DINAME 2011 - 14th International Symposium on Dynamic Problems of Mechanics



On The Modelling of Structural Dynamics of Risers Composed of Functionally Graded Materials

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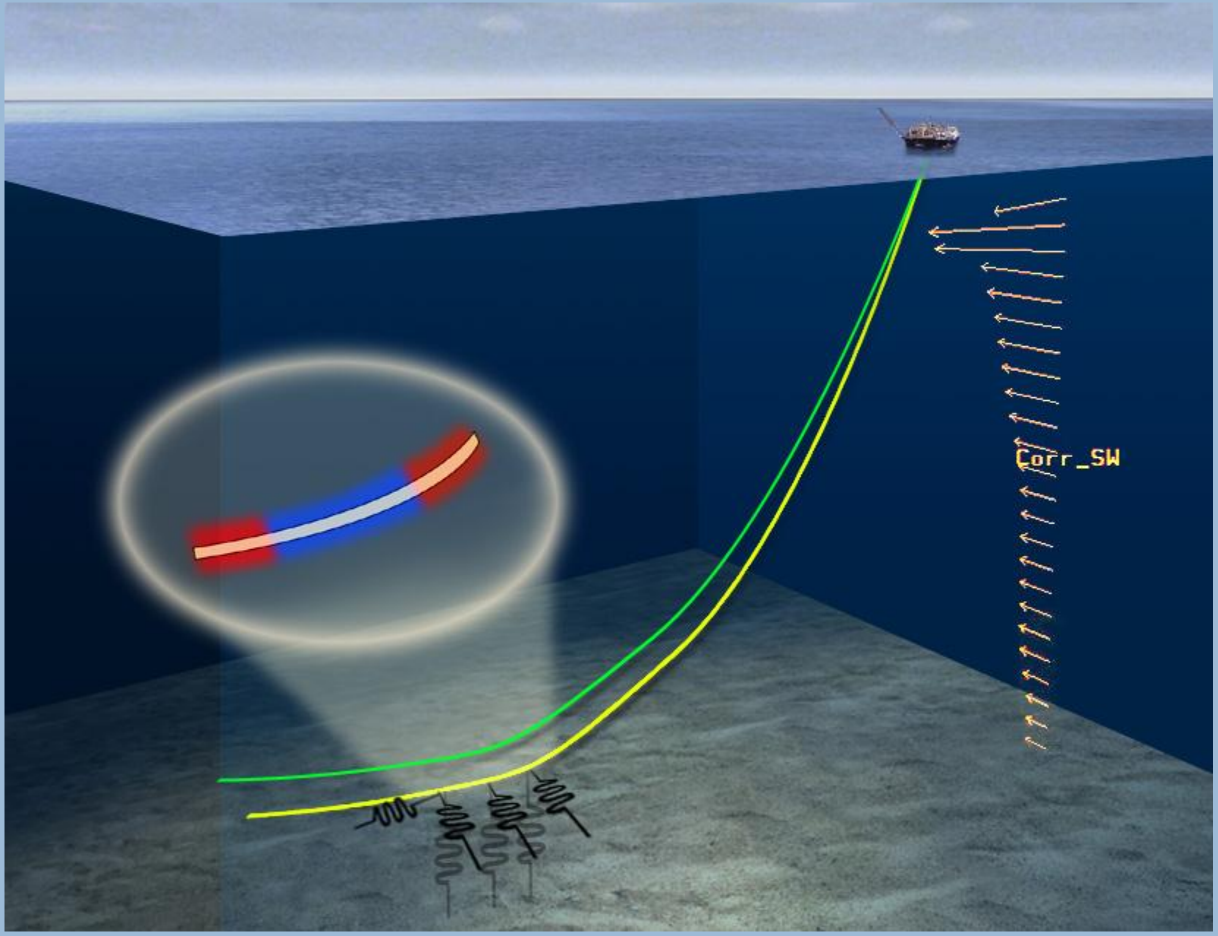
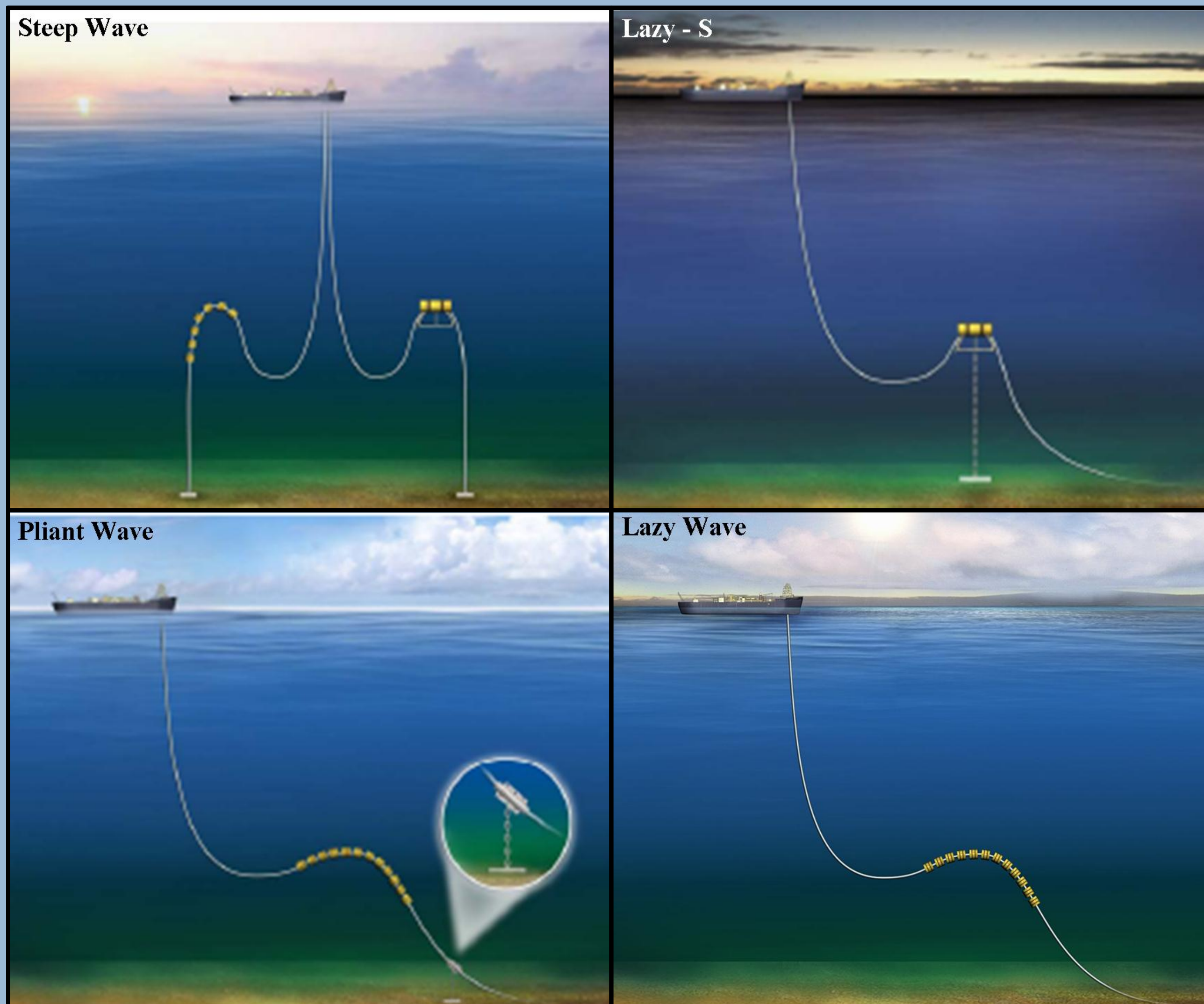
Paper Outline

- Introduction
- F E Formulation
- FGM Constitutive Law
- Sample Analyses
- Conclusions

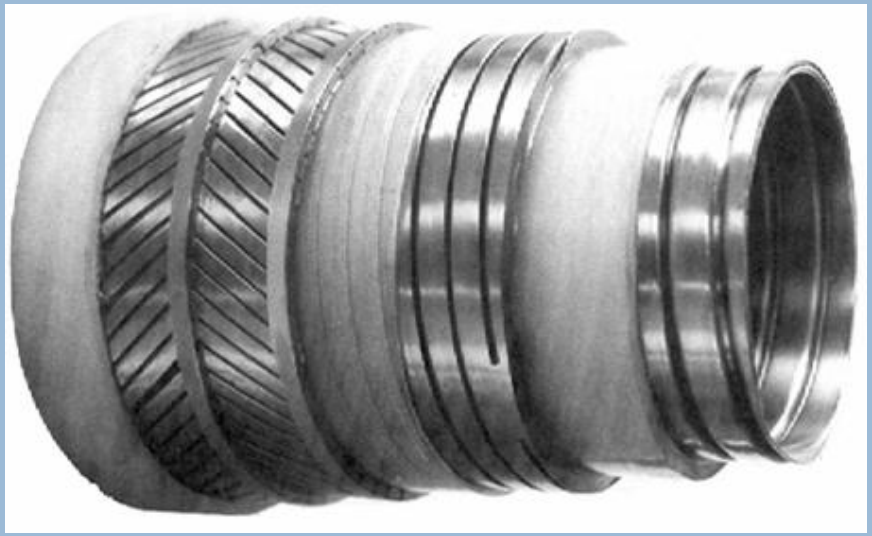
Introduction




Steep Wave
Lazy - S
Pliant Wave
Lazy Wave

Risers Should Provide:

- Easy and Effective Fluid and Gas Transport
- Signal Transmission Capabilities
- Power Supply to Sub-sea Components
- Compliance and High Deformability in Bending
- Strong and Stiff Response to: Internal and External Pressures, Tension and Torsion



Flexible Riser – Concentric Polymeric & Interlocking Metallic Layers

Rigid Riser – Inner Metallic Pipe w/ Sandwich Polymeric Layers

F E Formulation

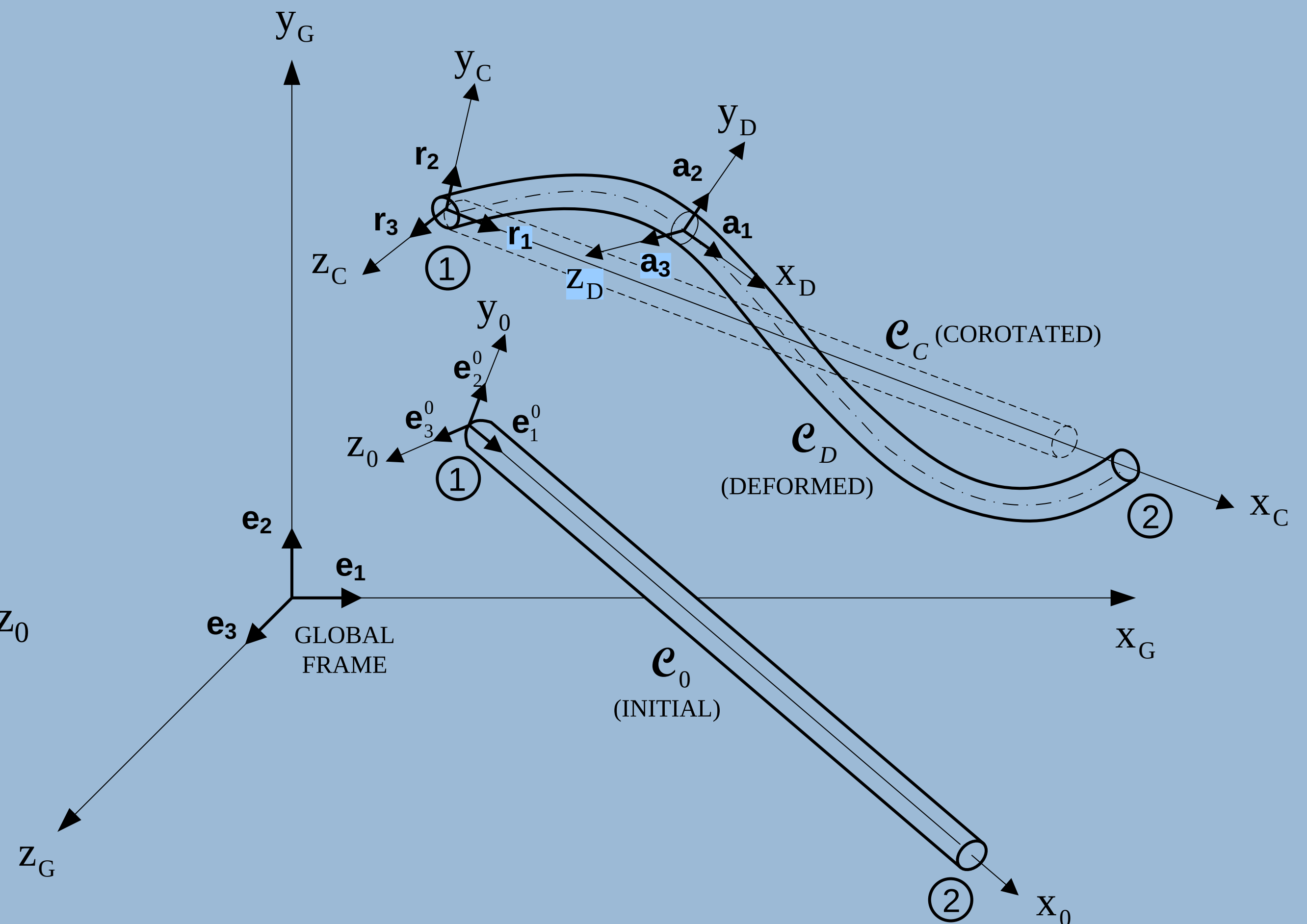
Co-rotational frame for a deformed beam element

Configurations:

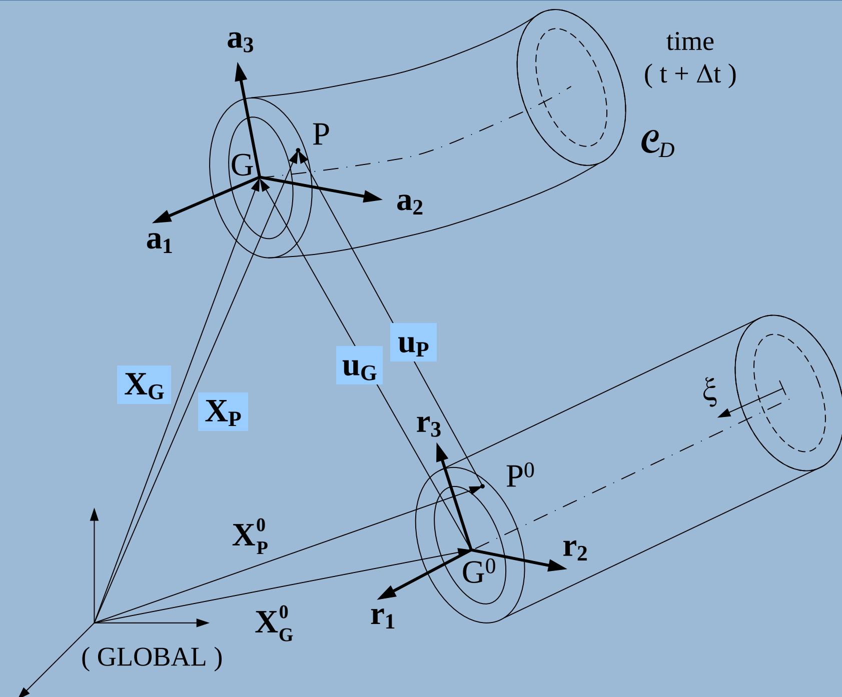
- Initial configuration $\mathbf{C}_0$.
- Co-rotated configuration $\mathbf{C}_C$.
- Deformed configuration $\mathbf{C}_D$.

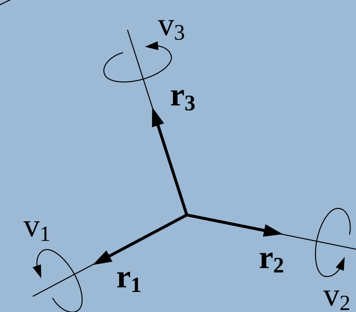
Coordinate systems:

- Global frame x_G, y_G, z_G
- Element base frame x_0, y_0, z_0
- Co-rotated frame x_C, y_C, z_C



Local Beam Kinematics





Displacement increment vector

$$\mathbf{u}_P = \mathbf{X}_P - \mathbf{X}_P^0$$

where

$$\mathbf{X}_P^0 = \mathbf{X}_G^0 + x_2\mathbf{r}_2 + x_3\mathbf{r}_3$$

$$\mathbf{X}_P = \mathbf{X}_G + x_2\mathbf{a}_2 + x_3\mathbf{a}_3$$

then

$$\mathbf{u}_P = \mathbf{X}_P - \mathbf{X}_P^0 = \mathbf{X}_G - \mathbf{X}_G^0 + x_2\left(\mathbf{a}_2 - \mathbf{r}_2\right) + x_3\left(\mathbf{a}_3 - \mathbf{r}_3\right)$$



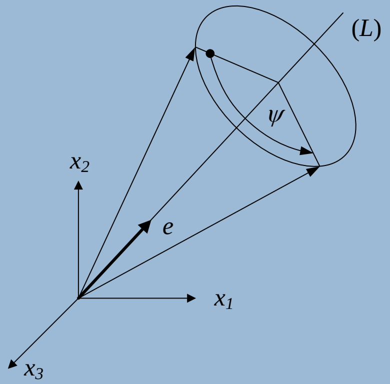
Rotation matrix

$\mathbf{a}_i = \mathbf{R}_i$, (i = 1,2,3)

$$\mathbf{\Psi} = v_1\mathbf{r}_1 + v_2\mathbf{r}_2 + v_3\mathbf{r}_3 = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \mathbf{e}\psi$$

Rotational vector

$$\psi = \sqrt{v_1^2 + v_2^2 + v_3^2}$$



In terms of $\mathbf{\Psi}$, the orthogonal matrix $\mathbf{R}$ admits the following representation:

$$\mathbf{R} = \mathbf{I} + \frac{sen\,\psi}{\psi}\mathbf{S}(\mathbf{\Psi}) + \frac{1}{2}\left[\frac{sen(\psi/2)}{\psi/2}\right]^2 \mathbf{S}(\mathbf{\Psi})\mathbf{S}(\mathbf{\Psi})$$

,where

$$\mathbf{S}(\mathbf{\Psi}) = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_3 & v_1 & 0 \end{bmatrix}$$

If the trigonometric functions in the above equation are expanded in Taylor series and using a second-order approximation of $\mathbf{R}$

$$\mathbf{R} = \mathbf{I} + \mathbf{S}(\mathbf{\Psi}) + \frac{1}{2}\mathbf{S}(\mathbf{\Psi})\mathbf{S}(\mathbf{\Psi}) \quad , \quad \mathbf{R} = \begin{bmatrix} 1 - \frac{v_2^2 + v_3^2}{2} & -v_3 + \frac{v_1 v_2}{2} & v_2 + \frac{v_1 v_3}{2} \\ v_3 + \frac{v_1 v_2}{2} & 1 - \frac{v_1^2 + v_3^2}{2} & -v_1 + \frac{v_2 v_3}{2} \\ -v_2 + \frac{v_1 v_3}{2} & v_1 + \frac{v_2 v_3}{2} & 1 - \frac{v_1^2 + v_2^2}{2} \end{bmatrix}$$



A second-order approximation of the displacement increment vector $\mathbf{u}_P$

$$
\begin{aligned}
u_{P_1} &= u_1 - x_2 v_3 + x_3 v_2 + \frac{1}{2} x_2 v_1 v_2 + \frac{1}{2} x_3 v_1 v_3 \\
u_{P_2} &= u_2 \qquad - x_3 v_1 - \frac{1}{2} x_2 \left(v_1^2 + v_3^2\right) + \frac{1}{2} x_3 v_2 v_3 \\
u_{P_3} &= \underbrace{u_3 + x_2 v_1}_{\text{linear}} + \underbrace{\frac{1}{2} x_2 v_2 v_3 - \frac{1}{2} x_3 \left(v_1^2 + v_2^2\right)}_{\text{não-linear}}
\end{aligned}
$$

With respect to the local system $\mathbf{r}_i$, the Green–Lagrange strain components which contribute to the strain energy of the beam are given by

$$
\begin{aligned}
\varepsilon_{11} &= u_{P_{1,1}} \qquad + \frac{1}{2}\left(u_{P_{1,1}}\right)^2 + \frac{1}{2}\left(u_{P_{2,1}}\right)^2 + \frac{1}{2}\left(u_{P_{3,1}}\right)^2 \\
\gamma_{12} &= u_{P_{1,2}} + u_{P_{2,1}} + u_{P_{1,1}} u_{P_{1,2}} + u_{P_{2,1}} u_{P_{2,2}} + u_{P_{3,1}} u_{P_{3,2}} \\
\gamma_{13} &= \underbrace{u_{P_{1,3}} + u_{P_{3,1}}}_{\text{linear}} + \underbrace{u_{P_{1,1}} u_{P_{1,3}} + u_{P_{2,1}} u_{P_{2,3}} + u_{P_{3,1}} u_{P_{3,3}}}_{\text{não-linear}}
\end{aligned}
$$

$$
\begin{aligned}
\varepsilon_{11} &= u_{1,1} - x_2 v_{3,1} + x_3 v_{2,1} + \frac{1}{2}\left(u_{2,1}^2 + u_{3,1}^2\right) + x_2\left[\frac{1}{2}\left(v_{1,1} v_2 + v_1 v_{2,1}\right) + u_{3,1} v_{1,1}\right] \\
&\quad + x_3\left[\frac{1}{2}\left(v_{1,1} v_3 + v_1 v_{3,1}\right) - u_{2,1} v_{1,1}\right] + \frac{1}{2}\left(x_2^2 + x_3^2\right) v_{1,1}^2 \\
&\quad + \frac{1}{2}\left(x_2^2 v_{3,1}^2 + x_3^2 v_{2,1}^2\right) - \left(x_2 x_3\right) v_{3,1} v_{2,1} \\
\gamma_{12} &= u_{2,1} - v_3 - x_3 v_{1,1} + \frac{1}{2} v_1 v_2 + u_{3,1} v_1 - \frac{1}{2} x_3 \left(v_{2,1} v_3 - v_2 v_{3,1}\right) \\
\gamma_{13} &= \underbrace{u_{3,1} + v_2 + x_2 v_{1,1}}_{\text{linear } (e_{ij})} + \underbrace{\frac{1}{2} v_1 v_3 - u_{2,1} v_1 + \frac{1}{2} x_2 \left(v_{2,1} v_3 - v_2 v_{3,1}\right)}_{\text{não-linear } (\eta_{ij})}
\end{aligned}
$$



Linearization of the Principle of Virtual Work

Principle of virtual displacements in the co-rotational updated Lagrangian formulation

$$\boxed{\int_{^tV} {}^{t+\Delta t}_{\;\;t}S_{ij}\; \delta\, {}^{t+\Delta t}_{\;\;t}\varepsilon_{ij}\; d\,{}^tV = {}^{t+\Delta t}\Re} \qquad \text{,where} \qquad {}^{t+\Delta t}\Re = \int_{^0S} {}^{t+\Delta t}_{\;\;0}f_i^S\; \delta u_i^S\; d\,{}^0S + \int_{^oV} {}^{t+\Delta t}_{\;\;0}f_i^B\; \delta u_i\; d\,{}^0V$$

The following incremental decompositions are used

$${}^{t+\Delta t}_{\;\;t}S_{ij} = {}^t\tau_{ij} + {}_tS_{ij}$$

$${}^{t+\Delta t}_{\;\;t}\varepsilon_{ij} = {}^t_t\varepsilon_{ij} + {}_t\varepsilon_{ij}\,; \qquad {}_t\varepsilon_{ij} = {}_te_{ij} + {}_t\eta_{ij}$$

then

$$\int_{^tV} \left({}^t\tau_{ij} + {}_tS_{ij} \right) \delta \left({}^t_t\varepsilon_{ij} + {}_te_{ij} + {}_t\eta_{ij} \right) d\,{}^tV = {}^{t+\Delta t}\Re$$

Using the approximations $\delta\,{}^t_t\varepsilon_{ij} = 0$, ${}_tS_{ij} = {}_tC_{ijrs}\;{}_te_{ij}$, we obtain as equation of motion

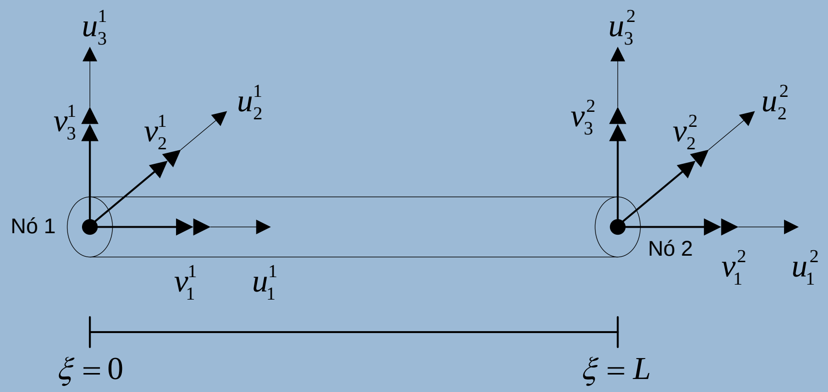
$$\int_{^tV} {}_tC_{ijrs}\;{}_te_{rs}\;\delta\,{}_te_{ij}\; d\,{}^tV + \int_{^tV} {}^t\tau_{ij}\;\delta\,{}_t\eta_{ij}\; d\,{}^tV = {}^{t+\Delta t}\Re - \int_{^tV} {}^t\tau_{ij}\;\delta\,{}_te_{ij}\; d\,{}^tV$$

Using d’Alembert’s principle and introducing the damping forces, we obtain the dynamic incremental equilibrium equation in the integral form

$$\int_{^oV} {}^0\rho\, {}^{t+\Delta t}\ddot{u}_i\; \delta u_i\; d\,{}^0V + \int_{^oV} c\, {}^{t+\Delta t}\dot{u}_i\; \delta u_i\; d\,{}^0V + \int_{^tV} {}_tC_{ijrs}\;{}_te_{rs}\;\delta\,{}_te_{ij}\; d\,{}^tV + \int_{^tV} {}^t\tau_{ij}\;\delta\,{}_t\eta_{ij}\; d\,{}^tV =$$

$$\int_{^0S} {}^{t+\Delta t}_{\;\;0}f_i^S\; \delta u_i^S\; d\,{}^0S + \int_{^oV} {}^{t+\Delta t}_{\;\;0}f_i^B\; \delta u_i\; d\,{}^0V - \int_{^tV} {}^t\tau_{ij}\;\delta\,{}_te_{ij}\; d\,{}^tV$$

Finite Element Discretization



Nodal displacement increment vector (translations and rotations)

$$\hat{\mathbf{u}}^T = \begin{bmatrix} u_1^1 & u_2^1 & u_3^1 & v_1^1 & v_2^1 & v_3^1 & | & u_1^2 & u_2^2 & u_3^2 & v_1^2 & v_2^2 & v_3^2 \end{bmatrix}$$

Using Hermite interpolation functions

$$u_1(\xi) = \phi_1(\xi)u_1^1 + \phi_2(\xi)u_1^2$$

$$u_2(\xi) = \phi_3(\xi)u_2^1 + \phi_4(\xi)u_2^2 + \phi_5(\xi)v_3^1 - \phi_6(\xi)v_3^2$$

$$u_3(\xi) = \phi_3(\xi)u_3^1 + \phi_4(\xi)u_3^2 - \phi_5(\xi)v_2^1 + \phi_6(\xi)v_2^2$$

$$v_1(\xi) = \phi_1(\xi)v_1^1 + \phi_2(\xi)v_1^2$$

$$v_2(\xi) = -\phi_7(\xi)u_3^1 + \phi_7(\xi)u_3^2 + \phi_8(\xi)v_2^1 - \phi_9(\xi)v_2^2$$

$$v_3(\xi) = \phi_7(\xi)u_2^1 - \phi_7(\xi)u_2^2 + \phi_8(\xi)v_3^1 - \phi_9(\xi)v_3^2$$

where,

$$\phi_1(\xi) = 1 - (\xi/\mathrm{L}), \quad \phi_2(\xi) = \xi/\mathrm{L}$$

$$\phi_3(\xi) = 1 - 3(\xi/\mathrm{L})^2 + 2(\xi/\mathrm{L})^3$$

$$\phi_4(\xi) = 3(\xi/\mathrm{L})^2 - 2(\xi/\mathrm{L})^3$$

$$\phi_5(\xi) = \left[(\xi/\mathrm{L}) - 2(\xi/\mathrm{L})^2 + (\xi/\mathrm{L})^3\right]\mathrm{L}$$

$$\phi_6(\xi) = \left[(\xi/\mathrm{L})^2 - (\xi/\mathrm{L})^3\right]\mathrm{L}$$

$$\phi_7(\xi) = \frac{6}{\mathrm{L}}\left[(\xi/\mathrm{L}) - (\xi/\mathrm{L})^2\right]$$

$$\phi_8(\xi) = 1 - 4(\xi/\mathrm{L}) + 3(\xi/\mathrm{L})^2$$

$$\phi_9(\xi) = 2(\xi/\mathrm{L}) - 3(\xi/\mathrm{L})^2 \qquad \text{for} \qquad (0 \le \xi/\mathrm{L} \le +1)$$

Finite Element Matrices

$$\int_{^0V} {}^0\rho\, {}^{t+\Delta t}\ddot{u}_i\, \delta u_i\, d\,{}^0V \qquad \mathbf{M}\,{}^{t+\Delta t}\ddot{\mathbf{U}} = \left[\sum_m \int_{^0V^{(m)}} \rho^{(m)} \mathbf{H}^{(m)T}\; {}^{t+\Delta t}_{\;0}\mathbf{H}^{B(m)}\, d\,{}^0V^{(m)}\right] {}^{t+\Delta t}\ddot{\mathbf{U}}$$

$$+ \qquad\qquad +$$

$$\int_{^0V} c\, {}^{t+\Delta t}\dot{u}_i\, \delta u_i\, d\,{}^0V \qquad \mathbf{C}\,{}^{t+\Delta t}\dot{\mathbf{U}} = \left[\sum_m \int_{^0V^{(m)}} c^{(m)} \mathbf{H}^{(m)T}\; {}^{t+\Delta t}_{\;0}\mathbf{H}^{B(m)}\, d\,{}^0V^{(m)}\right] {}^{t+\Delta t}\dot{\mathbf{U}}$$

$$+ \qquad\qquad +$$

$$\int_{^tV} {}_tC_{ijrs}\; {}_te_{rs}\; \delta\, {}_te_{ij}\; d\,{}^tV \qquad {}^t_t\mathbf{K}_L\mathbf{U} = \left[\sum_m \int_{^tV^{(m)}} {}^t_t\mathbf{B}_L^{(m)T}\; {}_t\mathbf{C}^{(m)}\; {}^t_t\mathbf{B}_L^{(m)}\, d\,{}^tV^{(m)}\right]\mathbf{U}$$

$$+ \qquad\qquad +$$

$$\int_{^tV} {}^t\tau_{ij}\; \delta\, {}_t\eta_{ij}\; d\,{}^tV \qquad {}^t_t\mathbf{K}_{NL}\mathbf{U} = \left[\sum_m \int_{^tV^{(m)}} {}^t_t\mathbf{B}_{NL}^{(m)T}\; {}^t\boldsymbol{\tau}^{(m)}\; {}^t_t\mathbf{B}_{NL}^{(m)}\, d\,{}^tV^{(m)}\right]\mathbf{U}$$

$$= \qquad\qquad =$$

$${}^{t+\Delta t}\Re = \int_{^0S} {}^{t+\Delta t}_{\;0}f_i^S\; \delta u_i^S\; d\,{}^0S + \int_{^oV} {}^{t+\Delta t}_{\;0}f_i^B\; \delta u_i\; d\,{}^0V \qquad {}^{t+\Delta t}\mathbf{R} = \sum_m \int_{^0S^{(m)}} \mathbf{H}^{S(m)T}\; {}^{t+\Delta t}_{\;0}\mathbf{f}^{S(m)}\, d\,{}^0S^{(m)} + \sum_m \int_{^0V^{(m)}} \mathbf{H}^{(m)T}\; {}^{t+\Delta t}_{\;0}\mathbf{f}^{B(m)}\, d\,{}^0V^{(m)}$$

$$- \qquad\qquad -$$

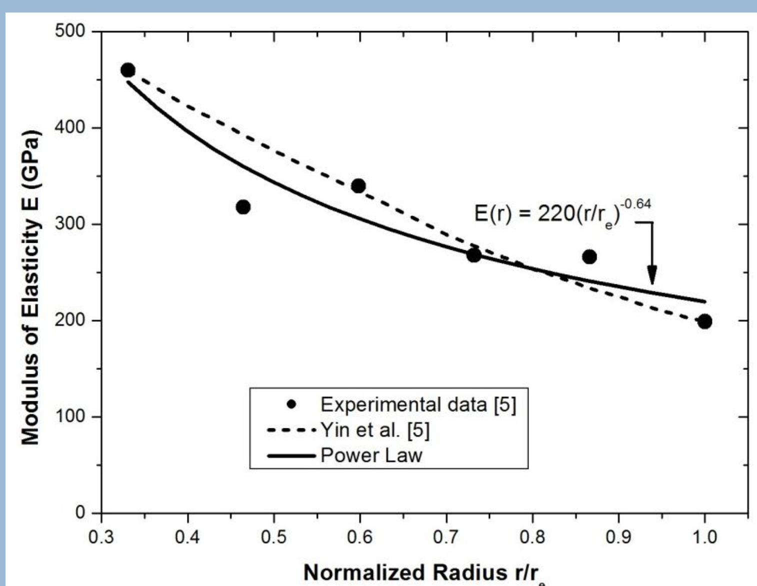
$$\int_{^tV} {}^t\tau_{ij}\; \delta\, {}_te_{ij}\; d\,{}^tV \qquad {}^t_t\mathbf{F} = \sum_m \int_{^tV^{(m)}} {}^t_t\mathbf{B}_L^{(m)T}\; {}^t\hat{\boldsymbol{\tau}}^{(m)}\; d\,{}^tV^{(m)}$$

Dynamic incremental equilibrium equation in the matrix form

$$\mathbf{M}\,{}^{t+\Delta t}\ddot{\mathbf{U}} + \mathbf{C}\,{}^{t+\Delta t}\dot{\mathbf{U}} + \left({}^t_t\mathbf{K}_L + {}^t_t\mathbf{K}_{NL}\right)\mathbf{U} = {}^{t+\Delta t}\mathbf{R} - {}^t_t\mathbf{F}$$

F G M Constitutive Law

$$
{}_t\mathbf{C}^{(m)} = \begin{bmatrix} E_0 r^{\beta} & 0 & 0 \\ 0 & \dfrac{E_0 r^{\beta}}{2(1+\nu)} & 0 \\ 0 & 0 & \dfrac{E_0 r^{\beta}}{2(1+\nu)} \end{bmatrix}
$$



Power Law Approximation for FGM (TiC – Ni_3Al) Along Riser Thickness

Estimate of the power law

$$E = E_0 r^{\beta}$$

$$\ln E = \beta \ln r + \ln E_0$$

$$
\left.\begin{aligned} y &= \ln E \\ m &= \beta \\ x &= \ln r \\ n &= \ln E_0 \end{aligned}\right\} y = mx + n
$$

Least squares

$$m = \frac{N S_{xy} - S_x S_y}{N S_{xx} - S_x S_x}$$

$$n = \frac{S_{xx} S_y - S_x S_{xy}}{N S_{xx} - S_x S_x}$$

where

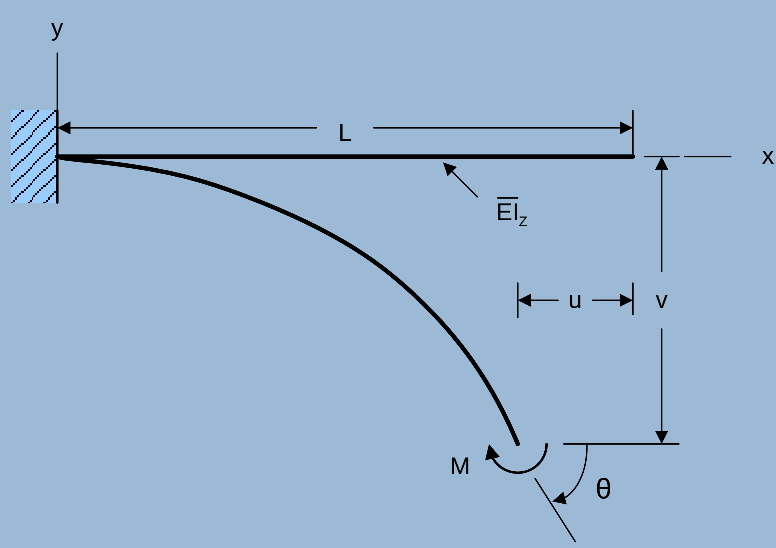
$$S_x = \sum_{i=1}^{N} x_i \qquad S_{xx} = \sum_{i=1}^{N} x_i^2$$

$$S_y = \sum_{i=1}^{N} y_i \qquad S_{xy} = \sum_{i=1}^{N} x_i y_i$$

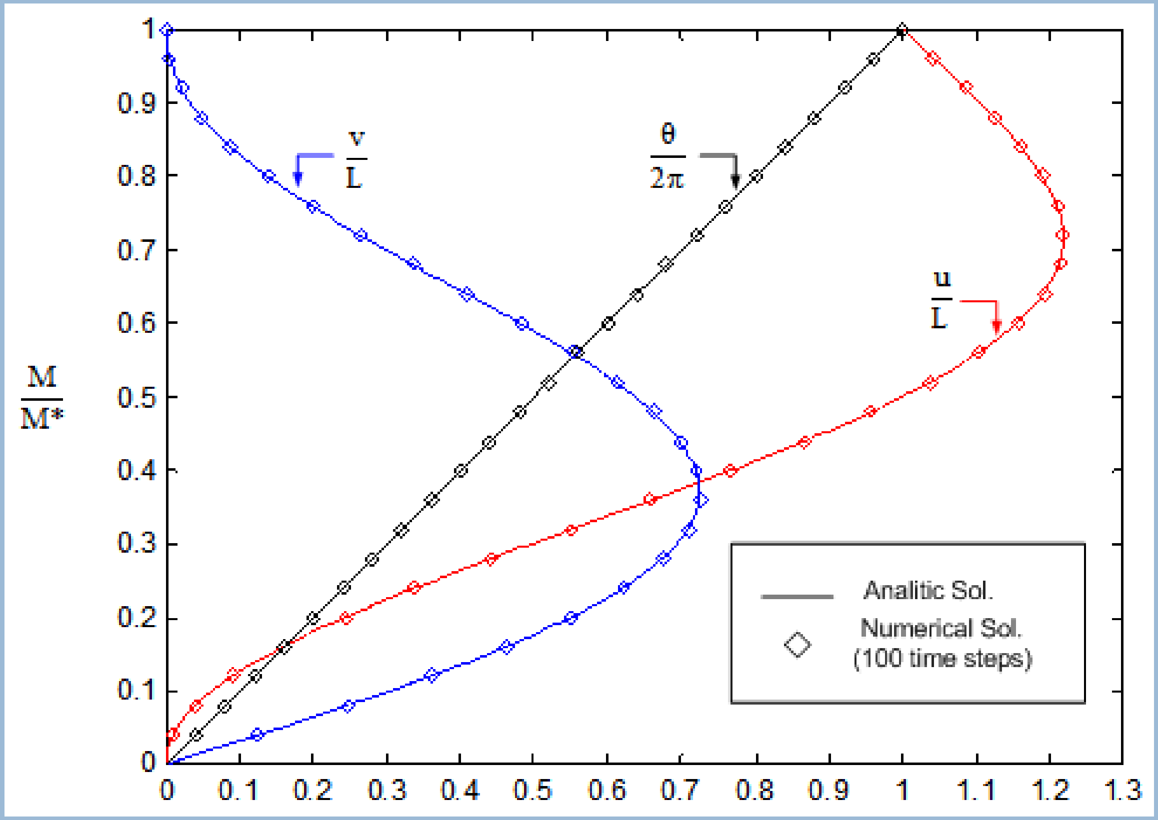
then $\beta = m, \quad E_0 = e^n$

Sample Analyses

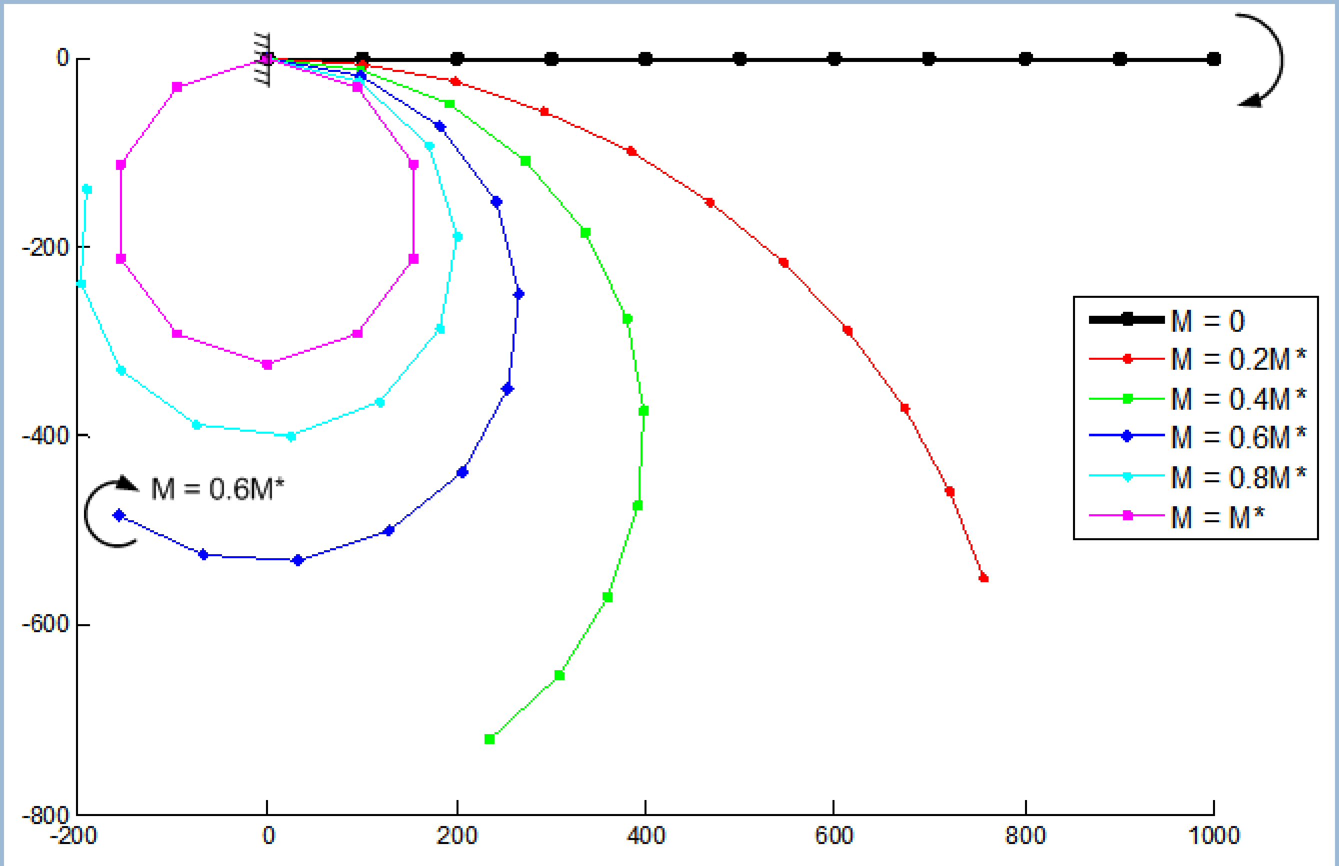
- Cantilever Pipe-Beam Under Constant Bending Moment in Large Displacements



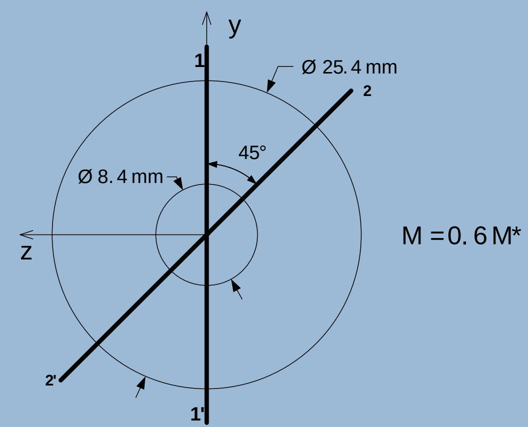
$$\phi_e = 25.4\ mm$$
$$t = 8.5\ mm$$
$$L = 1000\ mm$$
$$E = 220\left(r/r_e\right)^{-0.64}\ GPa$$
$$\overline{EI}_Z = 5.23\ kN.m^2$$
$$M^* = 2\pi\left(\frac{EI_Z}{L}\right) = 32.8\ kN.m$$

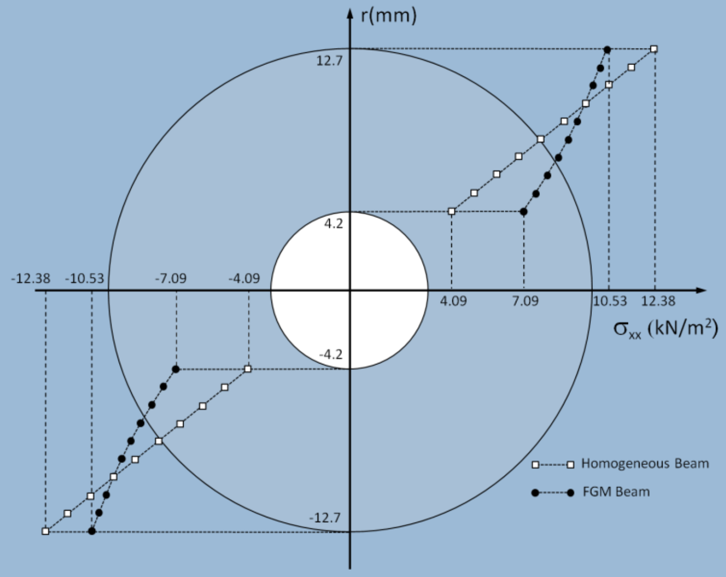


Tip displacements and rotations of cantilever beam undergoing large displacements

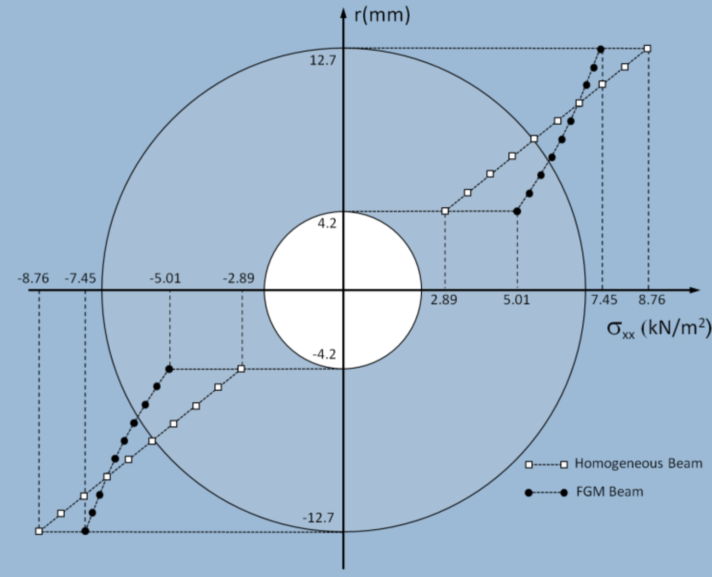


Cantilever pipe-beam undergoing large displacements and rotations

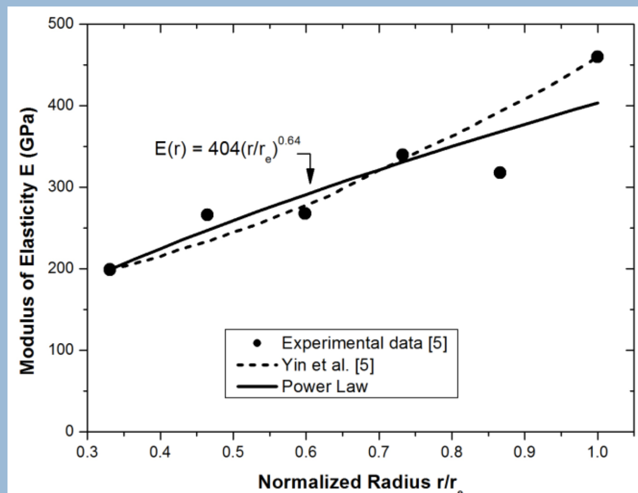


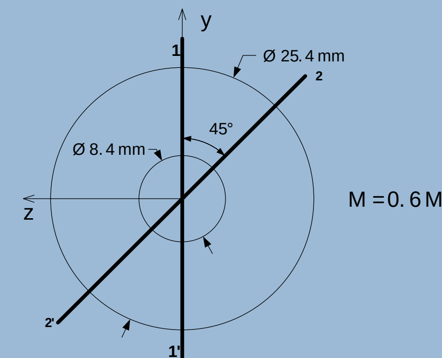


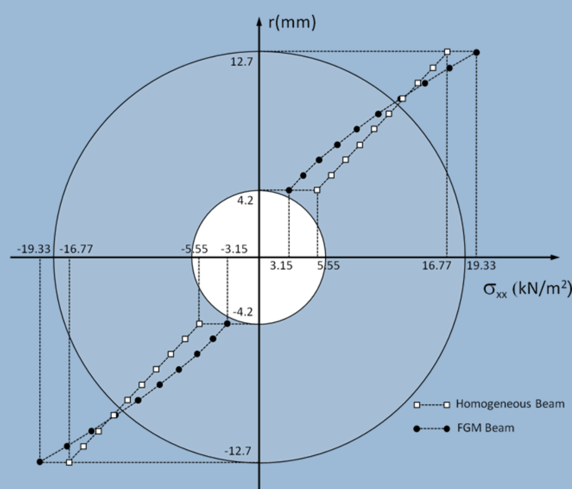
Section 1 – 1'



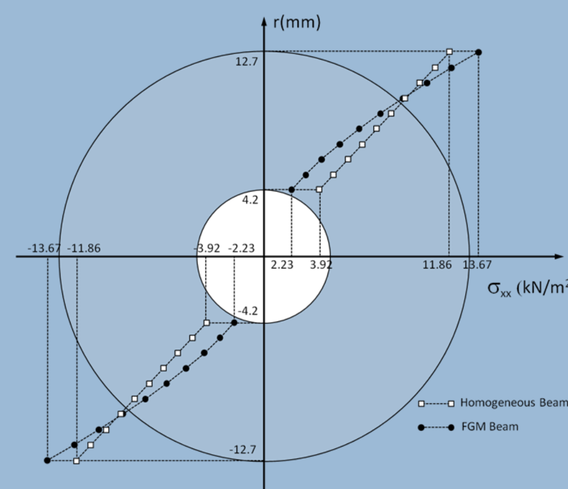
Section 2 – 2'





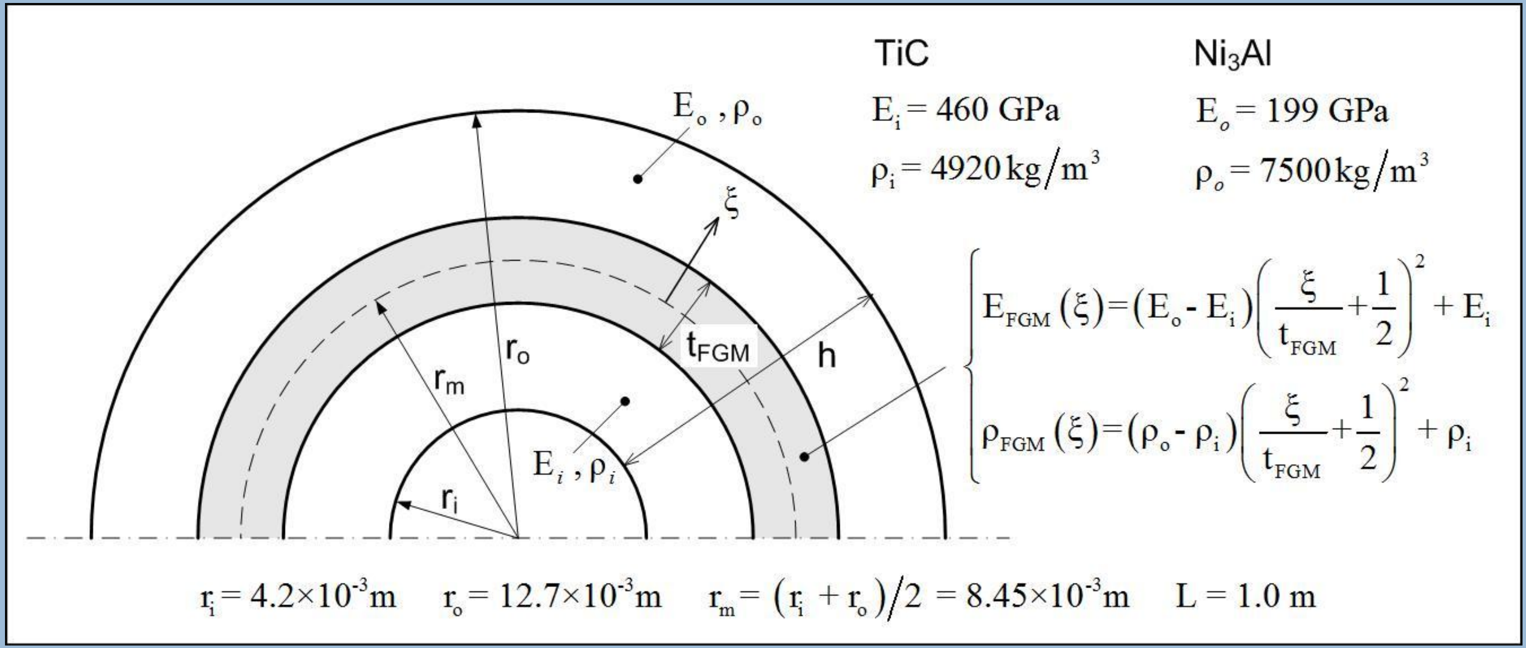


Section 1 – 1’

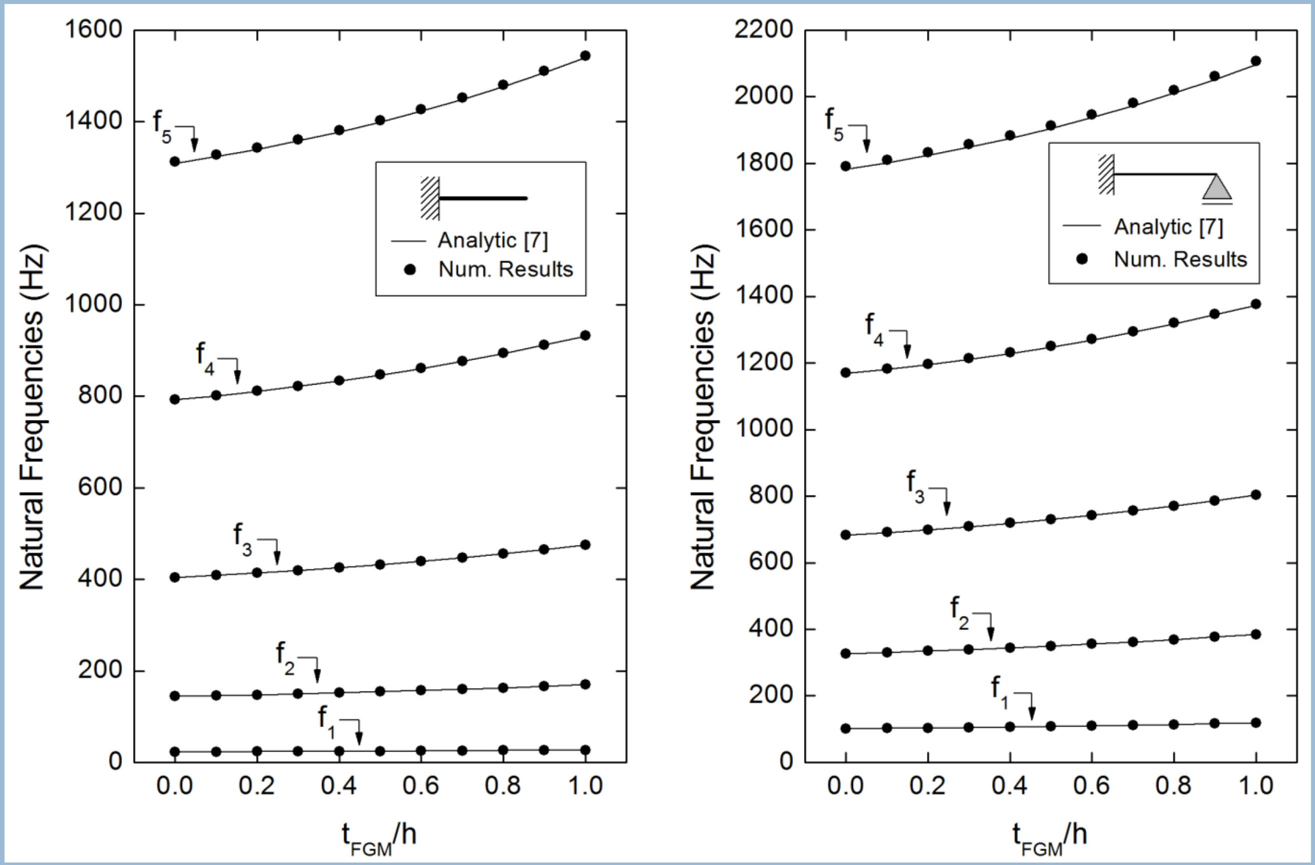


Section 2 – 2’

- Natural Frequency Evaluations of Composed Cross-Section Straight Beams

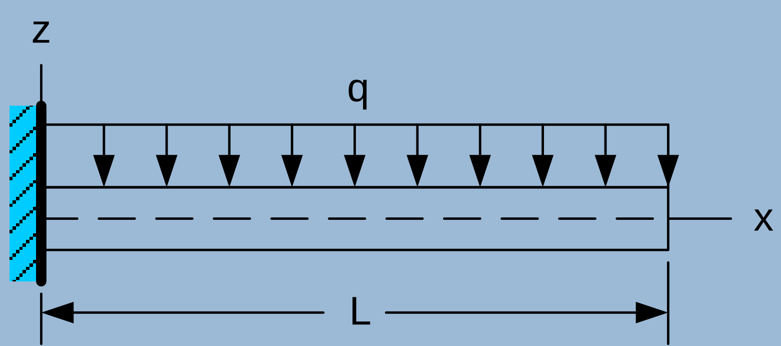


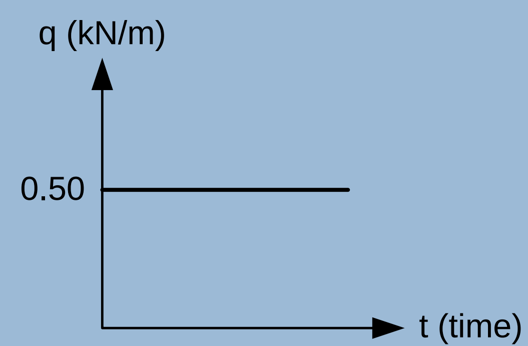
Composed cross-section details as considered in the numerical analysis



First five flexure natural frequencies obtained for the composed beams considered

• Large Displacement Dynamic Analysis of a Cantilever Pipe-Beam Under Uniformly Distributed Load





(Dynamic Analysis)

$L = 254\ mm$

$\phi_e = 29\ mm$

$t = 12.3\ mm$

$E = 82.74\ MPa$

$\nu = 0.2$

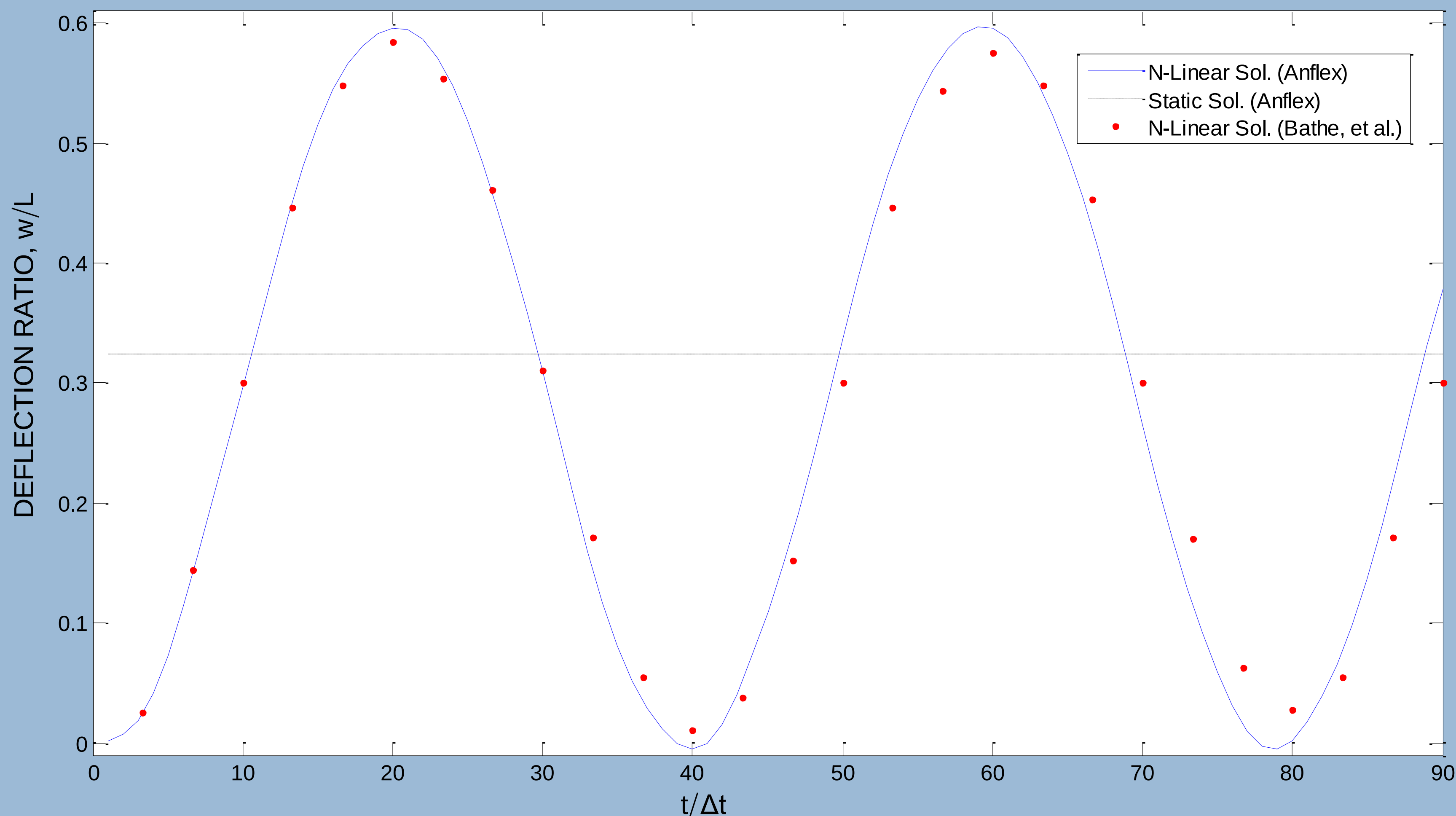
$\rho = 10.7\ kg/m^3$

$tol = 0.001\ (force\ /\ displc.)$

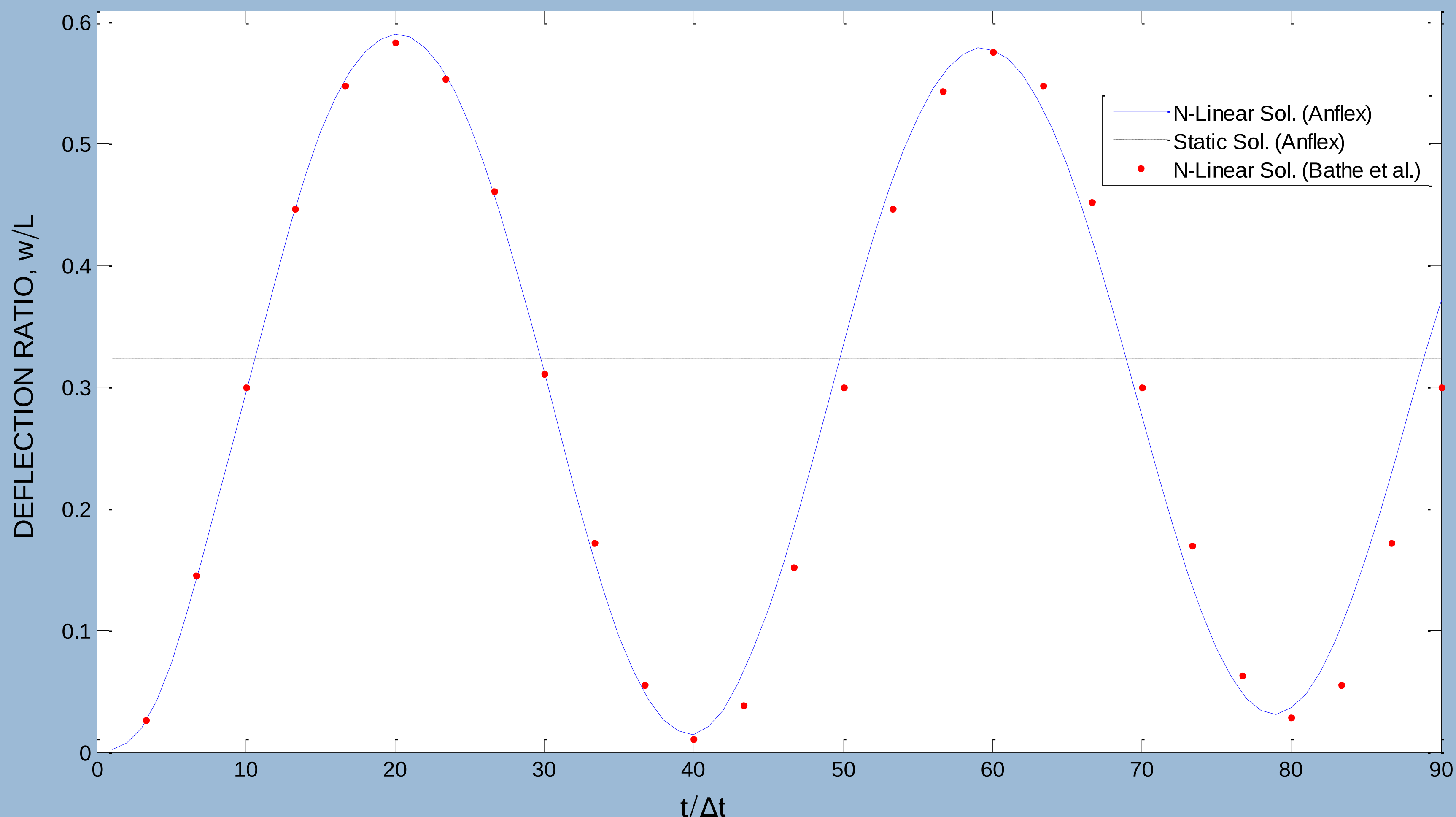
$\Delta t = 1.3643 \times 10^{-4}$ sec

HHT Time Integration Method

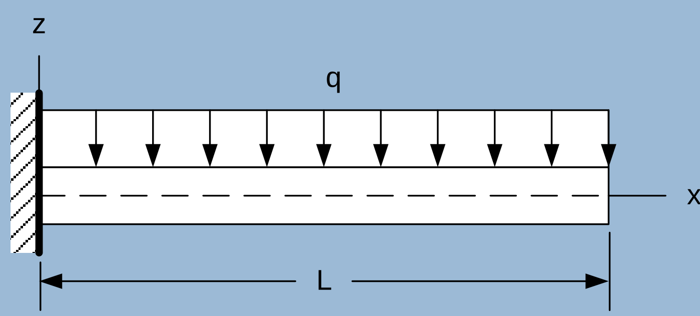




Large displacement dynamic response of a cantilever under uniformly distributed load



Large displacement dynamic response of a cantilever under uniformly distributed load

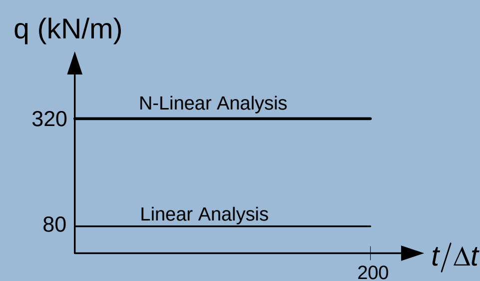


Dynamic Analysis

$L = 254\ mm$

$\phi_e = 25.4\ mm$

$t = 8.5\ mm$



$tol = 0.001\ (force\ /\ displ.)$

$\Delta t = 4.5 \times 10^{-5}$ sec

HHT Time Integration Method

Case A (TiC)

$E = 448\ GPa$

$\nu = 0.3$

$\rho = 4830\ kg/m^3$

Case B (FGM)

$E = 220\ r^{-0.643}\ GPa$

$\nu = 0.3$

$\rho = 7360\ r^{0.381}\ kg/m^3$

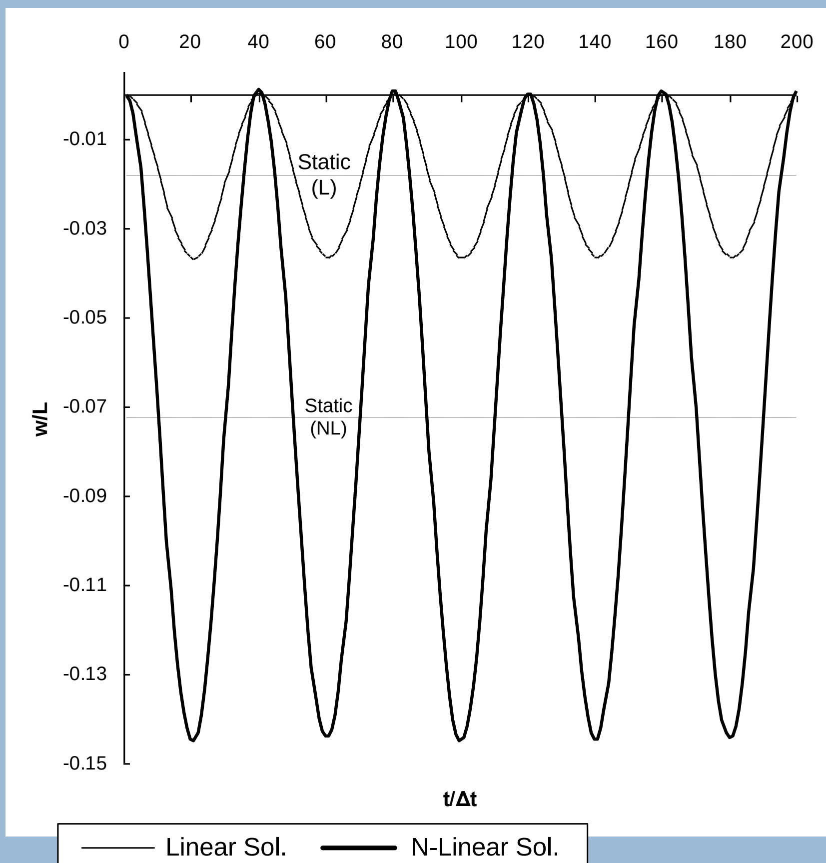
Case C (Ni_3Al)

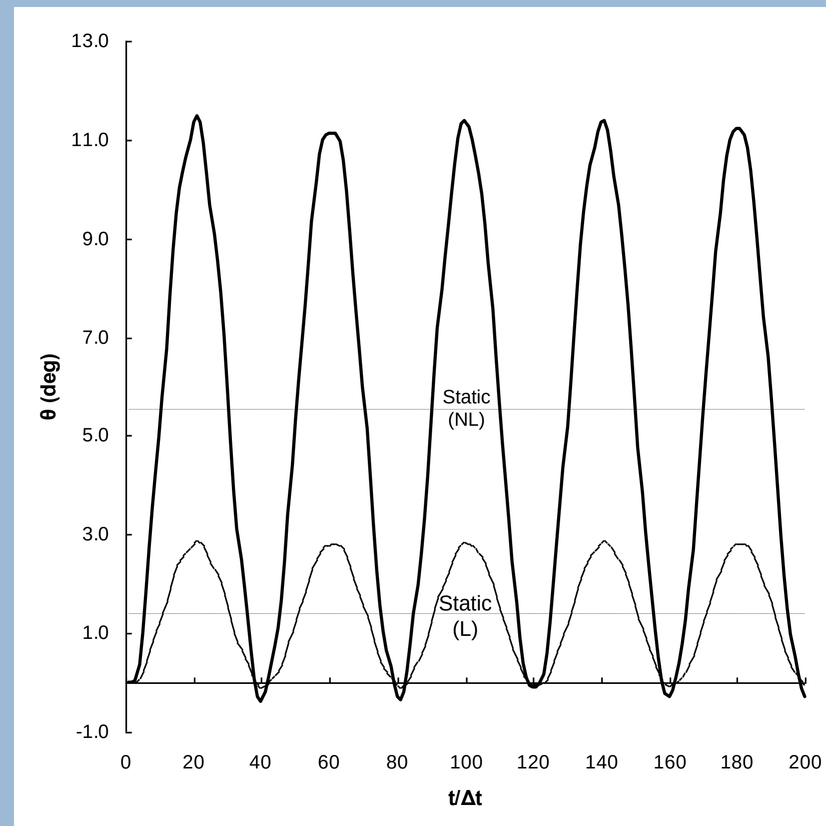
$E = 220\ GPa$

$\nu = 0.3$

$\rho = 7360\ kg/m^3$

Case A (TiC)





Small Displacement (Linear)

Period : 0.00182 s (0.00179 s, L. Meirovitch) Static Sol. : w/L = -0.0182

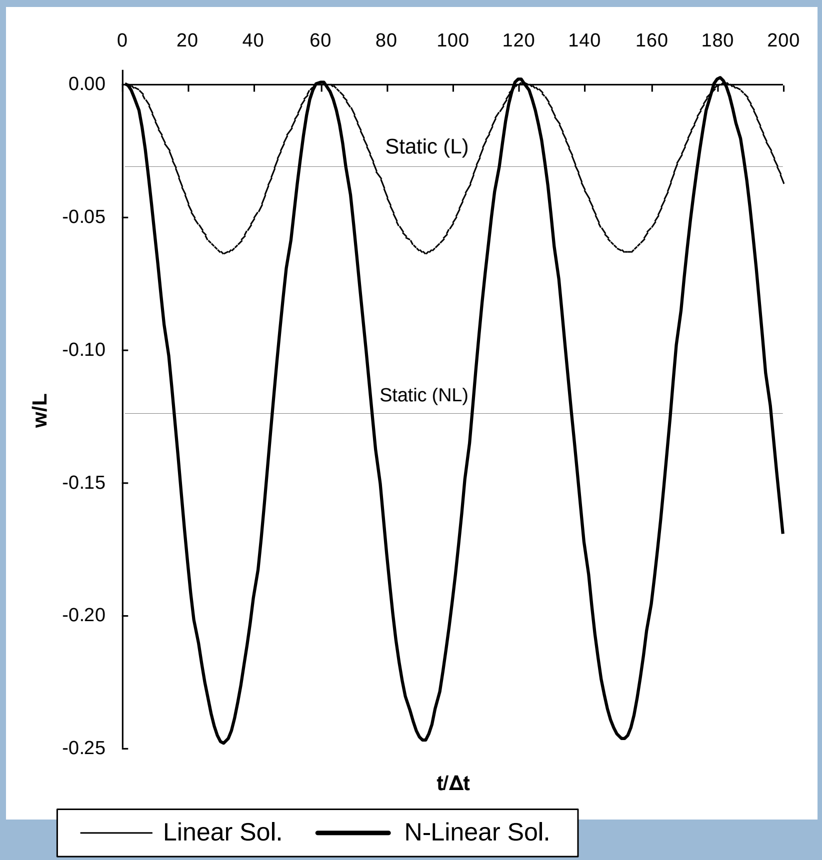
θ = 1.39 deg

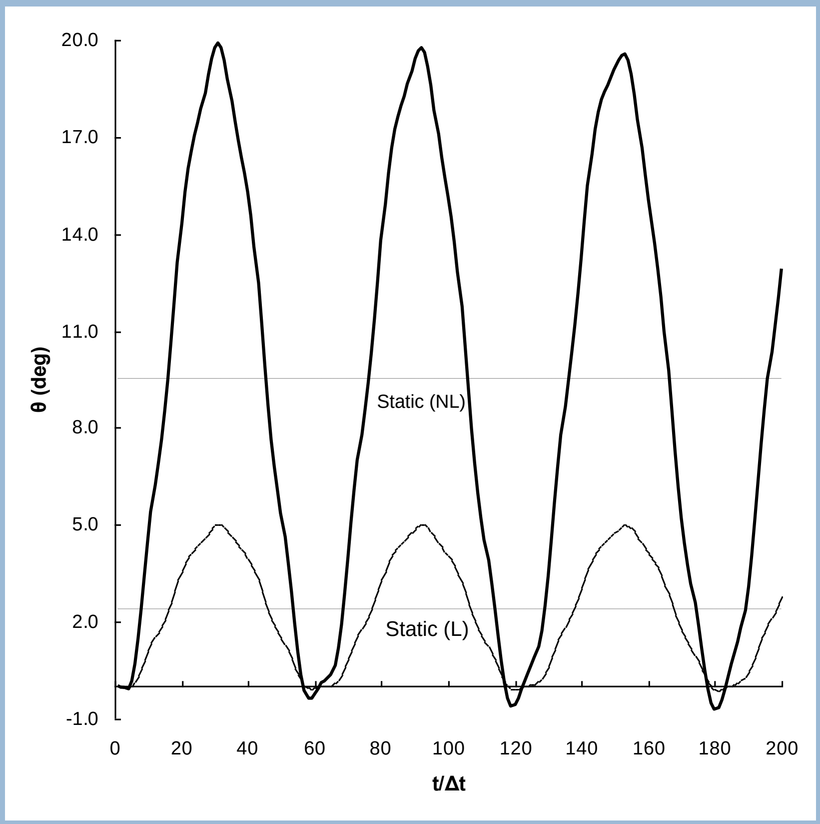
Large Displacement (N-Linear)

Period : 0.00180 s Static Sol. : w/L = -0.0724

θ = 5.54 deg

Case B (FGM)





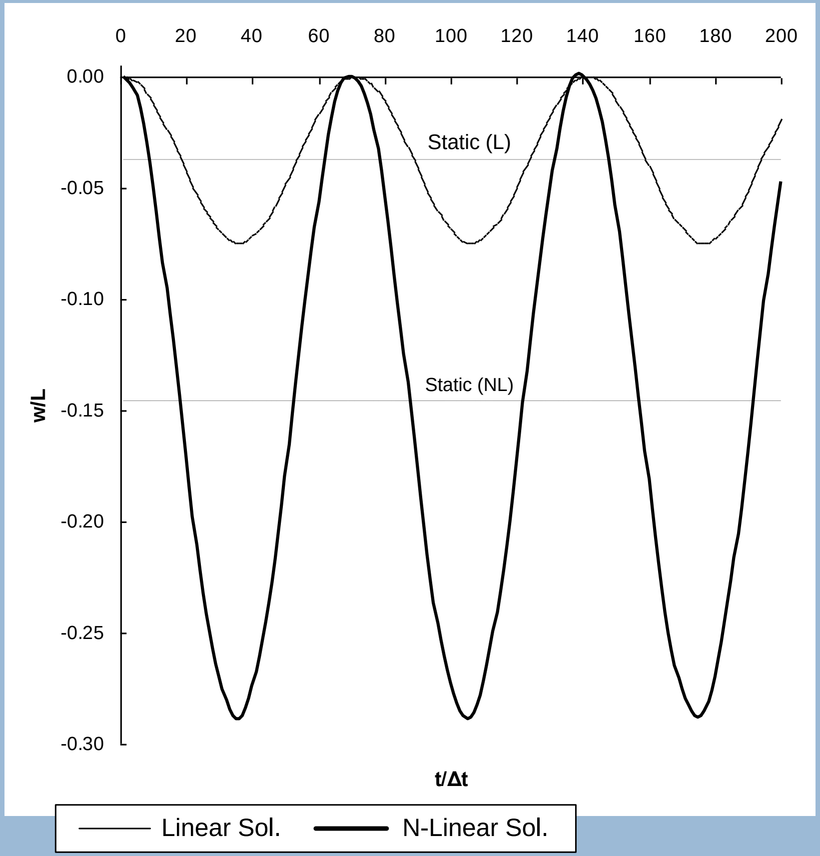
Small Displacement (Linear)

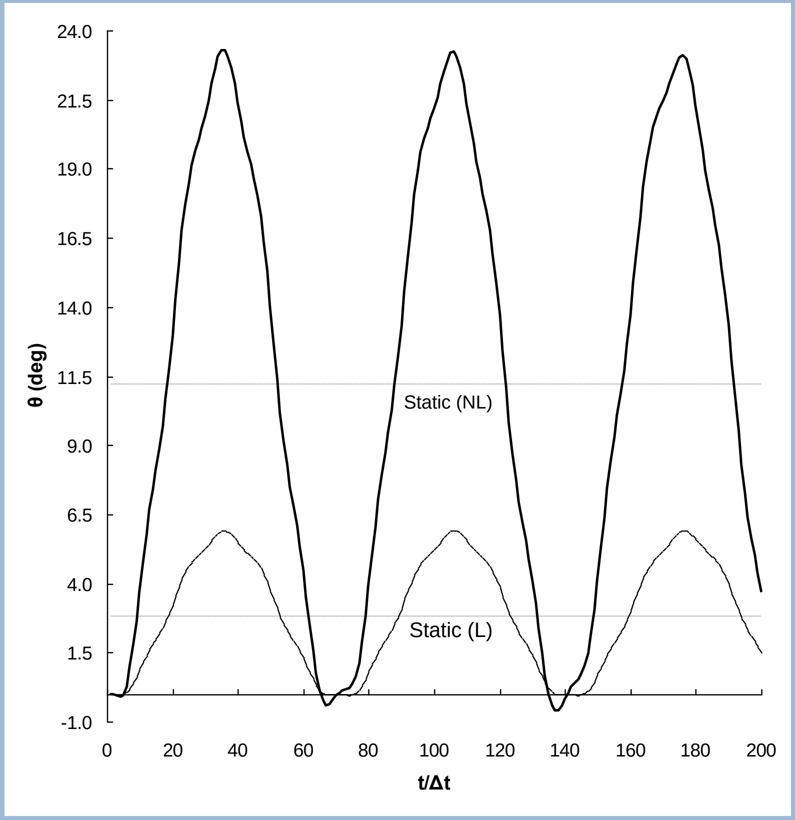
Period : 0.00272 s Static Sol. : w/L = -0.0314
θ = 2.41 deg

Large Displacement (N-Linear)

Period : 0.00272 s Static Sol. : w/L = -0.1243
θ = 9.54 deg

Case C (Ni_3Al)





Small Displacement (Linear)

Period : 0.00315 s (0.00315 s, L. Meirovitch) Static Sol. : w/L = -0.0370

θ = 2.83 deg

Large Displacement (N-Linear)

Period : 0.00315 s Static Sol. : w/L = -0.1456

θ = 11.19 deg

Conclusions

- General two-node beam element based on the corotational formulation for geometric non-linear analysis has been implemented.

- FGM composed cross-sections are accounted by adjusting the beam theory rigidity parameters considering cross-section evaluations in closed form.

- Significant differences in stress distributions are obtained as compared to homogeneous section beams, allowing for an effective use of FGM’s on riser design.

- Formulation applications to dynamic analyses has shown good result agreements with analytical parental theoretical results, both for the natural frequency evaluation as per the step-by-step incremental solutions.

Thank you!