Pattern Gradation and Repetition with Application to High-Rise Building Design

Lauren L. Stromberg, Glaucio H. Paulino, William F. Baker

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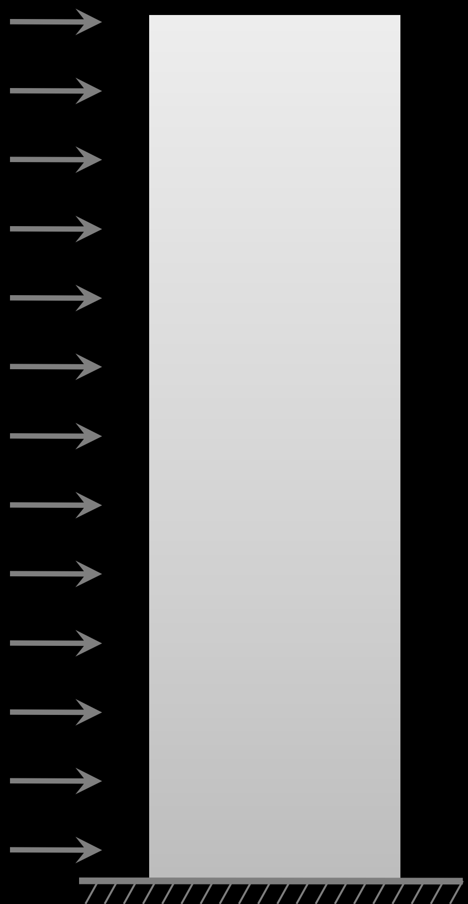
Department of Civil and Environmental Engineering
University of Illinois at Urbana-Champaign

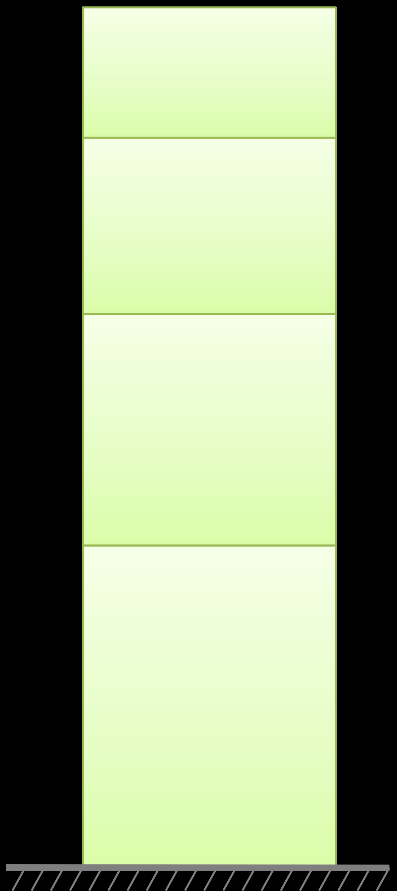
Skidmore, Owings & Merrill, LLP

Motivation: Functionally Graded Buildings



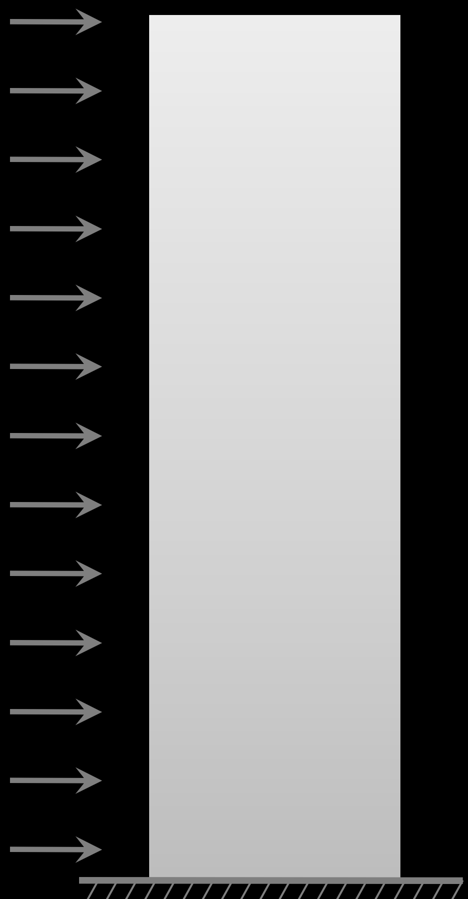
Unconstrained Problem

John Hancock Building



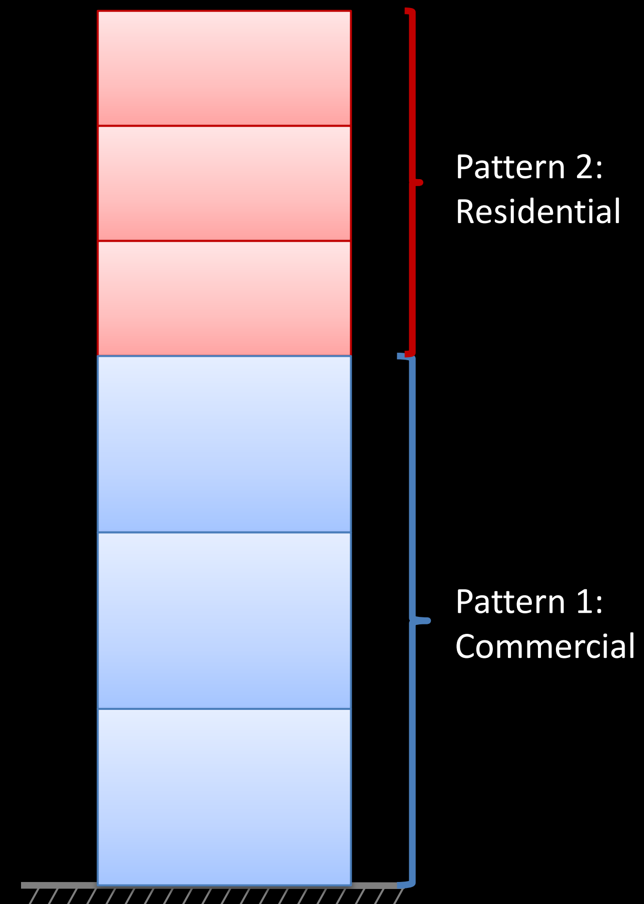
Patterns of Different Sizes

Motivation: Functionally Graded Buildings



Unconstrained Problem

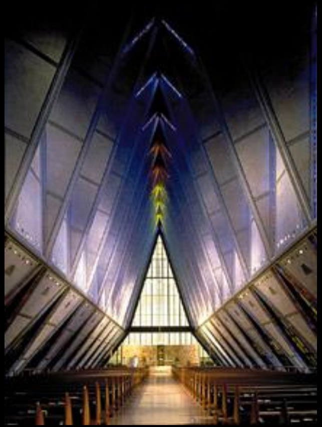
Trump Tower



Groups of Patterns

Outline

- Introduction and Motivation
- Topology Optimization Framework
  - Basic problem formulation
- Manufacturing constraints and pattern gradation
  - Uniform Density approach
  - CAMD approach
  - Projection scheme (length scale)
- Numerical Results
  - 2D
  - Graded thicknesses using Lagrange Multipliers
  - Building example in 3D
- Concluding Remarks



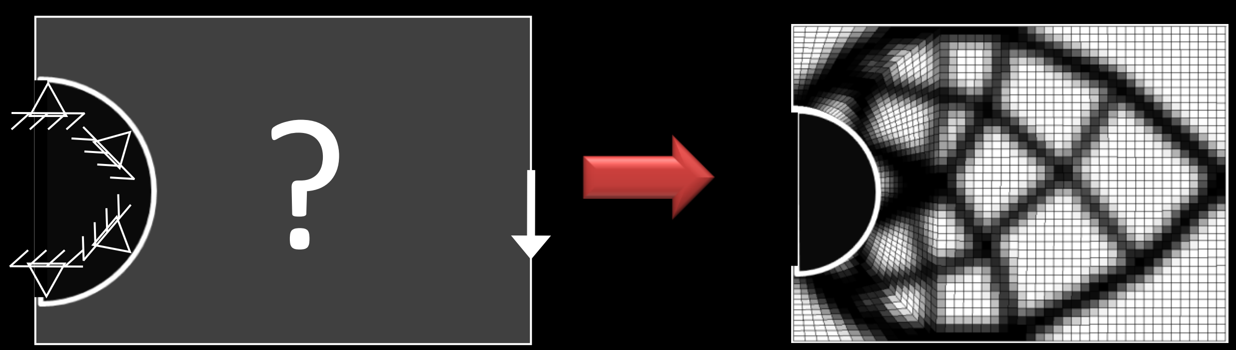
Topology Optimization Framework

- **Minimum compliance problem in discrete form:**

$$
\begin{aligned}
\min_{\rho,\boldsymbol{u}}: \quad & c(\rho,\boldsymbol{u}) && \text{Objective function} \\
s.t.: \quad & \boldsymbol{K}(\rho)\mathbf{u} = \mathbf{f} && \text{Equilibrium constraint} \\
& \int_{\Omega} \rho \, dV \leq V_s && \text{Volume constraint}
\end{aligned}
$$

- **Solid Isotropic Material with Penalization (SIMP) model:**

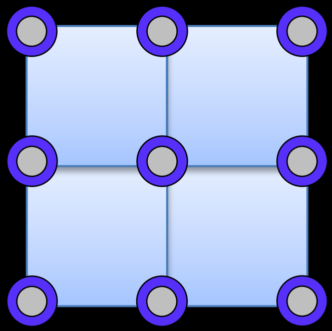
$$E(\boldsymbol{x}) = \rho(\boldsymbol{x})^p E^0, \quad p > 1$$



Manufacturing Constraints and Pattern Gradation

- **Uniform Element Density Approach: design variables are coincident with element centroids (or nodes)**

- **CAMD Approach: design variables are nodal densities and shape functions used to obtain density throughout design domain**



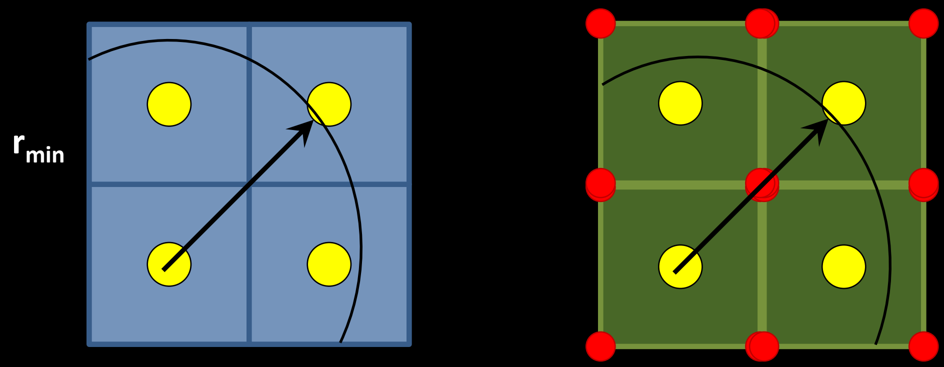
$$\rho(\boldsymbol{x}) = \sum_{e=1}^{n} \sum_{i=1}^{4} N_i^e(\boldsymbol{x}) \rho_i^e$$

- **Update sensitivities:**

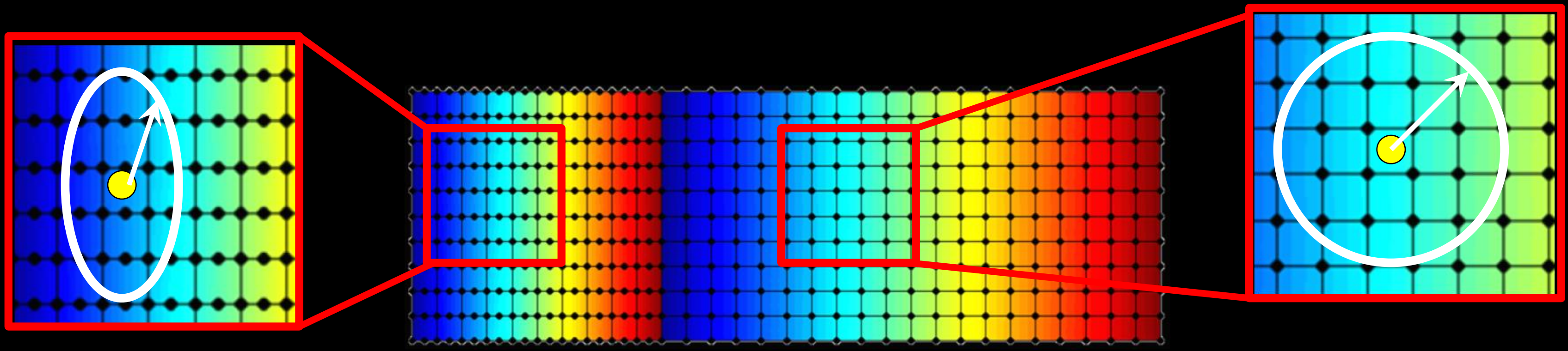
$$\frac{\partial c}{\partial \rho_d} = \sum \frac{\partial c}{\partial \rho_i^e} \frac{\partial \rho_i^e}{\partial \rho_d}$$

Projection scheme with graded patterns

- Projection using element centroids or nodal densities as design variables

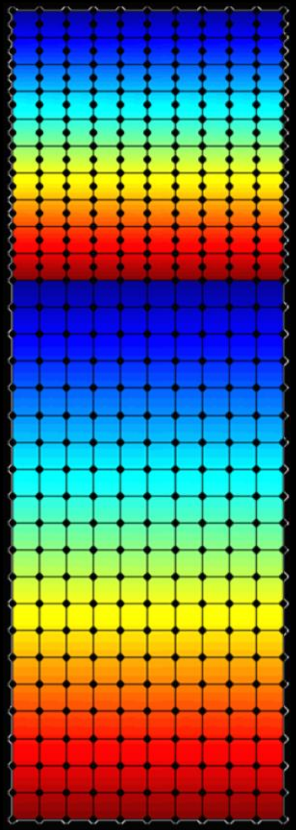


- Must use scaled projection for pattern gradation

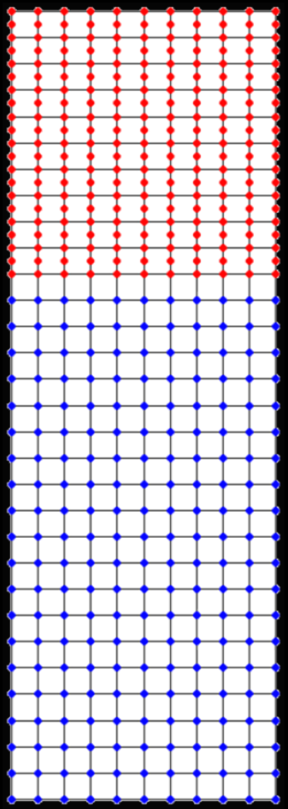


Numerical Examples

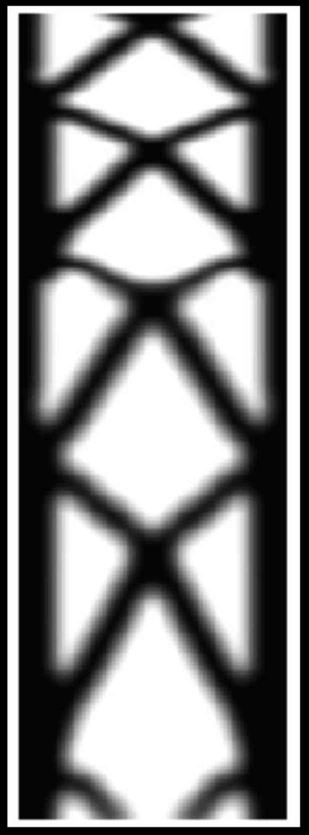
- **Graded cantilever building: Uniform Element Densities**



Pattern constraints



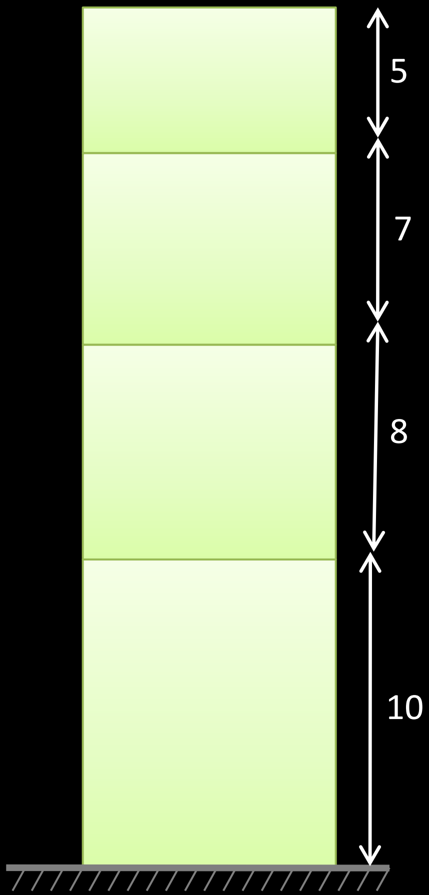
Design variables

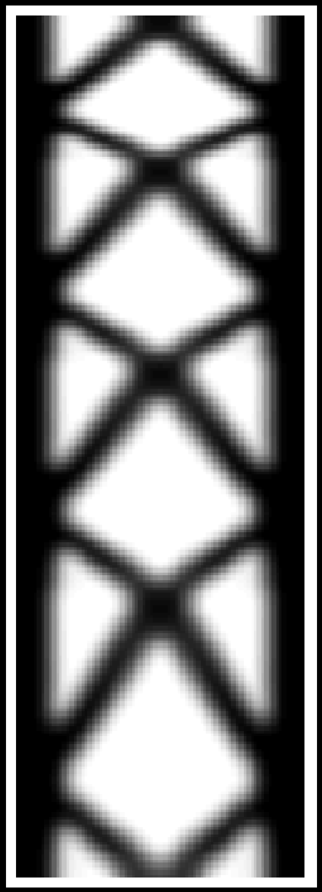


Mesh: 80 x 240
penal = 4, r_{min} = 1.2
volume = 50%

Numerical Examples

- **Patterns of Different Sizes: Uniform Element Densities**

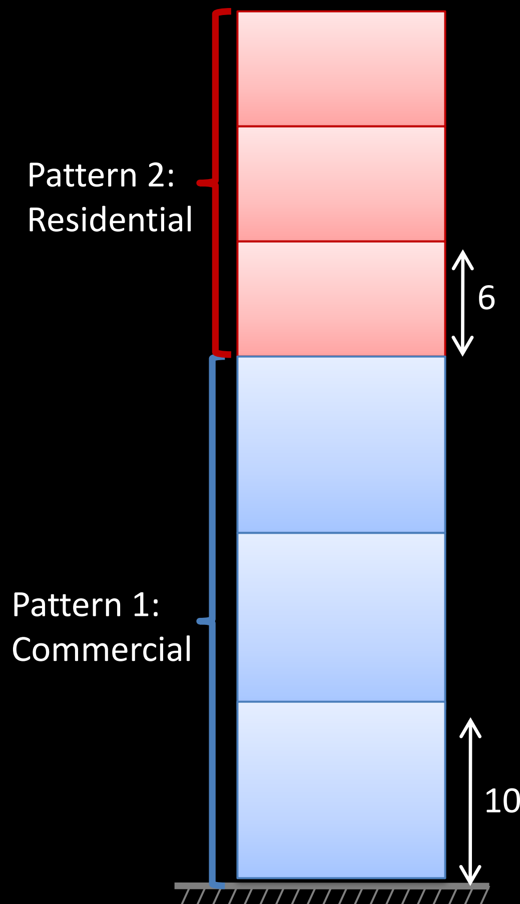




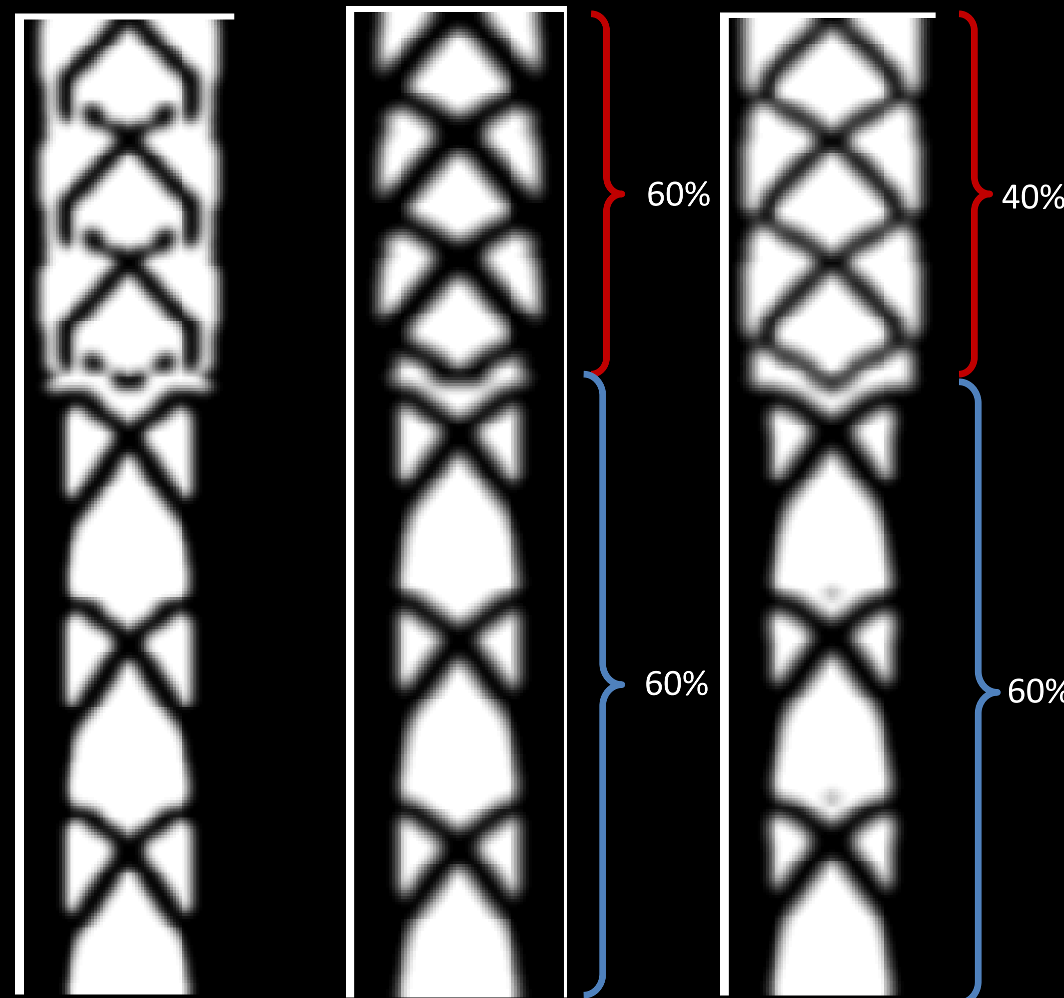
Mesh: 80 x 240
penal = 4, r_{min} = 1.2

Numerical Examples

- **Multiuse Building w/ volume gradation**



Mesh: 75 x 360
penal = 4, r_{min} = 1.2



Unconstrained Volume (50%)

With volume constraints

Virtual Work/Lagrange Multipliers for gradation in wall thicknesses

- **Virtual Work**

$$W_i = \int_A [\boldsymbol{N}^T \delta\boldsymbol{\eta} + \boldsymbol{M}^T \delta\boldsymbol{\chi} + \mathbf{V}^{\mathrm{T}} \delta\boldsymbol{\Gamma}]\,\mathrm{dA}$$

axial | flexural | shear

- **Lagrange Method**

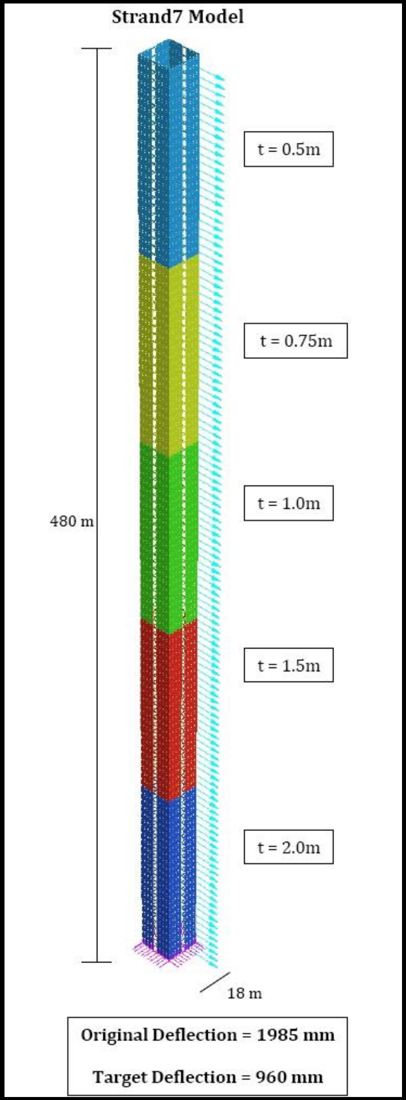
$$\Delta = \Sigma_{plates}\,\xi_j + \lambda(\Sigma t_j A_j - V)$$

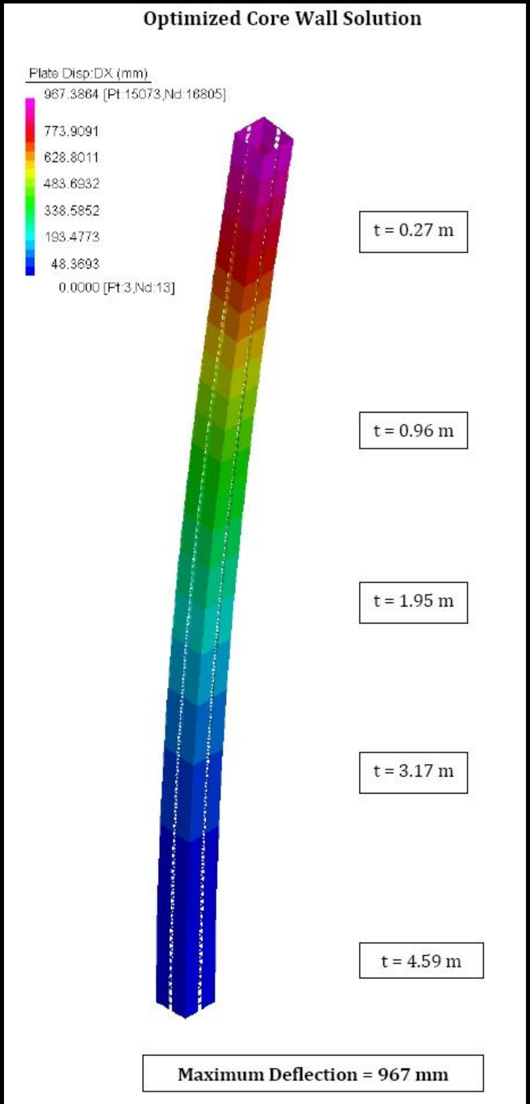
- **Optimal thickness**

$$t_i = \frac{1}{\Delta_{req}} \left(\frac{\nu_i}{A_i}\right)^{0.5} \Sigma_j \left(A_j \nu_j\right)^{0.5}$$

Baker, William. "Energy-Based Design of Lateral Systems." *Structural Engineering International* (1992).

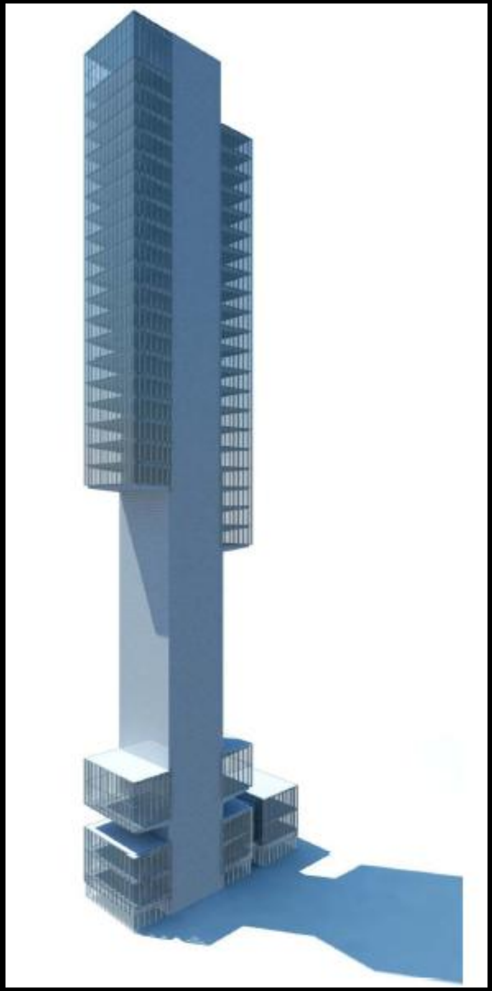
Virtual Work/Lagrange Multipliers for gradation in wall thicknesses



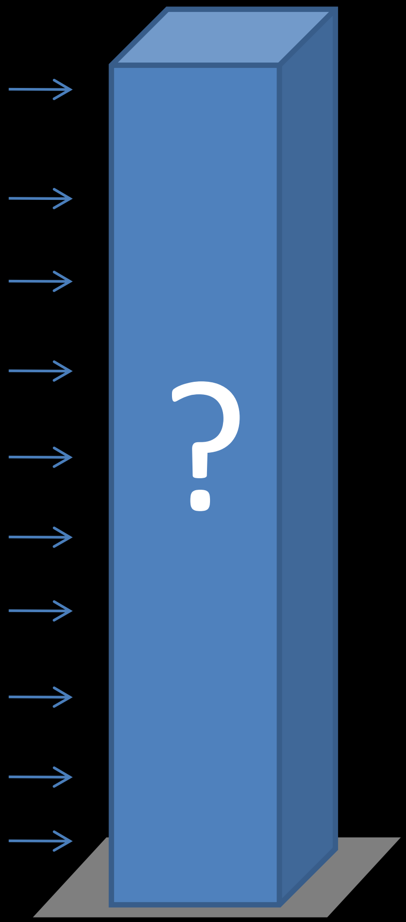


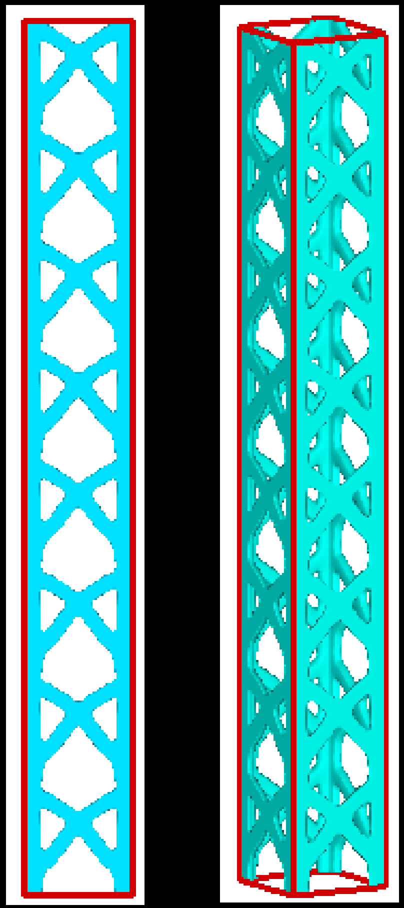
Building Design Example

N = 8
Mesh: 96 x 12 x 12
Volume = 50%



Skidmore, Owings & Merrill
Proposed Tower Design
Hong Kong





Proposed Topology Optimization
Design using Pattern Repetition
1,728,000 design variables

Nguyen, T., Paulino, G. H., Song, J., and Le, C. H. "Multilevel Topology Optimization (MTOP) – A Method to Obtain High Fidelity Solutions." JSMO (submitted).

Concluding Remarks

- **Manufacturing constraints in topology optimization allow for design of optimal buildings in terms of stiffness, cost, deflection, etc.**

- **Additional building design considerations, such as stability and nonlinear behavior are sources for future investigation**

Concluding Remarks

- The present approach may be extended for industry purposes by exploring computational expenses associated with non-coincident FEM displacement and design variable meshes to be used on a larger scale

- Future work includes optimization of large scale 3D problems using Topological Data Structure (TopS) integrated with finite element analysis and topology optimization

USNCCM X Presentation

- C. Talischi, G. Paulino, R. Espinha, A. Pereira, I. Menezes, W. Celes. "Topological embedding using a multilevel mesh representation for topology optimization" Section 63: New Advances in Topology Optimization, July 17, 10:10-10:30.