Large deflection analysis of planar solids based on the Finite Particle Method

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Outline

- Motivation of the Finite Particle Method (FPM)
- Fundamentals of the FPM
- Numerical Examples
- Concluding Remarks
- Future Work

Motivation of the FPM

- Structural nonlinear problems:

  — large deformation, large rotation

- Discontinuous problems:

  — fracture, collapse, fragmentation

Methods:

- Finite Element Method (FEM):

  — general method

- Mesh-free Methods: (DEM, SPH, and others)

  — suited for particulate material, such as sand and concrete

Motivation of the FPM

Finite Particle Method (FPM)

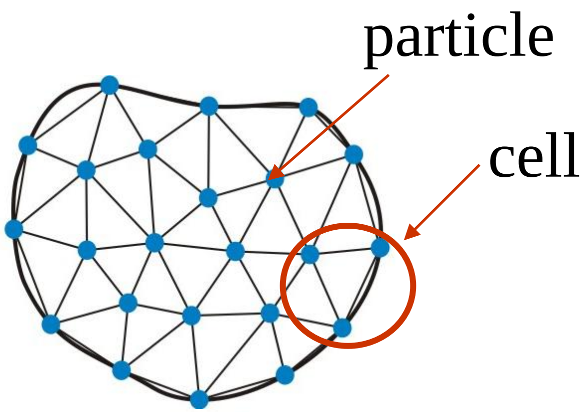
- Based on Vector Mechanics

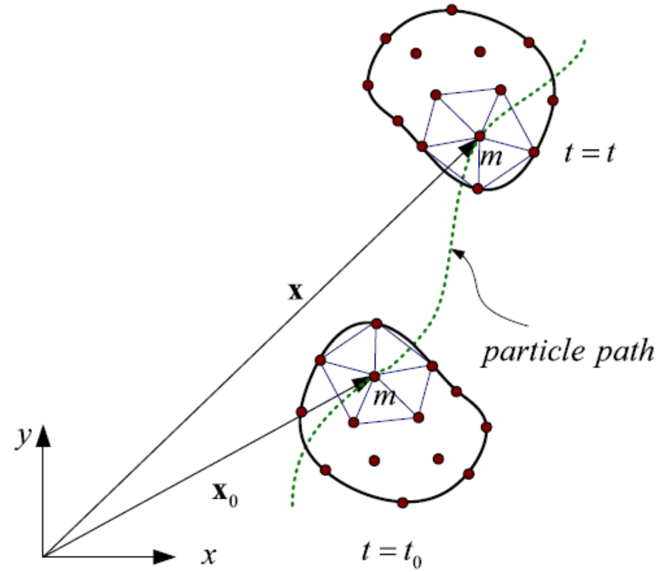
The procedure of this method is simple and unified. Without special considerations, strong nonlinear and discontinuous problems can be solved.

Ting, E.C., Shi, C., Wang, Y.K.: Fundamentals of a vector form intrinsic finite element: Part I. Basic procedure and a planar frame element. J. Mech. **20**, 113–122 (2004)

Fundamentals of the FPM

1) Discretization of structure.



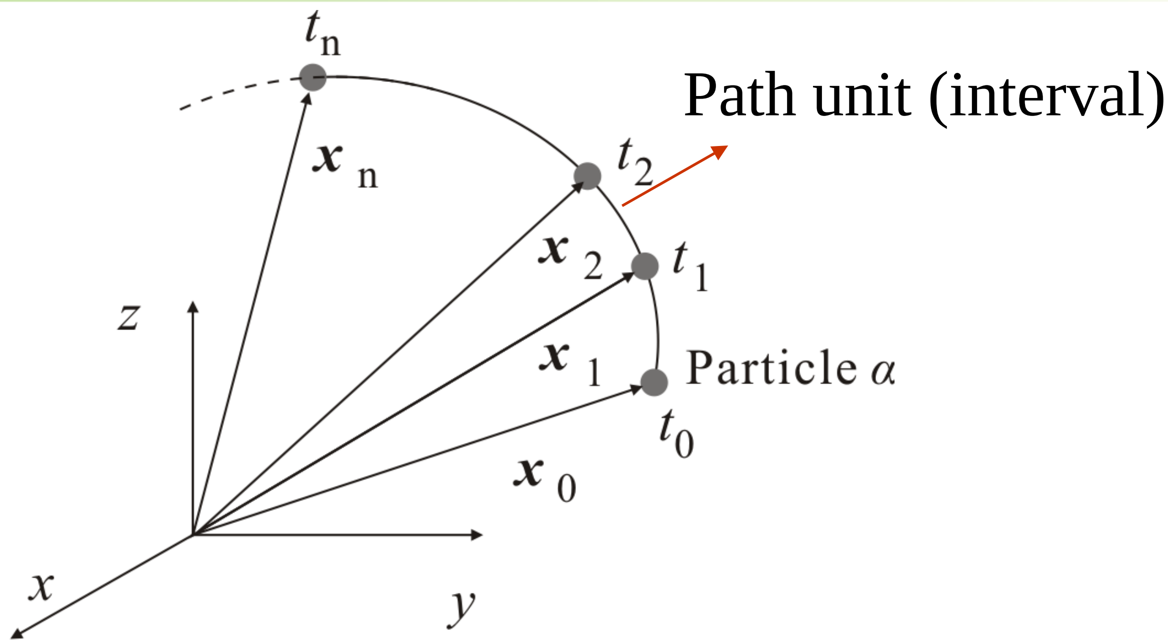


Assumptions:

- Particle
- Cell (“element” like)
- The relationship between particle and element
- The particle motion undergoes a time history.

Fundamentals of the FPM

2) Discrete Path



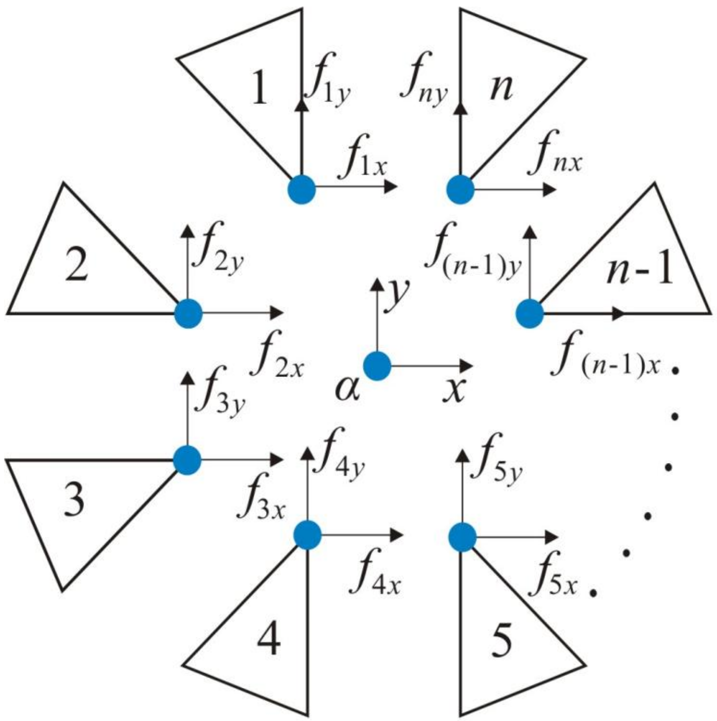
Assumptions:

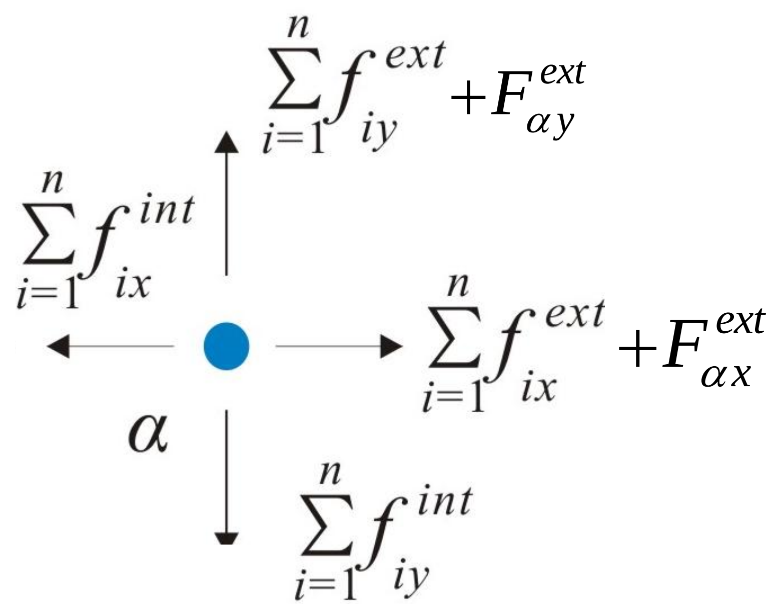
- The effect due to geometrical changes within the time interval t_1-t_2 can be neglected
- The use of infinitesimal strain and engineering stress for evaluating stresses and computing virtual work is feasible.

Fundamentals of the FPM

3) Particle Motion Equation

$$M_\alpha \ddot{d}_\alpha = F_\alpha^{ext} + \sum_{i=1}^{n} f^{ext} - \sum_{i=1}^{n} f^{int}$$





Fundamentals of the FPM

4)Evaluation of deformations and internal forces

a) Evaluate the rigid body motion

↓

b) Use reverse motion to remove rigid translation and rotation

↓

c) Use deformed coordinate to reduce element degree of freedom

↓ →

d) Calculate the internal force at the deformed coordinate system

↑

g) Transform the internal force back to global coordinate

↑

f) Use forward motion to get the element back to the original position

Fundamentals of the FPM

5) Time Integration

To avoid iteration, explicit time integration is used..

If a second order, explicit, central difference time integrator is adopted:

Velocity:

$$\dot{\boldsymbol{d}}_n = \frac{1}{2\Delta t}(\boldsymbol{d}_{n+1} - \boldsymbol{d}_{n-1}),$$

Acceleration:

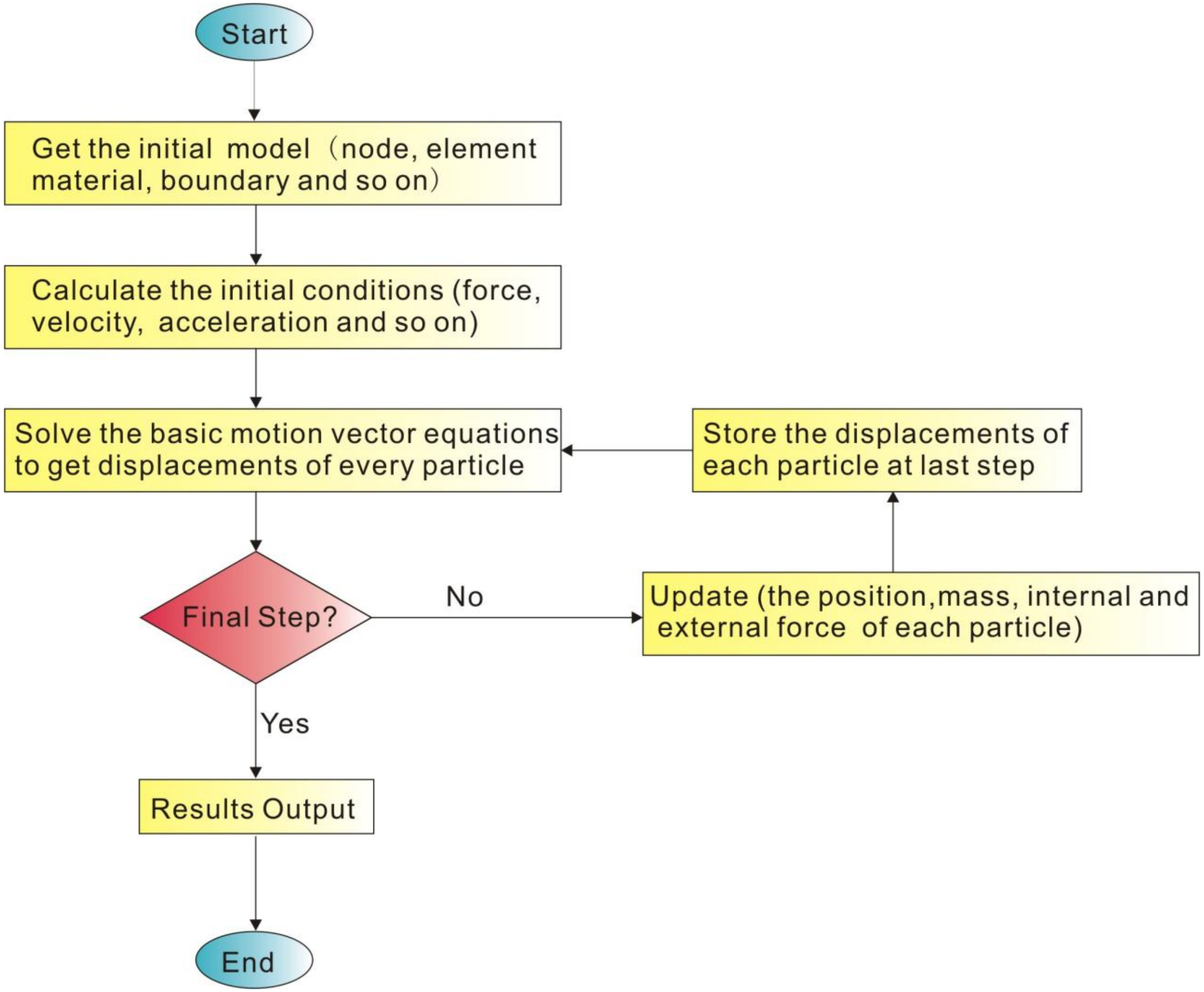
$$\ddot{\boldsymbol{d}}_n = \frac{1}{\Delta t^2}(\boldsymbol{d}_{n+1} - 2\boldsymbol{d}_n + \boldsymbol{d}_{n-1}).$$

Displacement:

$$\boldsymbol{d}_{n+1} = \left(\frac{2}{2+\mu\Delta t}\right)\frac{\Delta t^2}{m_\alpha}(\boldsymbol{F}_\alpha^{ext} + \sum_{i=1}^{n} f^{ext} - \sum_{i=1}^{n} f^{int}) + \left(\frac{4}{2+\mu\Delta t}\right)\boldsymbol{d}_n - \left(\frac{2-\mu\Delta t}{2+\mu\Delta t}\right)\boldsymbol{d}_{n-1}.$$

Fundamentals of the FPM

6) Flow Chart

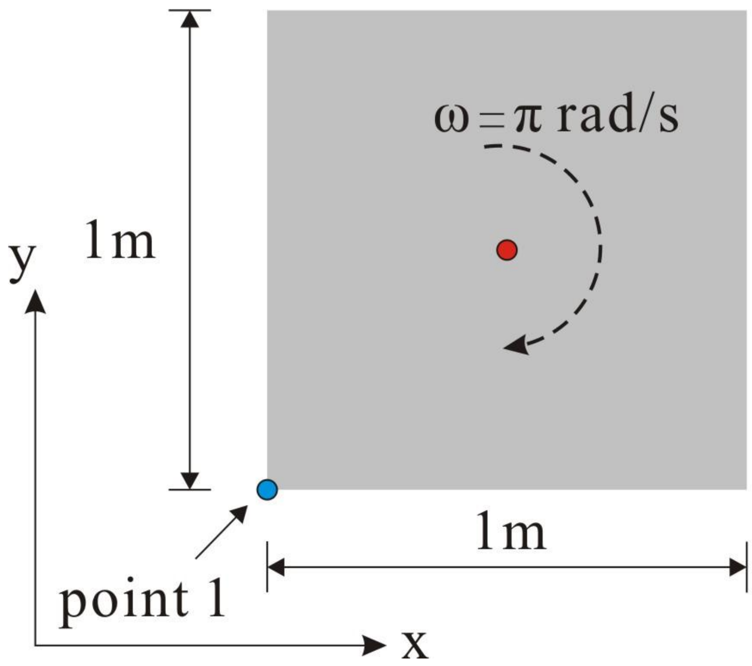


Numerical examples

1. A square plane subjected to an initial angular velocity

Goal: a) verify the accuracy of internal force evaluation

b) effect of the Young’s modulus



Mass density: $\rho = 1.0\ kg/m^3$

Young's modulus:

E1=10Mpa;

E2=1Mpa;

E3=0.5Mpa

Poisson's ratio: $\nu=0$

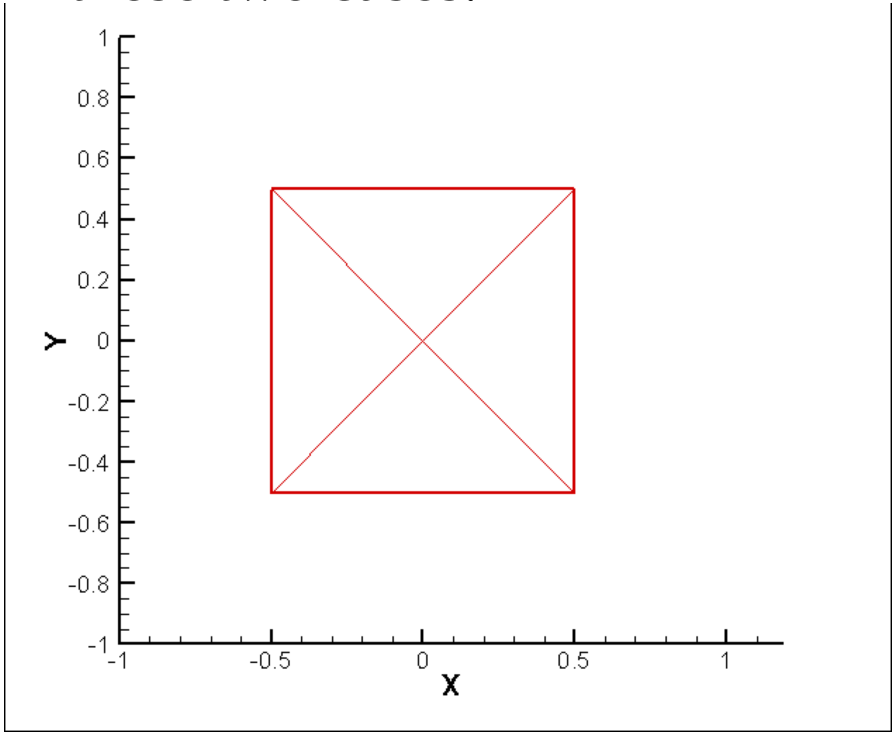
Thickness: h=0.1m

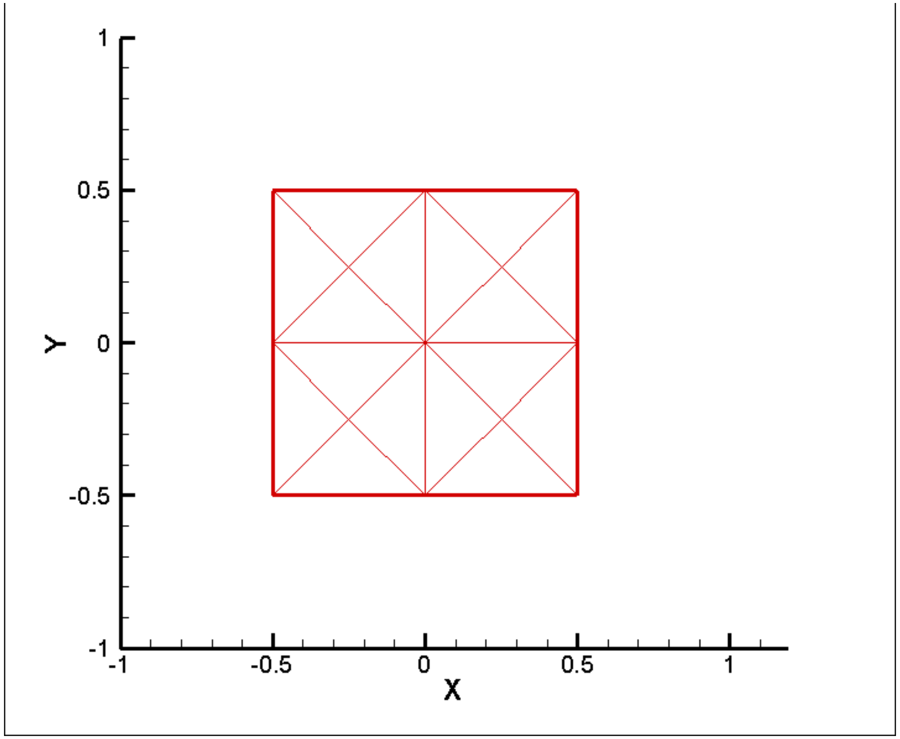
Numerical examples

1. A square plane subjected to an initial angular velocity

- Animation

The Young's modulus and initial angular velocity are the same in these two cases.

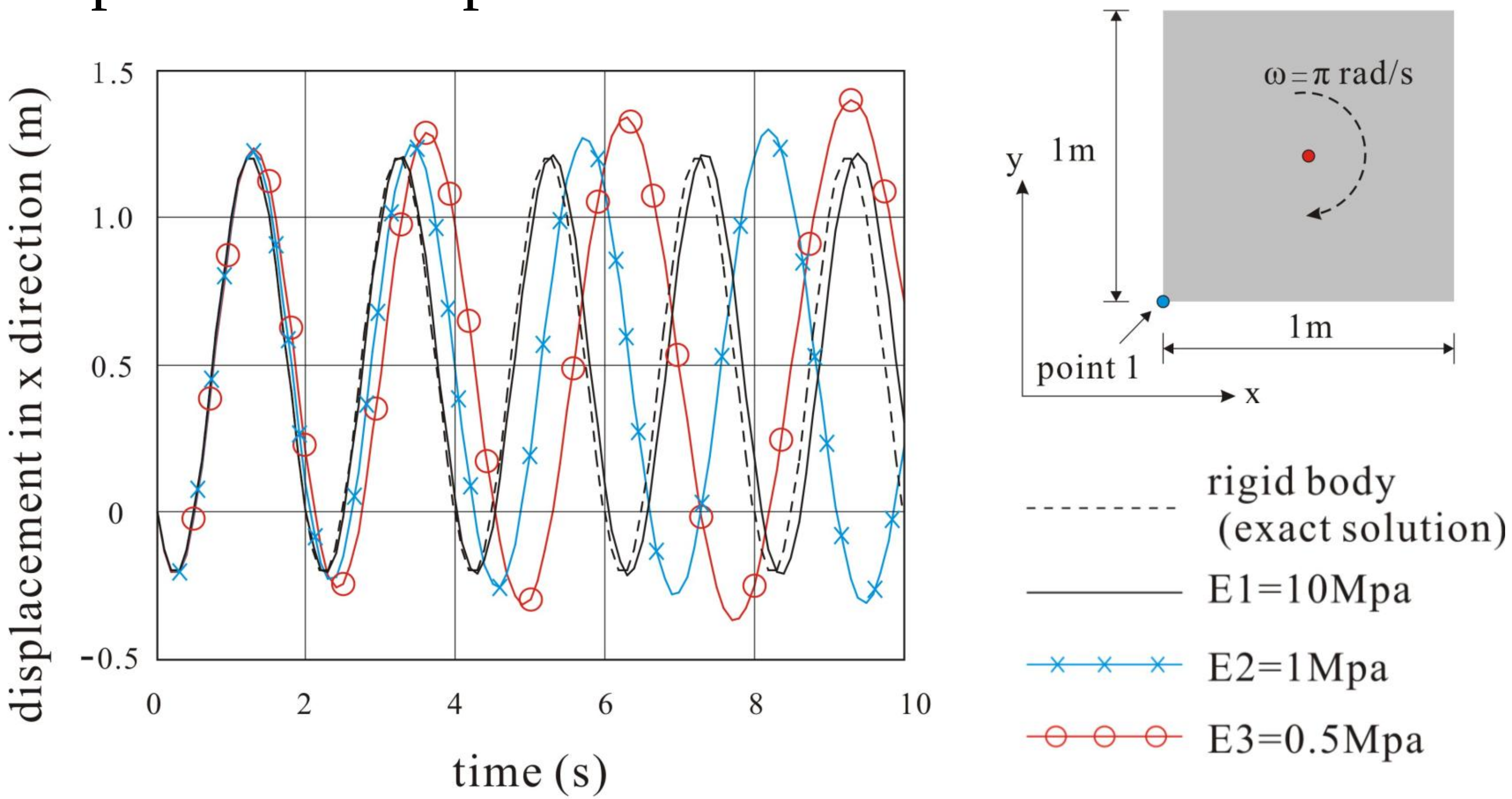




Numerical examples

1. A square plane subjected to an initial angular velocity

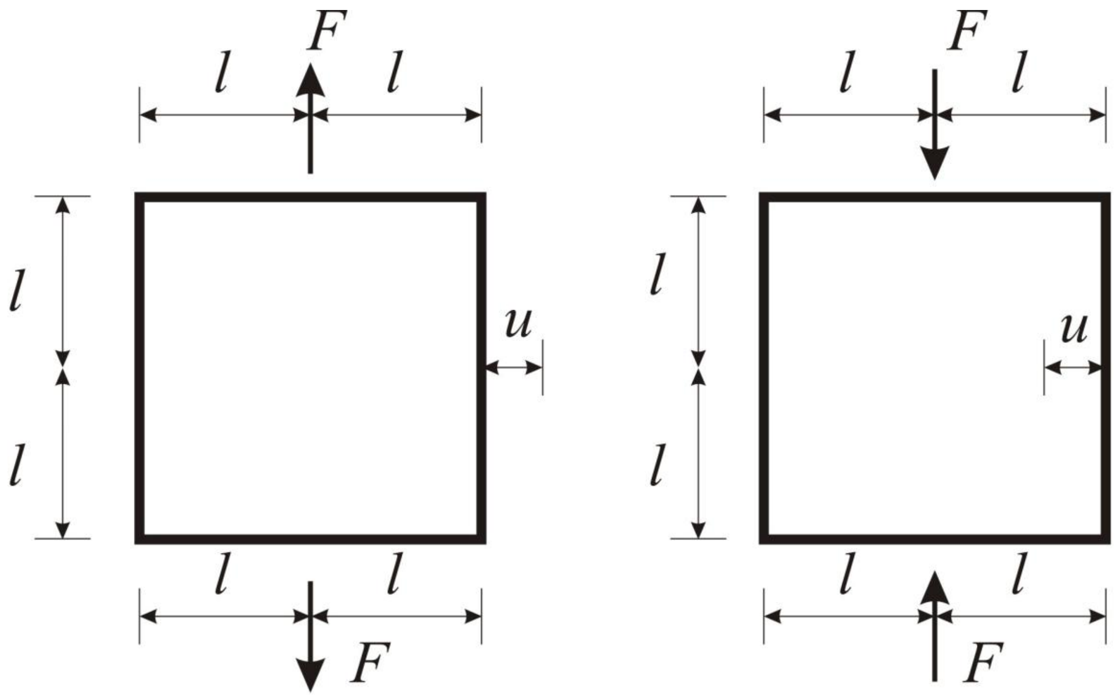
Displacement of point 1 in x direction



Numerical examples

2. A square frame under tension and compression

Goal: test the capability of FPM in simulating large deformation of planar solids



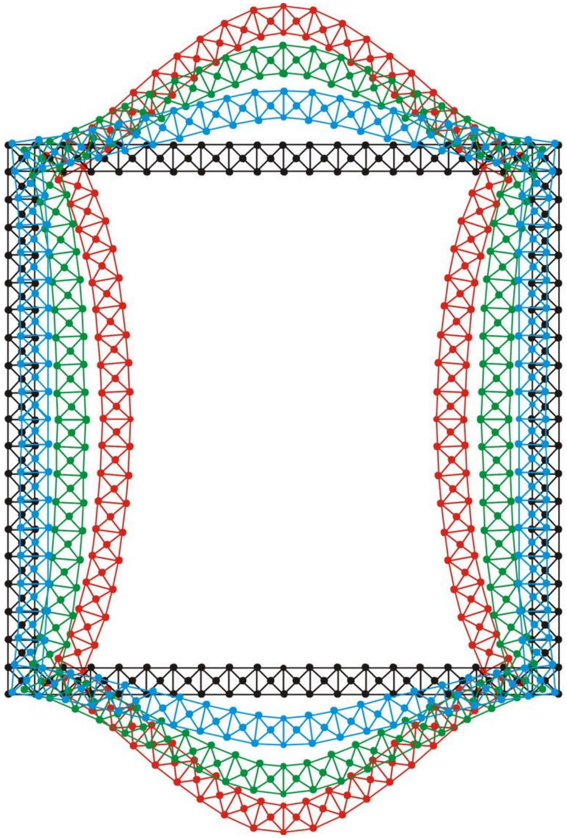
Mass density: $\rho = 1.0\ kg/m^3$ width: l=10m

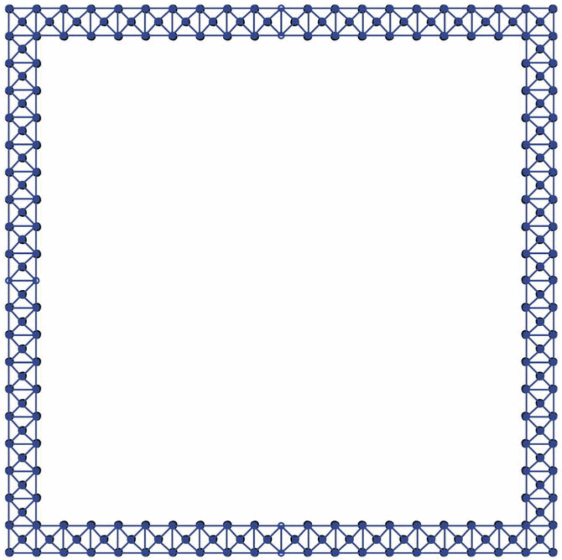
Young's modulus: E=10pa Poisson's ratio: v=0

Numerical examples

2. A square frame under tension

Modeled with 228 particles

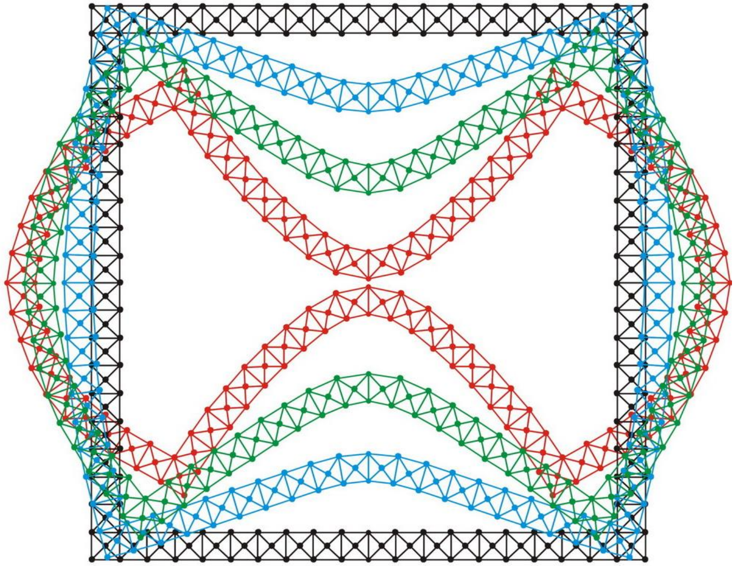


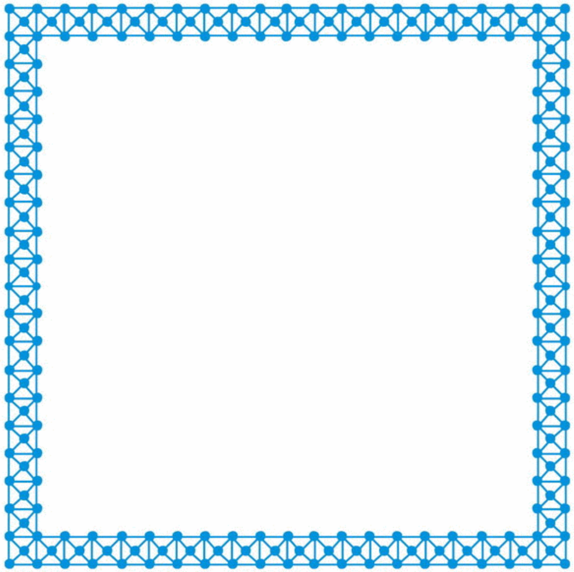


Numerical examples

2. A square frame under compression

Modeled with 228 particles

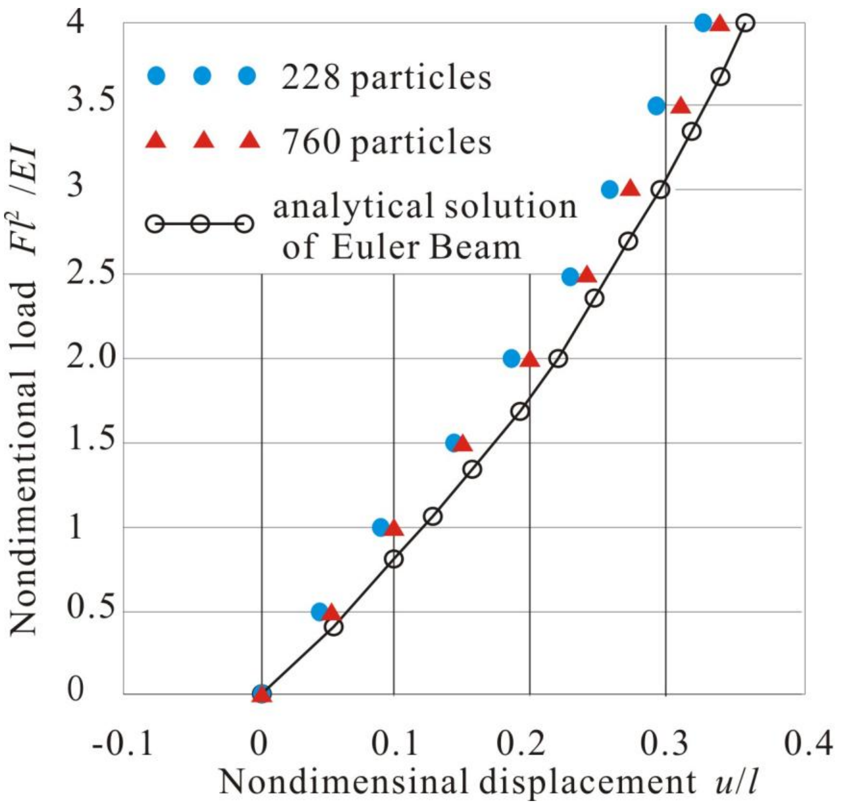




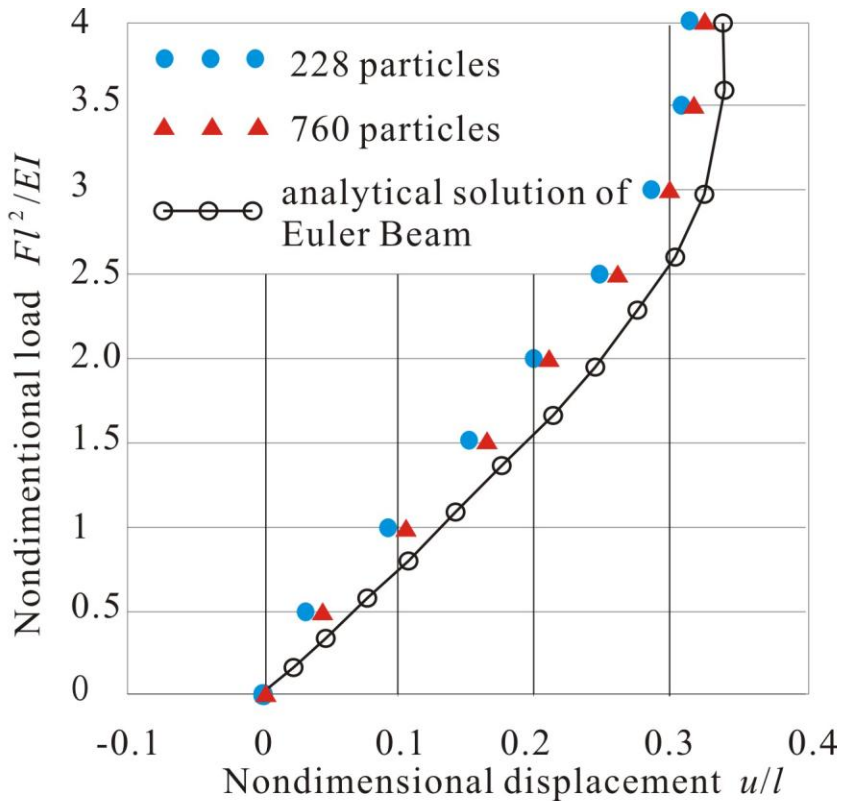
Numerical examples

2. A square frame under tension and compression

Compare with analytical solution of Euler beam



under tension



under compression

Remarks on the Finite Particle Method

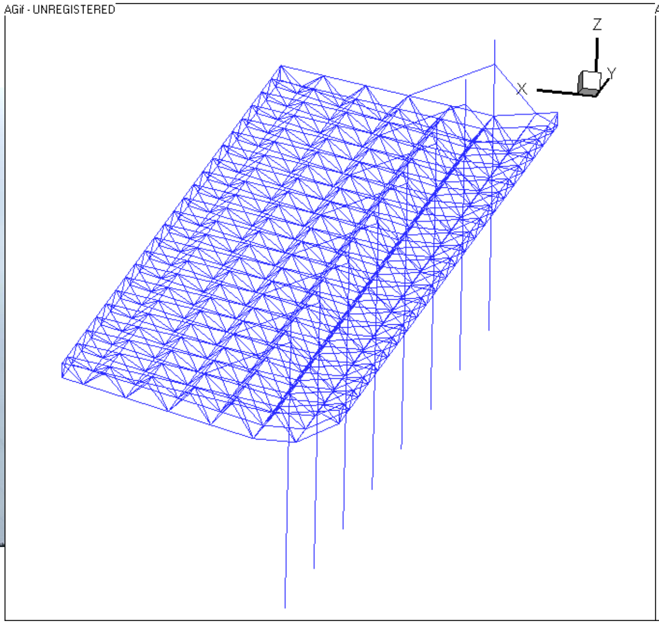
- 1. FPM is based on the combination of the vector mechanics and numerical calculations. It enforces equilibrium on each particle.

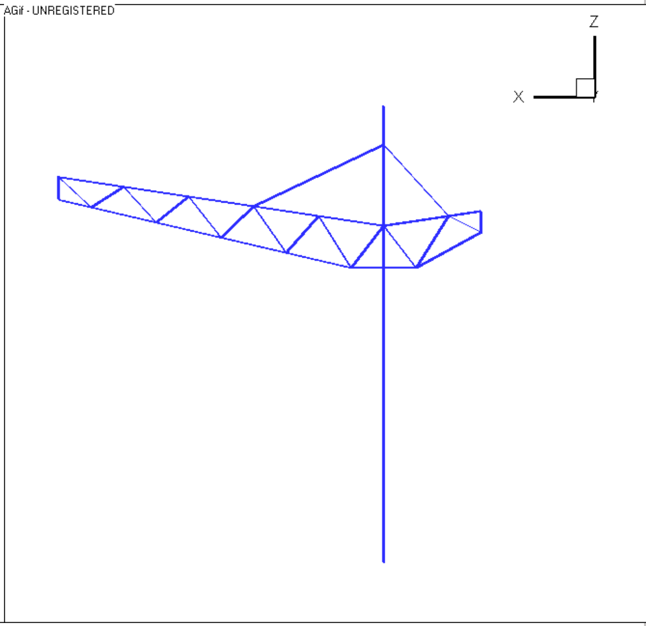
- 2. No iterations are used to follow nonlinear laws, and no matrices are formed. The procedures are quite simple and robust.

- 3. The examples demonstrate performance and applicability of the proposed method on large deflection analysis of planar solids.

Future work

- 1. Expand the present work into 3D;
- 2. Use FPM in failure and collapse simulation.





References

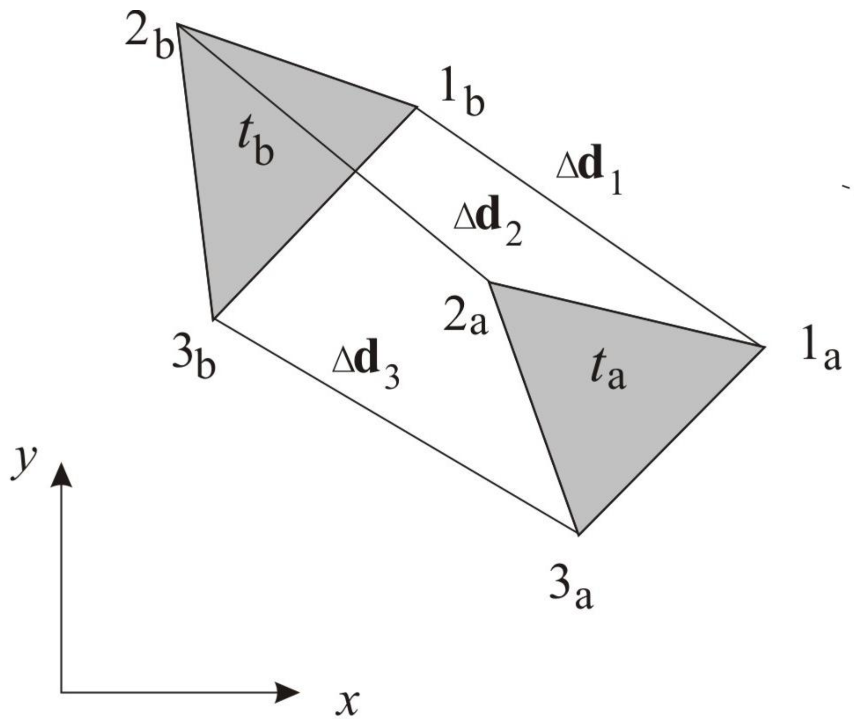
- Ying Yu, Yaozhi Luo. Motion analysis of deployable structures based on the rod hinge element by the finite particle method. *Proc. IMechE Part G: J. Aerospace Engineering*. 2009, 223: 1-10.

- Ying Yu, Yaozhi Luo. Finite particle method for kinematically indeterminate bar assemblies. *J Zhejiang Univ Sci A*. 2009, 10 (5): 667-676.

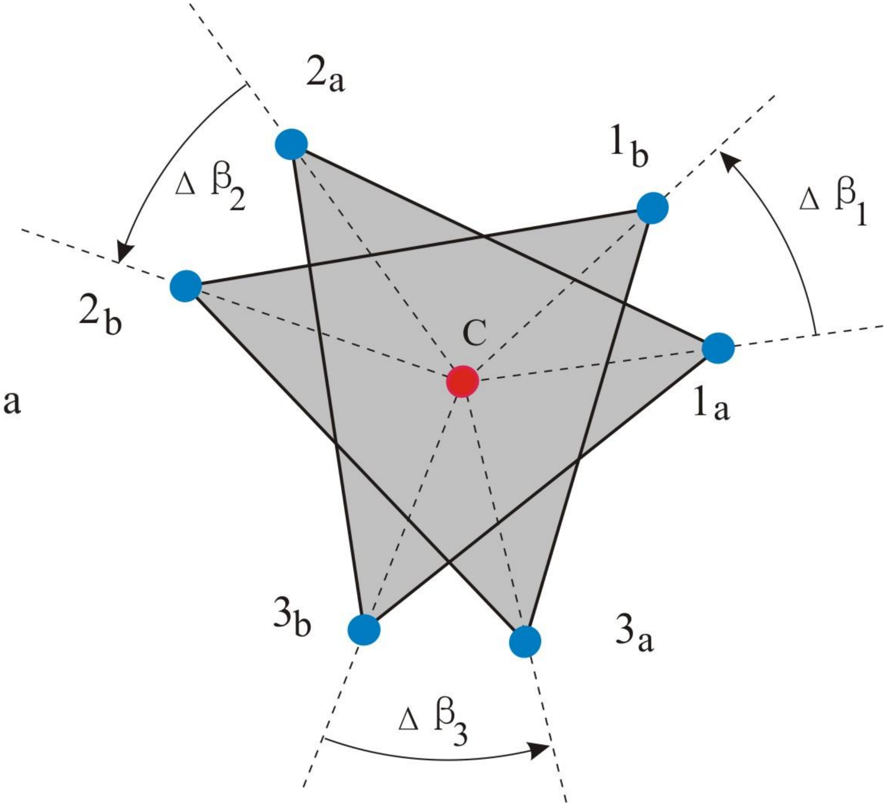
- Ying Yu, Glaucio Paulino, Yaozhi Luo. Finite particle method for progressive failure simulation of framed structures . (finished and will be submitted for publication)

Thank you!!

a) Evaluate the rigid body motion



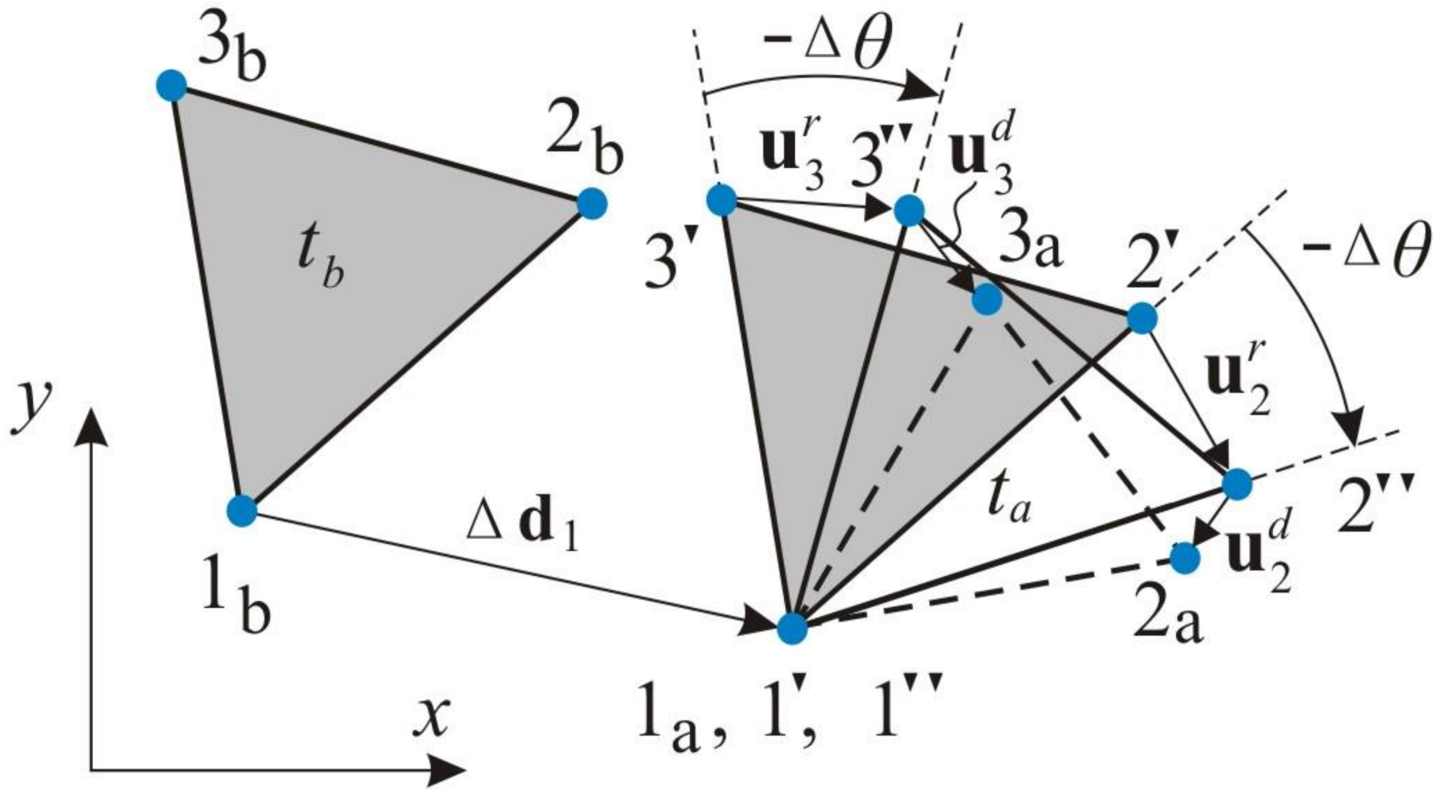
translation



rotation

$$\Delta\theta = \frac{1}{3}\sum_{i=1}^{3}\Delta\beta_i$$

b) Using reverse motion to remove rigid translation and rotation

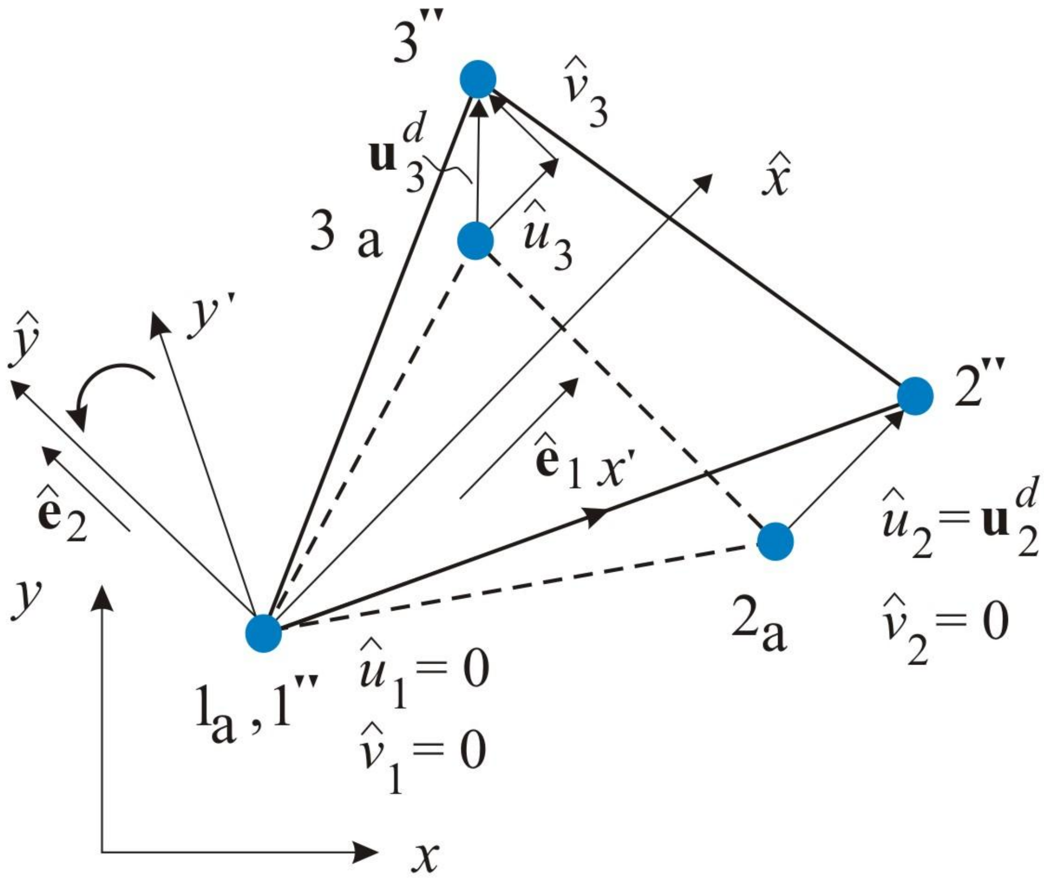


$$\mathbf{R}=\begin{bmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix}$$

$$\mathbf{u}_i^d=\mathbf{u}_i-\mathbf{u}_i^r$$

$$=\Delta\mathbf{d}_i-\Delta\mathbf{d}_1-(\mathbf{R}-\mathbf{I})(\mathbf{x}_i-\mathbf{x}_1),\quad (i=1,2,3)$$

c) Using deformation coordinate to reduce element degree of freedom



$$\hat{\mathbf{e}}_1 = \begin{Bmatrix} l_1 \\ m_1 \end{Bmatrix} = \frac{1}{\left|u_2^d\right|} \begin{Bmatrix} u_{2x}^d \\ u_{2y}^d \end{Bmatrix}$$

$$\hat{\mathbf{e}}_2 = \begin{Bmatrix} -m_1 \\ l_1 \end{Bmatrix}$$

$$\hat{\mathbf{x}} = \hat{\mathbf{Q}}(\mathbf{x} - \mathbf{x}_1)$$

$$\hat{\mathbf{Q}} = \begin{Bmatrix} \hat{\mathbf{e}}_1^T \\ \hat{\mathbf{e}}_2^T \end{Bmatrix} = \begin{bmatrix} l_1 & m_1 \\ -m_1 & l_1 \end{bmatrix}$$

d) Calculate the internal force at the deformation coordinate

$$\hat{u} = N_1\hat{u}_1 + N_2\hat{u}_2 + N_3\hat{u}_3$$
$$\hat{v} = N_1\hat{v}_1 + N_2\hat{v}_2 + N_3\hat{v}_3$$

$$\hat{x}_1 = \hat{y}_1 = 0$$
$$\hat{u}_1 = \hat{v}_1 = \hat{v}_2 = 0$$

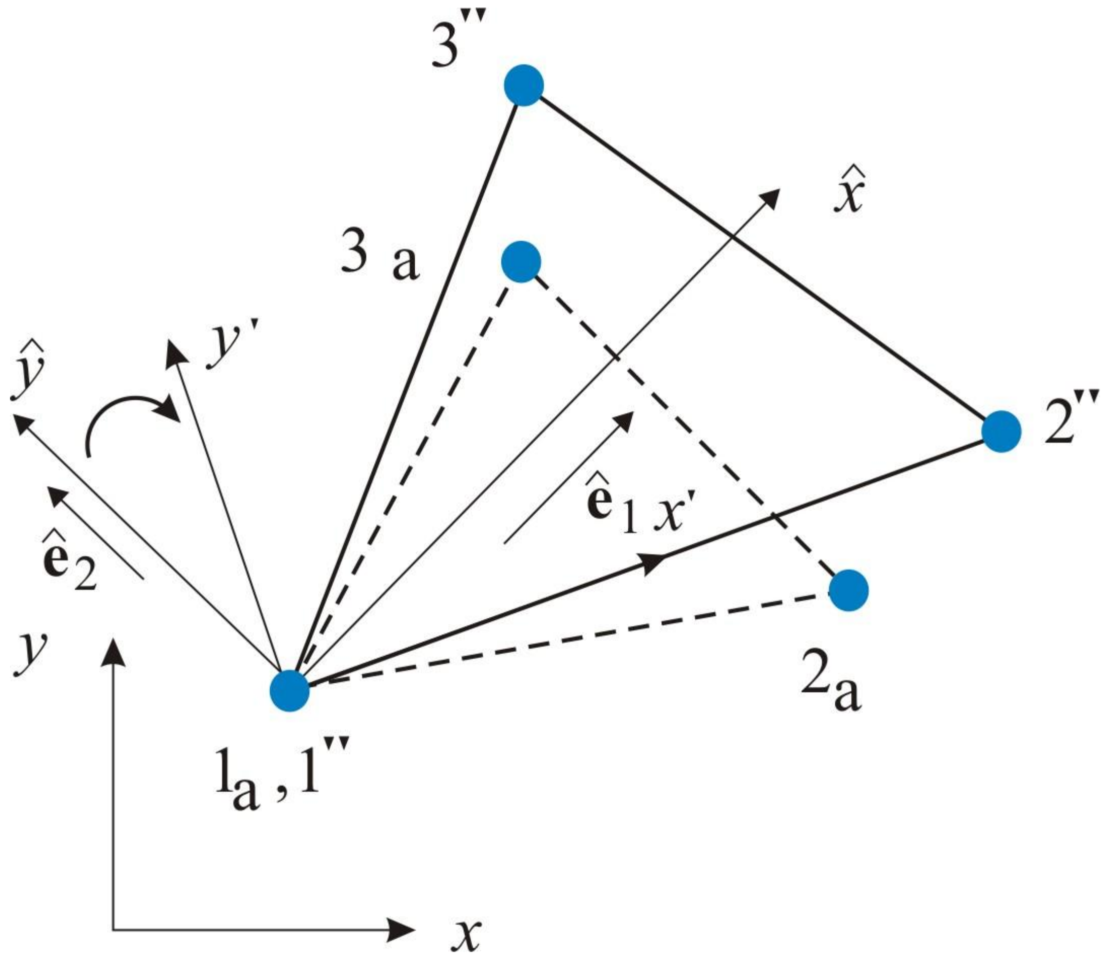
$$\hat{u} = N_2\hat{u}_2 + N_3\hat{u}_3$$
$$\hat{v} = N_3\hat{v}_3$$

Principle of virtual work:

$$\begin{Bmatrix} f'_{2x} \\ f'_{3x} \\ f'_{3y} \end{Bmatrix} = \begin{Bmatrix} f_{2xa} \\ f_{3xa} \\ f_{3ya} \end{Bmatrix} + \begin{Bmatrix} \Delta f'_{2x} \\ \Delta f'_{3x} \\ \Delta f'_{3y} \end{Bmatrix} = \begin{Bmatrix} f_{2xa} \\ f_{3xa} \\ f_{3ya} \end{Bmatrix} + [t\int_A \mathbf{B}^T \mathbf{E}\mathbf{B}\,dA] \begin{Bmatrix} \delta\hat{u}_{2x} \\ \delta\hat{u}_{3x} \\ \delta v_{3y} \end{Bmatrix}$$

$$\{ f'_{1x} \quad f'_{1y} \quad f'_{2x} \quad f'_{2y} \quad f'_{3x} \quad f'_{3y} \}^T$$

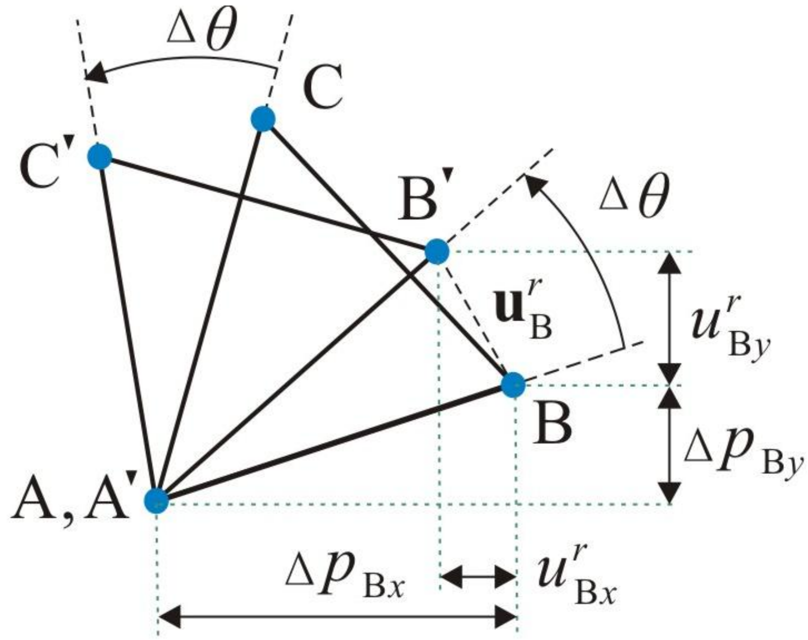
g) Transform the internal force back to global coordinate



$$\hat{\mathbf{Q}} = \begin{Bmatrix} \hat{\mathbf{e}}_1^T \\ \hat{\mathbf{e}}_2^T \end{Bmatrix} = \begin{bmatrix} l_1 & m_1 \\ -m_1 & l_1 \end{bmatrix}$$

$$\begin{Bmatrix} f'_{ix} \\ f'_{iy} \end{Bmatrix} = \mathbf{Q}^T \begin{Bmatrix} \hat{f}_{ix} \\ \hat{f}_{iy} \end{Bmatrix}, \qquad i = 1, 2, 3$$

f) Using forward motion to get the element back to the original position



Transform the internal force back to the original direction

$$\begin{Bmatrix} f_{ix} \\ f_{iy} \end{Bmatrix} = \mathbf{R} \begin{Bmatrix} f'_{ix} \\ f'_{iy} \end{Bmatrix}, \qquad i = 1, 2, 3$$