Topological embedding using a multilevel mesh representation for topology optimization

Cameron Talischi, Glaucio H. Paulino, Rodrigo Espinha, Anderson Pereira, Ivan Menezes, Waldemar Celes

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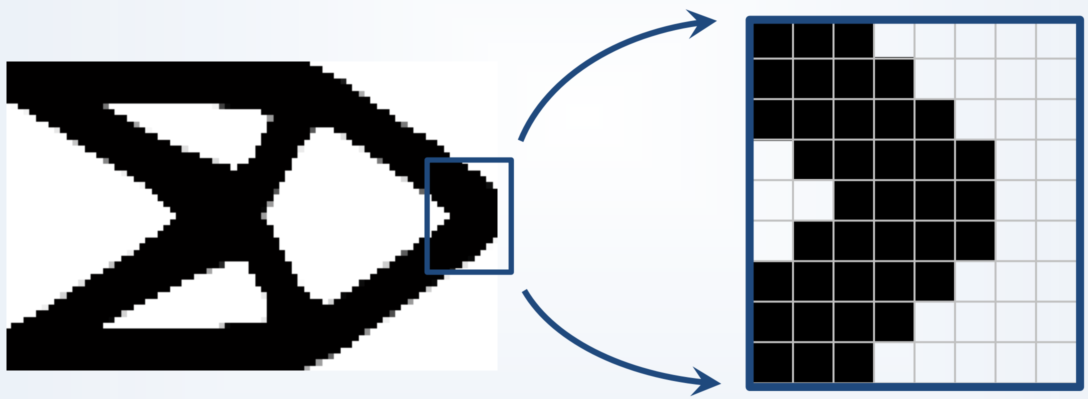
Civil and Environmental Engineering, University of Illinois at Urbana-Champaign
Tecgraf, Pontifical Catholic University of Rio de Janeiro

Motivation

- In topology optimization, the parameterization of *design* and approximation of its *response* are closely linked through the discretization process

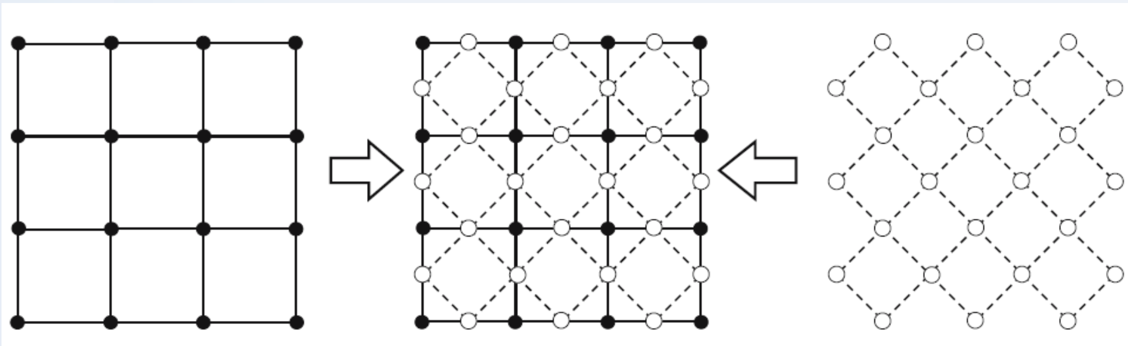
- For example, in the "element-based" approach, the design variables are the constant densities assigned to each displacement finite element:



- Such an approach:
  i. limits the resolution of topology and accuracy of response
  ii. can lead to numerical artifacts and instabilities
  iii. restricts the range of application of topology optimization

Motivation

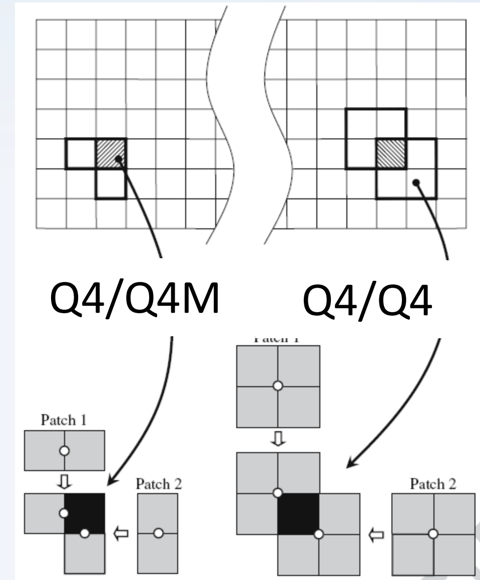
- Resolution of the topology



Displacement:
Q4 mesh

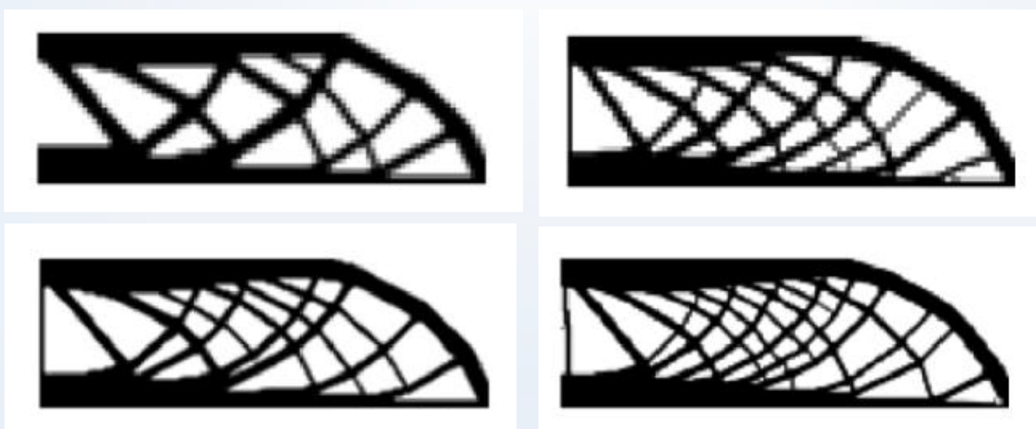
Q4/Q4M

Density:
Rotated Q4 mesh



90 x 30

150 x 50



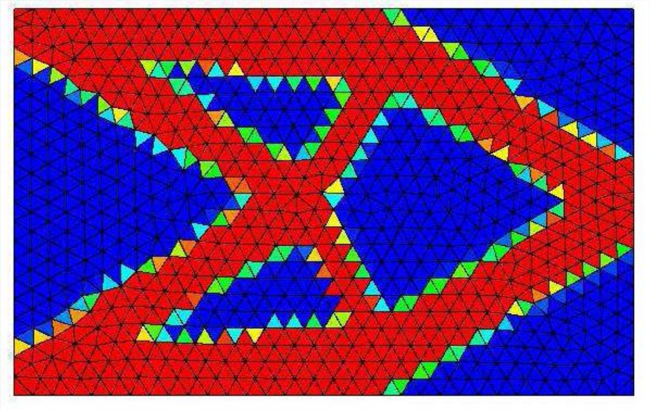
Q4/Q4 Q4/Q4M

The modified Q4/Q4 element produces solutions with higher resolution!

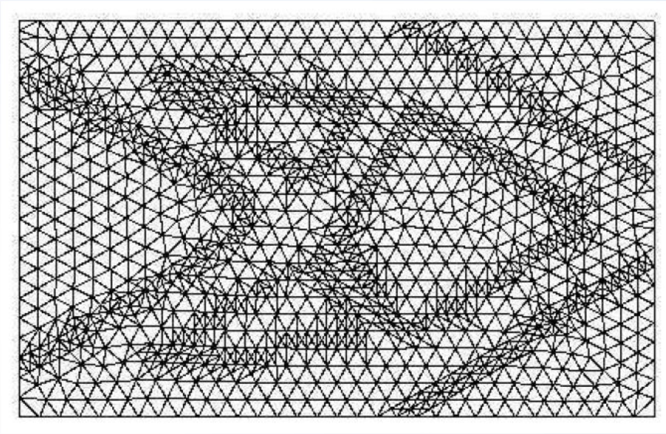
Paulino, Le (2009) A modified Q4/Q4 element for topology optimization, ***Struct Multidisc Optim***

Motivation

- In adaptive schemes, the rigid link between the two discretizations can lead to inefficiencies



Distribution of design variables at current iteration



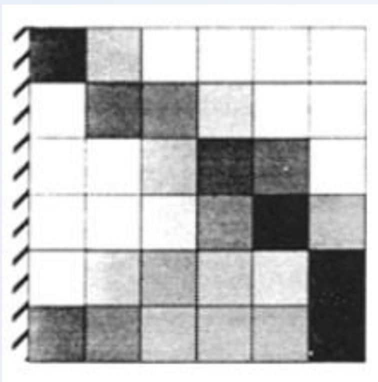
Refined finite element model constructed for next iteration

- For example, the finite element mesh can be made coarser in the void and interior regions without requiring the definition of design variables to change

Stainko (2006). An adaptive multilevel approach to the minimal compliance problem. ***Comm in Numer Methods in Eng***

Motivation

- In Maute and Ramm (1995), the design and analysis models are connected through a mapping procedure



Current design and underlying mesh

Finite element mesh based on the ‘outline’

Mapped design variables for the next iteration

Maute, Ramm (1995). Adaptive Topology Optimization. ***Struct Multidisc Optim***

Motivation

- We propose a framework that allows independent discretizations for design and analysis models

- The necessary communication between the two meshes are carried out using a topological data structure

Outline

- Motivation
- Setting for optimal design problem
- Topological data structure (Multi-level TopS)
- Computational framework
- Preliminary numerical results
- Concluding remarks

Problem setting

- Consider the following general optimal design problem:

$$\min_{\boldsymbol{d}} f(\boldsymbol{d}, \boldsymbol{u}) \quad \text{subject to} \quad g_i(\boldsymbol{d}, \boldsymbol{u}) = 0 \text{ for } i = 1, \cdots, k$$

  where $\boldsymbol{d}$ represents the *design* field and $\boldsymbol{u}$ is the *response* and they are related through the constraint functions g_i

- A canonical example is the minimum compliance problem:

$$\begin{array}{ll} \text{(Compliance)} & f(\boldsymbol{d}, \boldsymbol{u}) = \boldsymbol{p}^T \boldsymbol{u} \\ \text{(Equilibrium)} & \boldsymbol{g}_1(\boldsymbol{d}, \boldsymbol{u}) = \boldsymbol{K}(\boldsymbol{d})\boldsymbol{u} - \boldsymbol{p} \\ \text{(Volume Consraint)} & g_2(\boldsymbol{d}) = V(\boldsymbol{d}) - \overline{V} \end{array}$$

- The main difficulty is in the equilibrium constraint, in particular computing $\boldsymbol{K}(\boldsymbol{d})$ since this is the quantity relating the two fields

Topological Data Structure: TopS

- TopS is a topological data structure for representing two- or three-dimensional models with two important properties:

  1. It is a complete data structure in that it provides fast access to all adjacency information (e.g. list of elements adjacent to an edge)

  2. It relies on a reduced representation which leaves a small memory footprint, making it appropriate for storing large models

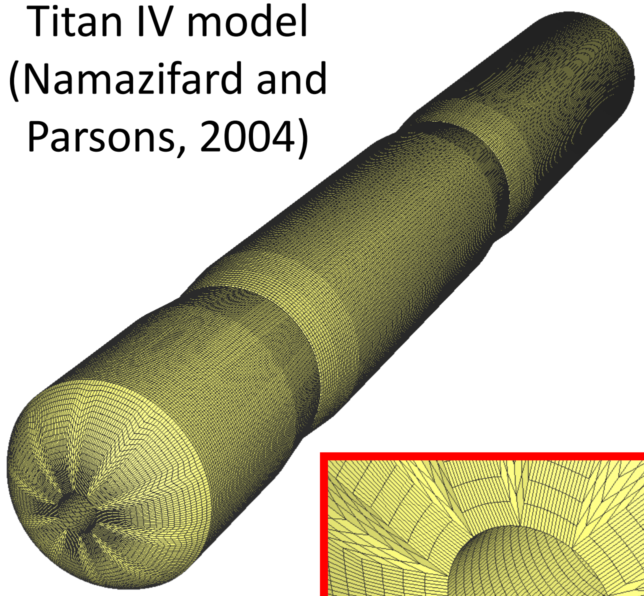
- TopS is especially powerful for adaptive analysis since it offers framework for remeshing operations (e.g. insertion of cohesive element for fragmentation simulation)

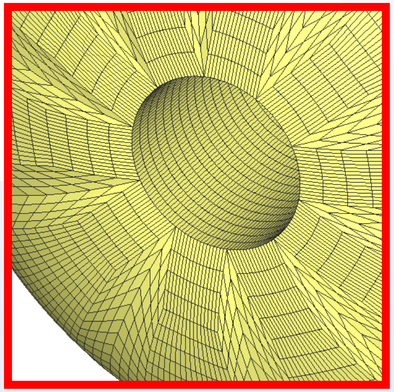
Celes, Paulino, Espinha(2005), A compact adjacency-based topological data structure. *Int Num Method Eng*
Paulino, Celes, Espinha, Zhang (2008), A general topology-based framework for adaptive insertion, *Eng W Compt*

Topological Data Structure: TopS

❖ Example of entity enumeration of a large model using TopS:

Titan IV model (Namazifard and Parsons, 2004)



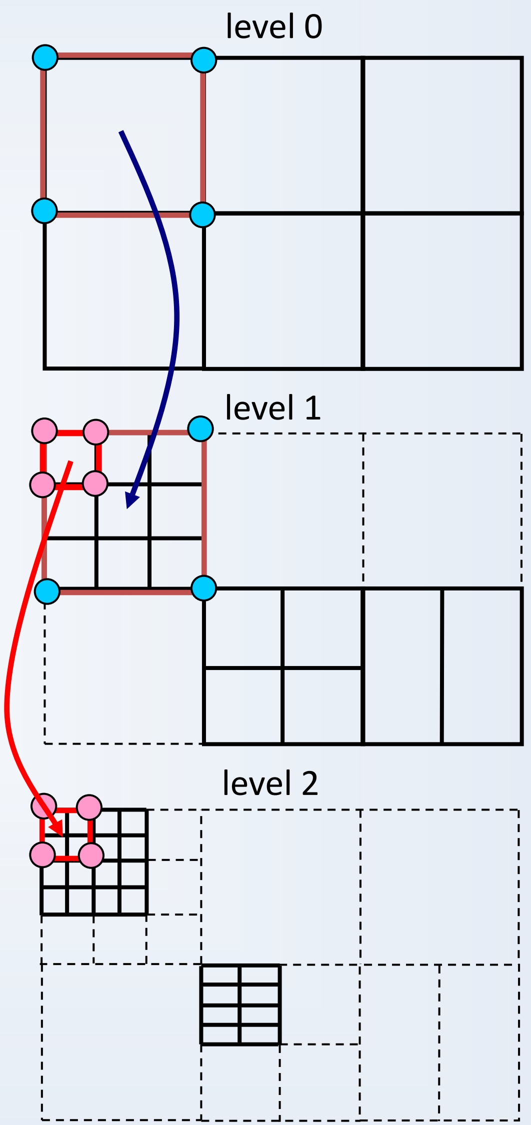


Entity information		*Elapsed time (s)*
Topological entity	*Number of entities*	
Element	1,738,240	**0.097**
Node	1,845,640	**0.046**
Facet	5,321,600	**0.219**
Edge	5,429,000	**0.292**
Vertex	1,845,640	**0.186**

Celes, Paulino, Espinha(2005), A compact adjacency-based topological data structure. *Int Num Method Eng*

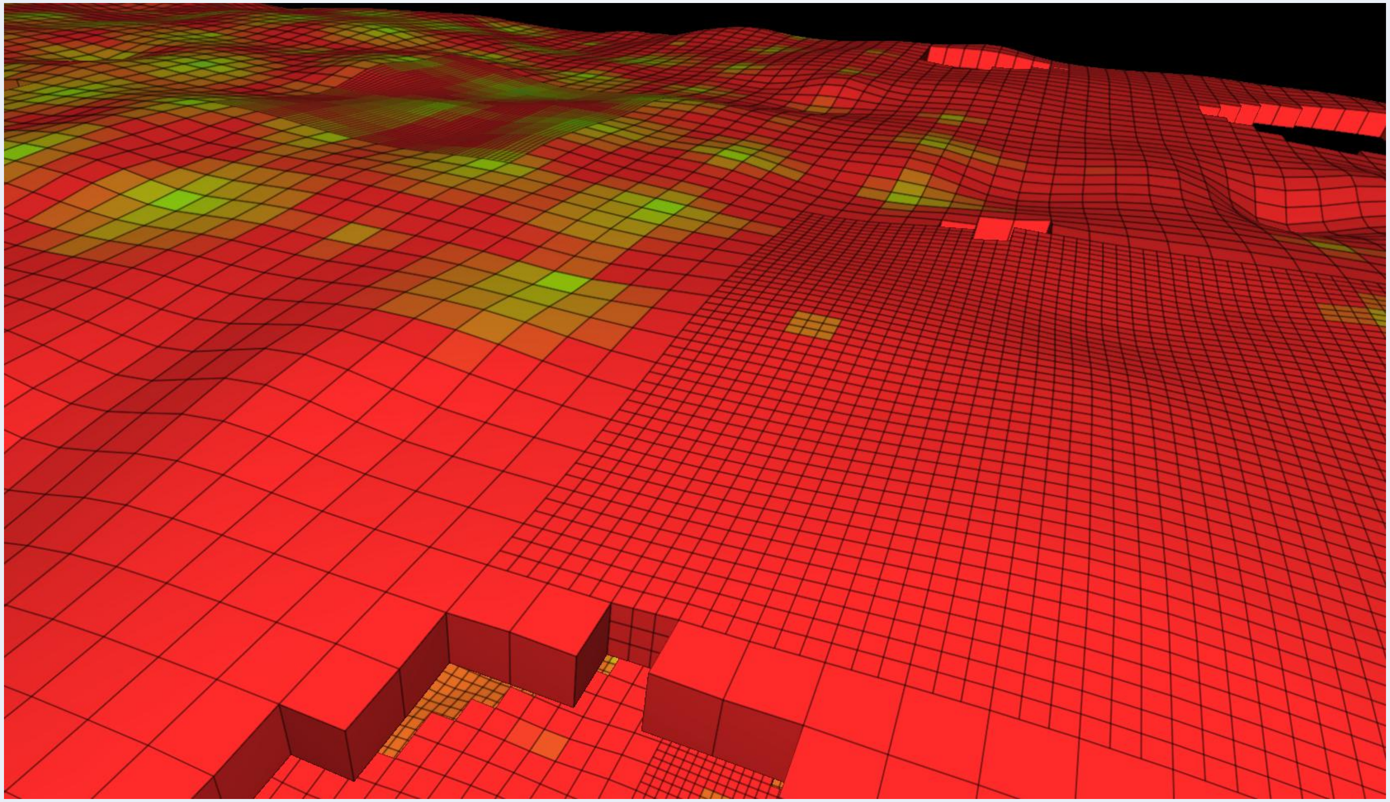
Multi-level TopS

- Multi-Level TopS supports the representations of multiple discretization, each of which are stored in a TopS model
- The mesh at each level is an embedded refinement of previous level mesh
- The topological information at each level is retrieved as usual from the entities existing at that level
- The link between the various levels are provided by parent/child/sibling references



Multi-level TopS

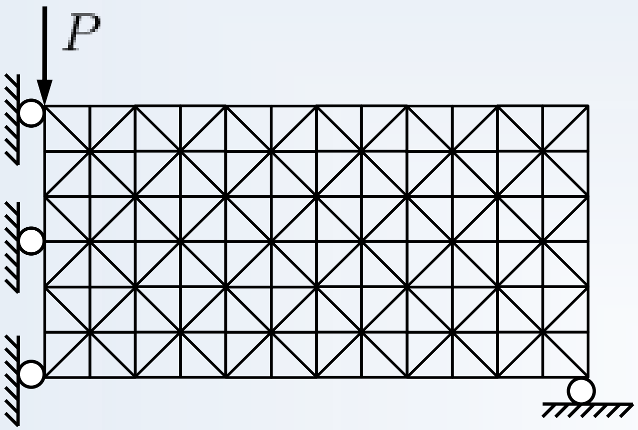
- Oil reservoir models (Tecgraf)



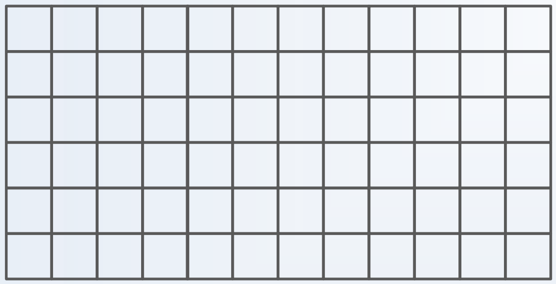
Implementation approach

- The computation on different levels are kept separate:
  - For example, the finite element code is only aware of the TopS model representing the response mesh
- During each iteration, Multi-level TopS will provide the necessary information to each level of computation via its topological operator
  - In this manner, the two meshes can change during the optimization without the need to reconstruct any explicit maps
- The optimization algorithm and the analysis routine are decoupled
  - This framework can lead to an object-oriented implementation of topology optimization

Example problem

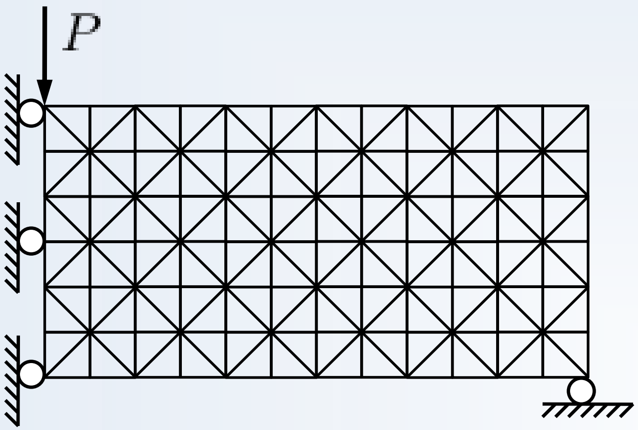


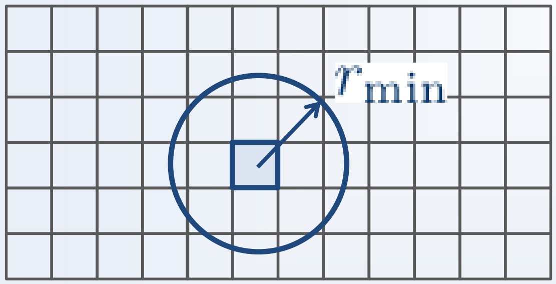
Finite element mesh
(associated to u)



Design variable mesh
(associated to d)

Example problem

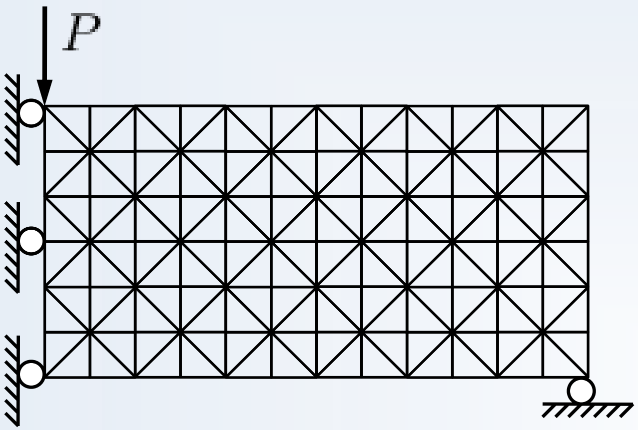


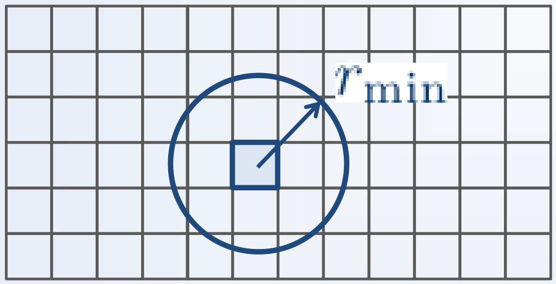


For the sake of illustration, we consider a formulation consisting of an intermediate filtered density variable:

$$\rho_j = \sum_i P_{ji}(r_{\min}) d_i$$

Example problem





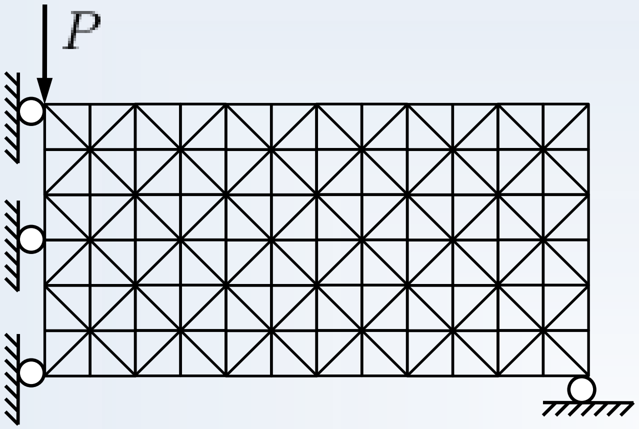
The stiffness is related to density according to the given material model, e.g. SIMP:

$$E = \rho^p E_0, \quad 0 < \rho \leq 1$$

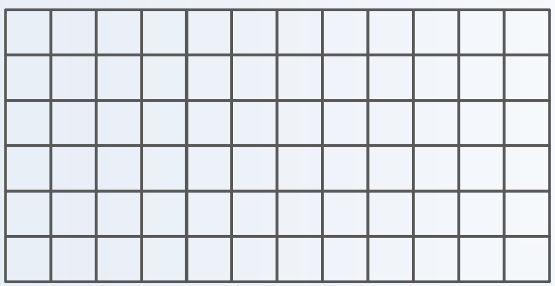
$$\rho_j = \sum_i P_{ji}(r_{\min}) d_i$$

Example problem

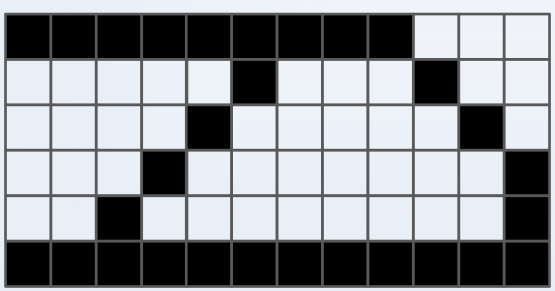
analysis (FEM)



density (topology!)



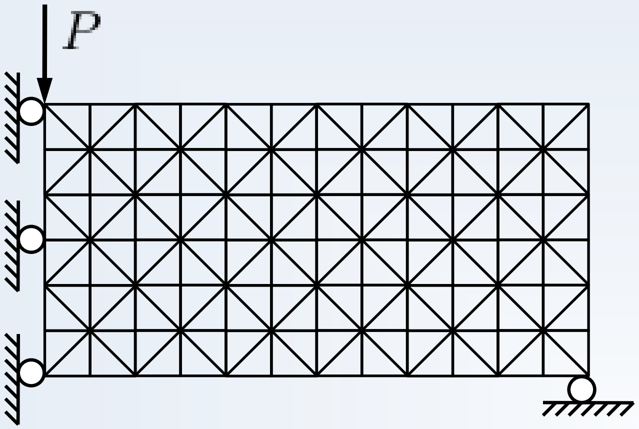
design



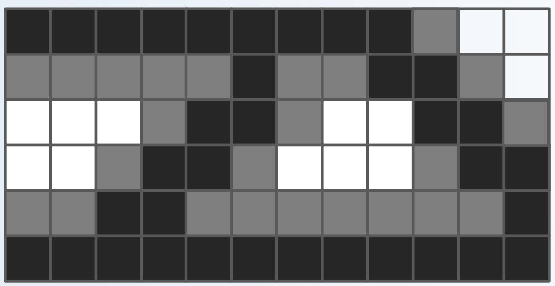
← Design at current iteration: $d^{(\nu)}$

Example problem

analysis (FEM)



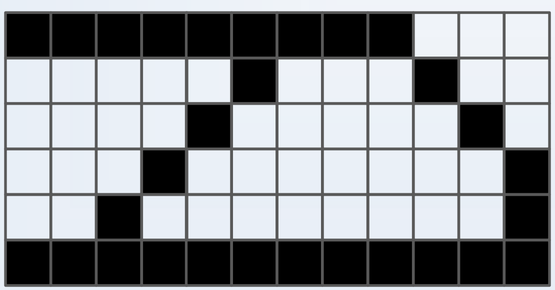
density (topology!)



Compute the densities using the TopS model representing design mesh:

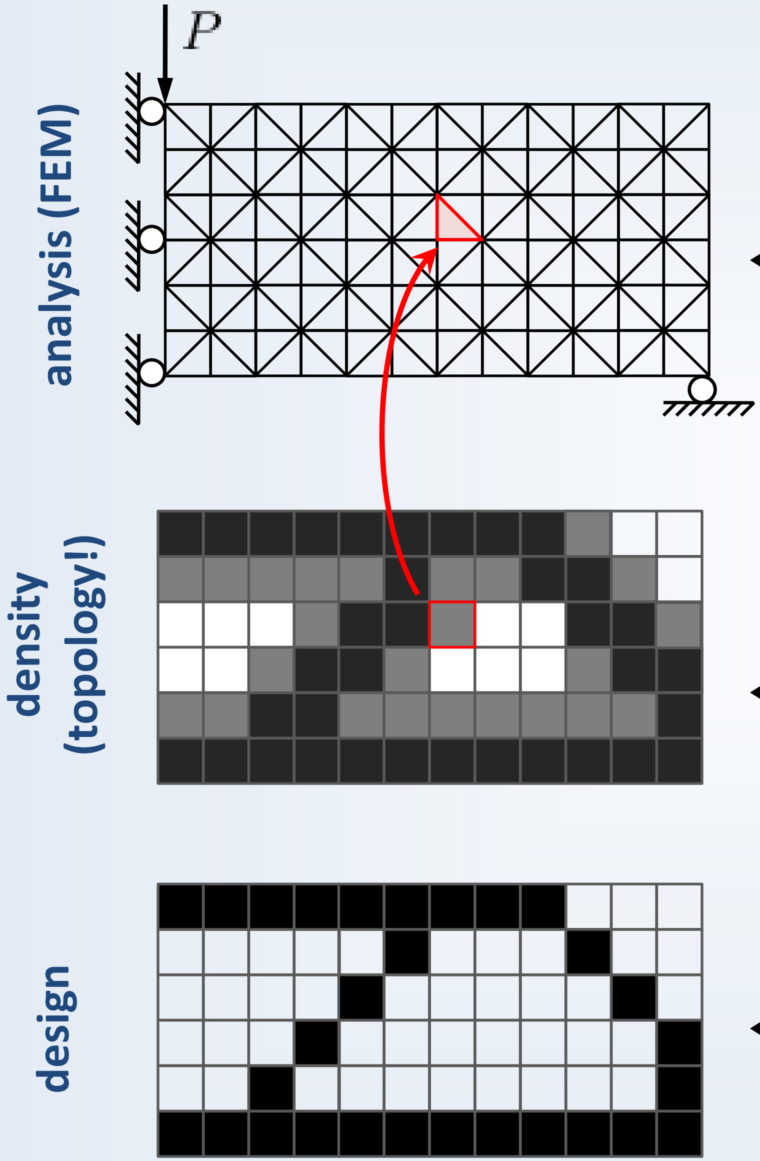
$$\rho_j = \sum_i P_{ji} d_i$$

design



Design at current iteration: $\boldsymbol{d}^{(\nu)}$

Example problem



Obtain the element properties from the design mesh (this is an interface routine)

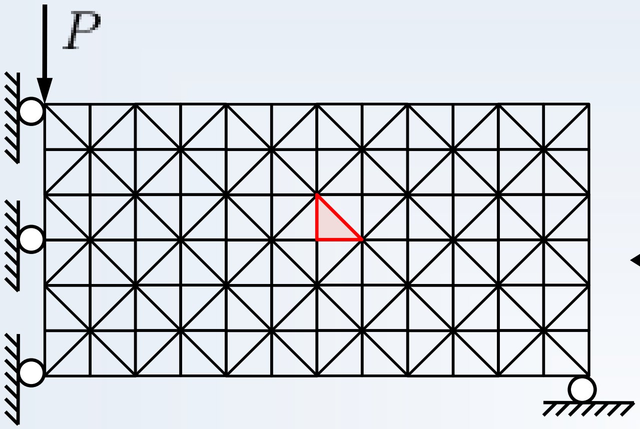
$$E_e = \sum_j M_{ej} (\rho_j)^p E_0$$

$$\rho_j = \sum_i P_{ji} d_i$$

Design at current iteration: $\boldsymbol{d}^{(\nu)}$

Example problem

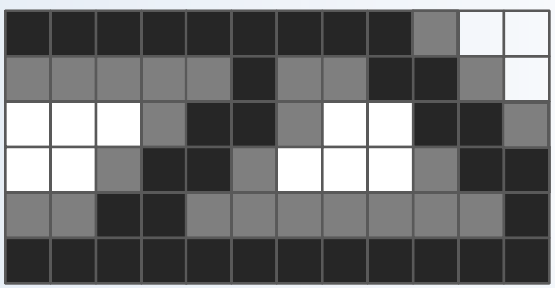
analysis (FEM)



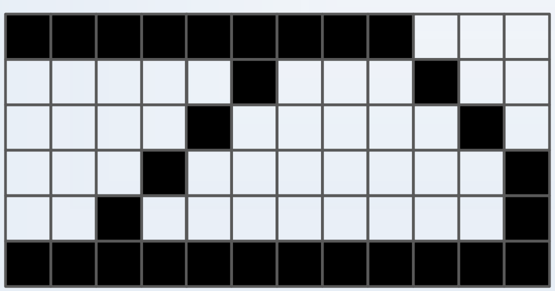
Compute $\boldsymbol{K}$ and $\boldsymbol{u}$ as well as the sensitivities in the analysis routine:

$$\frac{\partial f}{\partial E_e} = -\boldsymbol{u}^T \frac{\partial \boldsymbol{K}}{\partial E_e} \boldsymbol{u}$$

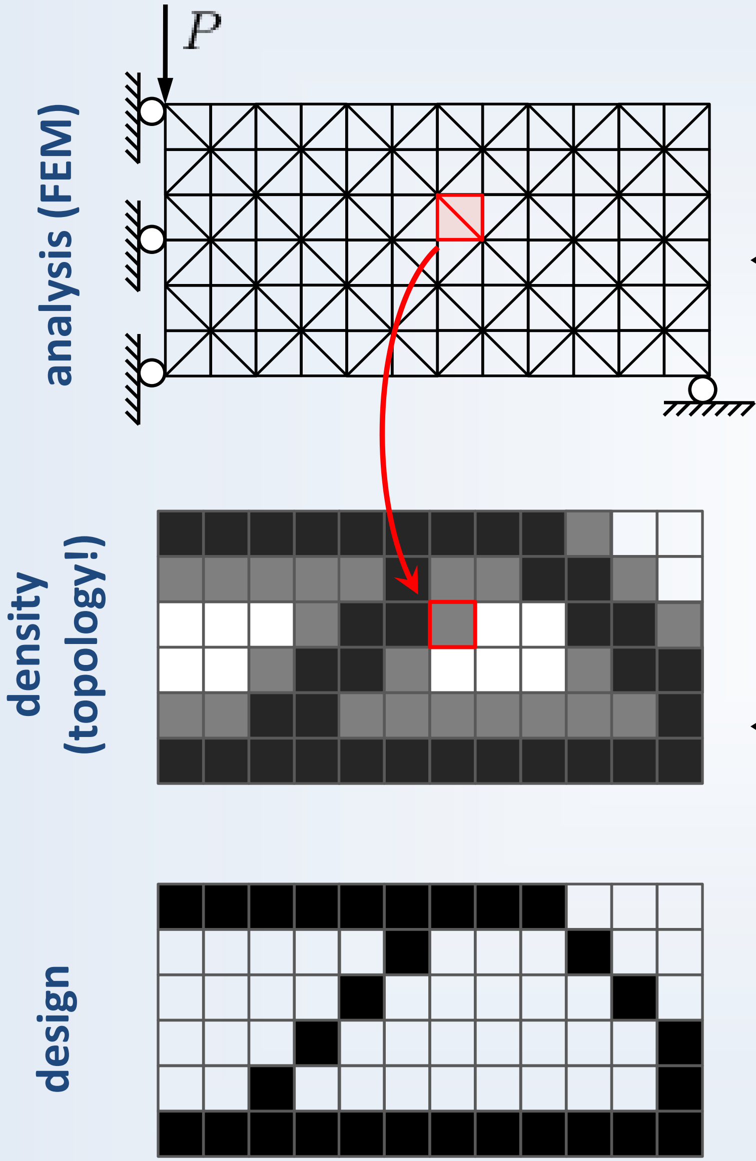
density (topology!)



design



Example problem

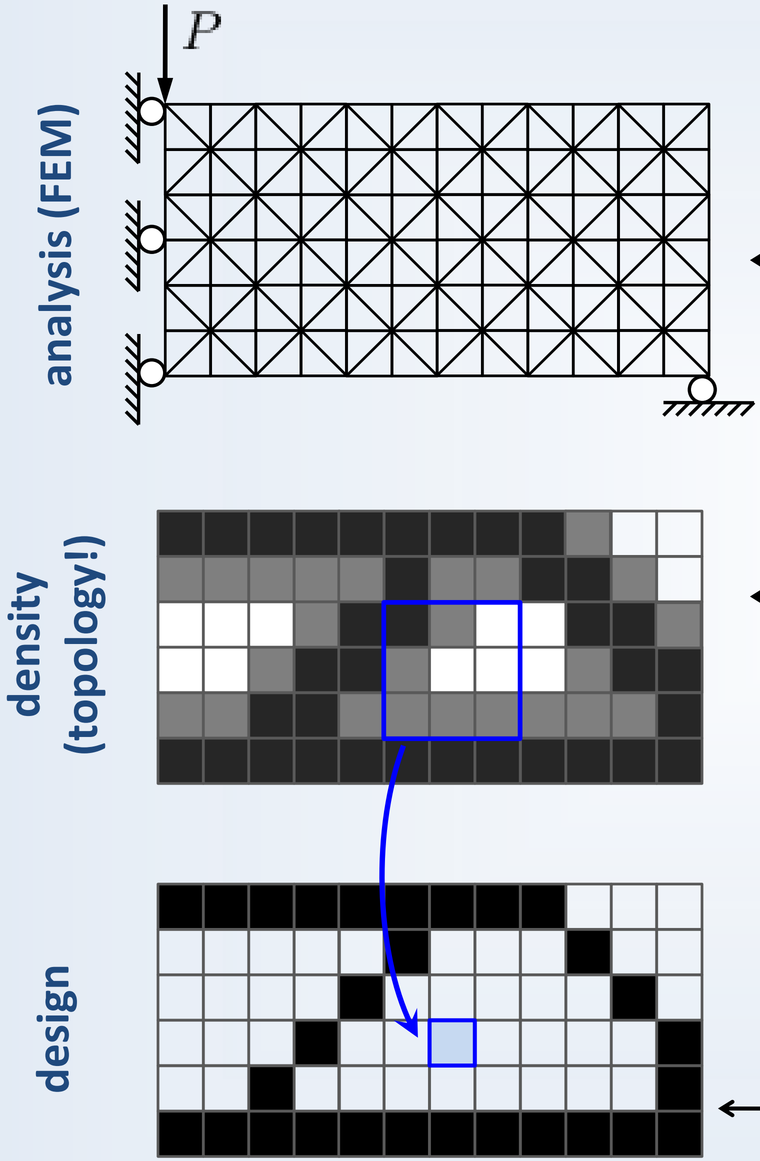


$$\frac{\partial f}{\partial E_e} = -\boldsymbol{u}^T \frac{\partial \boldsymbol{K}}{\partial E_e} \boldsymbol{u}$$

Find which finite elements were affected by the given densities (interface routine):

$$\frac{\partial f}{\partial \rho_j} = \sum_e \frac{\partial f}{\partial E_e} \frac{\partial E_e}{\partial \rho_j}$$

Example problem



$$\frac{\partial f}{\partial E_e} = -\boldsymbol{u}^T \frac{\partial \boldsymbol{K}}{\partial E_e} \boldsymbol{u}$$

$$\frac{\partial f}{\partial \rho_j} = \sum_e \frac{\partial f}{\partial E_e} \frac{\partial E_e}{\partial \rho_j}$$

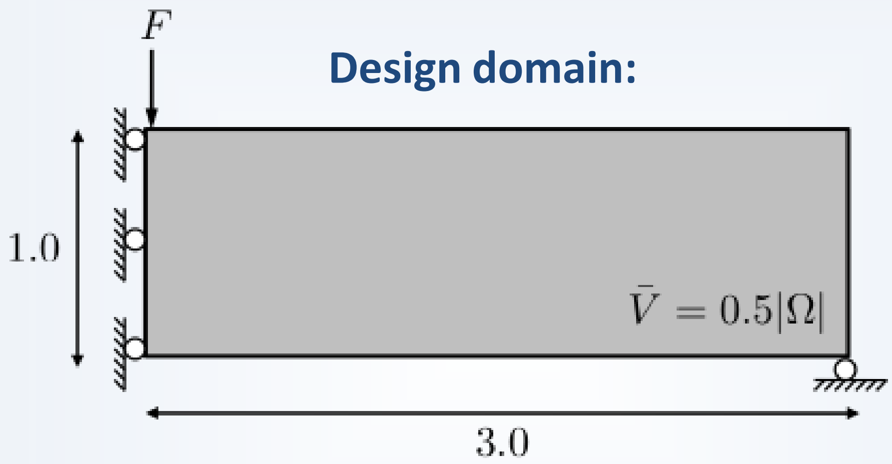
This step along with computing $\partial g_2 / \partial d_i$ is performed in the design routine:

$$\frac{\partial f}{\partial d_i} = \sum_j \frac{\partial f}{\partial \rho_j} \frac{\partial \rho_j}{\partial d_i}$$

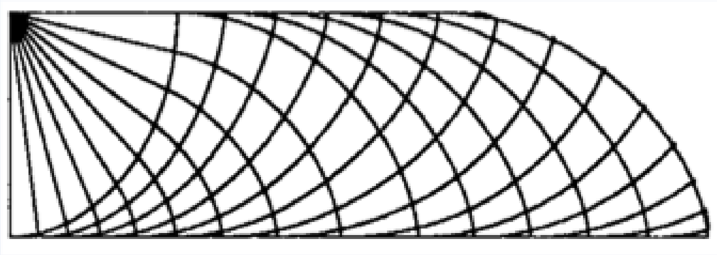
Preliminary results: Superelements

- Messerschmitt-Bolkow-Blohm (MBB):

Design domain:

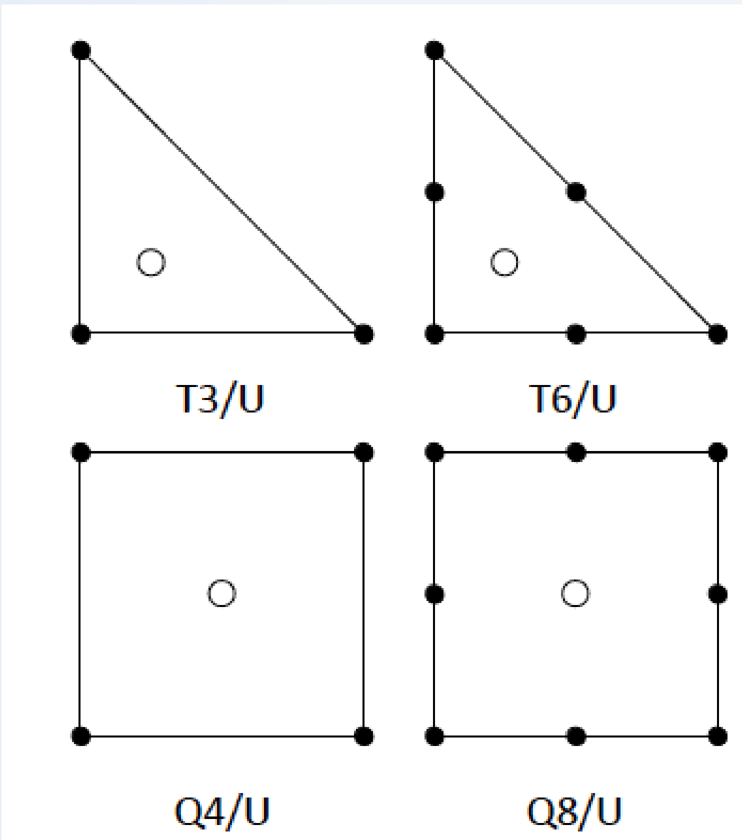


Michell-type solution:

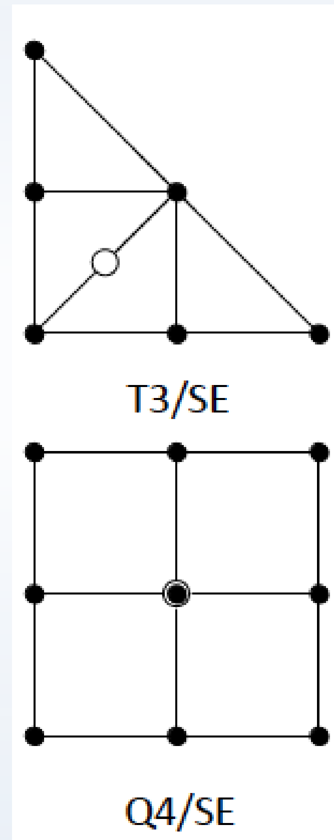


Preliminary results:

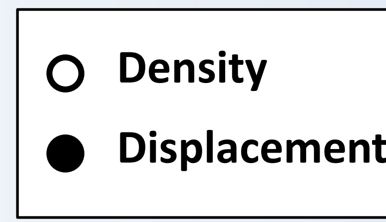
- Definition of design variable and displacement discretizations:



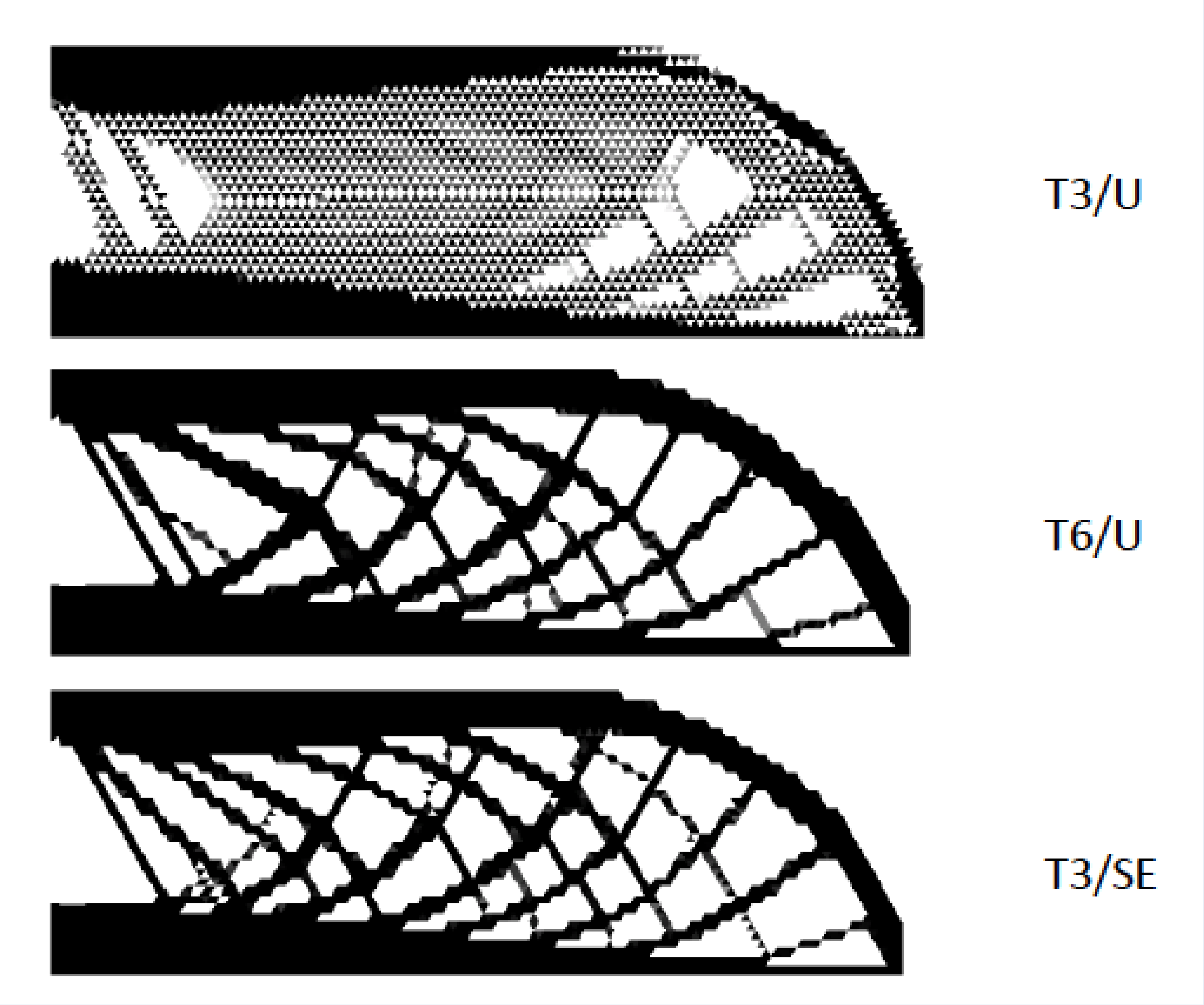
Element-based



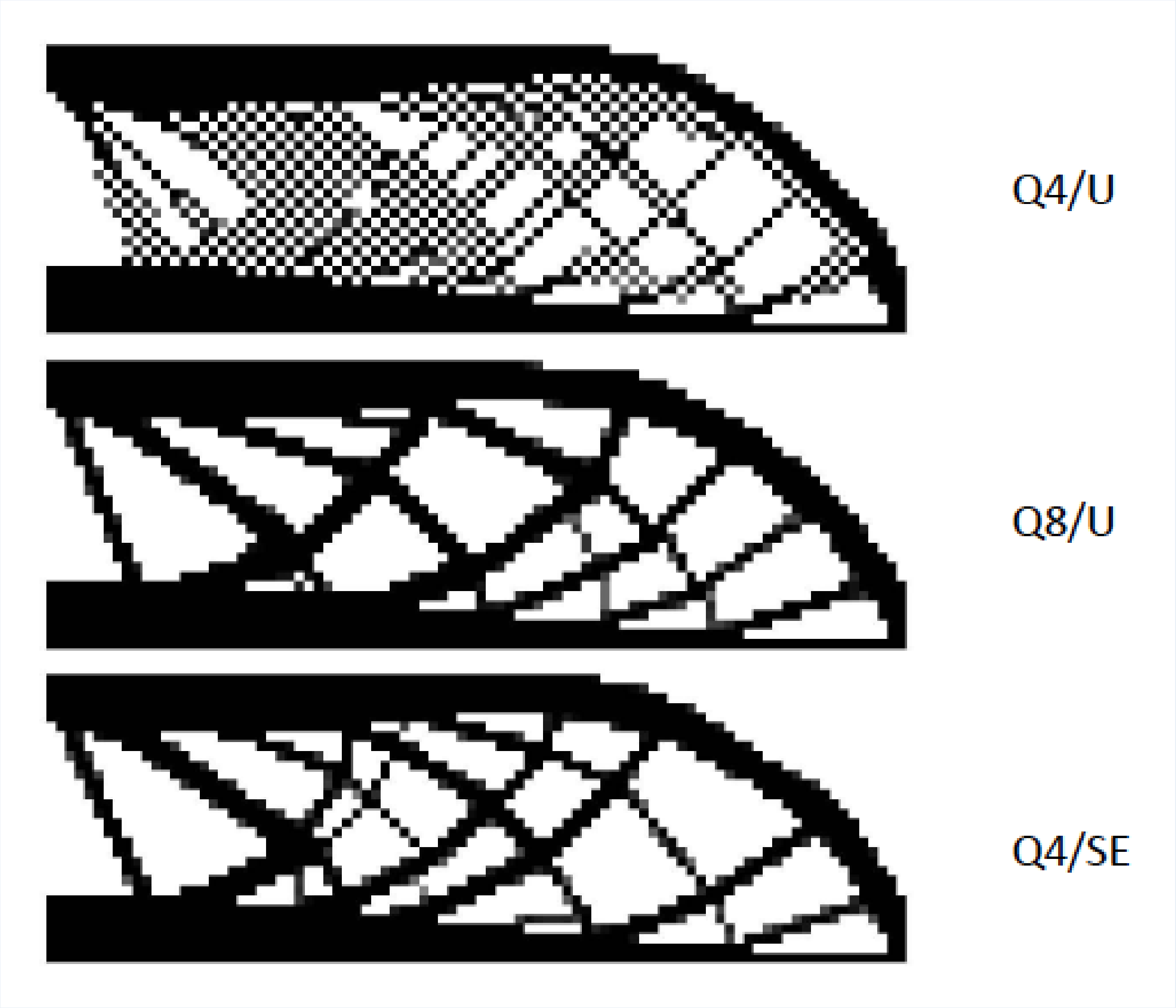
"Superelements"



Preliminary results: MBB beam



Preliminary results: MBB beam



Conclusions

- The separation of finite element and topology optimization discretizations can offer several advantages in obtaining high-fidelity solutions
- This work proposes the use of a multilevel mesh representation involving analysis and design variables using a compact topological data structure
- An adaptive scheme in which the design and analysis variables change during the optimization iterations can be supported in this framework
- Framework offers linkage for multiscale topology optimization
- Implementation is currently under development

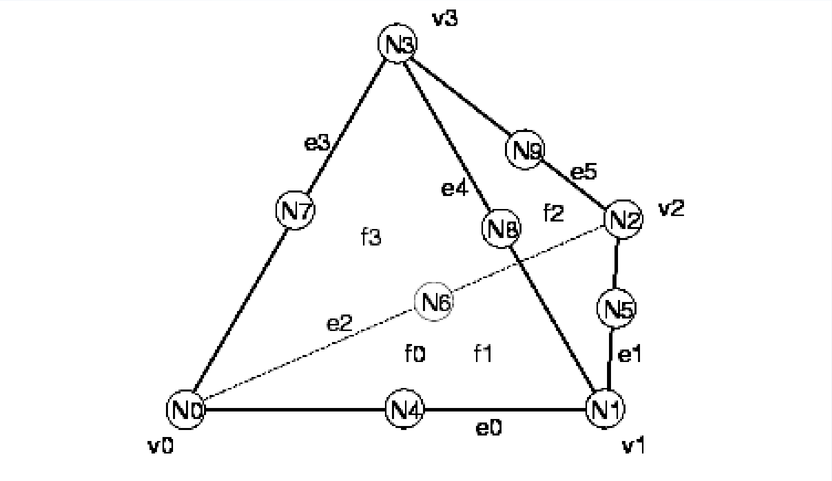
Topological Data Structure: TopS

- TopS defines 5 topological entities:

1. **Element**: Finite (or boundary) element
2. **Node**: Corner or midside node
3. **Facet**: Interface between two elements (2D or 3D)
4. **Edge**: Connection of two vertices
5. **Vertex**: Corresponds to corner nodes

Explicitly stored (1–2)

Implicitly represented (3–5)



Celes, Paulino, Espinha(2005), A compact adjacency-based topological data structure. ***Int Num Method Eng***

Preliminary results: Cantilever