USNCCM-XI

11th US National Congress
on Computational Mechanics
Minneapolis
July 25-29, 2011

Identification of fracture properties for a cohesive model using digital image correlation

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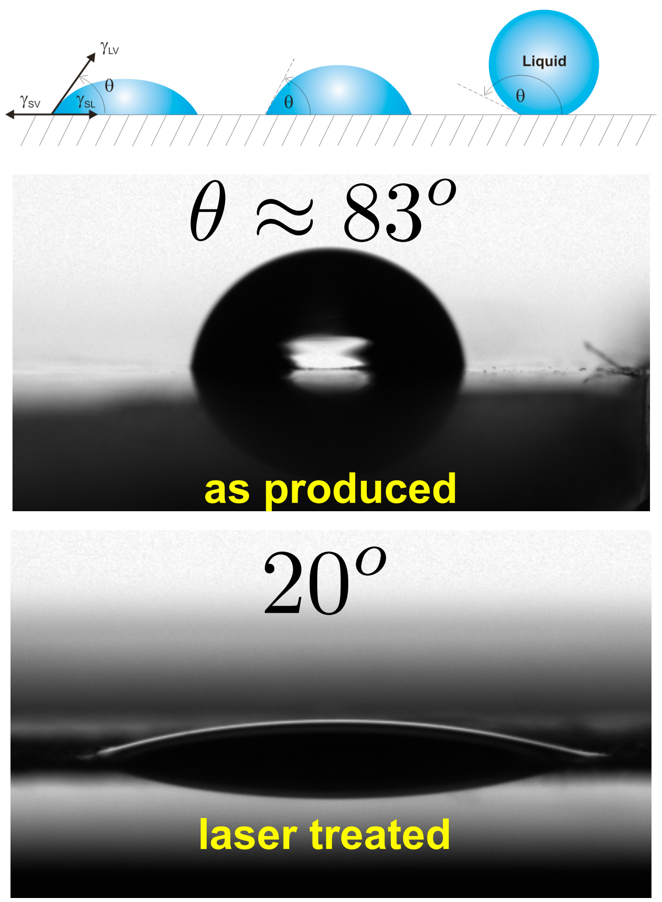
Minneapolis, July 25th 2011

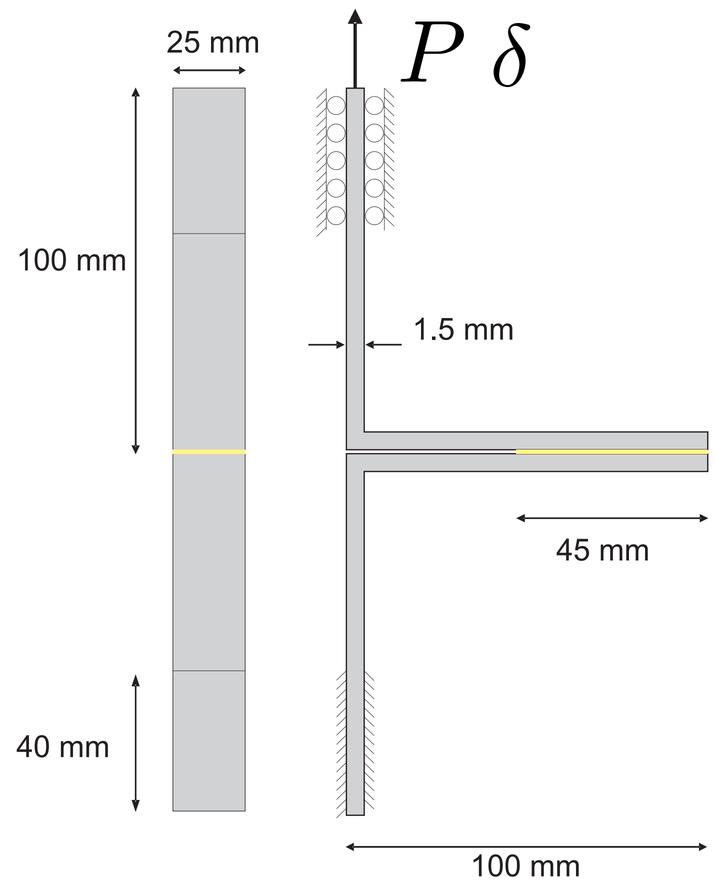
Outline

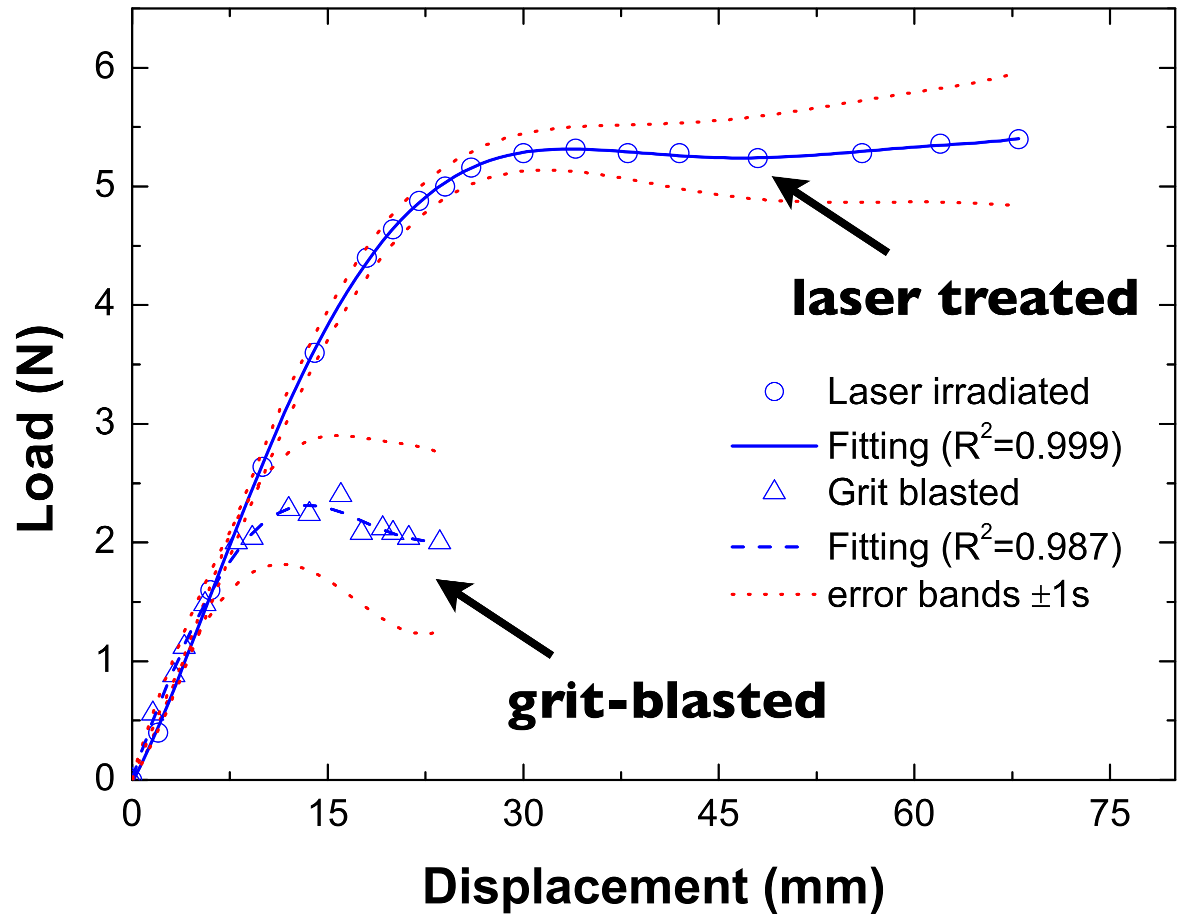
1. Introduction and motivation
2. Objective
3. Proposed approach
4. Algorithm and implementation of the inverse procedure
5. Target applications and experimental set-up (current status)
6. Follow-up work

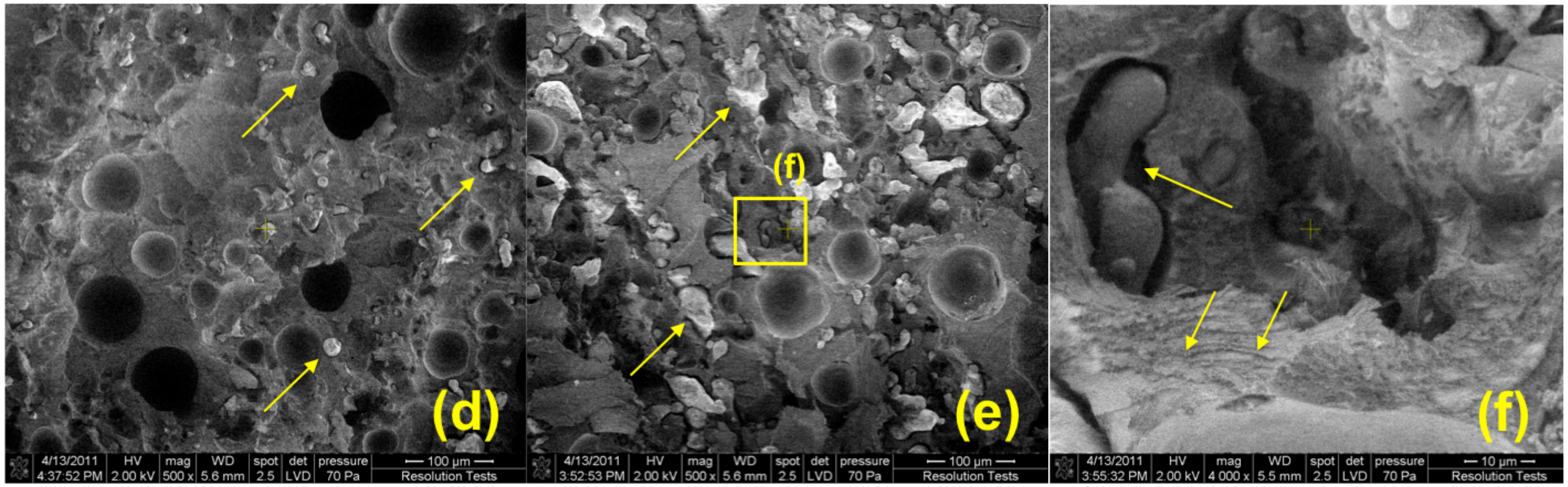
Introduction and motivations (1/4)

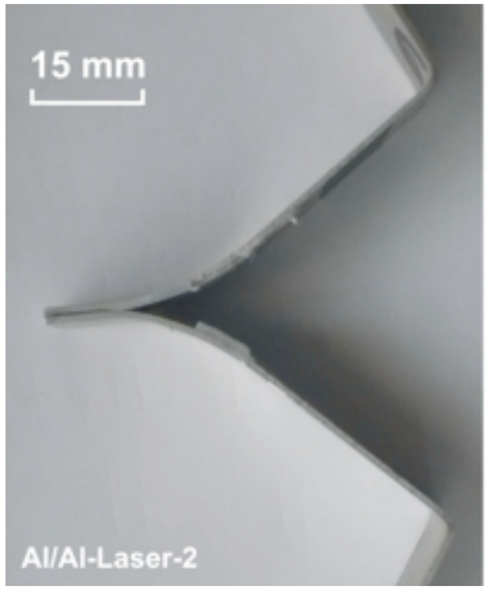
Mechanical testing







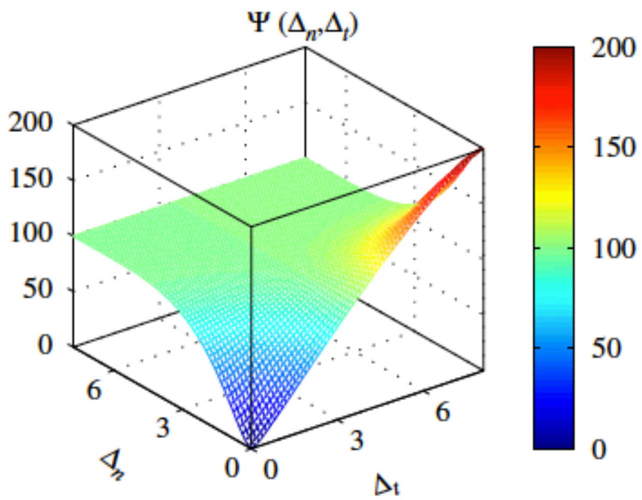




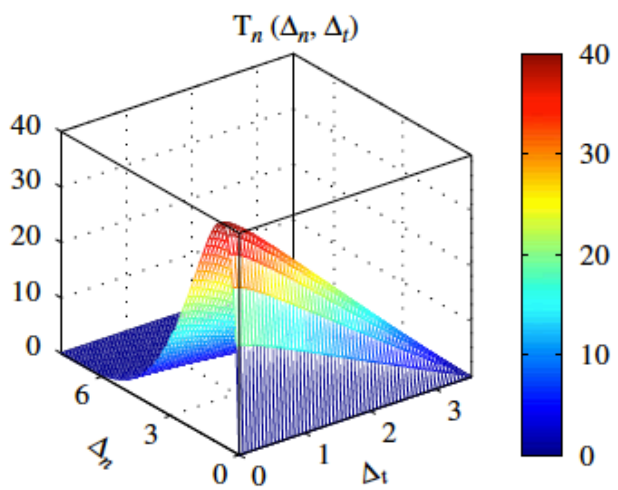
Alfano M, Furgiuele F, Lubineau G, Paulino GH. Role of laser surface preparation on damage and decohesion of Al/epoxy joints. *Submitted for Journal publication*.

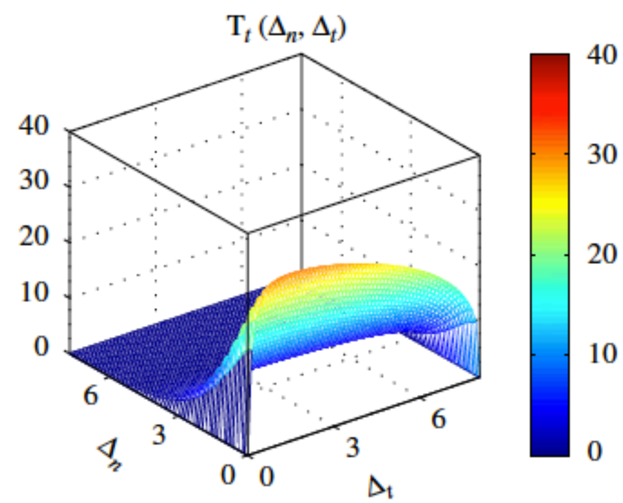
$$\Psi(\Delta_n,\Delta_t)=\min(\phi_n,\phi_t)+\left[\Gamma_n\left(1-\frac{\Delta_n}{\delta_n}\right)^{\alpha}\left(\frac{m}{\alpha}+\frac{\Delta_n}{\delta_n}\right)^{m}+\langle\phi_n-\phi_t\rangle\right] \times\left[\Gamma_t\left(1-\frac{|\Delta_t|}{\delta_t}\right)^{\beta}\left(\frac{n}{\beta}+\frac{|\Delta_t|}{\delta_t}\right)^{n}+\langle\phi_t-\phi_n\rangle\right].$$

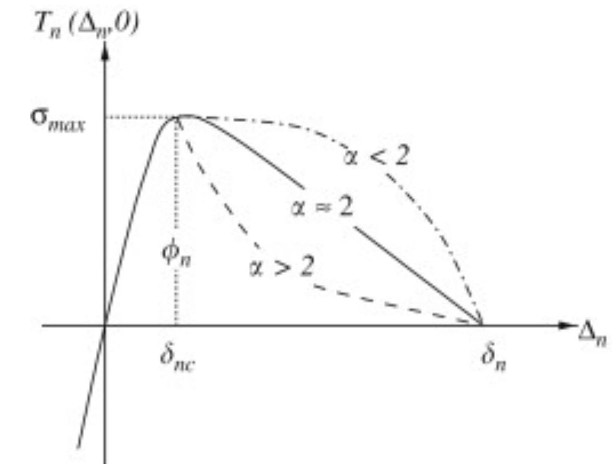
δ_n, δ_t	characteristic length scale parameters
$\bar{\delta}_n$, $\bar{\delta}_t$	normal and tangential conjugate final crack opening widths
λ_n, λ_t	initial slope indicators
σ_{max}, τ_{max}	normal and tangential cohesive strengths
ϕ_n, ϕ_t	modes I and II fracture energies
Ψ	potential function for cohesive fracture
α, β	shape parameters in the PPR model

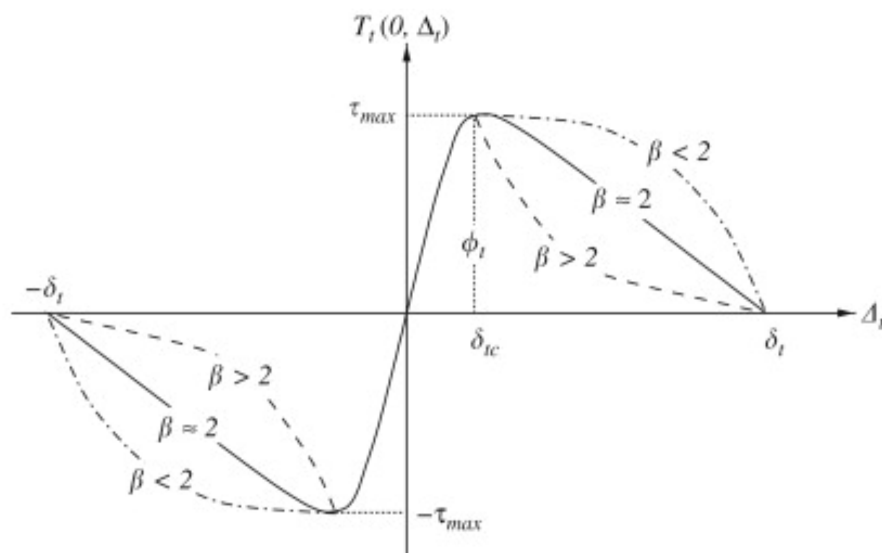


Axis	Units
Ψ	N/m
T_n, T_t	MPa
Δ_n, Δ_t	μm









Park K, Paulino GH, Roesler JR. A unified potential-based cohesive model of mixed-mode fracture. *Journal of the Mechanics and Physics of Solids*. 2009;57(6):891-908.

$$\Psi(\Delta_n) = \phi_n + \Gamma_n \left(1 - \frac{\Delta_n}{\delta_n}\right)^{\alpha} \left(\frac{m}{\alpha} + \frac{\Delta_n}{\delta_n}\right)^{m}$$

$$T(\Delta_n) = \frac{\partial \Psi}{\partial \Delta_n} =$$
$$= \frac{\Gamma_n}{\delta_n}\left[m\left(1 - \frac{\Delta_n}{\delta_n}\right)^{\alpha}\left(\frac{m}{\alpha} + \frac{\Delta_n}{\delta_n}\right)^{m-1} - \alpha\left(1 - \frac{\Delta_n}{\delta_n}\right)^{\alpha-1}\left(\frac{m}{\alpha} + \frac{\Delta_n}{\delta_n}\right)^{m}\right]$$

$$\Gamma_n = -\phi_n \left(\frac{\alpha}{m}\right)^{m}$$ **energy constant;**

$$m = \frac{\alpha(\alpha-1)\lambda_n^2}{1-\alpha\lambda_n^2}$$ **non-dimensional exponent;**

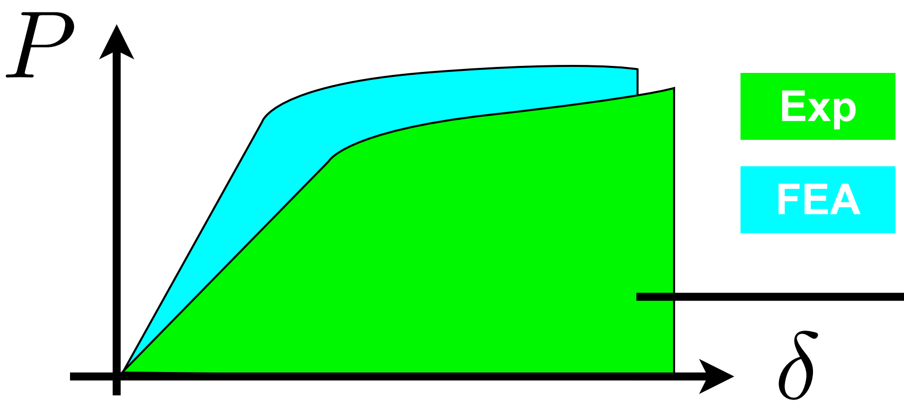
$$\delta_n = \frac{\phi_n}{\sigma_{max}} \alpha\lambda_n (1-\lambda_n)^{\alpha-1} \left(\frac{\alpha}{m} + 1\right) \cdot \left(\frac{\alpha}{m}\lambda_n + 1\right)^{m-1}$$ **final crack opening width;**

$$\mathbf{X} = \{\phi_n, \sigma_{max}, \lambda_n, \alpha\}$$ **unknown properties to be identified**

Park K, Paulino GH, Roesler JR. A unified potential-based cohesive model of mixed-mode fracture. *Journal of the Mechanics and Physics of Solids*. 2009;57(6):891-908.

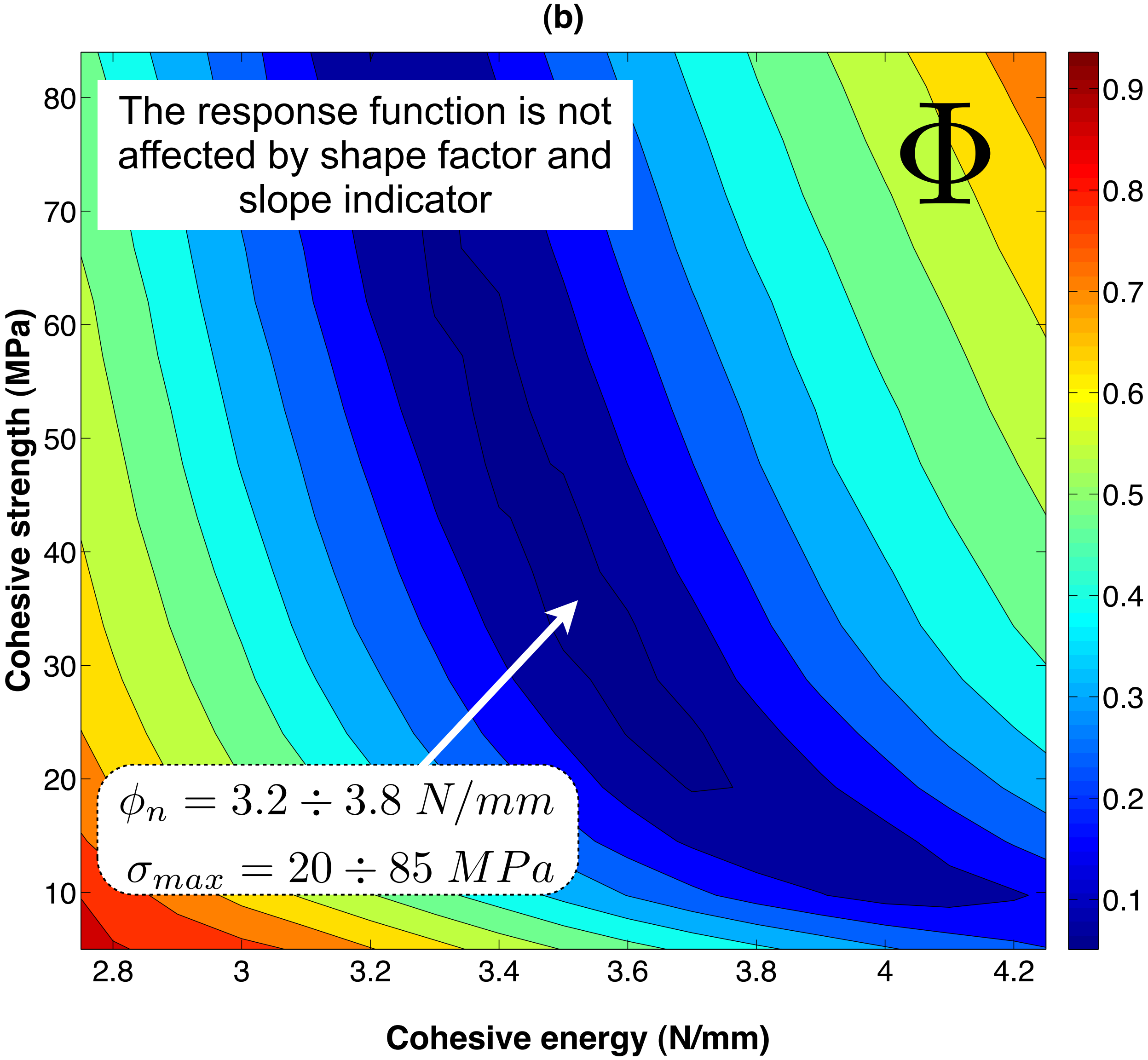
Introduction and motivations (4/4)

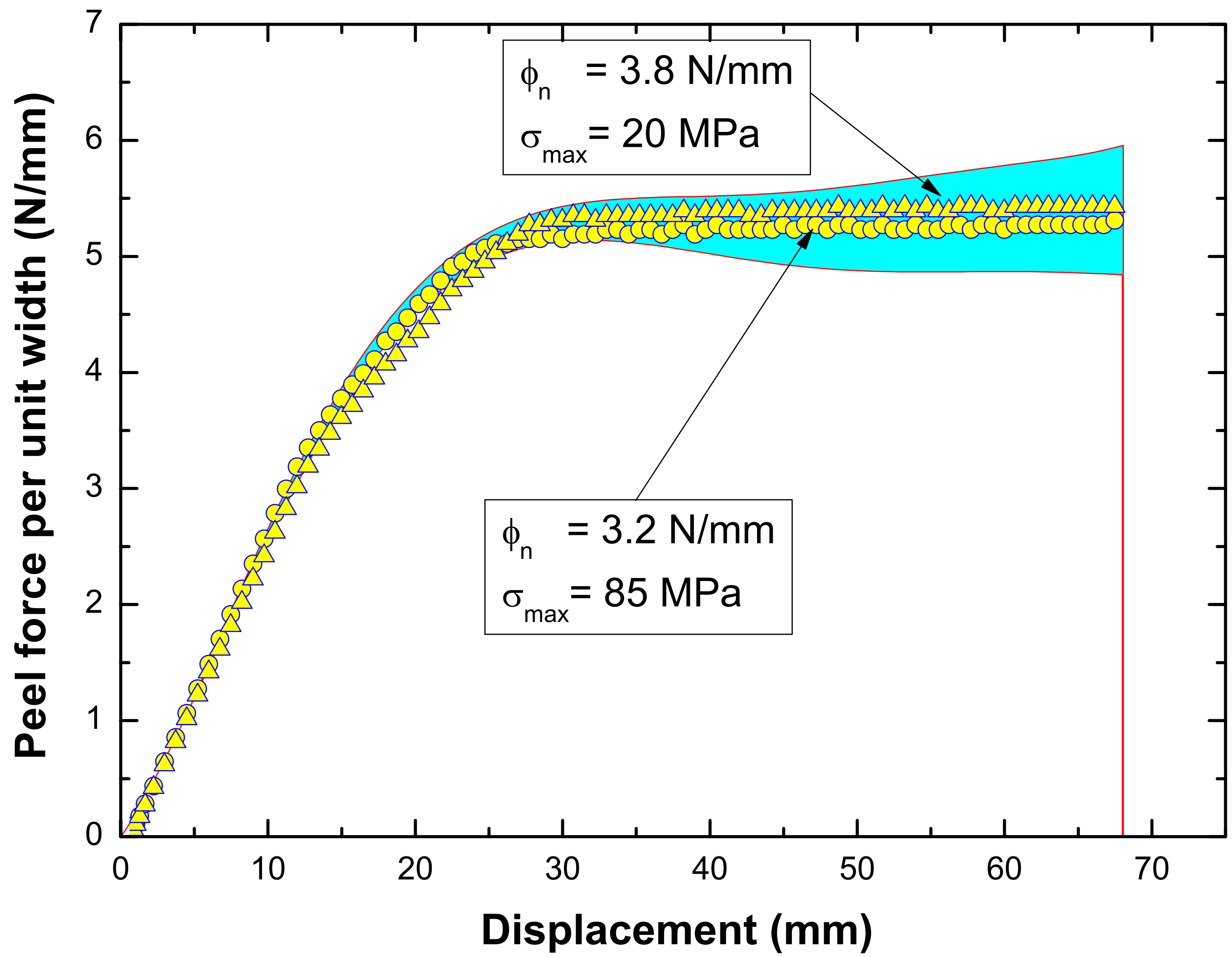
Identification of bond toughness using the CZM



$$\Phi = \| U(\delta)_{EXP} - U(\delta)_{FE} \|_2$$

$$U(\delta) = \int_{\delta_{i-1}}^{\delta_i} P(\delta)d\delta = (\delta_i - \delta_{i-1}) \times \frac{P(\delta_i) + P(\delta_{i-1})}{2}$$





Alfano M, Furgiuele F, Lubineau G, Paulino GH. Identification of mode-I cohesive zone parameters of Al / epoxy T-peel joints with laser treated substrates. *Submitted for Journal publication*.

Remarks and objective of the work

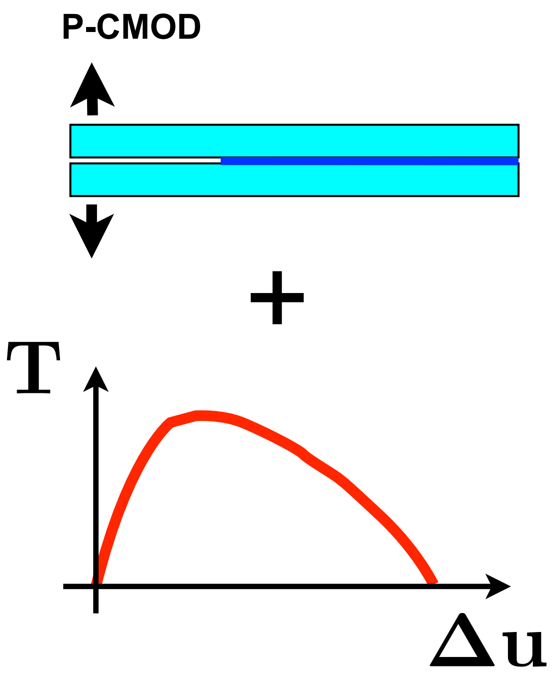
➡ A global response is often obtained from experiments, however, it may have low sensitivity to certain cohesive properties.

➡The uniqueness of the obtained cohesive zone model is not guaranteed.

Although the cohesive models obtained using global data can yield satisfactory predictive capabilities in FEA simulations of fracture, the **development of an alternative procedure is needed**, e.g. to determine cohesive strength.

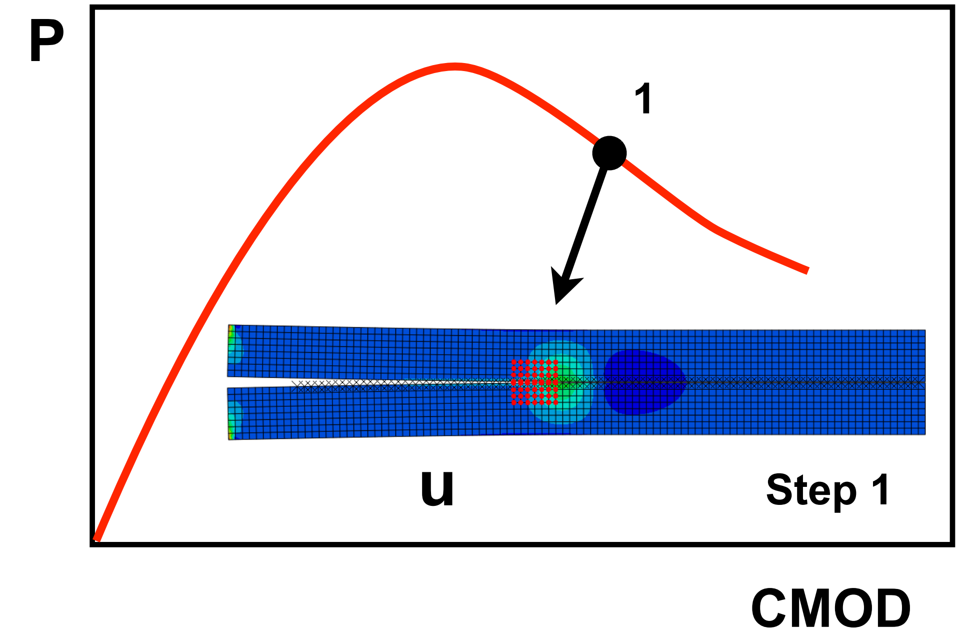
A solution may be provided by the original combination of experimental **full-field measurements** techniques and **inverse problems**.

Gain AL, Carroll J, Paulino GH, Lambros J. A hybrid experimental / numerical technique to extract cohesive fracture properties for mode-I fracture of quasi-brittle materials. *International Journal of Fracture*. 2011;169:113-131.
Shen B, Paulino GH. Direct Extraction of Cohesive Fracture Properties from Digital Image Correlation: A Hybrid Inverse Technique. *Experimental Mechanics*. 2011;51(2):143-161.



Forward problem

$$(\mathbf{K}_b + \mathbf{K}_c(\mathbf{u}, \mathbf{X}))\mathbf{u} = \mathbf{F}_{ext}$$



$$\mathbf{K}_b = \int_\Omega \mathbf{B}^T \mathbf{E} \mathbf{B} \, d\Omega$$

$$\mathbf{K}_{coh} = \int_{\Gamma_{coh}} \mathbf{N}^T \frac{\partial \mathbf{T}}{\partial \boldsymbol{\Delta} \mathbf{u}} \mathbf{N} \, d\Gamma_{coh}$$

$$\mathbf{F}_{ext} = \int_{\Gamma_{ext}} \mathbf{T}_{ext} d\Gamma_{ext}$$

Principle of virtual work

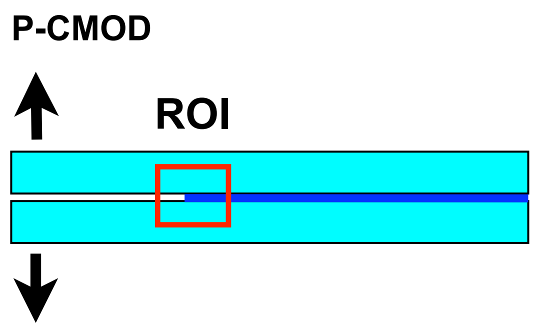
$$\int_\Omega \boldsymbol{\sigma} : \delta \boldsymbol{\epsilon} \, d\Omega - \int_{\Gamma_{ext}} \mathbf{T}_{ext} \cdot \delta \boldsymbol{\Delta u} \, d\Gamma_{ext} +$$

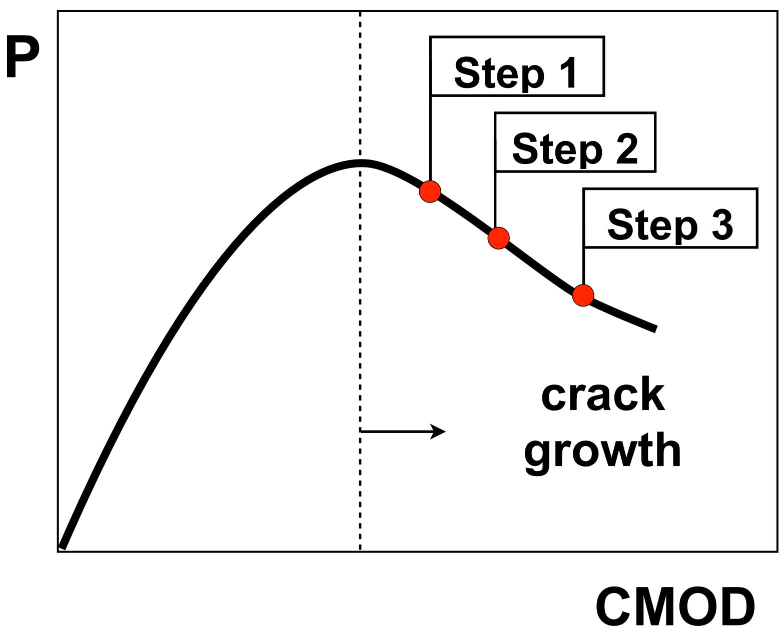
$$- \int_{\Gamma_{coh}} \mathbf{T}_{coh} \cdot \delta \boldsymbol{\Delta u} \, d\Gamma_{coh} = 0$$

Ω : specimen domain
Γ_{ext} : external boundary
Γ_{coh} : cohesive surfaces
$\boldsymbol{\Delta u}$: cohesive surfaces opening displacement
$\boldsymbol{\sigma}$: stress tensor
$\boldsymbol{\epsilon}$: strain tensor

Forward *versus* inverse problem

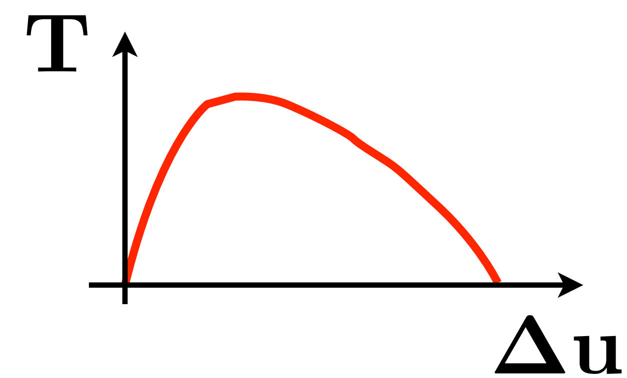
Inverse problem (objective of the work)





Inverse problem
(optimization)

$$\mathbf{X} = \{\phi_n \; , \sigma_{max} \; , \alpha \; , \lambda_n\}$$



$$\hat{\mathbf{X}} = \underset{\mathbf{X} \in \mathbb{R}^M}{\arg\min} \{\Pi = \sum_{i=1}^{m} \omega_i \left(\mathbf{X}\right)\}$$

$$\omega_i \left(\mathbf{X}\right) = \frac{1}{\left(U_{max,i}\right)^2} \sum_{j=1}^{n_n} \left[u_{exp} - u\left(\mathbf{X}\right)\right]_j^2$$

m : available measurement instants (load levels)
n_n : nodal displacements in the ROI

$$\mathbf{u}(\mathbf{X}) = \mathbf{K}_b^{-1} \hat{\mathbf{F}}^{ext} (\mathbf{u}_{exp} \; ; \mathbf{X})$$
$$\hat{\mathbf{F}}^{ext} (\mathbf{u}_{exp} \; ; \mathbf{X}) = \mathbf{F}^{ext} - \mathbf{K}_c (\mathbf{u}_{exp} \; ; \mathbf{X}) \mathbf{u}_{exp}$$

Optimization algorithm?

Genetic algorithm

Fundamentals concepts

Exploration algorithm based on the mechanism of natural selection and genetics: the strongest **individuals (chromosomes)** in a **population** survive and generate offsprings.

A **chromosome** represents a generic solution of the problem, in our context a set of cohesive fracture parameters (**X**):

X	Φ_n	σ_{max}	λ_n	α

Basic steps of the GA

1. Random generation of the **initial population** (individuals **X**) satisfying suitable restraint conditions (e.g. fracture energy must not be negative);

2. The chromosomes are evaluated, using some measures of **fitness**. We defined the following **objective function (or cost function)**:

$$\omega_i(\mathbf{X}) = \frac{1}{(U_{max,i})^2} \sum_{j=1}^{n_n} [u_{exp} - u(\mathbf{X})]_j^2$$

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X} \in \mathbb{R}^M} \{\Pi = \sum_{i=1}^{m} \omega_i(\mathbf{X})\}$$

m : available measurement instants (load levels)

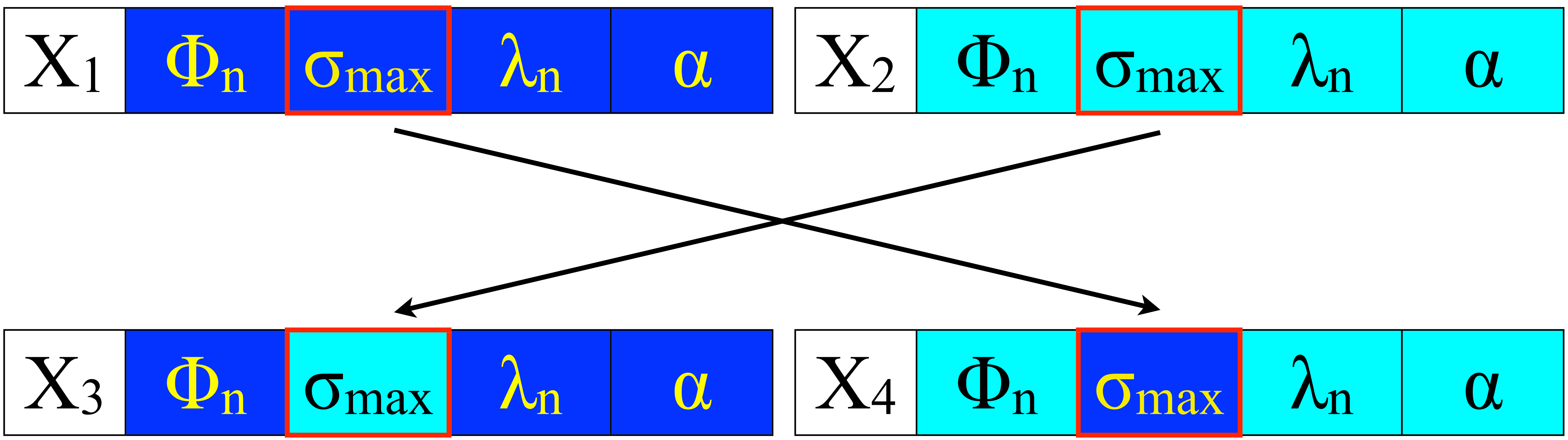
n_n : nodal displacements in the ROI

3. Individuals for **reproduction** are firstly chosen based on their fitness

4. and some of them are processed by means of **genetic operators (crossover and mutation)** to create a new populations

5. New chromosomes, called **offspring**, are formed by merging two chromosomes from current generation

Crossover (type 1)



Crossover (type 2)

$$\mathbf{X}_3 = a \cdot \mathbf{X}_1 + (1-a) \cdot \mathbf{X}_2$$
$$\mathbf{X}_4 = (1-a) \cdot \mathbf{X}_1 + a \cdot \mathbf{X}_2$$

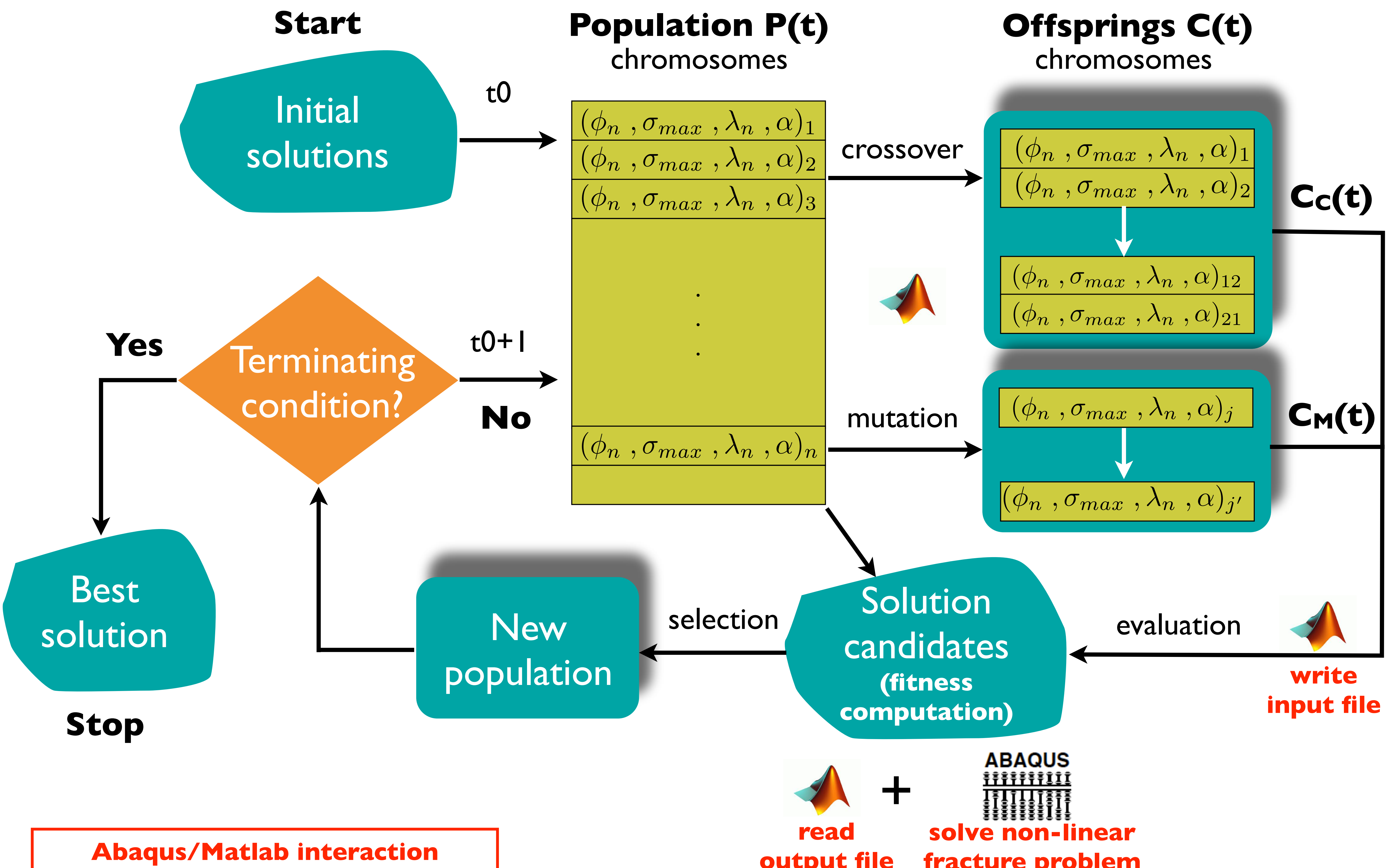
$$a \in [0, 1]$$

5. New chromosomes are also formed by modifying a chromosome using a mutation operator

$$\sigma'_{max} = \sigma_{max} + r \cdot \Delta\sigma_{max}$$

$$r \in [-1, 1], \Delta\sigma_{max} = cost.$$

6. The newly created population replace the old one and the process restarts.



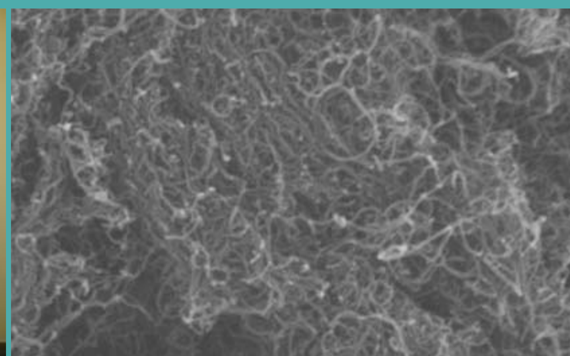
Abaqus/Matlab interaction by means of Linux shell scripts

Target applications and experimental set-up

current status

Material processing - DCB with metal and composites substrates

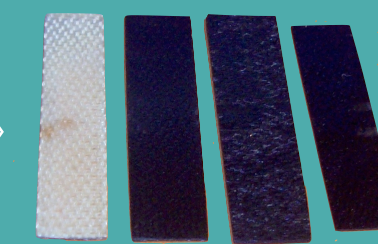
Ultrasonic Dispersion



MWCNT Dispersion

Hot press 15T/300°C

Specialized cutting devices

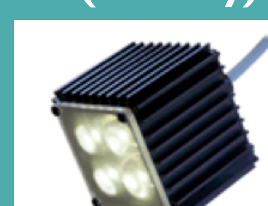


Composite laminates

Full-field experimental measurements

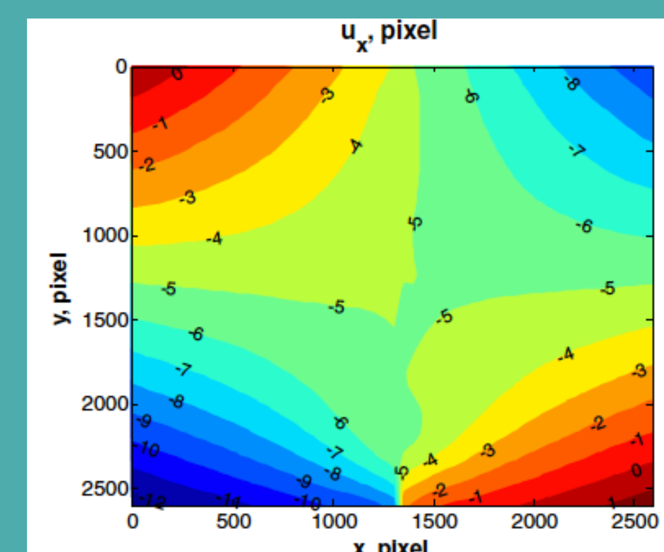
INSTRON Testing machine

High resolution CCD camera (PixelFly)



High intensity Led spotlight

QM100 long distance microscope (QUESTAR)



In-house developed DIC algorithm

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