Topology Optimization for Millifluidics

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Motivation

- Microfluidics consists of handling and analyzing fluids in structures at the micro scale. The microscale offers new design opportunities as the existing devices may not work properly.

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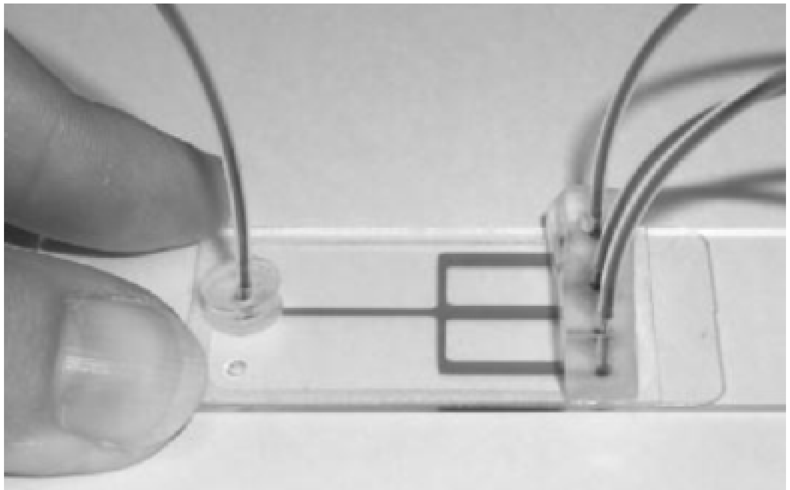
  - biological systems,



A device constructed using in situ construction techniques shows a channel network with external fluidic connections

Beebe, D.J., G.A. Mensing, e G.M. Walker. "Physics and applications of microfluidics in biology." Annual Review of Biomedical Engineering 4, 2002: pp. 261-286

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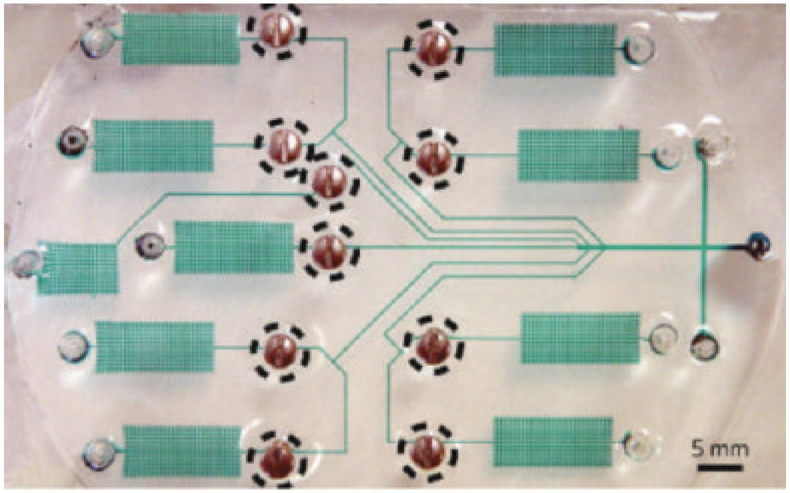

Device for micro channel network for biochemical analysis

Weibel, DB, M Kruithof, S Potenta, SK Sia, A Lee, e GM Whitesides. "Torque-actuated valves for microfluidics." Analytical Chemistry, 77 (15) , 2005: pp. 4726-4733.

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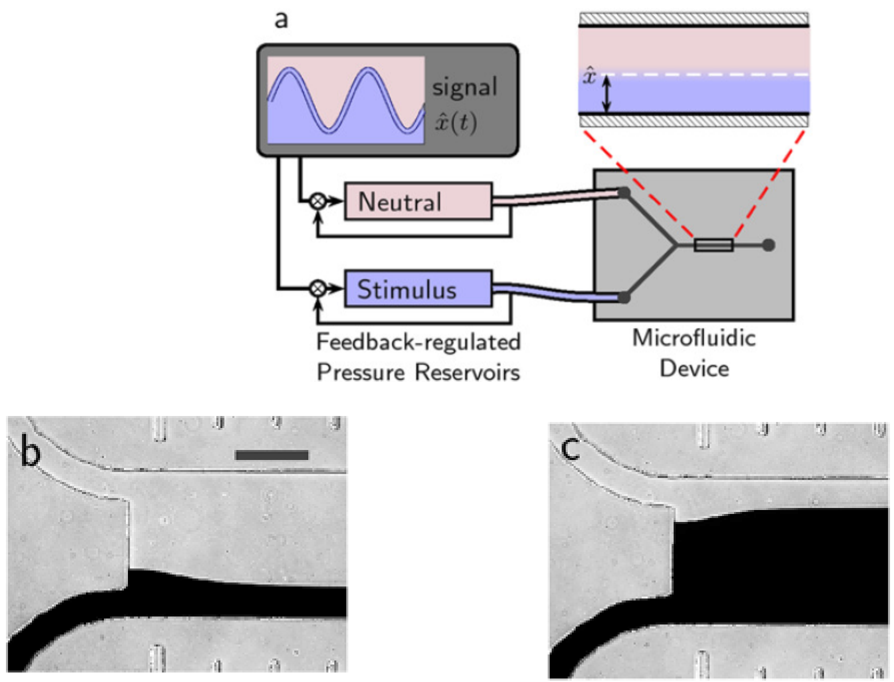

Mixing system with a feedback control loop

Kuczenski, B., W.C. Ruder, W.C. Messner, e P.R. LeDuc. "Probing Cellular Dynamics with a Chemical Signal Generator." *PLoS ONE 4(3): e4847, 2009*

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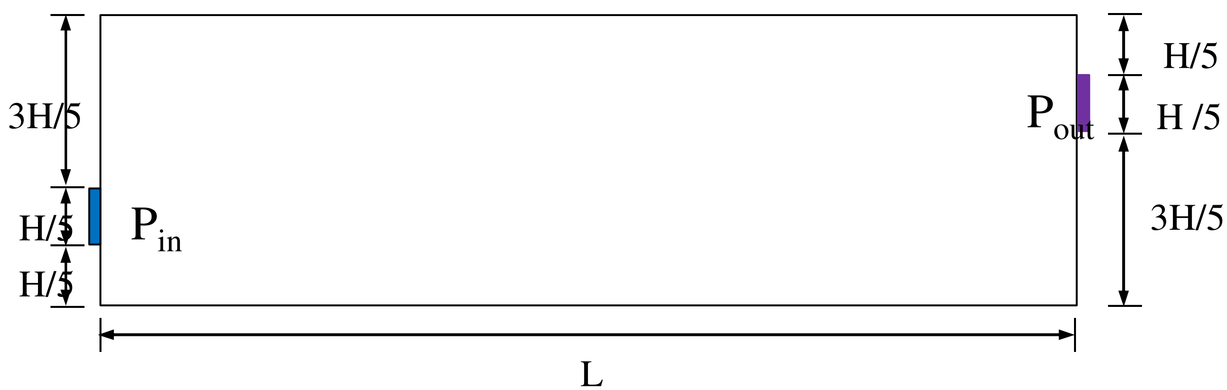
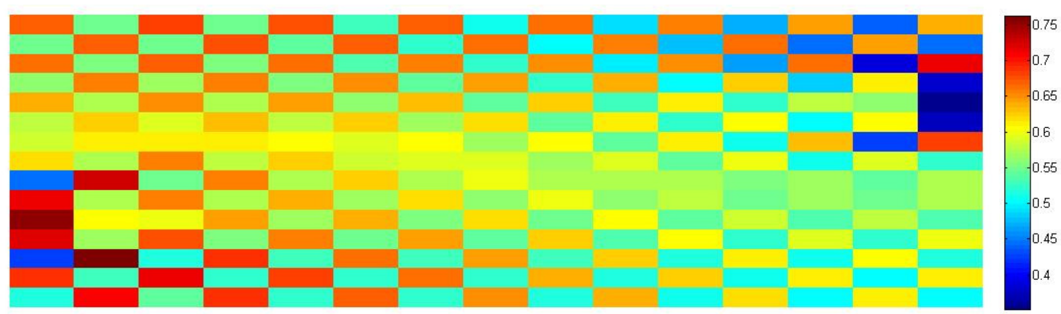
Motivation

- Numerical instabilities such the “checkerboard” problem could appear in mixed variational formulation (pressure-velocity) of the Stokes flow problems.

Checkerboard on pressure distribution:

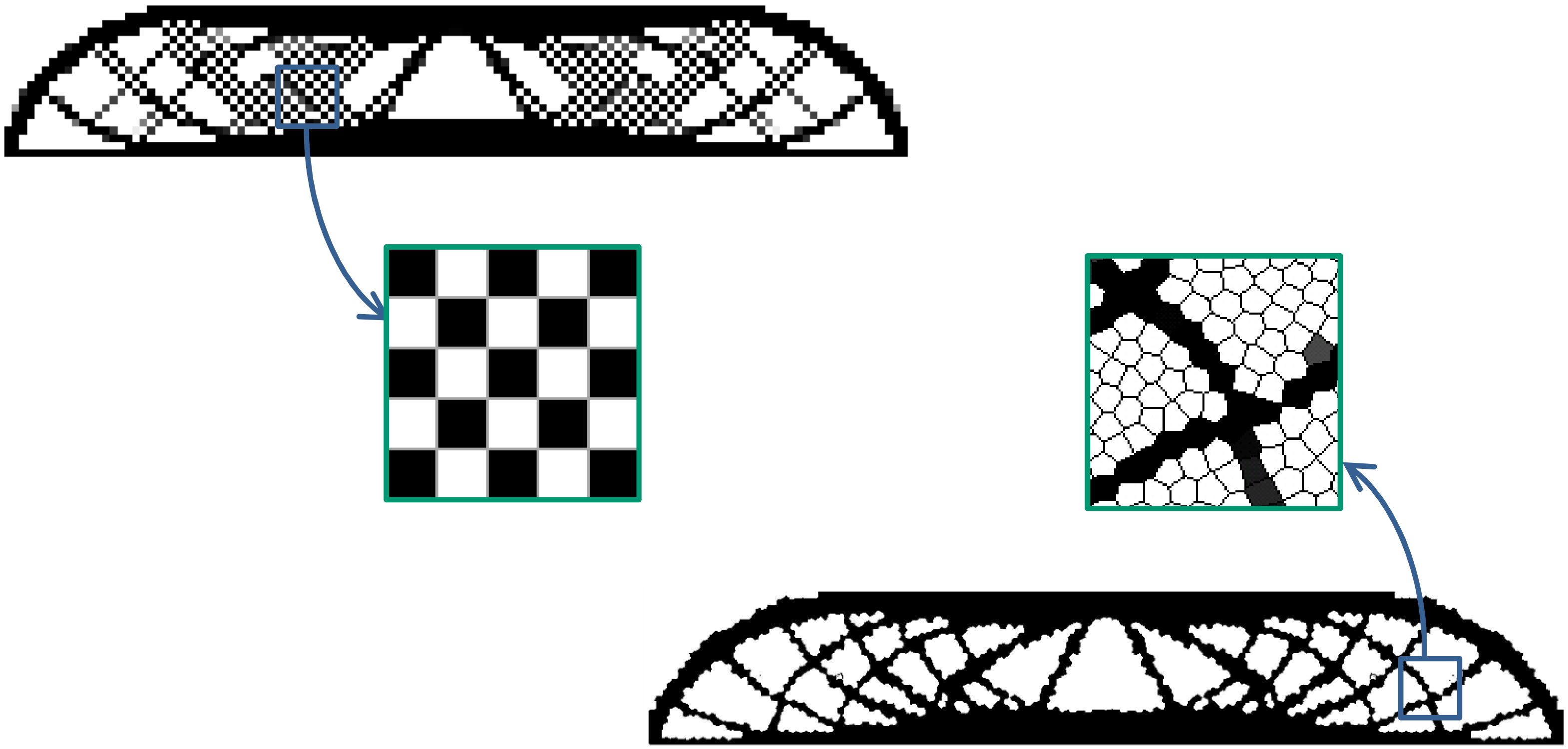




linear velocity
constant pressure

Motivation

- The “checkerboard” problem also appears in topology optimization depending on the choice of dicretizations for design and response fields.



Motivation

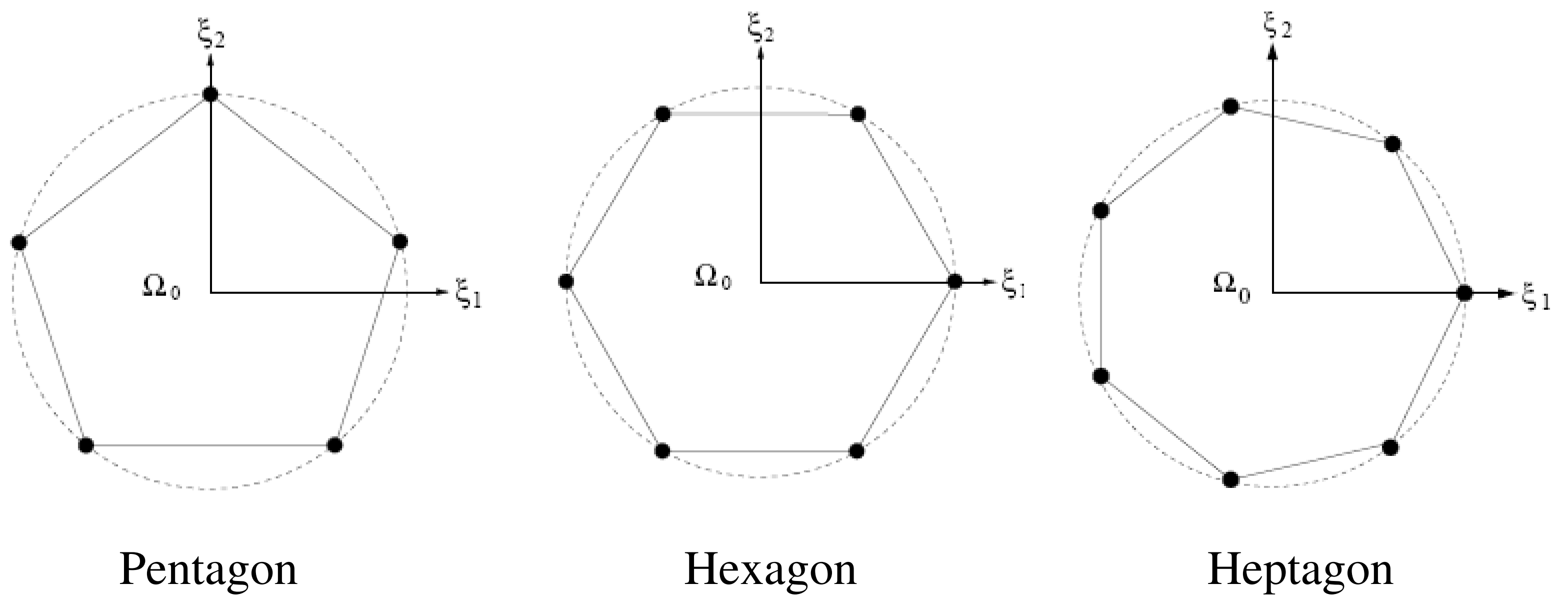
- In this work, we examine the use of polygonal discretization for solving the fluid flow problem.
- Our developments are based on PolyMesher/Polytop framework. A general topology optimization framework using unstructured polygonal finite element meshes.
- In particular, we consider constant pressure and velocity based on isoparametric polygonal elements.

Outline

- Polygonal Finite Element
- Stokes flow problems
- Implementation aspects
- Numerical Results
- Concluding remarks
- Ongoing work

Polygonal Finite Element

- Isoparametric finite element formulation constructed using Laplace shape function.

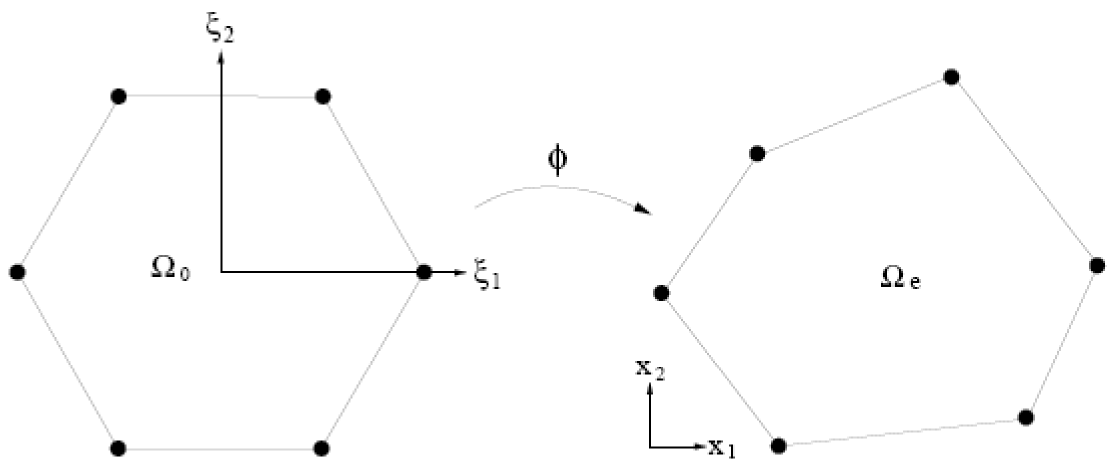


- The reference elements are regular n-gons inscribed in unit circles.

N. Sukumar and E. A. Malsch. Recent advances in the construction of polygonal finite element interpolants. 2006. Archives of Computational Methods in Engineering, 13(1):129--163

Polygonal Finite Element

- Isoparametric finite element formulation constructed using Laplace shape function.

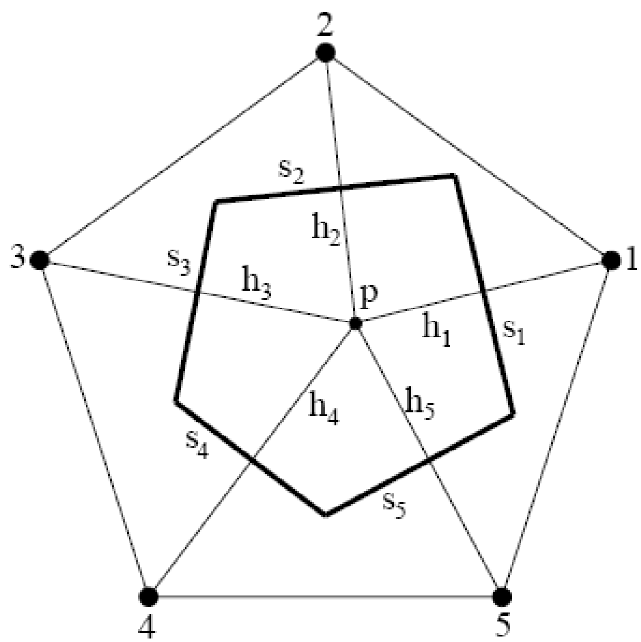


- Isoparametric mapping

N. Sukumar and E. A. Malsch. Recent advances in the construction of polygonal finite element interpolants. 2006. Archives of Computational Methods in Engineering, 13(1):129--163

- Laplace shape function

$$\phi_i(\boldsymbol{\xi}) = \frac{\alpha_i(\boldsymbol{\xi})}{\sum_{j=1}^{n} \alpha_j(\boldsymbol{\xi})}, \quad \alpha_j(\boldsymbol{\xi}) = \frac{s_j(\boldsymbol{\xi})}{h_j(\boldsymbol{\xi})}, \quad \boldsymbol{\xi} \in \Omega_0,$$



Non-negative

$$0 \le \phi_i(\boldsymbol{\xi}) \le 1, \quad \phi_i(\boldsymbol{\xi}_j) = \delta_{ij}, \quad \sum_{i=1}^{n} \phi_i(\boldsymbol{\xi}) = 1,$$

Linear completeness

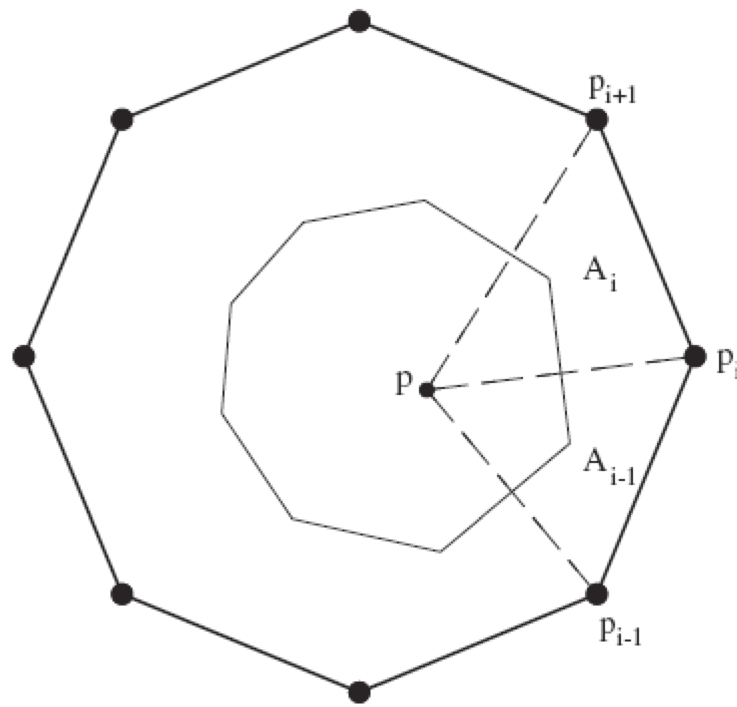
$$\mathbf{x}(\boldsymbol{\xi}) = \sum_{i=1}^{n} \phi_i(\boldsymbol{\xi}) \mathbf{x}_i,$$

Polygonal Finite Element

- Laplace shape function for regular polygons

$$\phi_i(\boldsymbol{\xi}) = \frac{\alpha_i(\boldsymbol{\xi})}{\sum_{j=1}^{n} \alpha_j(\boldsymbol{\xi})},$$

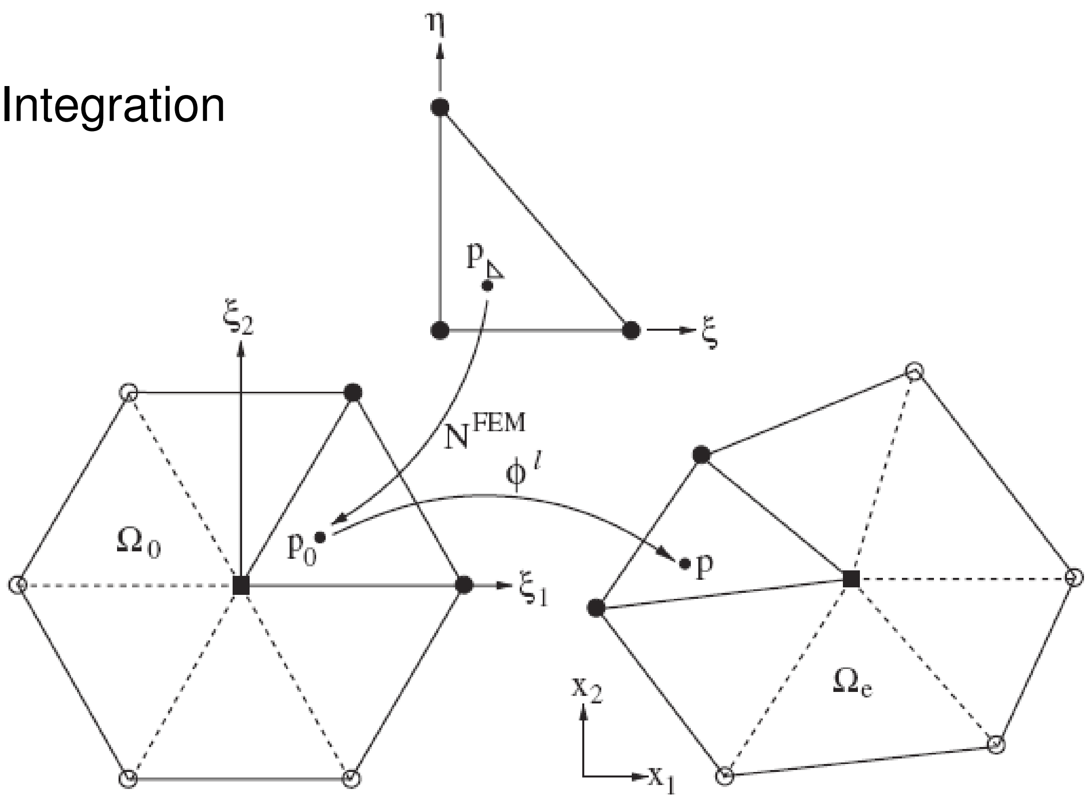
$$\alpha_i(\boldsymbol{\xi}) = \frac{2(1-\xi_1^2-\xi_2^2)\sin^3\dfrac{\pi}{n}\cos\dfrac{\pi}{n}}{A_{i-1}A_i}.$$



- Closed-form expressions can be obtained by employing a symbolic program such as Maple.

Polygonal Finite Element

- Numerical Integration



$$\int_{V_i} f \, d\Omega = \int_{\Omega_0} f \, |\mathbf{J}_2| \, d\Omega = \sum_{j=1}^{n} \int_{\Omega_0^{\Delta j}} f \, |\mathbf{J}_2| \, d\Omega = \sum_{j=1}^{n} \int_0^1 \int_0^{1-\xi} f \, |\mathbf{J}_1^j| |\mathbf{J}_2| \, d\xi \, d\eta$$

Stokes flow problems

The dynamic properties of velocity and pressure for incompressible fluidic flows can be expressed using the incompressible Navier–Stokes equations as:

$$\begin{cases} \rho \mathbf{u} \cdot \nabla \mathbf{u} = \nabla \cdot \mathbf{T} \\ \nabla \cdot \mathbf{u} = \mathbf{0} \end{cases}$$

where

$$\mathbf{T} = -p\mathbf{I} + \mu\left[\nabla \mathbf{u} + \nabla \mathbf{u}^T\right]$$

$\mathbf{u}$	velocity field
p	fluidic pressure
ρ	fluidic density
μ	fluidic viscosity

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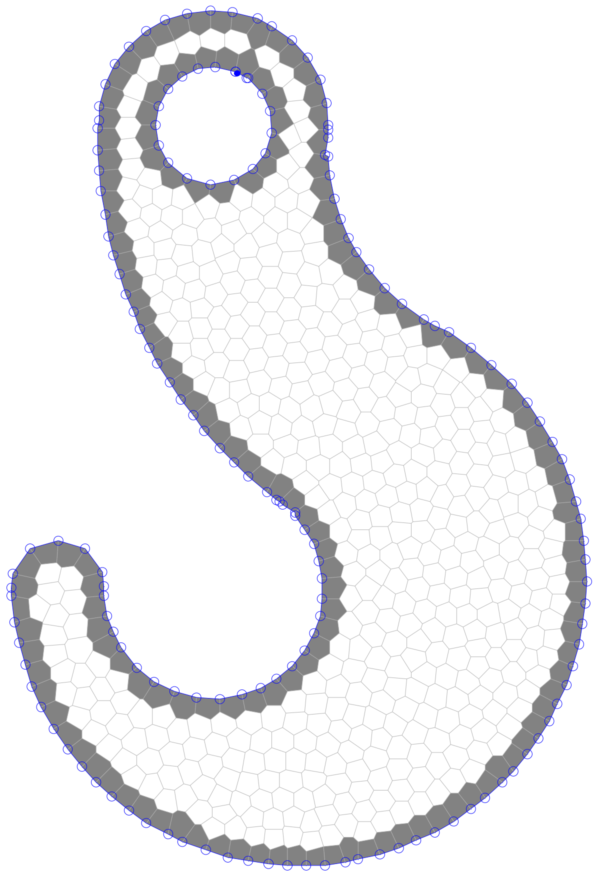
Implementation aspects

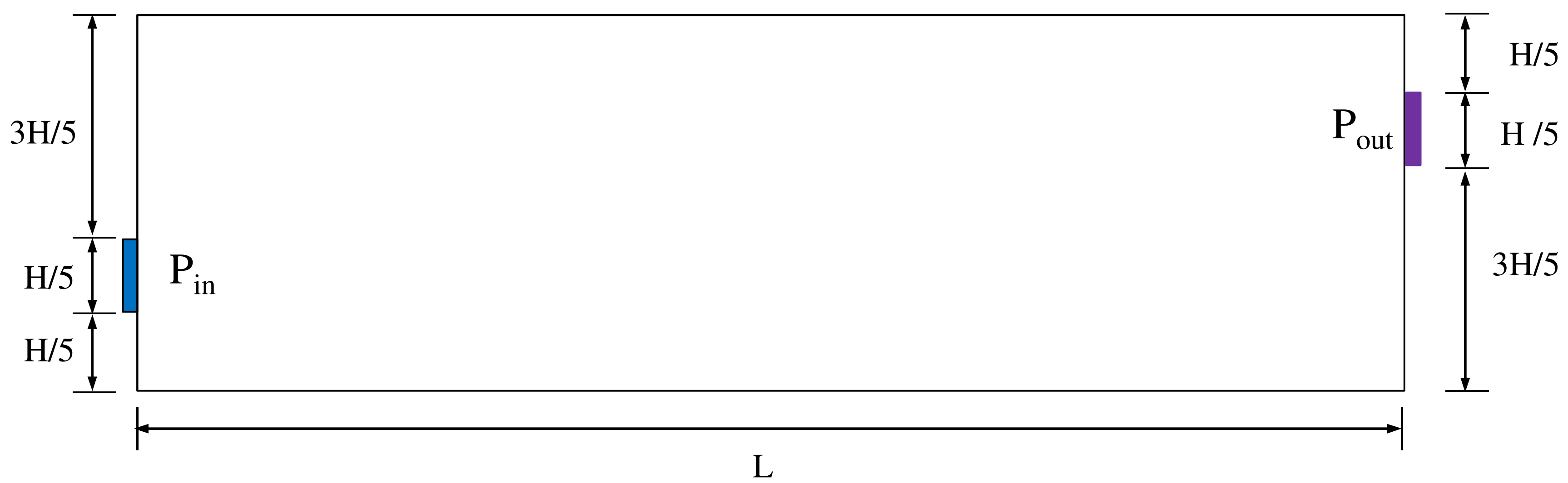
•Our implementation is based on PolyMesher and PolyTop.



- Automatic identification of boundary nodes and elements
- Simplify the task of application of the boundary conditions

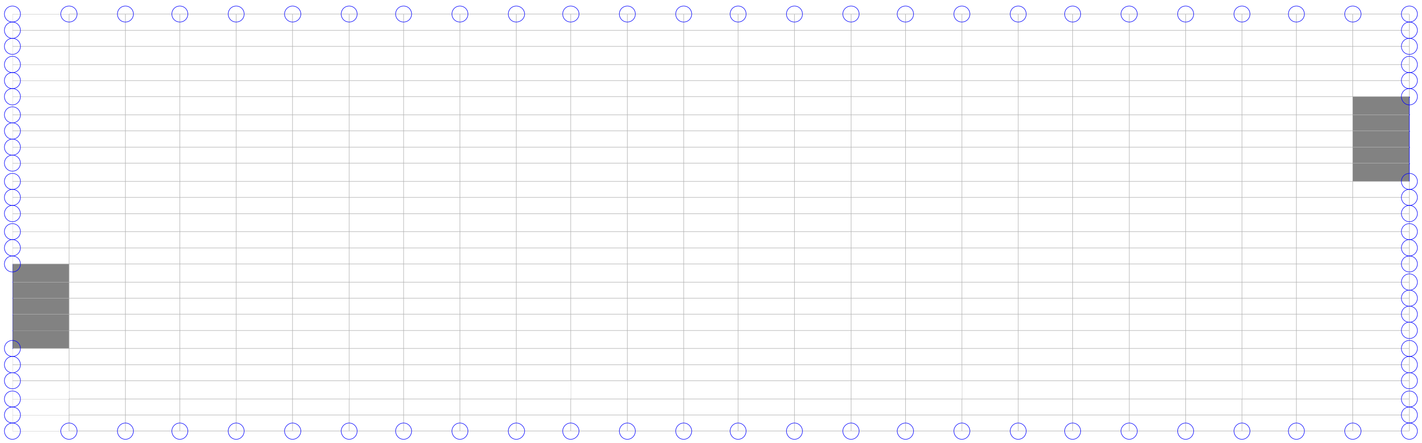
Numerical results: In/Out Flow Problem

Geometry Description

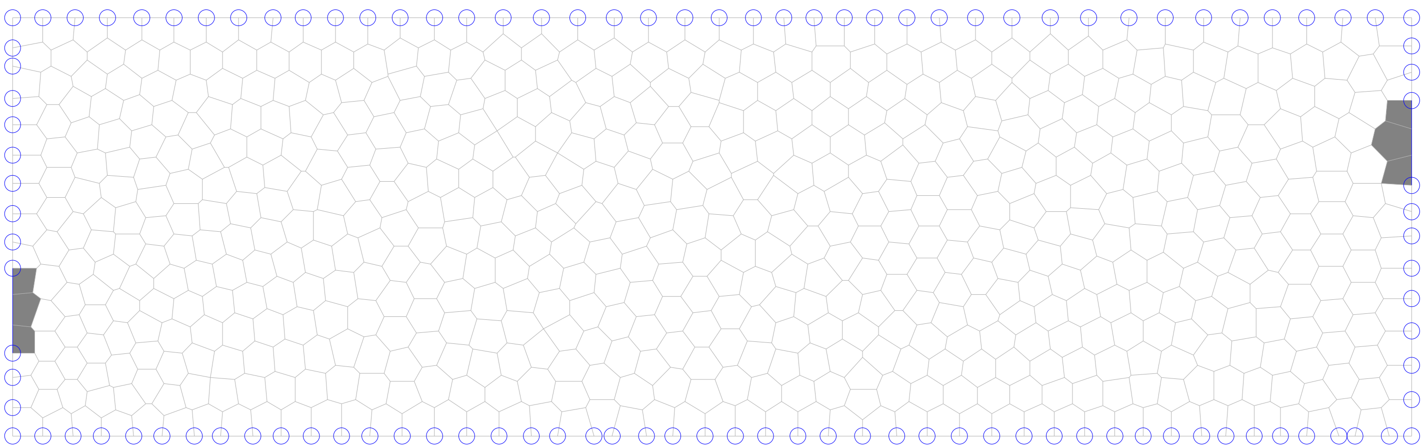


Example: In/Out Flow Problem (cont.)

Finite Element Meshes



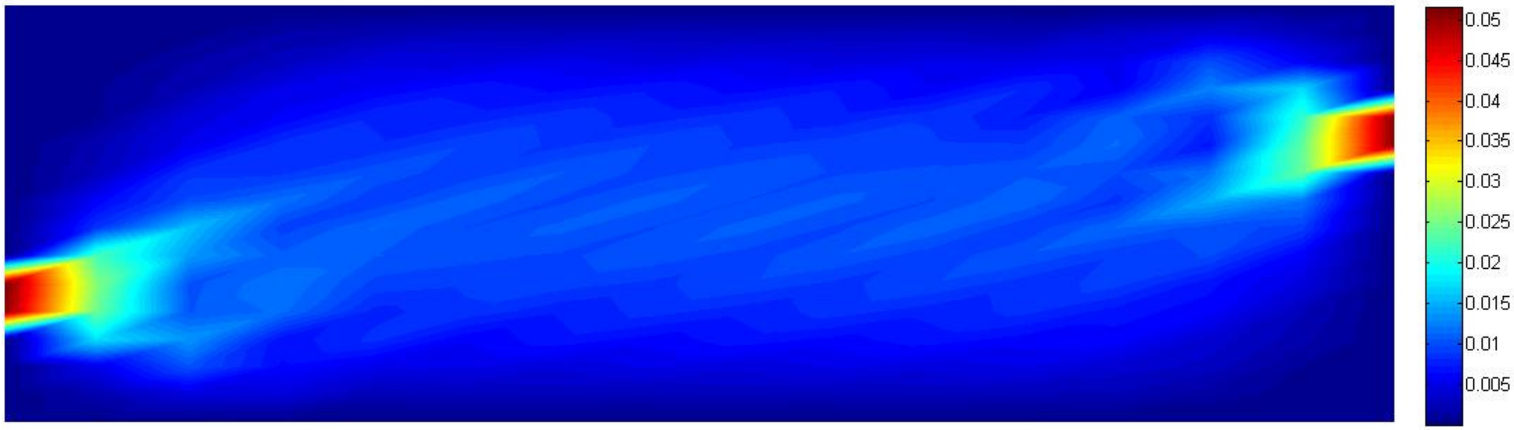
25x25 = 625 elements (4-gons)



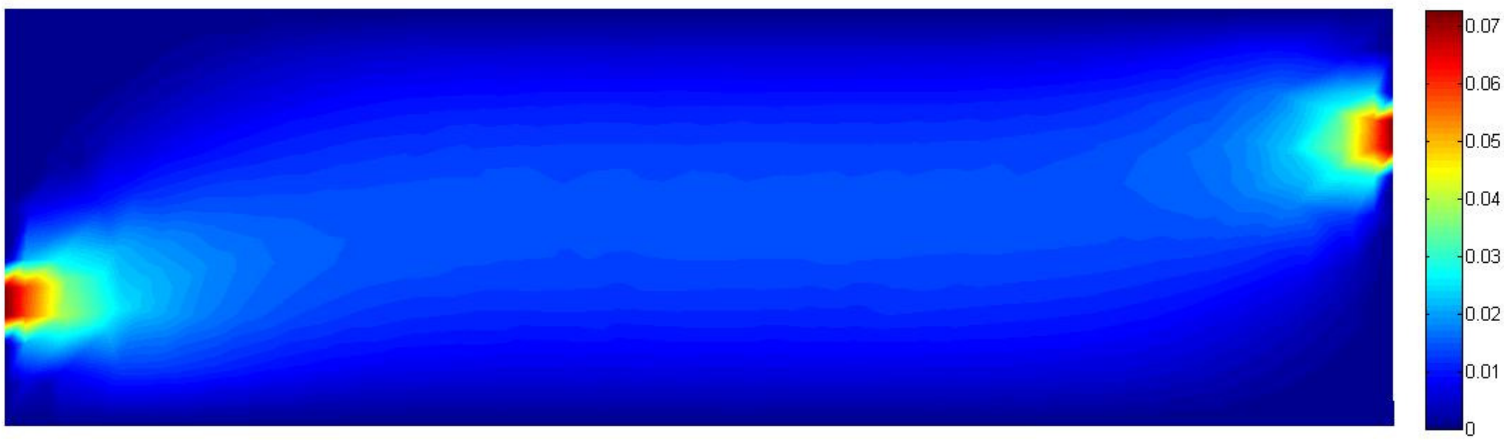
625 elements (n-gons)

Example: In/Out Flow Problem (cont.)

Contour Plot: Velocity Field (15x15 grid, 225 elements)



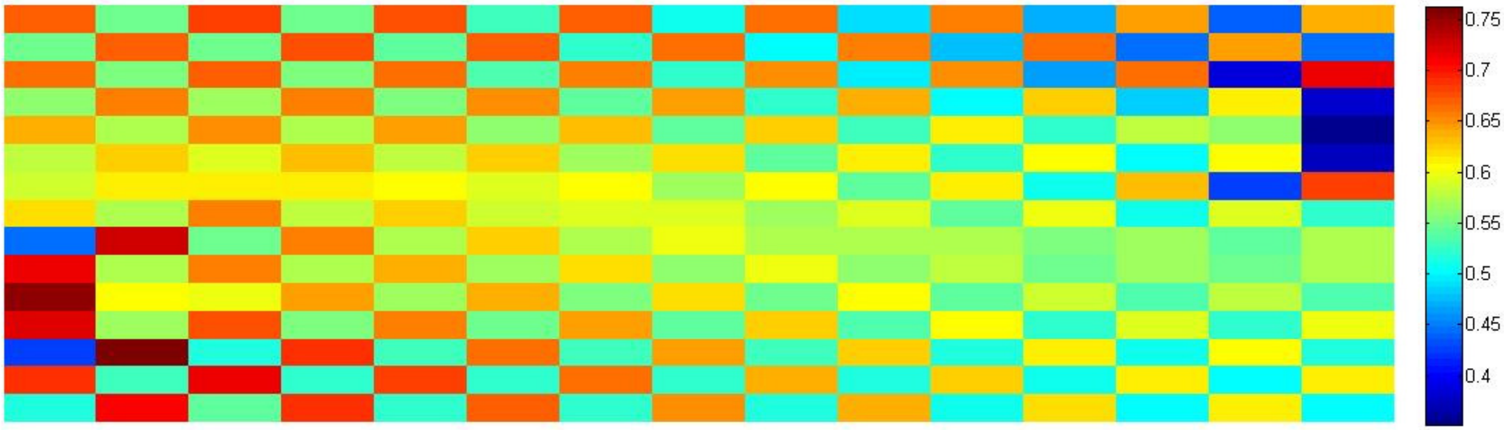
(a) Q4/1D



(b) n-gons/1D

Example: In/Out Flow Problem (cont.)

Contour Plot: Pressure Field (15x15 grid, 225 elements)

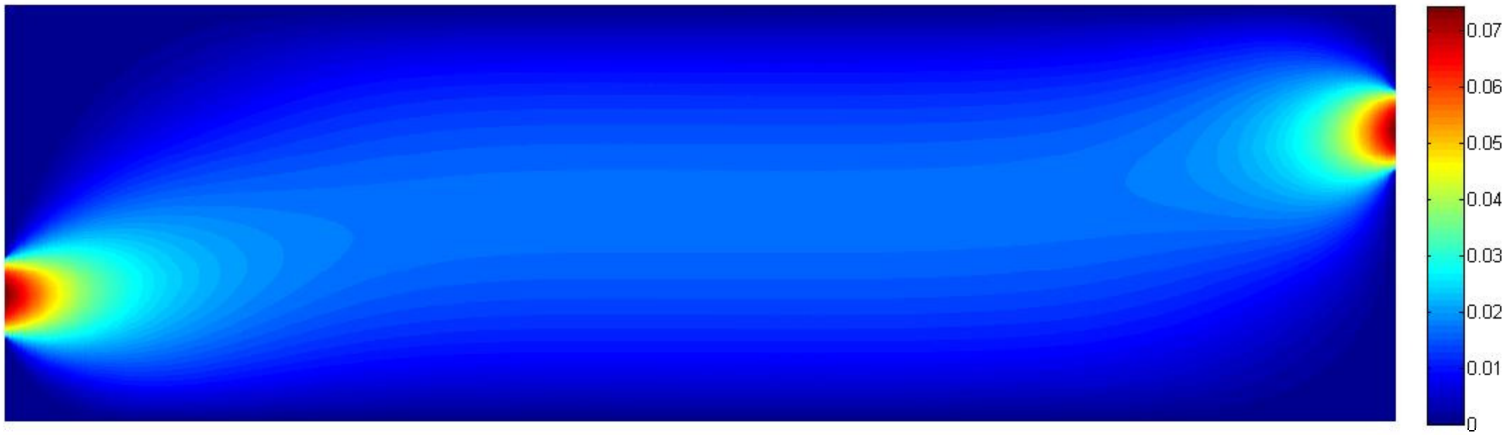


(a) Q4/1D

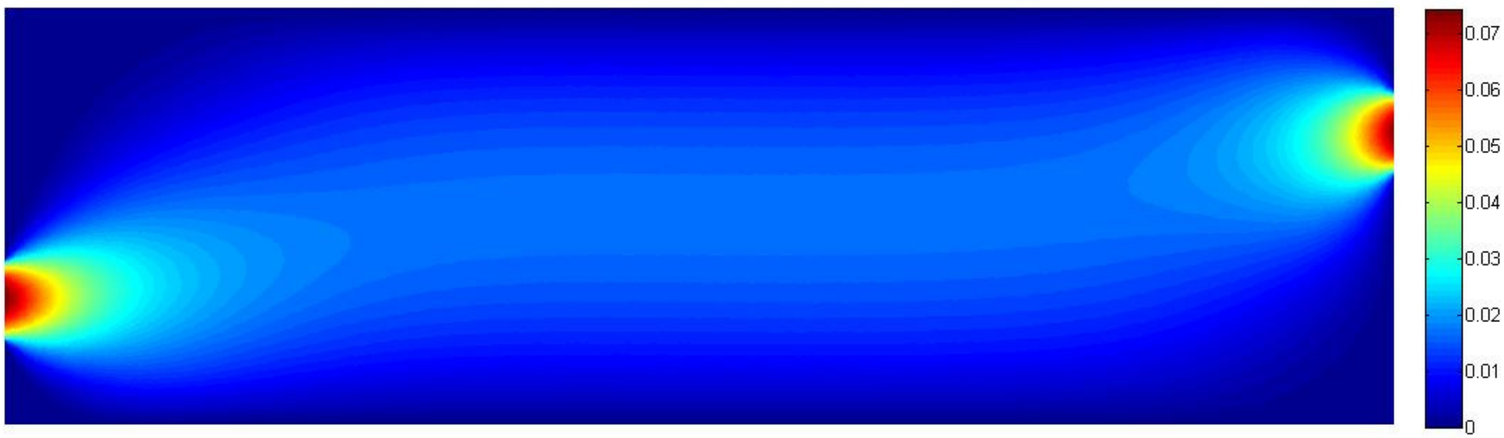
(b) n-gons/1D

Example: In/Out Flow Problem (cont.)

Contour Plot: Velocity Field (150x150 grid, 22500 elements)



(a) Q9/3D



(b) n-gons/1D

Example: In/Out Flow Problem (cont.)

Contour Plot: Pressure Field (150x150 grid, 22500 elements)

(a) Q9/3D

(b) n-gons/1D

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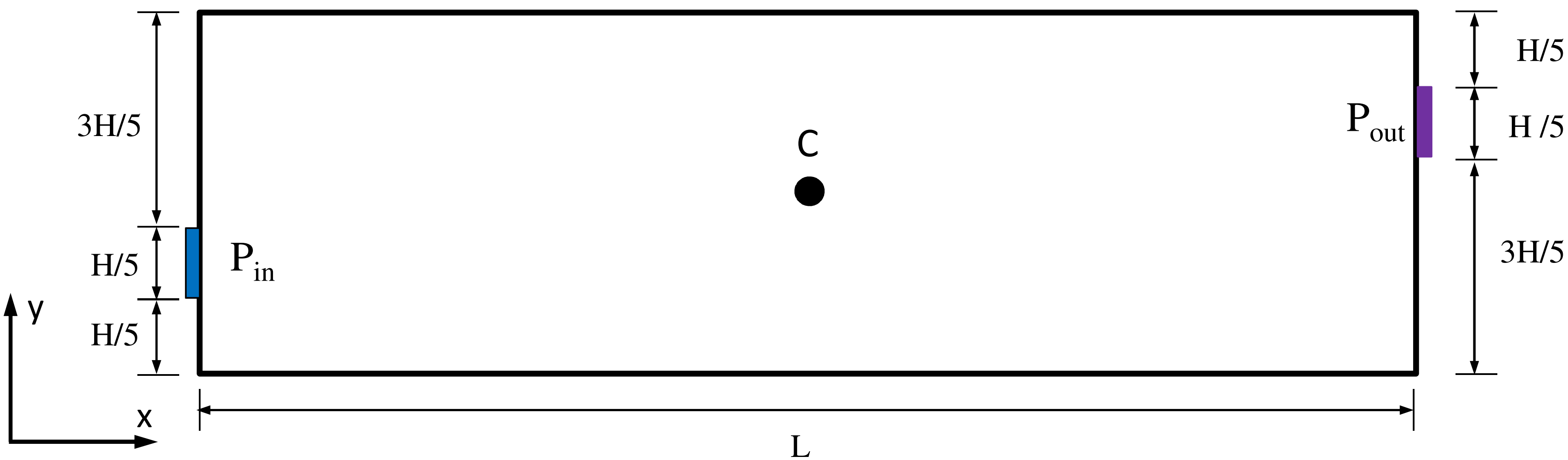
Problem Description:

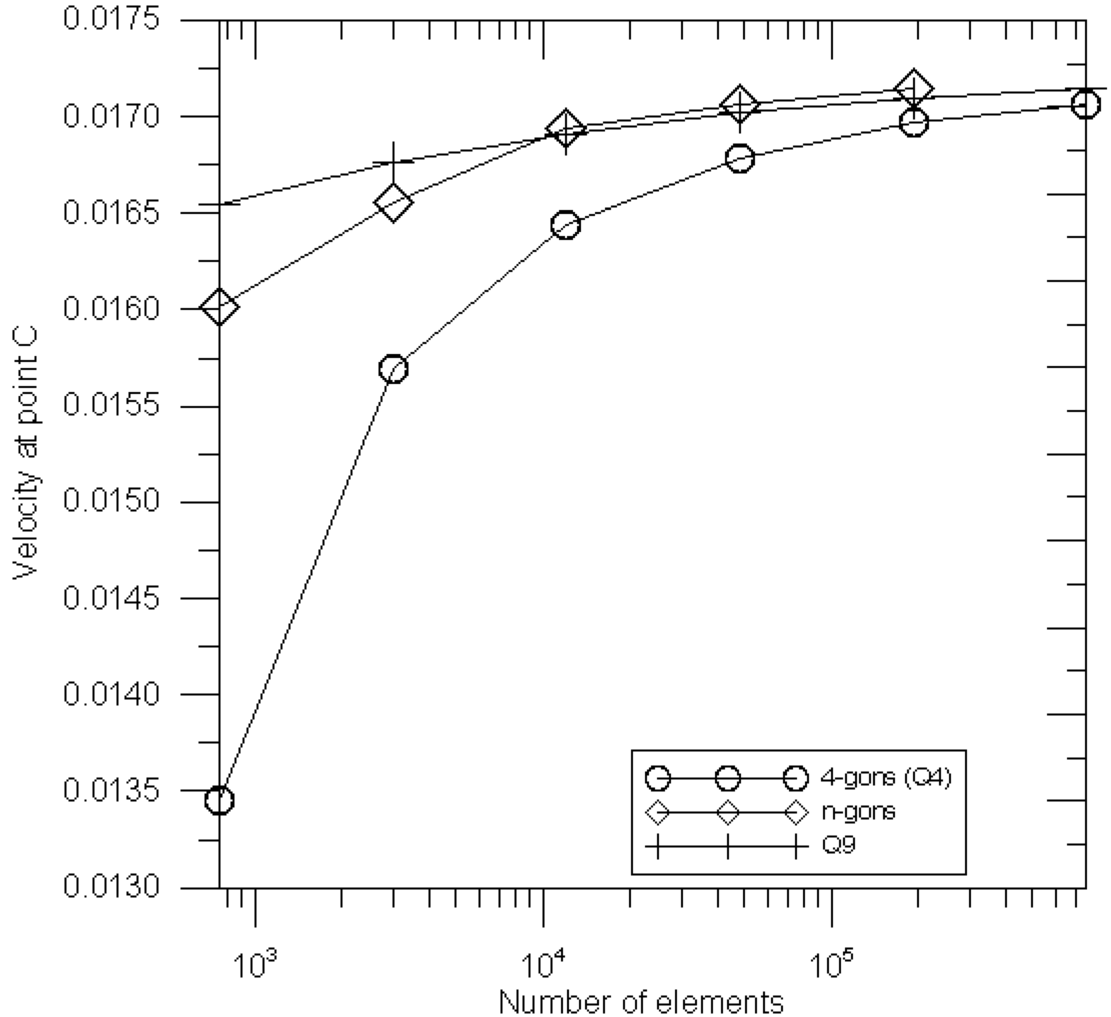


Coordinates of Point C: (L/2, H/2)

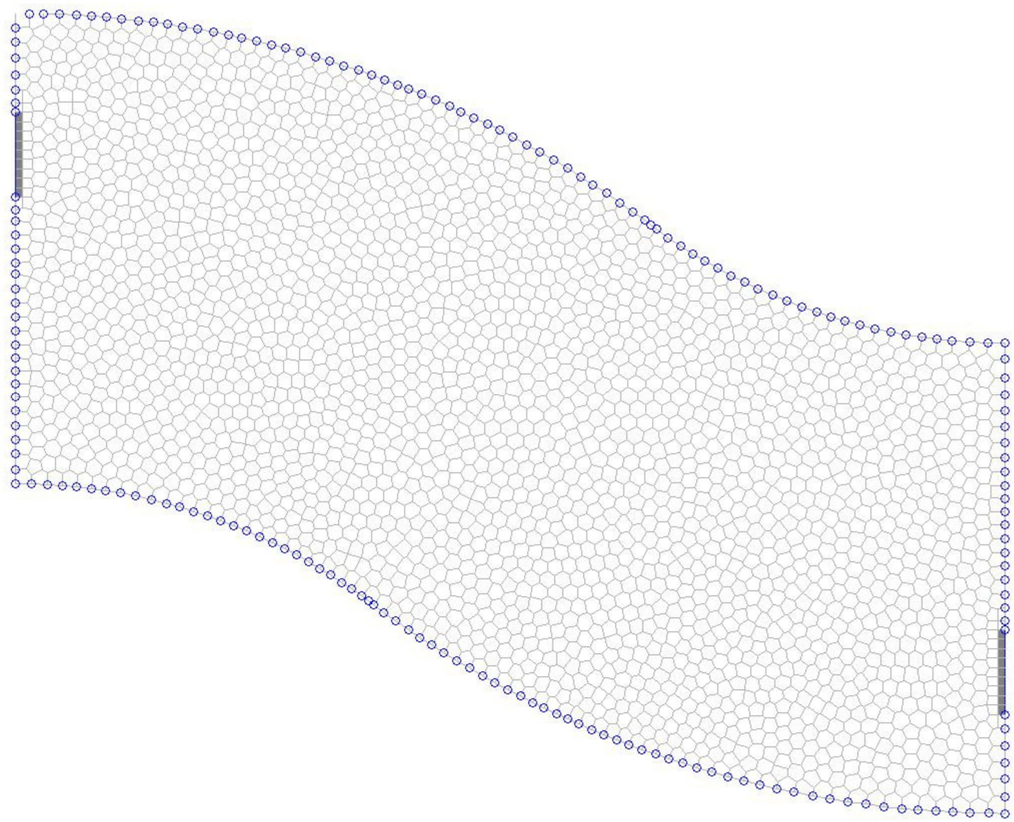
Convergence Analysis: Compute Velocity "u_x" at Point C

Convergence Analysis: In/Out Flow Problem (cont.)

Results using Q4 (*treated as 4-gons*), Q9 and n-gons:

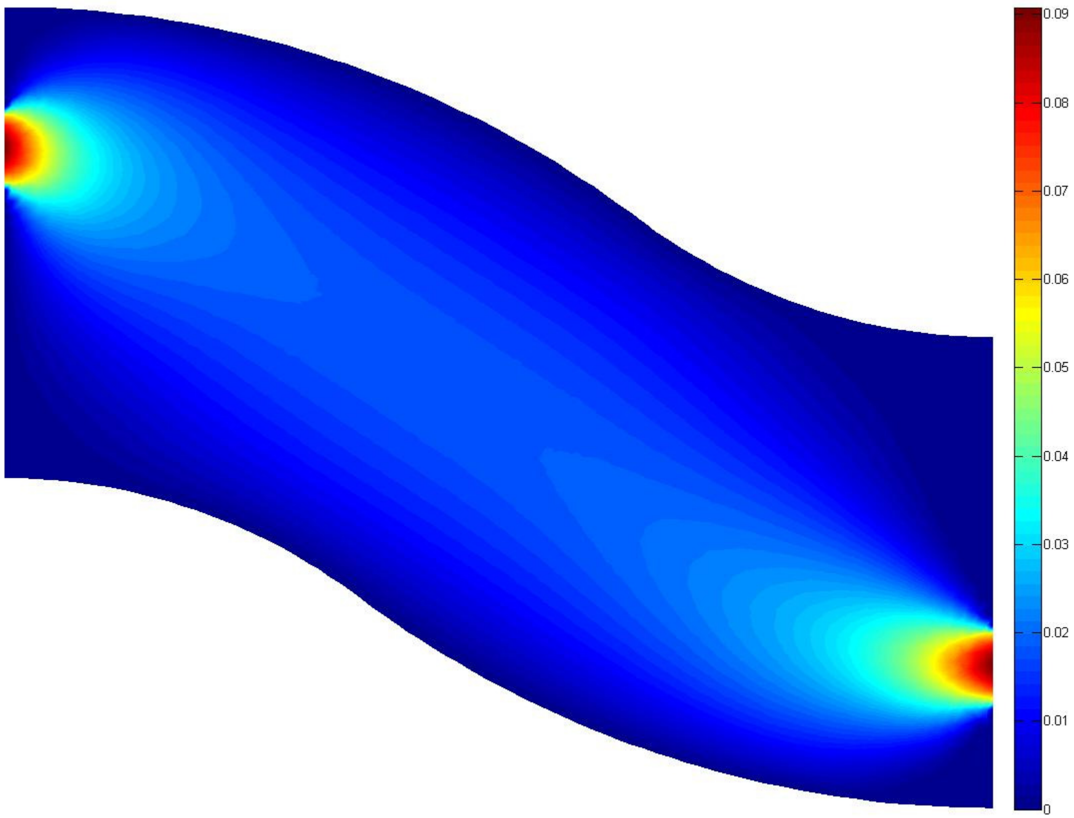


Examples with “More Complex” Domains



Curved Boundary

Examples with “More Complex” Domains



Velocity Field

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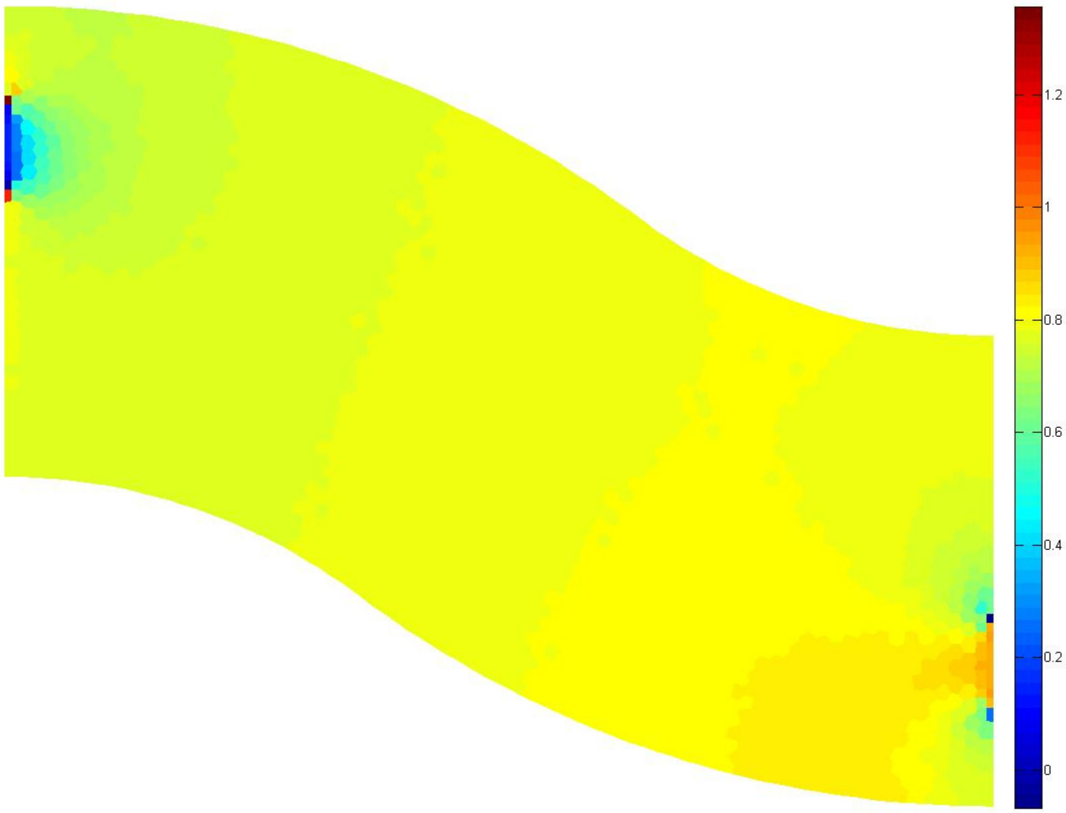
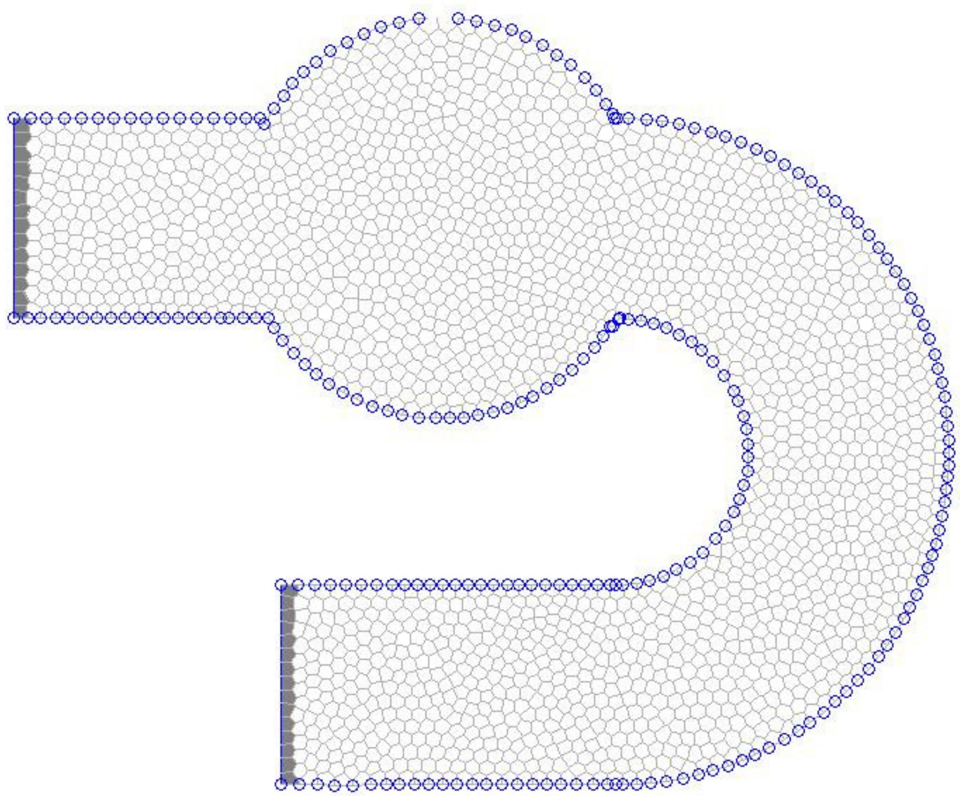


Pressure Field

Examples with “More Complex” Domains



Curved Domain

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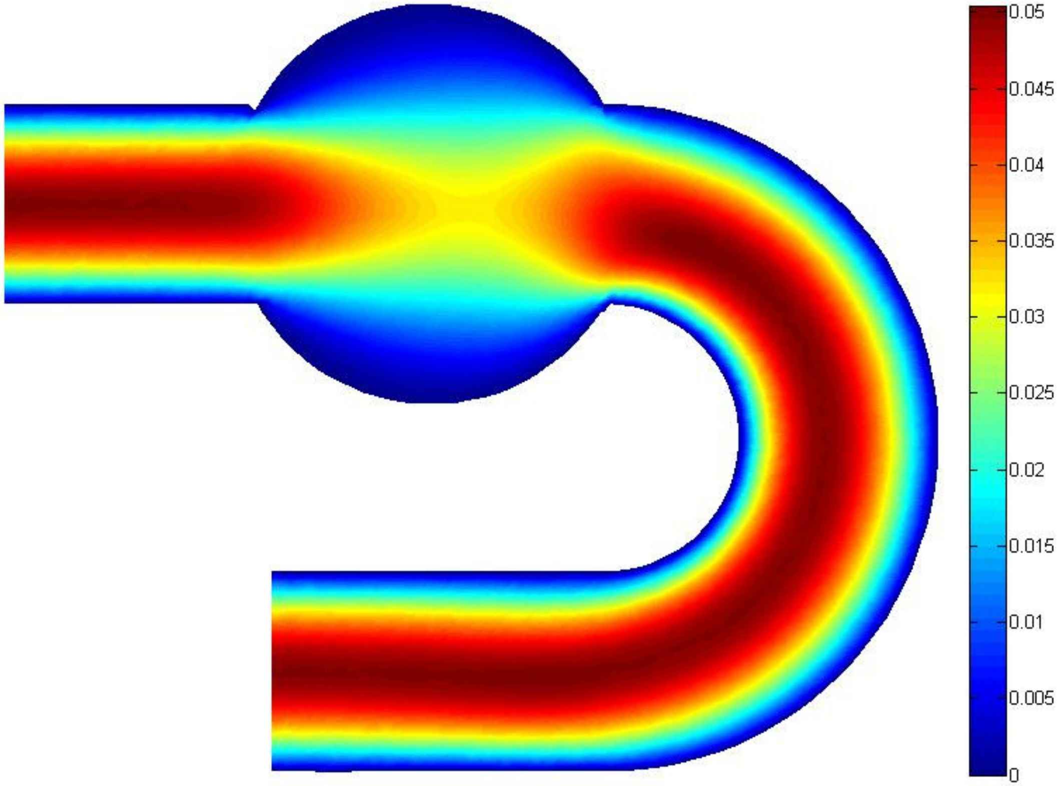

Curved Domain: Velocity Field

Examples with "More Complex" Domains

Curved Domain: Pressure Field

Concluding remarks

- Solutions of fluid flows problems may suffer from numerical instabilities depending on the choice of finite element approximation;
- The topology optimization formulation for design of milifluidics is part of ongoing work
- Polygonal elements offer an attractive avenue for FEM formulation of fluids
- The present goal is to extend the PolyMesher/PolyTop framework for topology optimization of special fluids

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Acknowledgment

Tecgraf - Computer Graphics Technology Group at the Pontifical Catholic University of Rio de Janeiro - PUC-Rio - Brazil

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