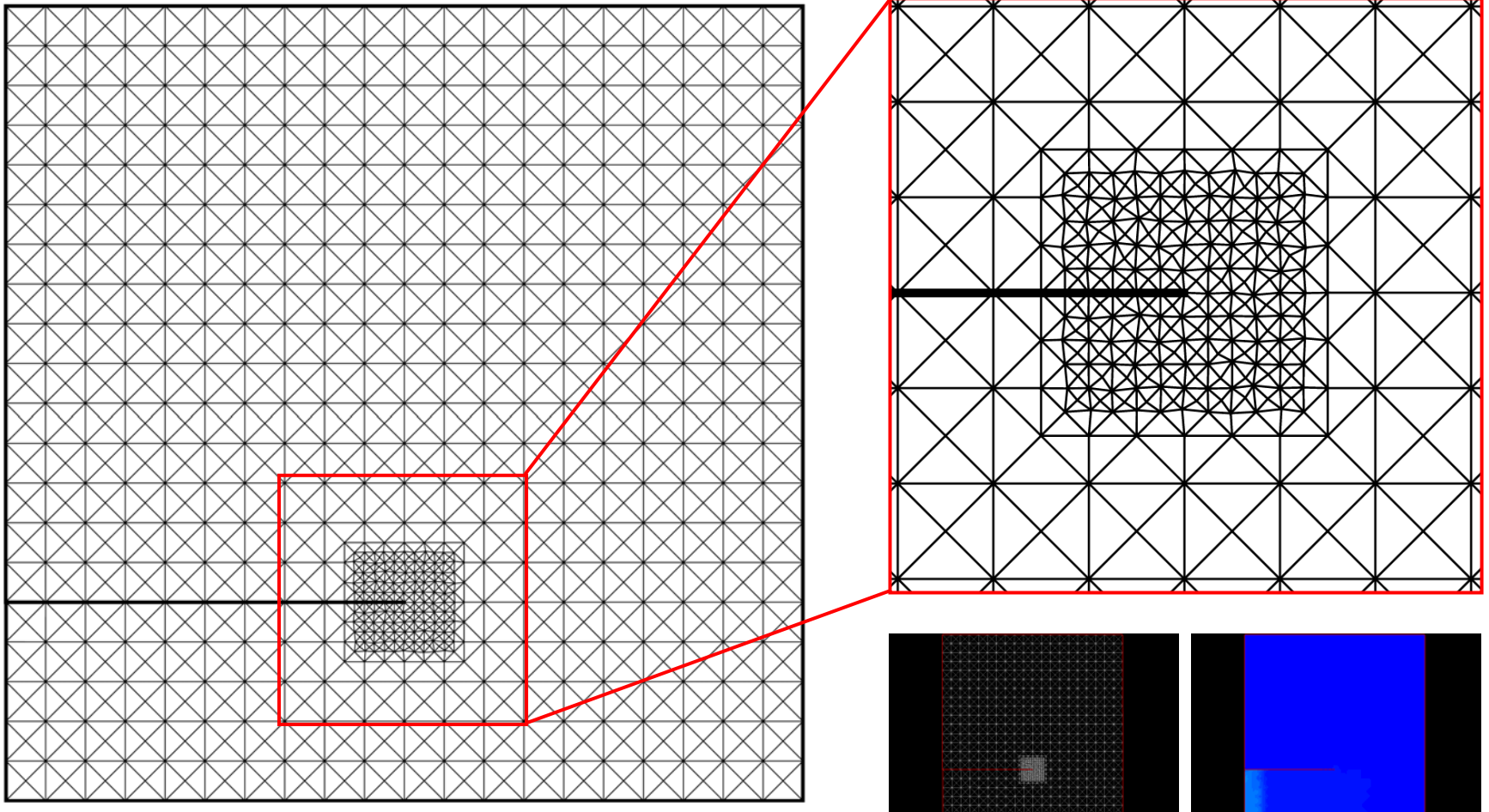
## □ Kalthoff-Winkler's Experiments



Kalthoff, J. F., Winkler, S., 1987. Failure mode transition at high rates of shear loading. International Conference on Impact Loading and Dynamic Behavior of Materials 1, 185–195.

# Finite Element Mesh

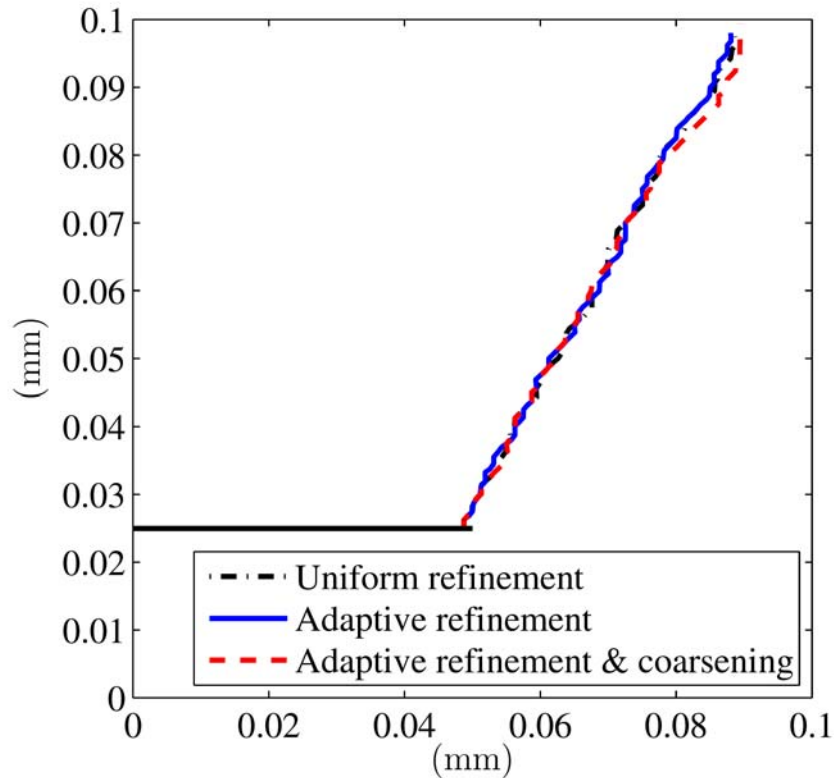
## □ Initial Discretization



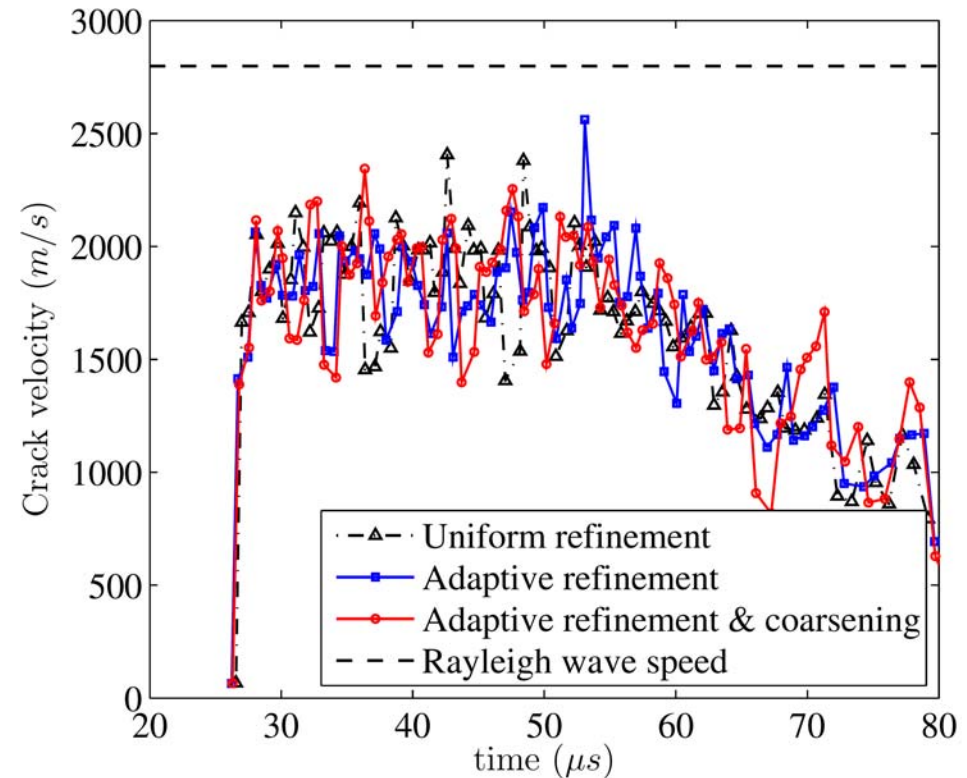
Animations (FE Mesh & Strain energy)

# Computational Results

## □ Crack Path

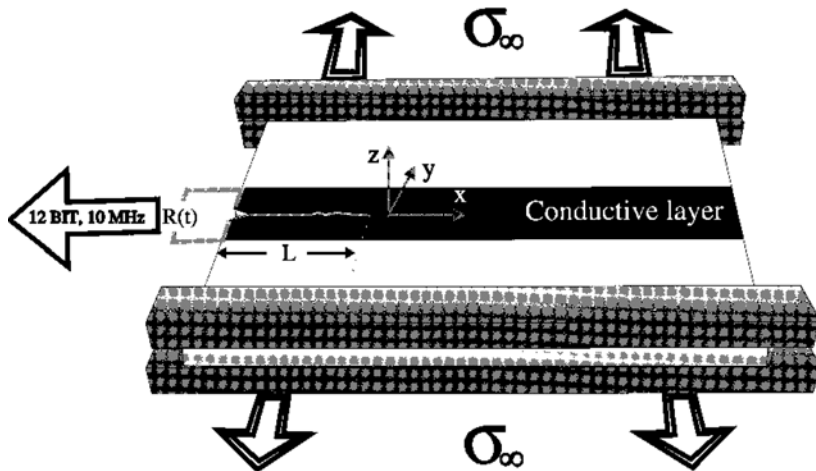


## □ Crack Velocity

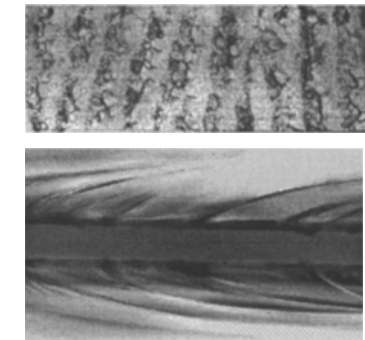
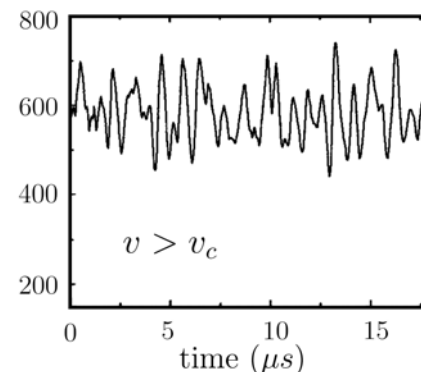
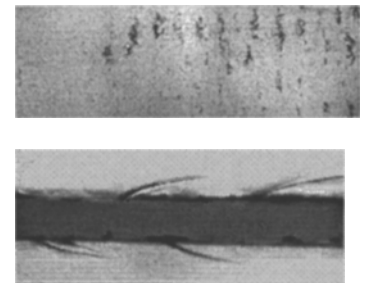
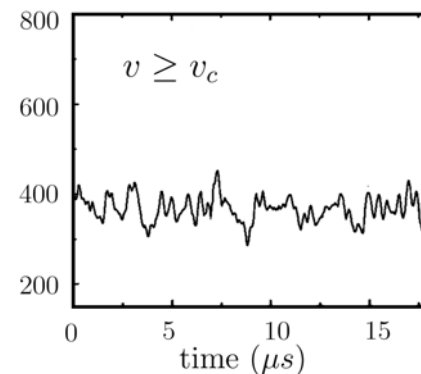
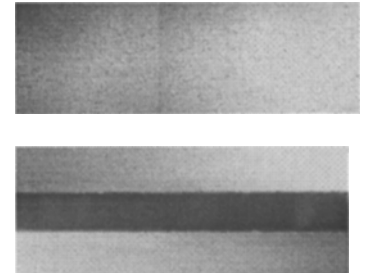
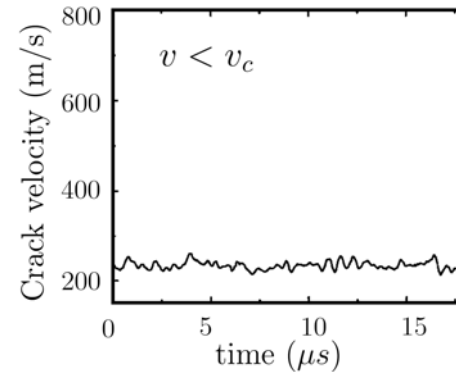
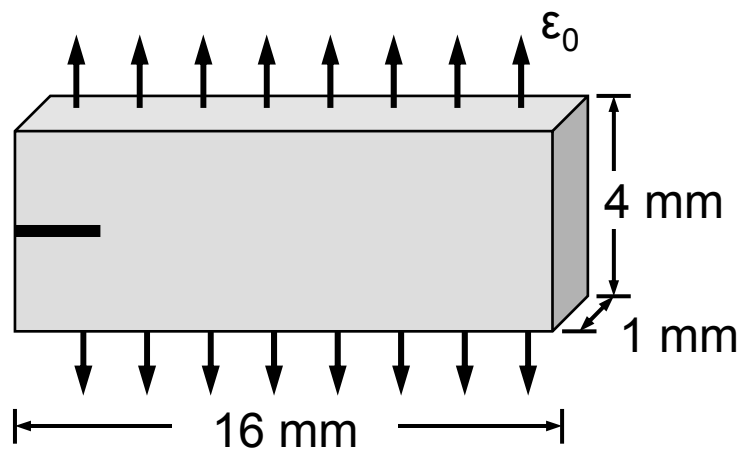




# Micro-Branching Experiment

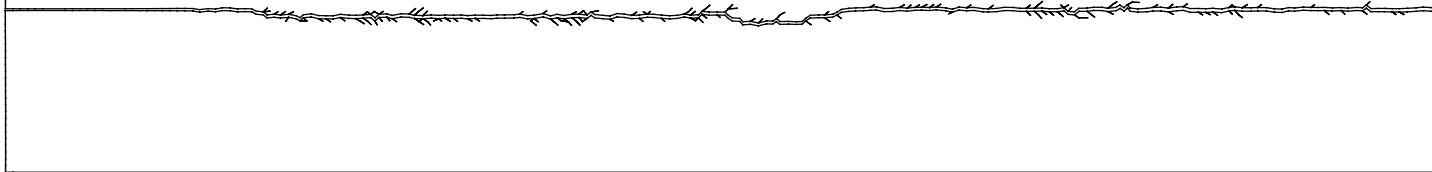


Sharon E, Fineberg J. Microbranching instability and the dynamic fracture of brittle materials. *Physical Review B* 1996; 54(10):7128–7139.

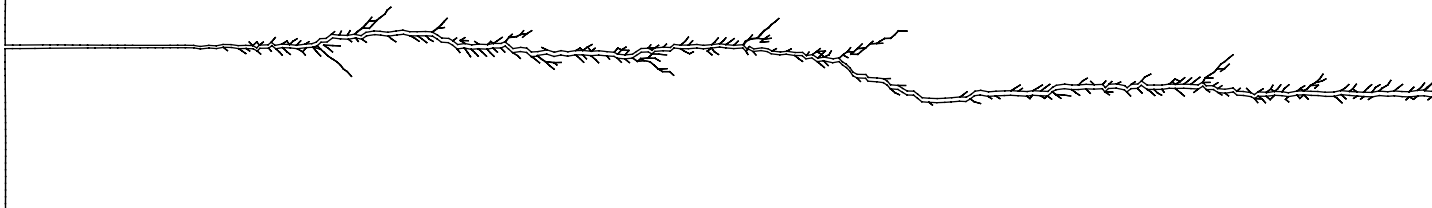


# Computational Results

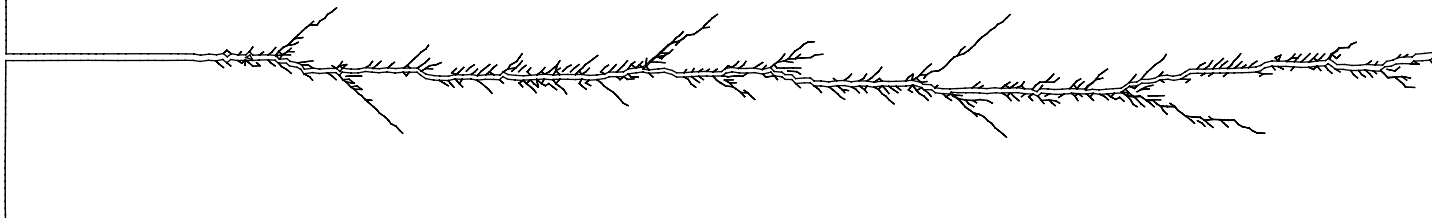
$$\varepsilon_0 = 0.010$$



$$\varepsilon_0 = 0.012$$

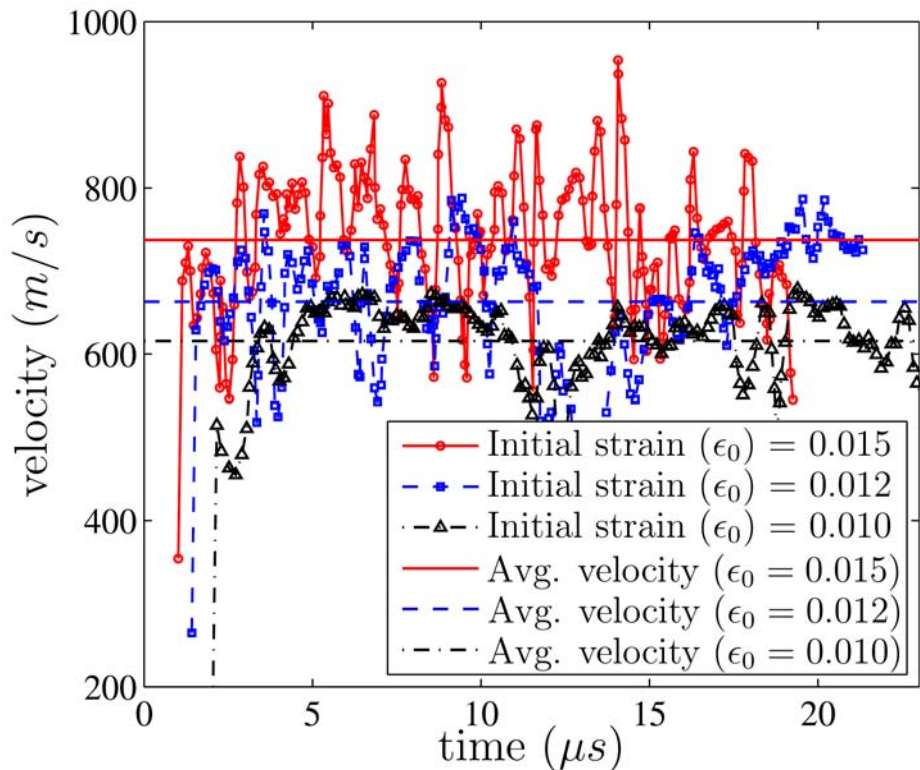


$$\varepsilon_0 = 0.015$$

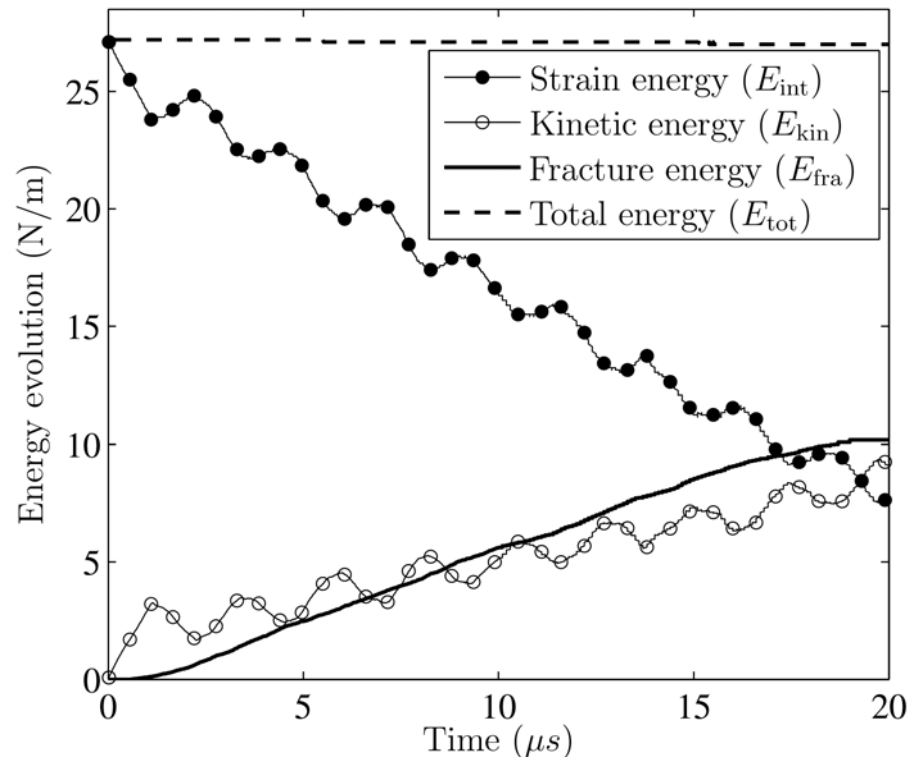


# Computational Results

## □ Crack Velocity



## □ Energy Evolution ( $\epsilon_0=0.015$ )



# Summary

- **The potential-based constitutive model**
- **Adaptive operators**
  - Nodal perturbation, Edge-swap
  - Edge-split, Vertex-removal
- **Effective and efficient computational framework to simulate physical phenomena associated with fracture.**
- **The computational results of the adaptive mesh refinement and coarsening is consistent with the results of the uniform mesh refinement.**



**Thank you for your attention !**

