VISCOELASTIC FUNCTIONALLY GRADED FINITE ELEMENT METHOD USING RECURSIVE TIME INTEGRATION WITH APPLICATIONS TO THIN BONDED ASPHALT OVERLAYS

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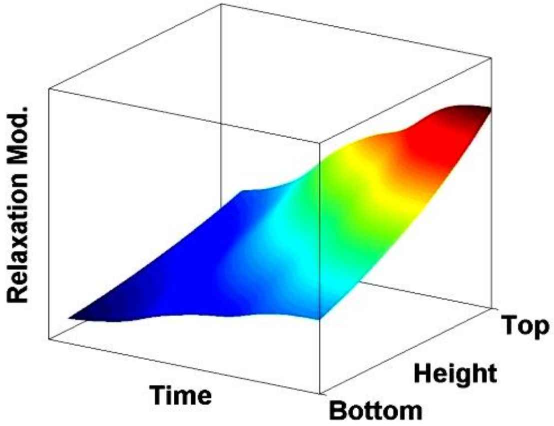
11th US National Congress on Computational Mechanics
Minneapolis, MN

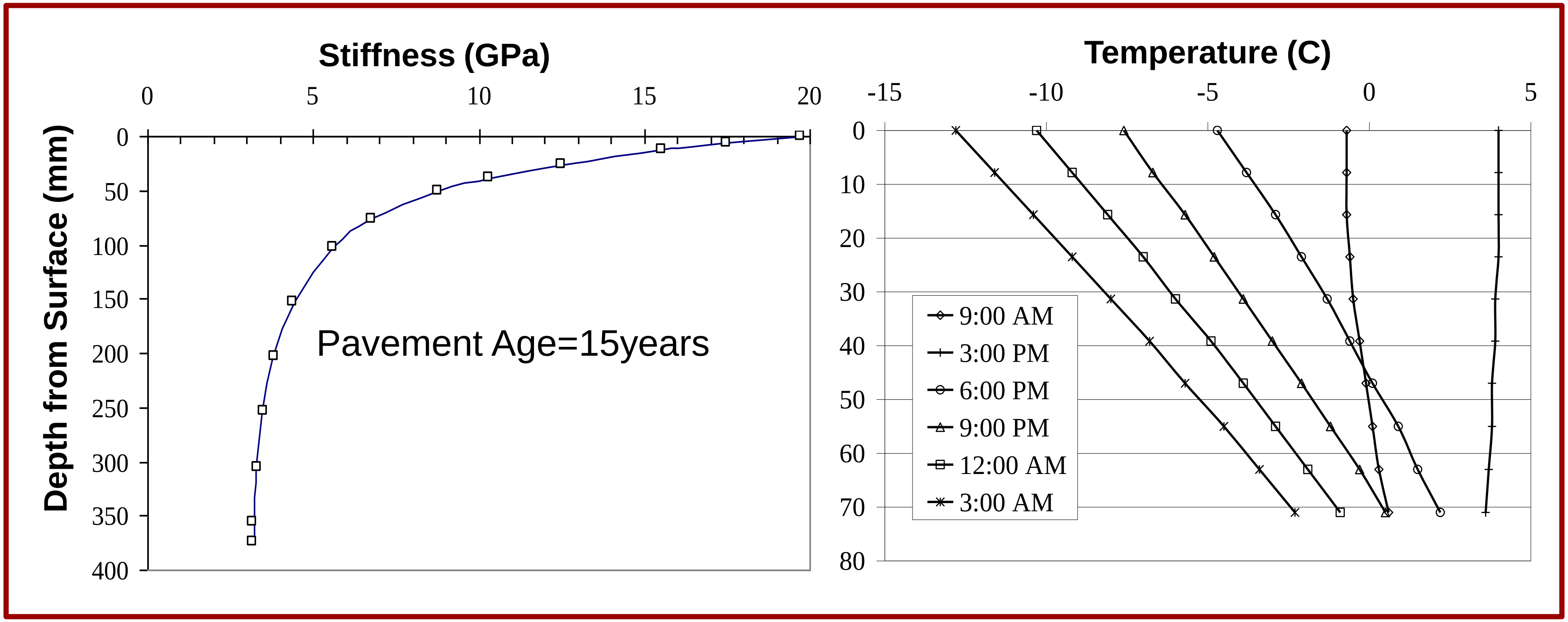
Outline

- Asphalt Pavements and Overlays as Functionally Graded Systems
- Functionally Graded Viscoelastic Finite Elements
  - Formulation
  - Verification
- Numerical Simulations of Thin Bonded Asphalt Overlays
  - Determination of Graded Properties
  - Simulation Results
- Summary and Future Extensions



Property Gradients in Asphalt Pavements

- Asphalt concrete exhibits heterogeneous behavior
- Smooth gradients can be approximated for certain effects:
  - Oxidative Aging
  - Temperature Non-uniformity

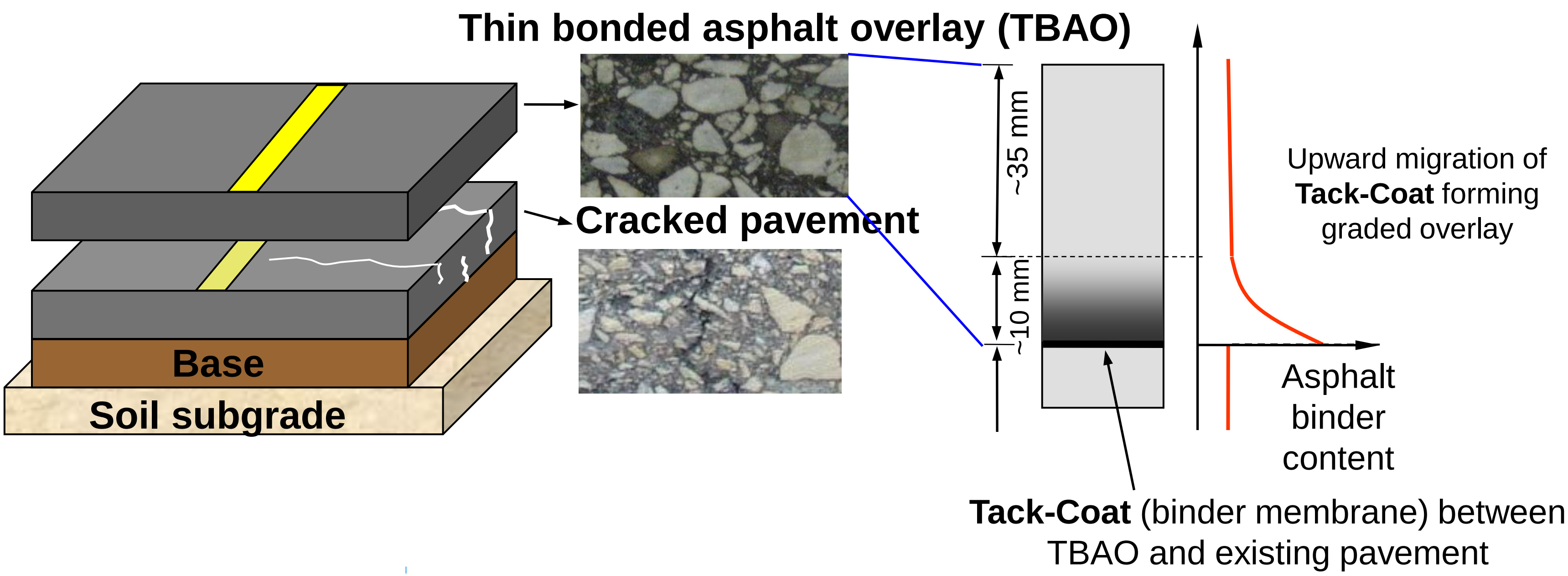


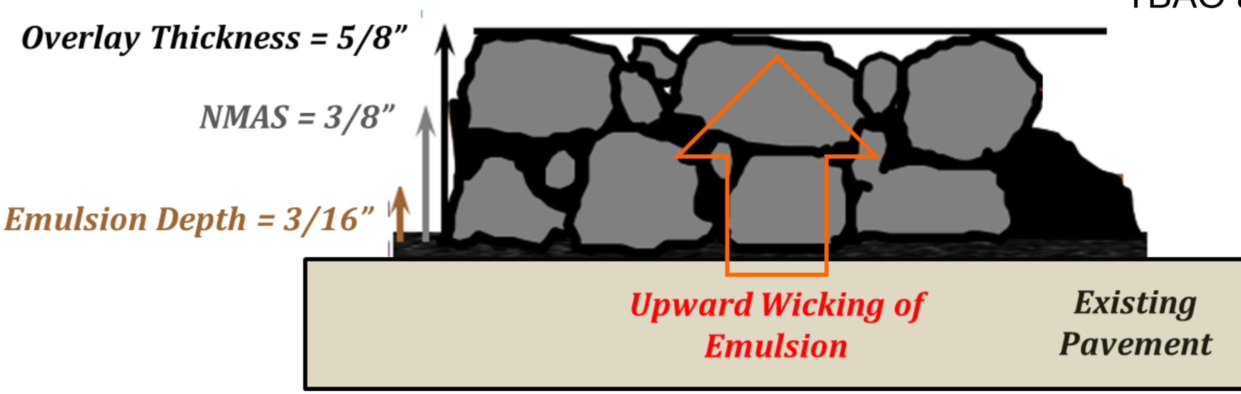
Aging gradient generated using "Global aging model" by Mirza and Witczak (1996)

Temperature profiles generated using "EICM" from AASHTO MEPDG (2002)

Thin Bonded Asphalt Overlays (TBAO)

- Use of specialized paving equipment for construction of TBAO → Graded System





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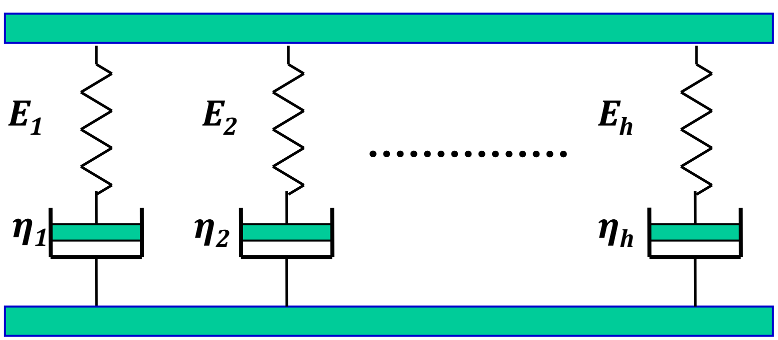
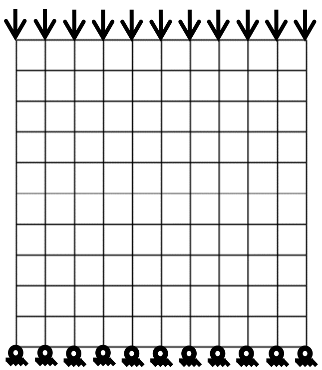
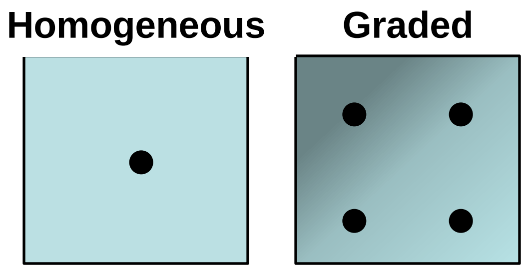




Brief Overview of Graded Finite Elements

- Graded Elements: Account for material non-homogeneity within elements unlike conventional (homogeneous) elements



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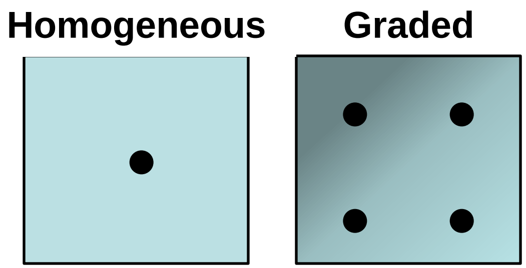

- Lee and Erdogan (1995) and Santare and Lambros (2000)
  - Direct Gaussian integration (properties sampled at integration points)

Y.D. Lee, and F. Erdogan, (1995) "Residual/thermal stresses in FGM and laminated thermal barrier coatings," International Journal of Fracture, 69:145-65.

M.H. Santare, and J. Lambros, (2000) "Use of graded finite elements to model the behavior of nonhomogeneous materials," Journal of Applied Mechanics, 67:819-22.

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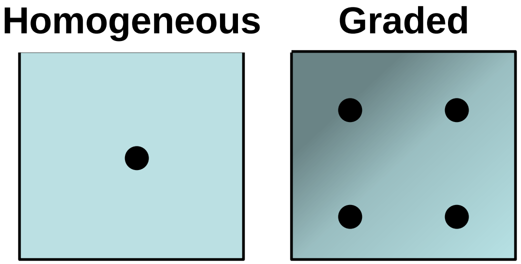

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- Kim and Paulino (2002)
  - Generalized isoparametric formulation (GIF)
- Paulino and Kim (2007) and Silva et al. (2007) further explored GIF graded elements
  - Proposed patch tests
  - GIF elements should be preferred for multiphysics applications

J.H. Kim, and G.H. Paulino, (2002) "Isoparametric graded finite elements for nonhomogeneous isotropic and orthotropic materials," Journal of Applied Mechanics, 69:502-14.

G.H. Paulino, and J.H. Kim, (2007) "The weak patch test for nonhomogeneous materials modeled with graded finite elements," Journal of the Brazilian Society of Mechanical Sciences and Engineering, 29:63-81.

E.C.N. Silva, R.C. Carbonari, and G.H. Paulino, (2007) "On graded elements for multiphysics applications," Smart Materials and Structures, 16:2408-2428.

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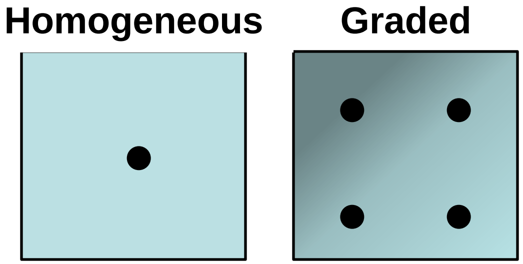

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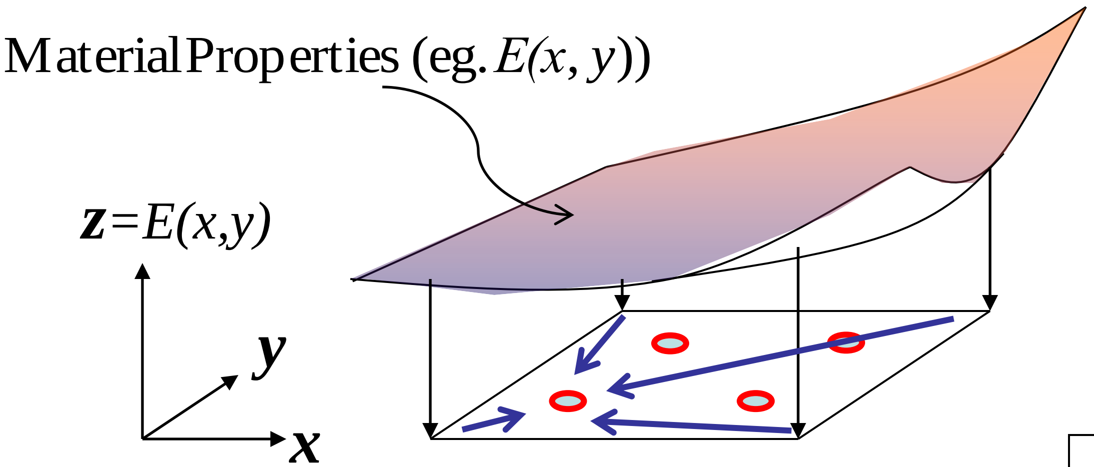
  - Proposed patch tests
  - GIF elements should be preferred for multiphysics applications
- Buttlar et al. (2006) demonstrated need of graded FE for asphalt pavements (elastic analysis)
- Dave et al. (2011) presented viscoealstic graded elements using correspondence principle

W.G. Buttlar, G.H. Paulino, and S.H. Song, (2006) "Application of graded finite elements for asphalt pavements," Journal of Engineering Mechanics, 132:240-249.

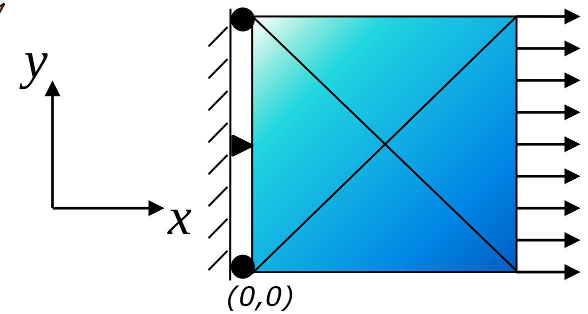
E.V. Dave, G.H. Paulino and S.H. Song, (2011) "Viscoelastic Functionally Graded Finite-Element Method Using Correspondence Principle," Journal of Materials in Civil Engineering, 23:39-48

Generalized Isoparametric Formulation for Finite Element Analysis of VFGMs

- Material properties are sampled at the element nodes
- Iso-parametric mapping provides material properties at integration points
- Natural extension of the conventional isoparametric formulation



$$E(x,y) = E_0 Exp[3x - 2y]$$

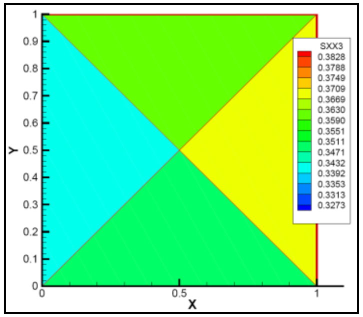


$$E(t) = \sum_{i=1}^{m} N_i \left[E(t) \right]_i$$

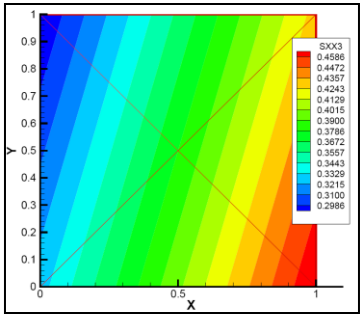
N_i = Shape function corresponding to node, i

m = Number of nodes per element

Conventional Homogeneous



GIF (FGM)



Constitutive Model

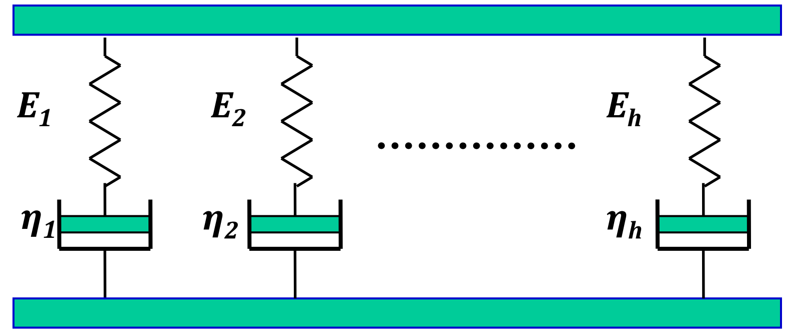
- Constitutive Relationship:

$$\sigma(x,t)=\int_{t'=-\infty}^{t'=t} C\left(x,\xi-\xi'\right)\frac{\partial\varepsilon\left(x,t'\right)}{\partial t'}dt'$$

σ : Stresses, $C(x,\xi)$: Relaxation Modulus, ε: Strains

- Material Representation: Generalized Maxwell Model

$$C(x,t)=\sum_{h=1}^{N}E_h(x)Exp\left[-\frac{t}{\tau_h(x)}\right]$$



Relaxation Time, $\tau_h=\dfrac{\eta_h}{E_h}$

- Time-Temperature Superposition

Reduced Time, $\xi(x,t)=\int_0^{t'} a(T,x,t)dt'$

Functionally Graded Viscoelastic Finite Element

- Problem Description:

$$K_{ij}\left(x,\xi(t)\right)u_j(0)+\int_{0^+}^{t}K_{ij}\left(x,\xi(t)-\xi(t^{'})\right)\frac{\partial u_j(t^{'})}{\partial t^{'}}dt^{'}=F_i(x,t)$$

- Solution approaches:

1. Correspondence Principle (CP)

$$\left[K^0(x)s\tilde{K}^t(s)\right]_{ij}\tilde{u}_j(s)=\tilde{F}_i(x,s)$$

$\tilde{a}(s)$ is Laplace transform of $a(t)$; s is transformation variable

$$\tilde{a}(s)=\int_0^\infty a(t)\,Exp[-st]dt$$

2. Time-Integration Schemes

- *Direct Integration*
- *Recursive Formulation*
- *Recursive-Incremental Formulation*

Time Integration Approach

$$K_{ij}\left(x,\xi\right)u_j\left(0\right)+\int_{0^+}^{t}K_{ij}\left(x,\xi-\xi'\right)\frac{\partial u_j\left(t'\right)}{\partial t'}dt'=F_i\left(x,t\right)$$

- Above could be solved sequentially using Newton-Cotes expansion (material history effect needs to be considered)

$$u_j\left(t_n\right)=\left[K_{ij}\left(x,0\right)+K_{ij}\left(\xi_n-\xi_{n-1}\right)\right]^{-1}\left\{\begin{aligned}&2F_i\left(t_n\right)-\left[K_{ij}\left(\xi_n\right)-K_{ij}\left(\xi_n-\xi_1\right)\right]u_j\left(0\right)\\&-\sum_{m=1}^{n-1}\left[K_{ij}\left(\xi_n-\xi_{m-1}\right)-K_{ij}\left(\xi_n-\xi_{m+1}\right)\right]\boxed{u_j\left(t_m\right)}\end{aligned}\right\}$$

- Alternatively, recursive formulation could be developed that requires only few previous solutions

Time-Integration Analysis

Recursive Formulation (extension from Yi and Hilton, 1994):

$$\left[\sum_{h=1}^{m}\left(K_{ij}^{e}(x)\right)_h\cdot\left[\left(v_{ij}^{1}(x,t_n)\right)_h\Delta t-\left(v_{ij}^{2}(x,t_n)\right)_h\right]\frac{2}{\Delta t^{2}}\right]u_j(t_n)=F_i(t_n)$$

$$+\sum_{h=1}^{m}\left[\left[\left(K_{ij}^{e}(x)\right)_h\cdot Exp\left[-\frac{\xi(t_n)}{\left(\tau_{ij}(x)\right)_h}\right]\right]\left\{\left(v_{ij}^{1}(x,t_{n-1})\right)_h\left[u_j(t_{n-1})\frac{2}{\Delta t}+\dot{u}_j(t_{n-1})\right]\right.\right.$$

$$\left.\left.-\frac{2}{\Delta t^{2}}\left(v_{ij}^{2}(x,t_{n-1})\right)_h\left[u_j(t_{n-1})+\dot{u}_j(t_{n-1})\Delta t\right]-u_i(t_0)+\left(v_{ij}^{1}(x,t_0)\right)\dot{u}_j(t_0)\right\}+\left(R_i(t_n)\right)_h\right]$$

Where,

$$\left(v_{ij}^{1}(x,t_n)\right)_h=\int_{0}^{t_n}Exp\left[-\xi(t')/\left(\tau_{ij}(x)\right)_h\right]dt';\ \left(v_{ij}^{2}(x,t_n)\right)_h=\int_{t_{n-1}}^{t_n}\left(v_{ij}^{1}(x,t')\right)_h dt'$$

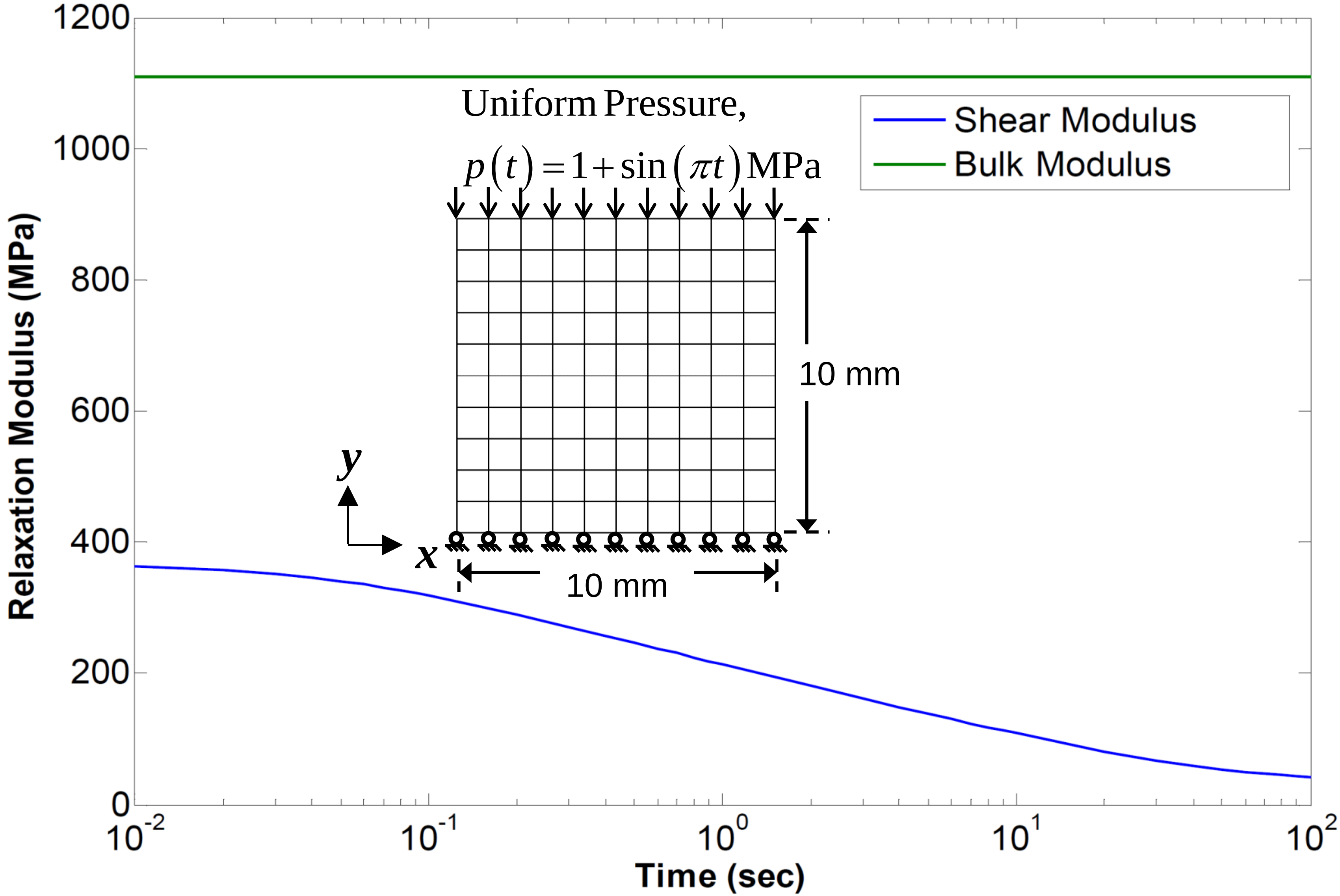
$$\left(R_i(t_n)\right)_h=K_{ij}^{e}\cdot Exp\left[-\xi(t')/\left(\tau_{ij}(x)\right)_h\right]\cdot\left(v_{ij}^{2}(x,t_n)\right)_h\ddot{u}_j(t_{n-1})$$

$$+Exp\left[-\xi(t')/\left(\tau_{ij}(x)\right)_h\right]\left(R_j(t_{n-1})\right)_h$$

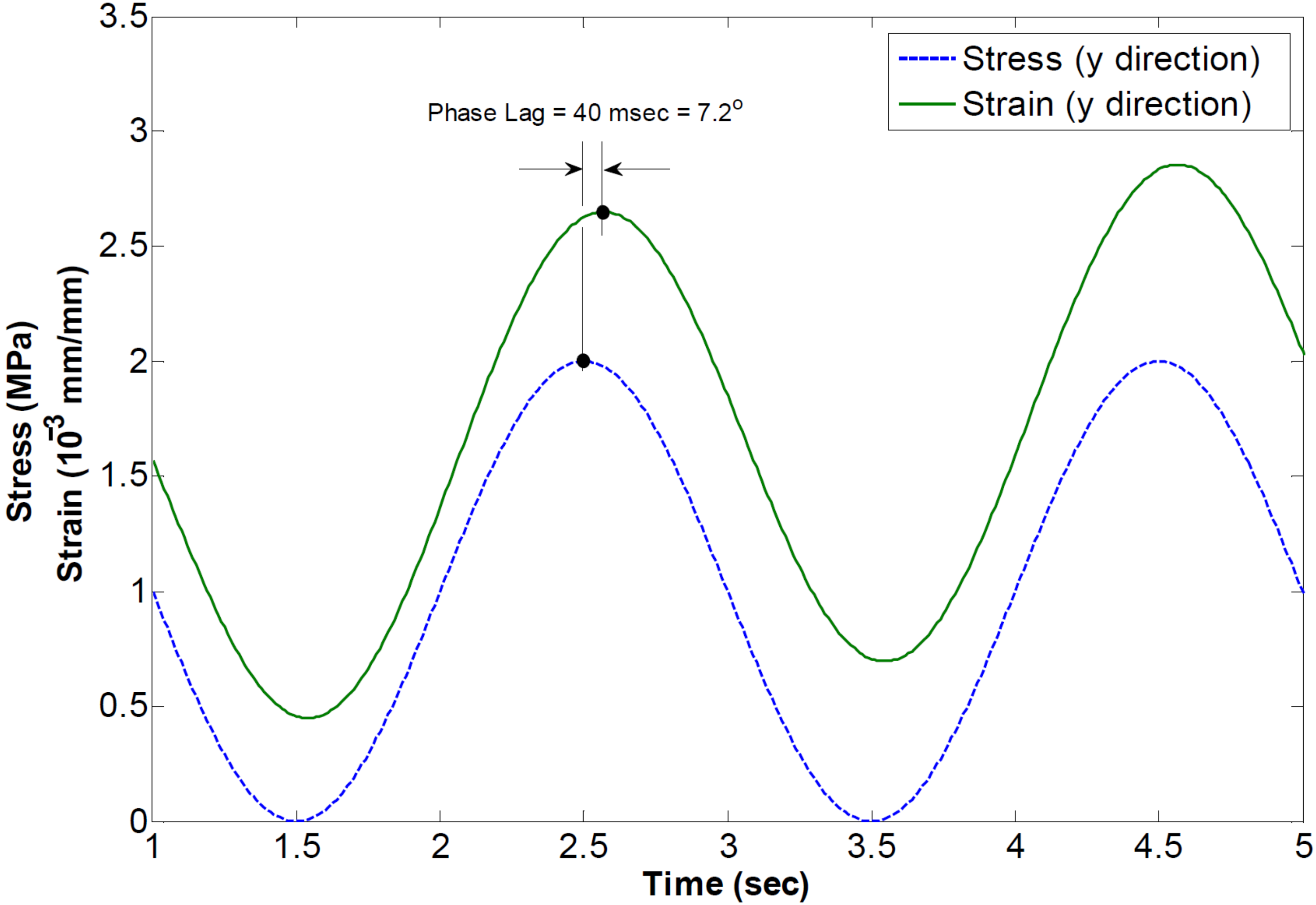
S. Yi, and H.H. Hilton, (1994) "Dynamic finite element analysis of viscoelastic composite plates in the time domain," International Journal for Numerical Methods in Engineering, 37:4081-96.

Efficiency and Accuracy Comparison

- Example: Cyclic Loading of Viscoelastic Body



Response of VE Body to Sinusoidal Loading



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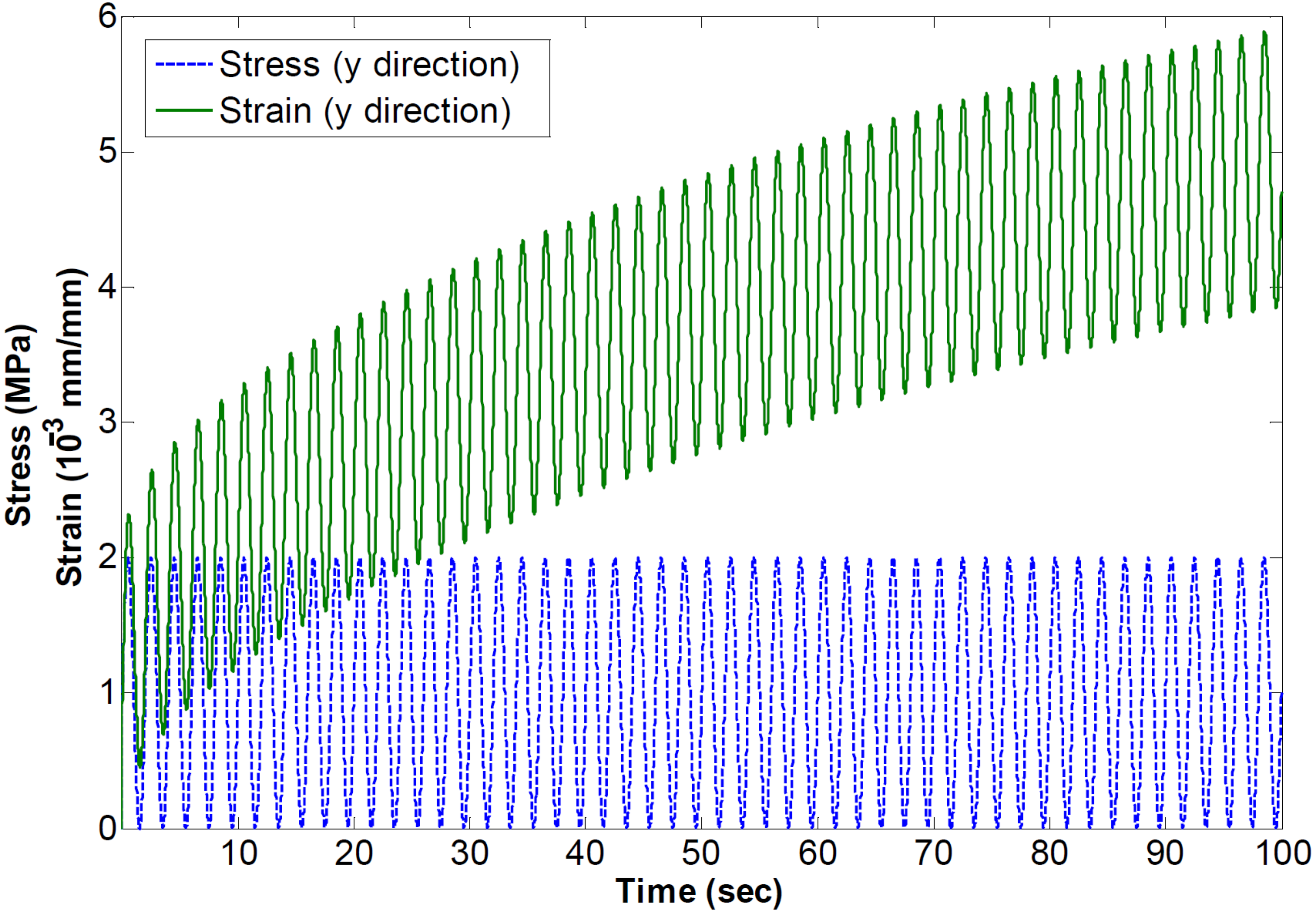
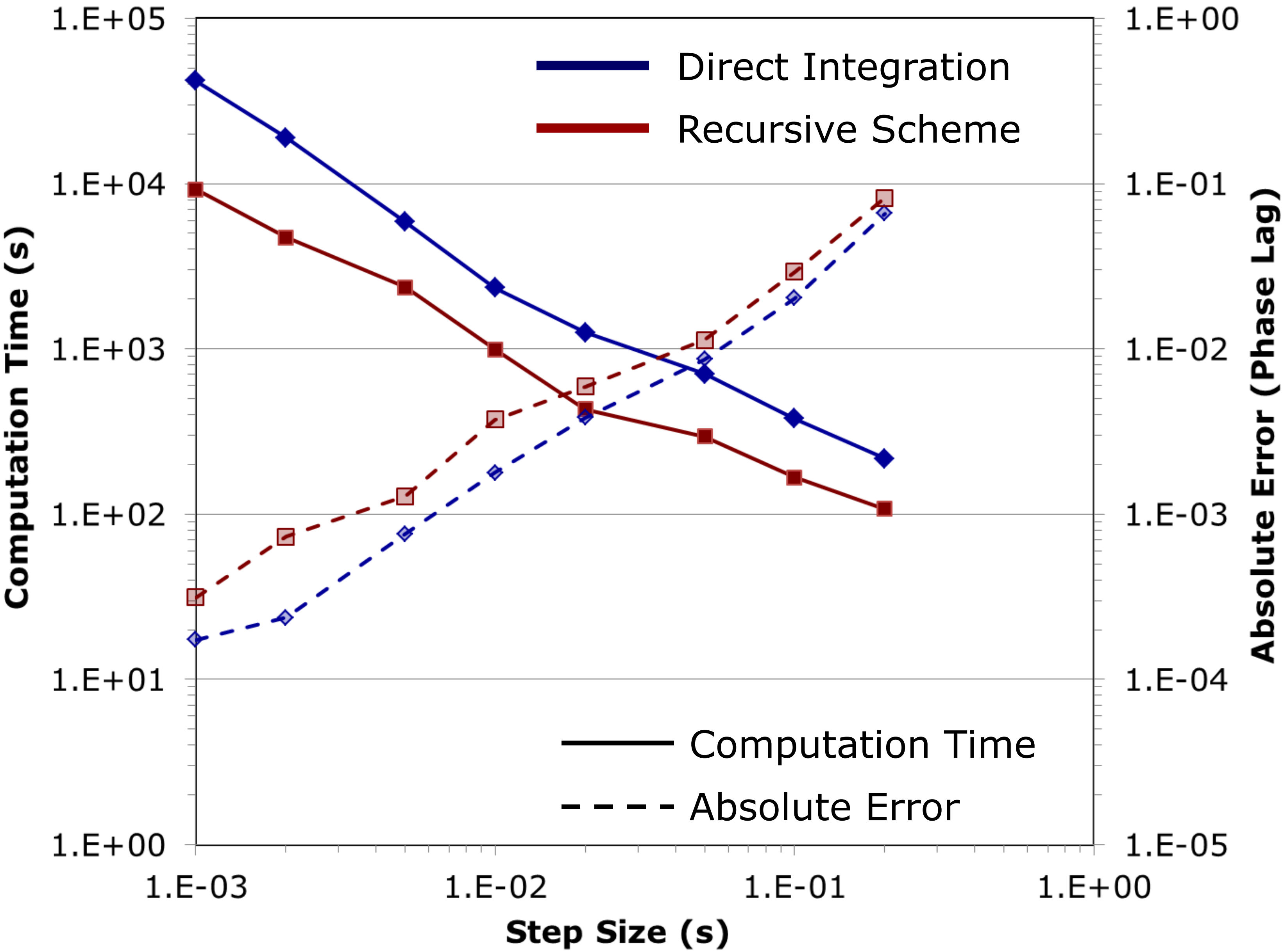


Absolute Error and Simulation Time



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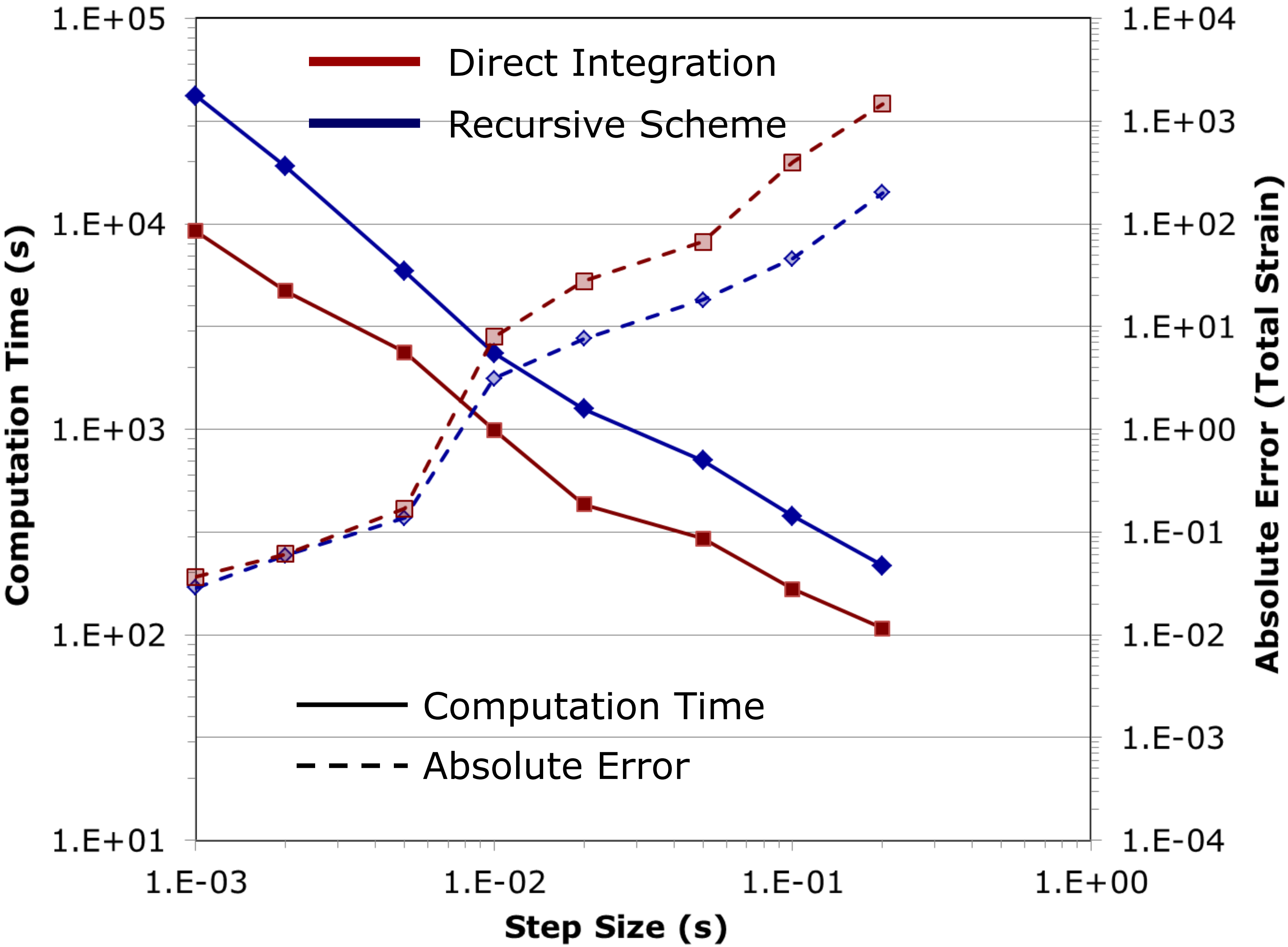
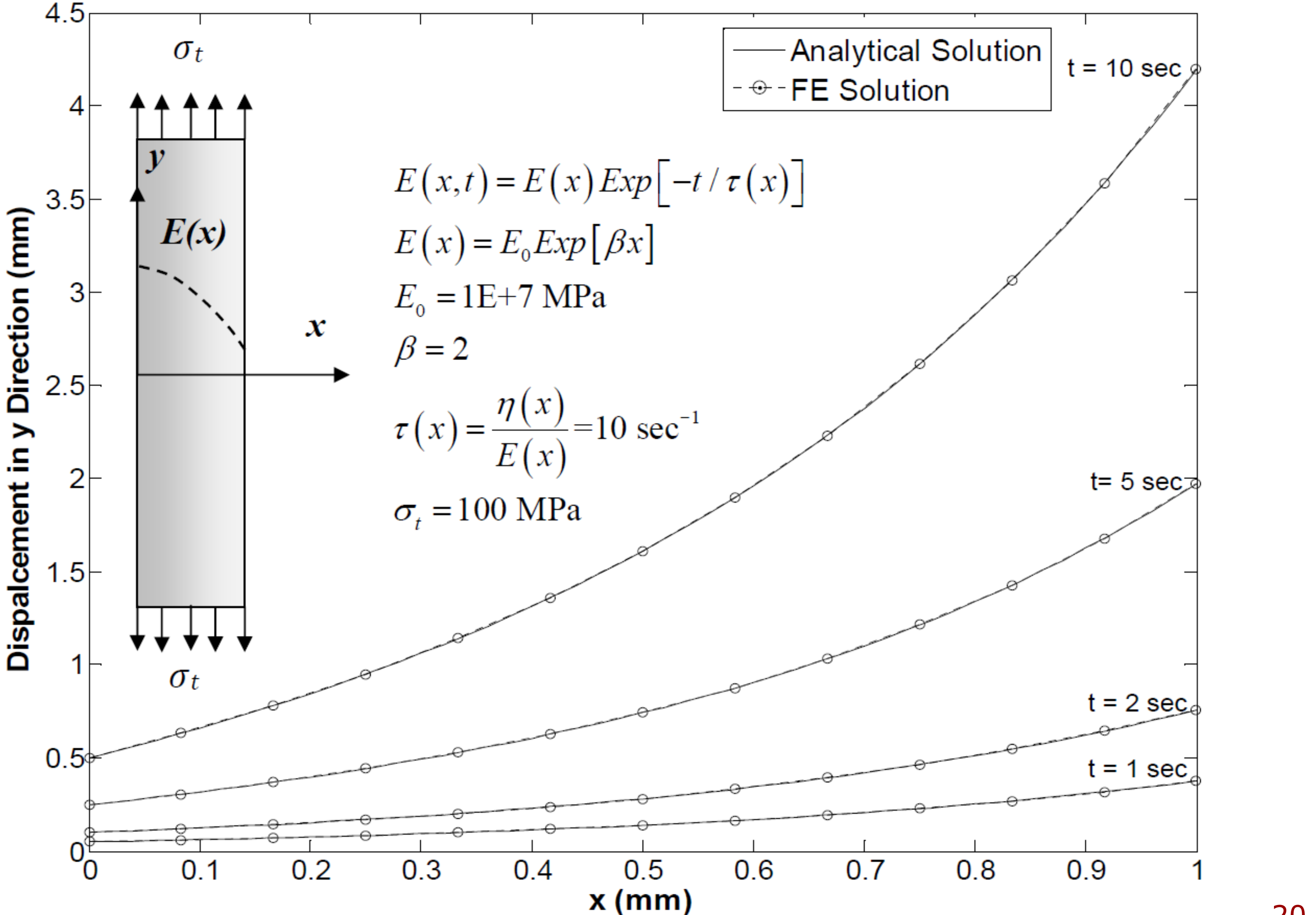


Verification Example



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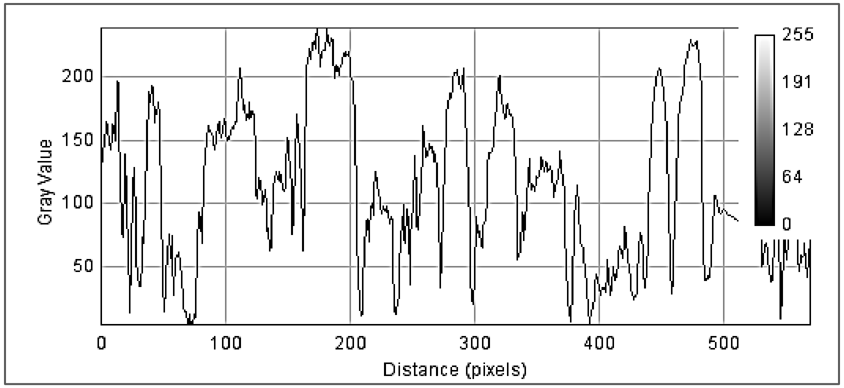
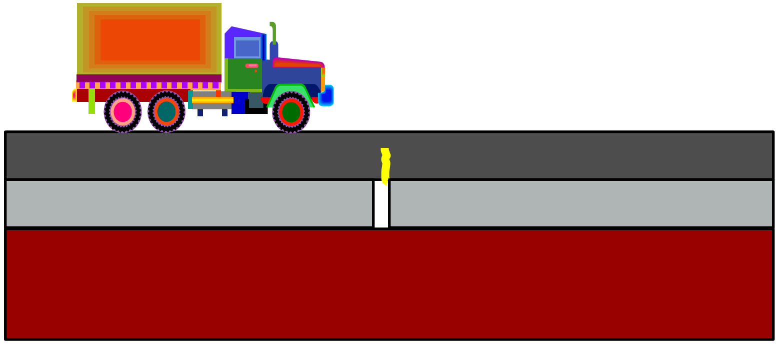
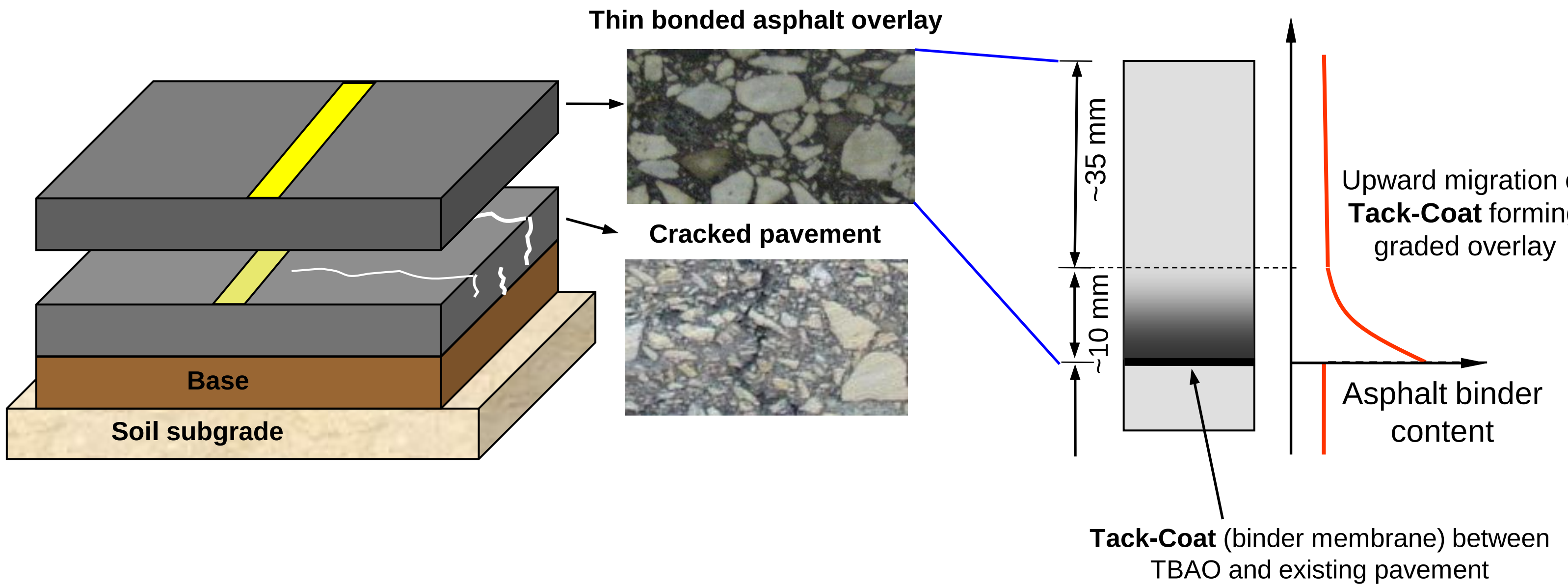




Evaluation of Graded Properties of TBAO

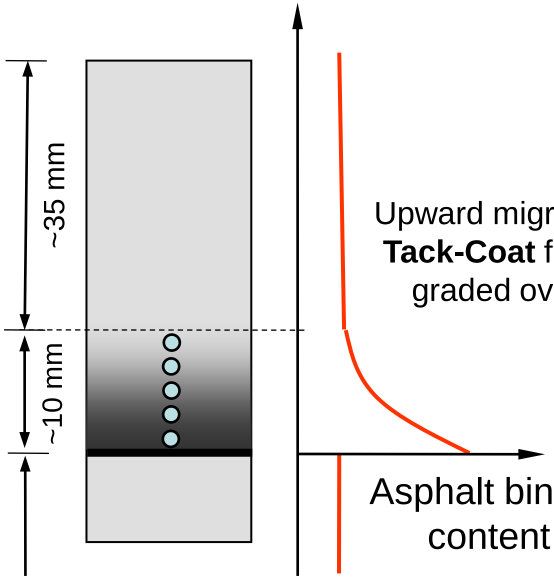
- Objective is to determine graded properties



- Needed Information:
  - Quantification of tack-coat permeation into TBAO
  - Effect of tack coat on properties

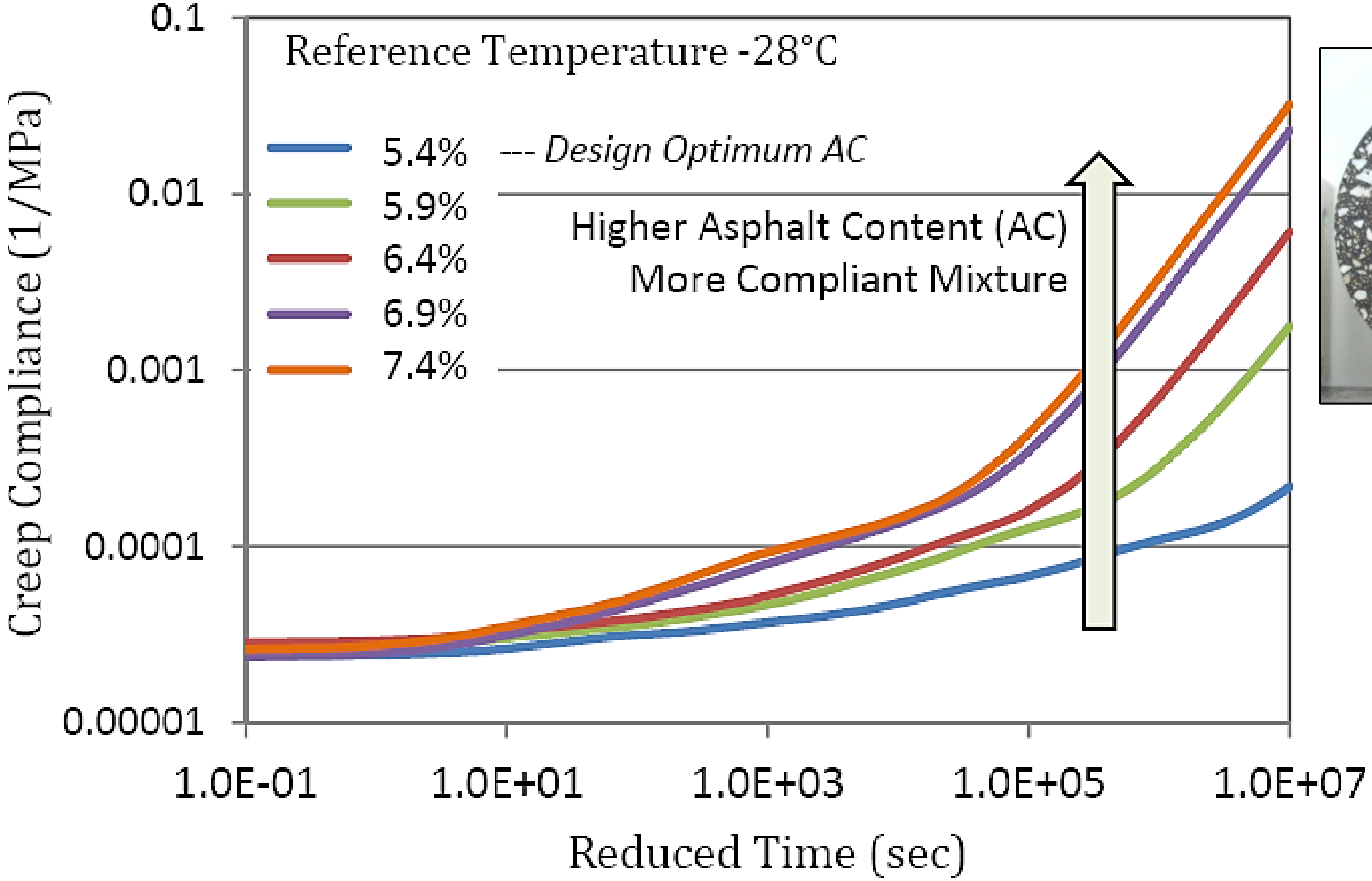
Graded Property Determination

- Research Approach:
  - Test asphalt concrete samples with different amounts of tack-coat emulsion
  - Use imaging technique to characterize tack-coat permeation



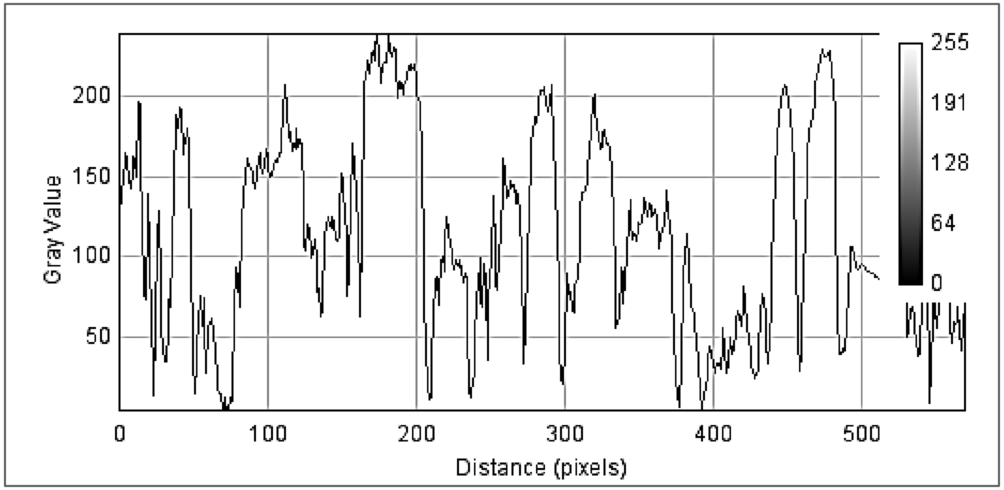
- Experiment Matrix:
  - 2 Mixes
    - *Gap Graded and Dense Graded*
  - Emulsion Added at 0, 0.5, 1.0, 1.5 and 2.0%

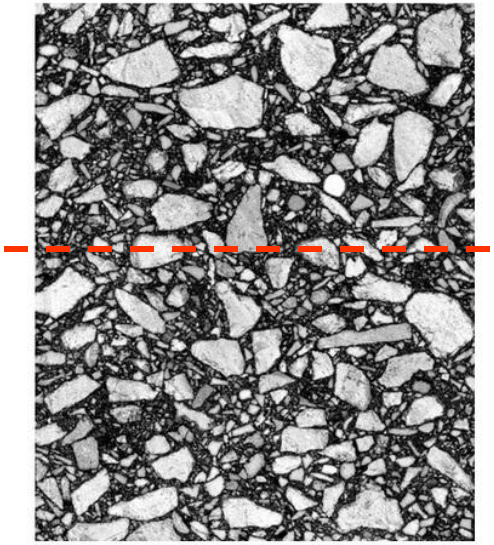
Effect of Tack Coat Viscoelastic Properties

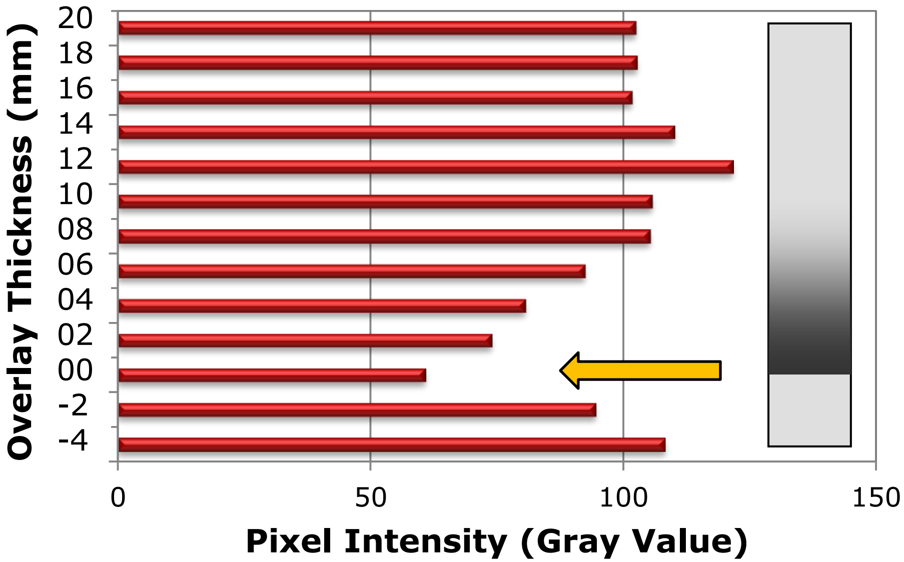


Tack Coat Permeation

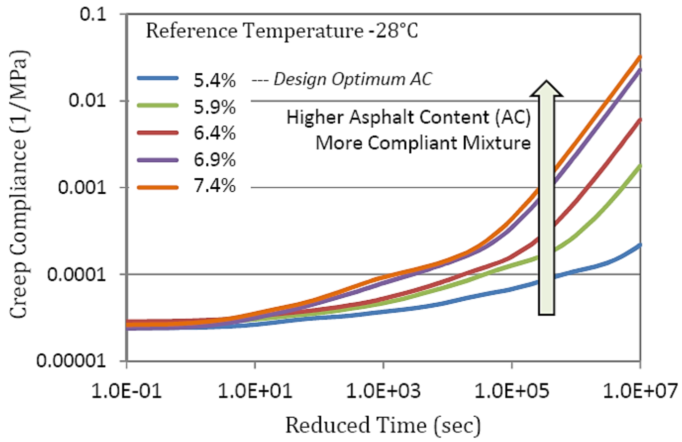
- Scanned images from sliced field cores were utilized
- Gray-scale intensities were determined using open-source software "*Image J*"

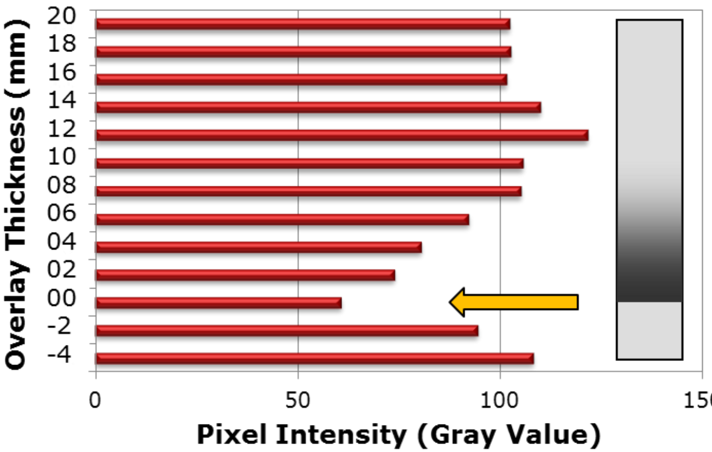


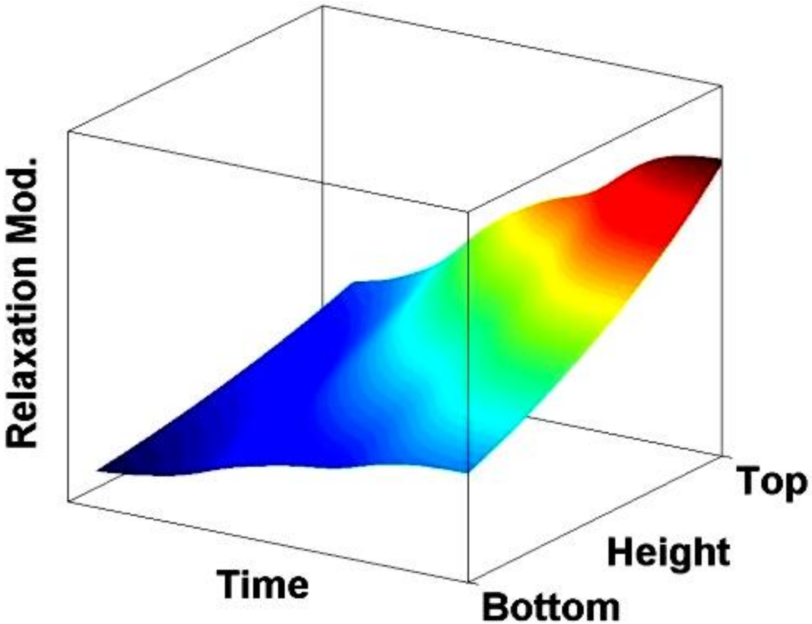




Graded Properties of TBAO







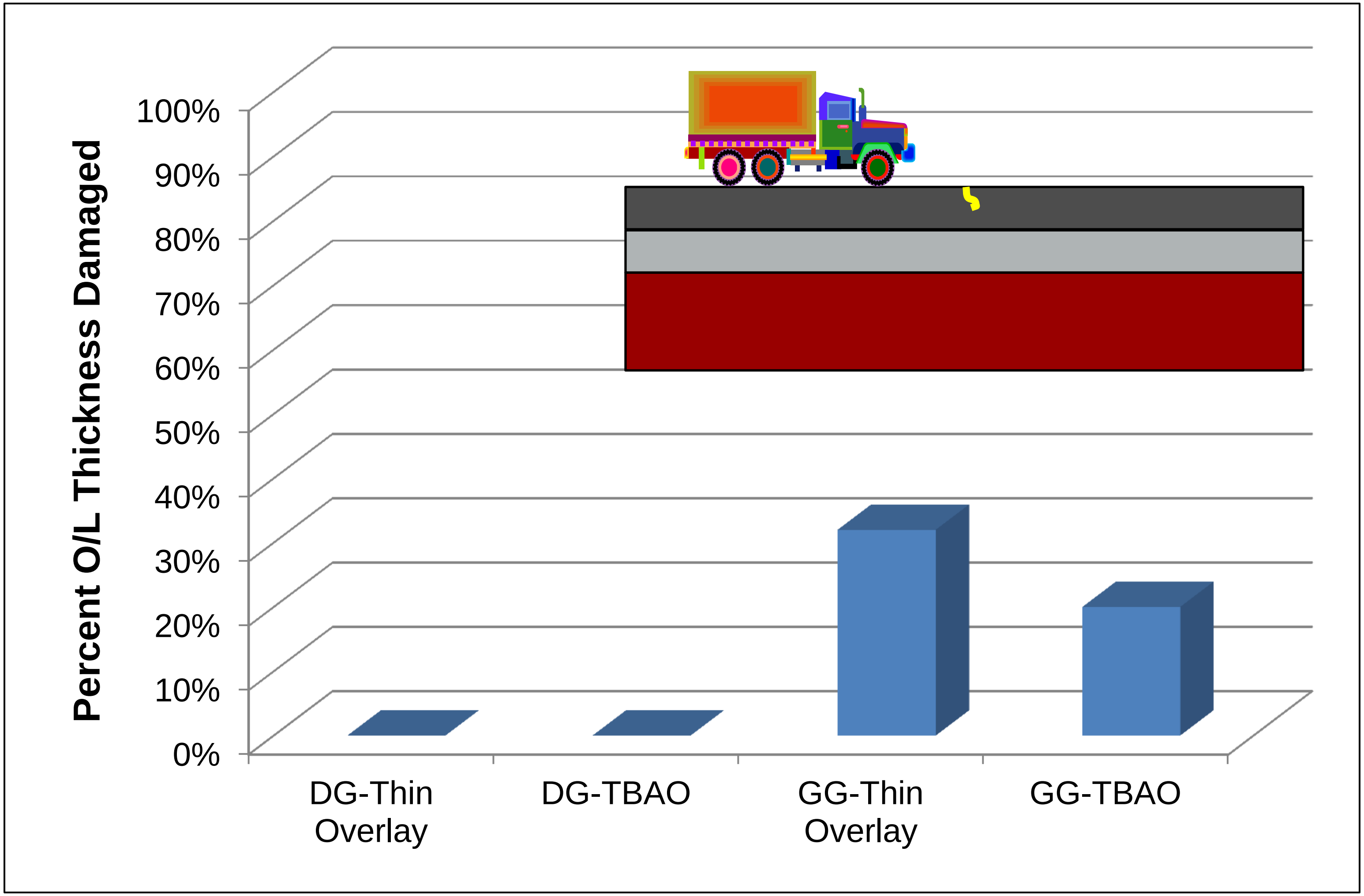
FE Pavement Model: Features

- *Asphalt*: Viscoelastic FGM (Time, Temperature and Space Dependent)
- *Other Layers (PCC, Gran. Base, Subgrade): Elastic*
- *Interfaces*: Finite Slip Frictional Contact Interface
- *Fracture:* Cohesive Zone Model
- Pavement Temperature Variation
  - EICM (MEPDG)

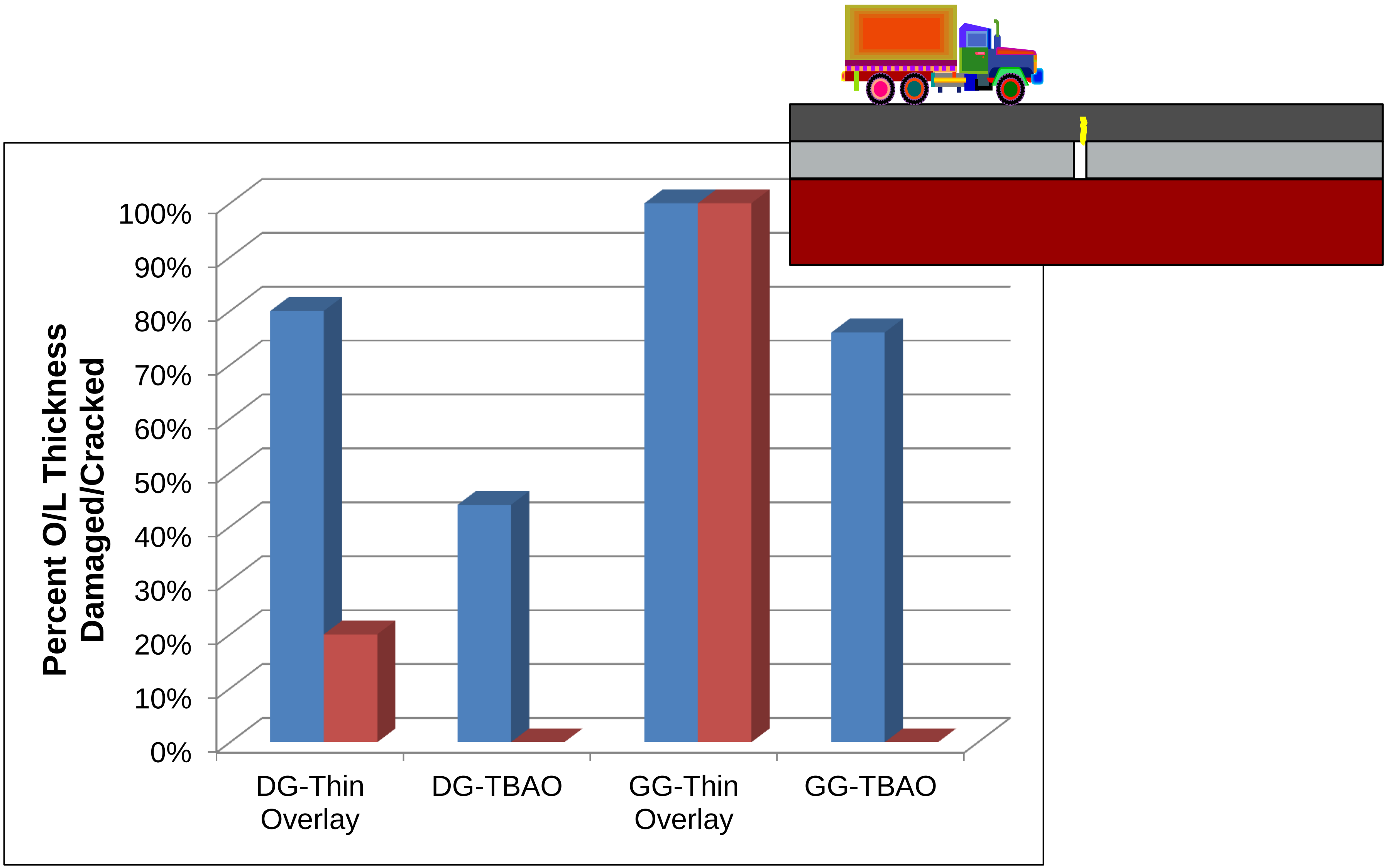
Modeling Scenarios

- Total four overlay types are simulated
  - Dense Graded
    - *Homogeneous (Thin-Overlay)*
    - *Graded (TBAO)*
  - Gap Graded
    - *Homogeneous (Thin-Overlay)*
    - *Graded (TBAO)*
- Two loading scenario
  - Thermal cracking (critical event)
  - Reflective cracking (thermal + tire loading)

Simulation Results: Thermal Cracking



Simulation Results: Reflective Cracking



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Summary and Findings

- Viscoelastic functionally graded finite elements using recursive time integration scheme are proposed
  - Efficiency and accuracy is briefly demonstrated
  - Formulation and implementation is verified
- Graded viscoelastic properties of thin bonded asphalt overlays have been estimated
  - VFGM finite elements were utilized for conducting simulations
- TBAOs show better cracking resistance compared to thin overlays with same mixes and thickness
  - Testing and field data also supports this claim
- Functionally graded material properties and models are needed for realistic analysis of TBAOs

Future Extensions

- Improvement on evaluation of functionally graded bulk and fracture properties
  - NSF Project 1031218: A Hybrid Failure Approach using Digital Image Correlation for Functionally Graded Thin-Bonded Overlays
- Improvements upon current testing and modeling approaches
  - Use of micromechanics to predict VFGM properties
  - Effect of material heterogeneity

Thank you for your attention!!!

Questions?

Acknowledgements:

- **Road Science LLC**

Reference:

E. V. Dave, G. H. Paulino and W. G. Buttlar, “Viscoelastic Functionally Graded Finite Element Method for Flexible Pavements – A Recursive Time Integration Approach,” *International Journal of Analytical and Numerical Methods in Geomechanics*. (Available Online, Article in Press)