A unified library of nonlinear solution schemes

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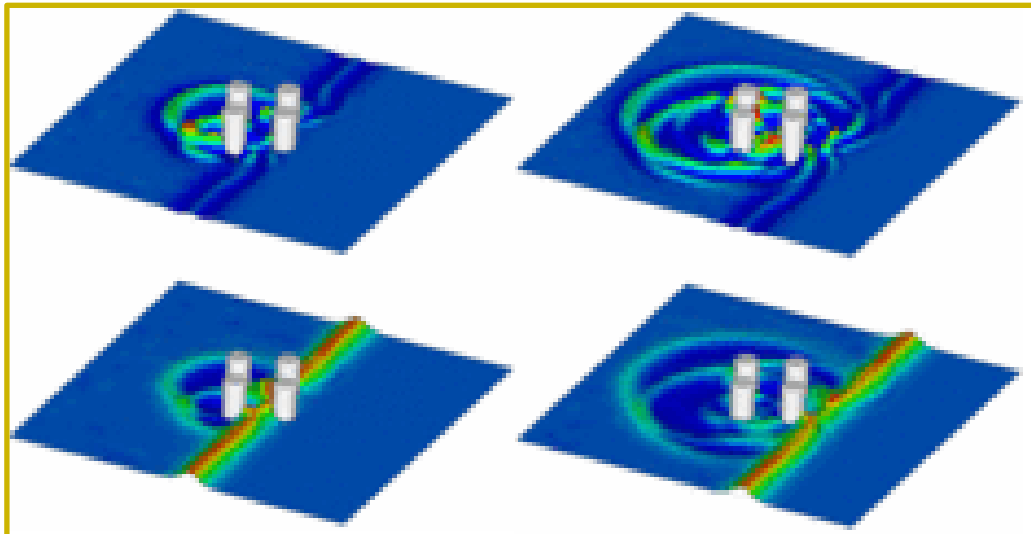
Motivation

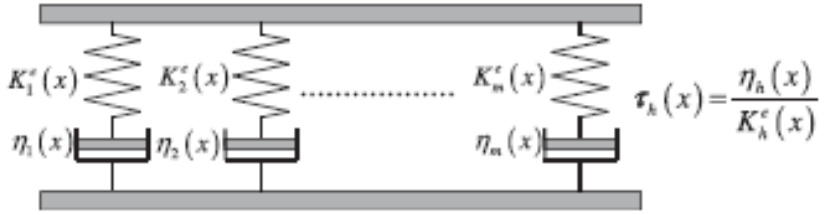
- Nonlinear problems are prevalent in structural, fluid, continuum, etc. mechanics



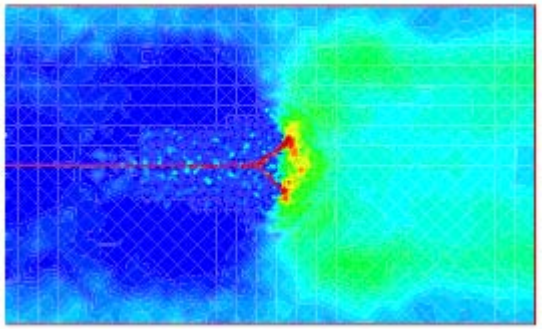
Nonlinear wave interaction simulated with libMesh. libMesh wiki.

Simulation of the electronic behavior of the heart performed with PETSc.

Antaki, J., et al., Procceedings of SC2000, 2000.



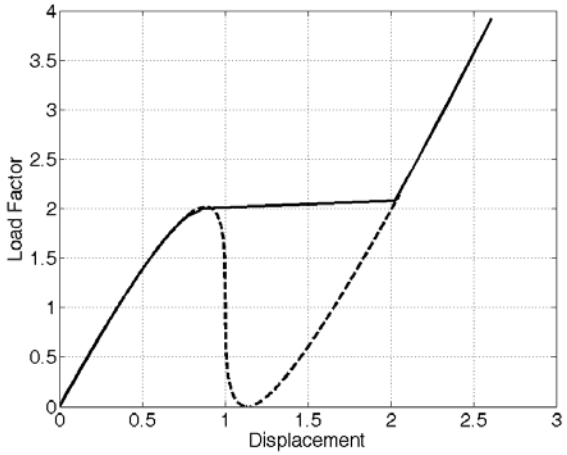
Maxwell model used for viscoelasticity in asphalt pavements. Dave et al. *IJNME*, 2011

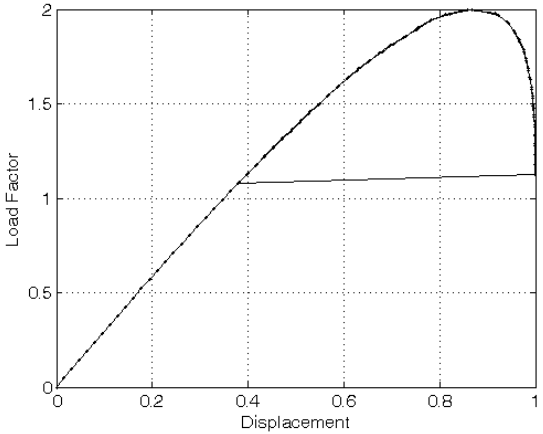


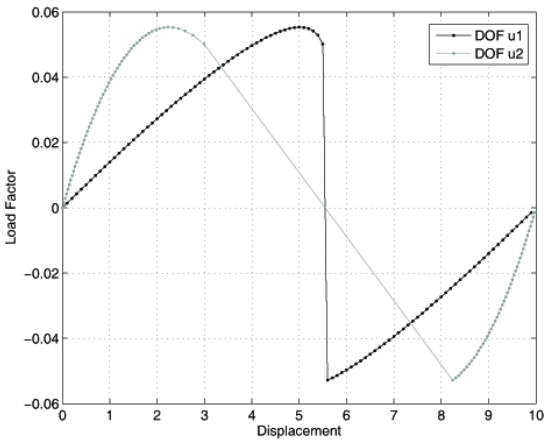
Nonlinear fracture process zone in dynamic fracture processes. Park et al. IJNME, 2010

Motivation

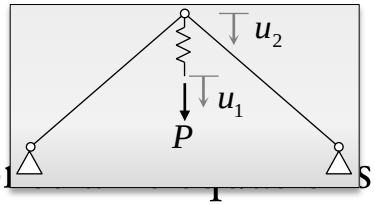
- Many algorithms have been developed to solve such problems, but no single algorithm is capable of solving any and all nonlinear problems







$$f(u) = -3\text{sign}(u)|u|^{\frac{1}{3}} + 4u + 1$$



- A library of nonlinear solution schemes, defined by unique co[illegible]s, is unified into a single space

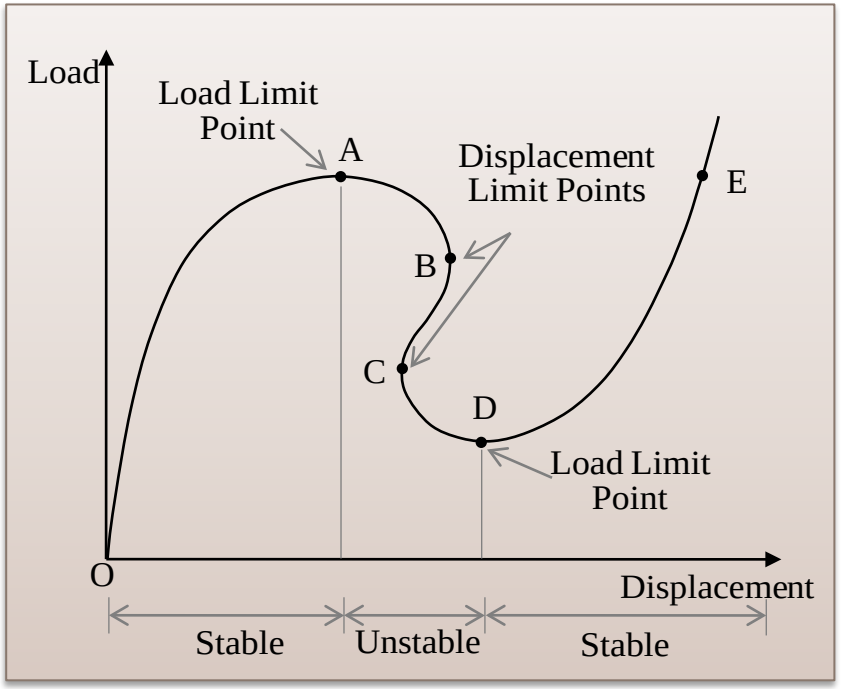
Bergan, P. G., et al. *IJNME* 1978. | Yang, Y.B. and Sheih, M.S. *AIAA Journal* 1990. | Rezaiee-Pajand, M., et al. *International Journal of Engineering, Transactions B: Applications*. 2009. | Salinger, A. G. et. al., "LOCA 1.0 Library of continuation algorithms", Sandia National Laboratories (2002). | Balay, S. et al Modern Software Tools in Scientific Computing (Birkhäuser Press, 1997)

Presentation Outline

- Nonlinearity overview
- N+1 dimensional space formulation
- Solution schemes
- Numerical results
- Evaluation and conclusions

Nonlinearity overview

- Geometric
  - Large strains and/or rotations
  - Linear or nonlinear constitutive relation
  - Loads have an effect on deformed configuration or configuration can have an effect on the load
- Material
  - i.e. inelasticity, strain-softening
- Critical points
  - Structure loses stability or bifurcation occurs



N+1 dimensional space formulation

- External loads and displacements

$$\mathbf{u}_j = \mathbf{u}^{\text{prev}} + \Delta\mathbf{u}_{j-1} + \delta\mathbf{u}_j$$
$$\mathbf{p}_j = \mathbf{p}^{\text{prev}} + \Delta\mathbf{p}_{j-1} + \delta\mathbf{p}_j$$

- Residual

$$\mathbf{r}_j = \mathbf{p}^{\text{prev}} + \Delta\mathbf{p}_j - \mathbf{q}(\mathbf{u}^{\text{prev}} + \Delta\mathbf{u}_j)$$

- Governing equation

$$\mathbf{K}_{j-1}\delta\mathbf{u}_j = \mathbf{p}_j - \mathbf{q}_{j-1}$$

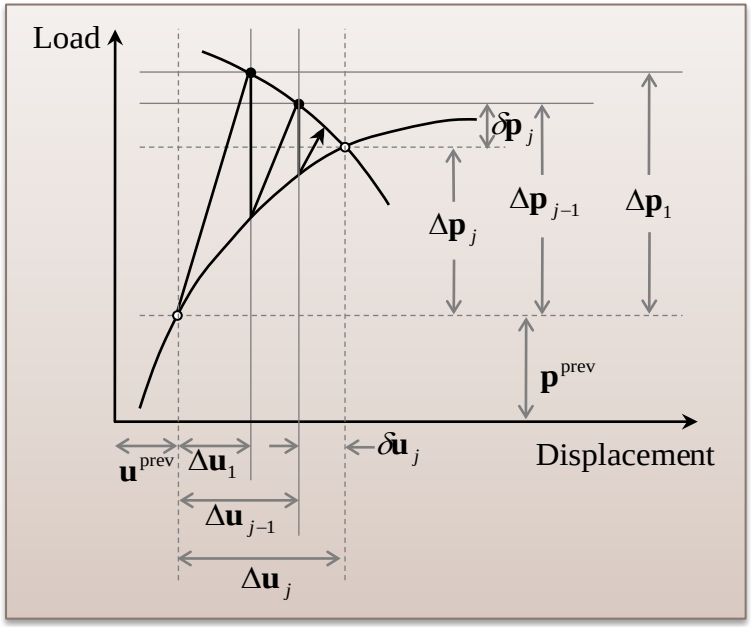
- Load parameter and reference load vector

$$\mathbf{p}_j = \mathbf{p}^{\text{prev}} + \Delta\mathbf{p}_{j-1} + \delta\lambda_j\bar{\mathbf{p}}$$

- N+1 space

$$\mathbf{a}_j \cdot \delta\mathbf{u}_j + b_j\delta\lambda_j = \mathrm{c}_j$$

$$\begin{bmatrix} \mathbf{K}_{j-1} & -\bar{\mathbf{p}} \\ (\mathbf{a}_j)^T & b_j \end{bmatrix} \begin{Bmatrix} \delta\mathbf{u}_j \\ \delta\lambda_j \end{Bmatrix} = \begin{Bmatrix} \mathbf{r}_{j-1} \\ \mathrm{c}_j \end{Bmatrix}$$



- Decomposition

$$\delta\mathbf{u} = \delta\mathbf{u}_I + \delta\lambda\delta\mathbf{u}_{II}$$

$$\mathbf{K}_{j-1}\delta\mathbf{u}_{j_I} = \bar{\mathbf{p}}$$
$$\mathbf{K}_{j-1}\delta\mathbf{u}_{j_{II}} = \mathbf{r}_{j-1}$$

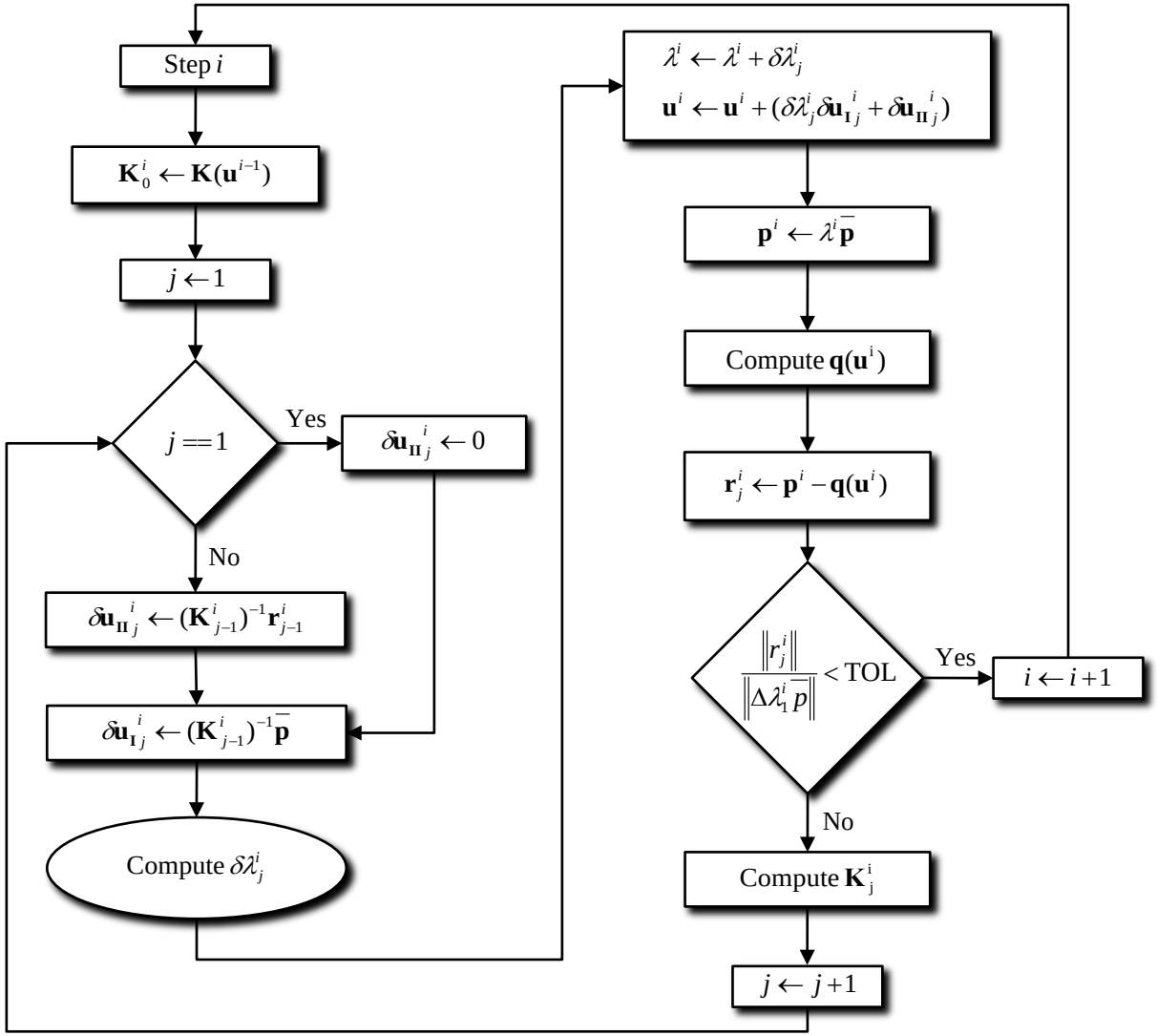
- Load parameter

$$\delta\lambda_j = \frac{\mathrm{c}_j - \mathbf{a}_j \cdot \delta\mathbf{u}_{j_{II}}}{\mathbf{a}_j \cdot \delta\mathbf{u}_{j_{II}} + b_j}$$

Batoz JJ and Dhatt G. *IJNME*, 1979.

Unified scheme – NLS++

- Incremental-iterative scheme is the same for any algorithm implemented in the N+1 space
- Only the computation of the load factor changes for each algorithm



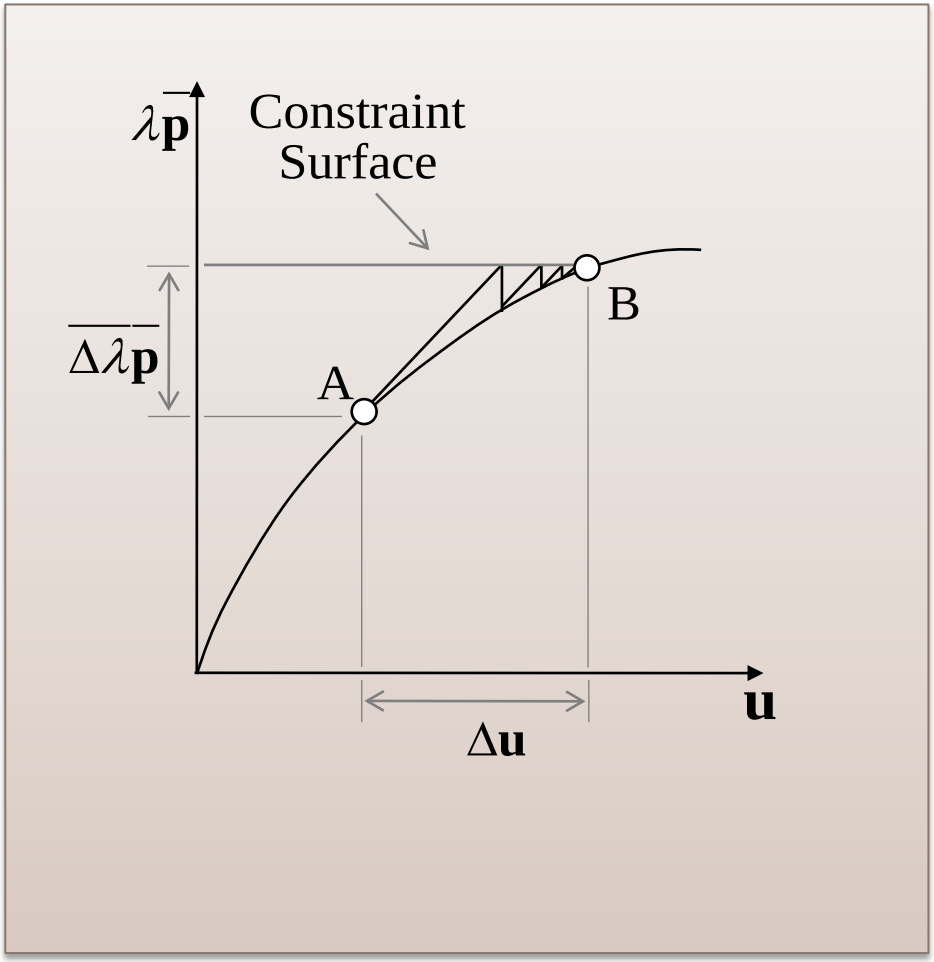
Nonlinear solution schemes

- $\delta\lambda$ is computed for each nonlinear solution scheme based on its unique constraint
  - Load control method (LCM)
  - Displacement control method (DCM)
  - Arc-Length control method (ALCM)
  - Work control method (WCM)
  - Generalized displacement control method (GDCM)
  - Orthogonal residual procedure (ORP)

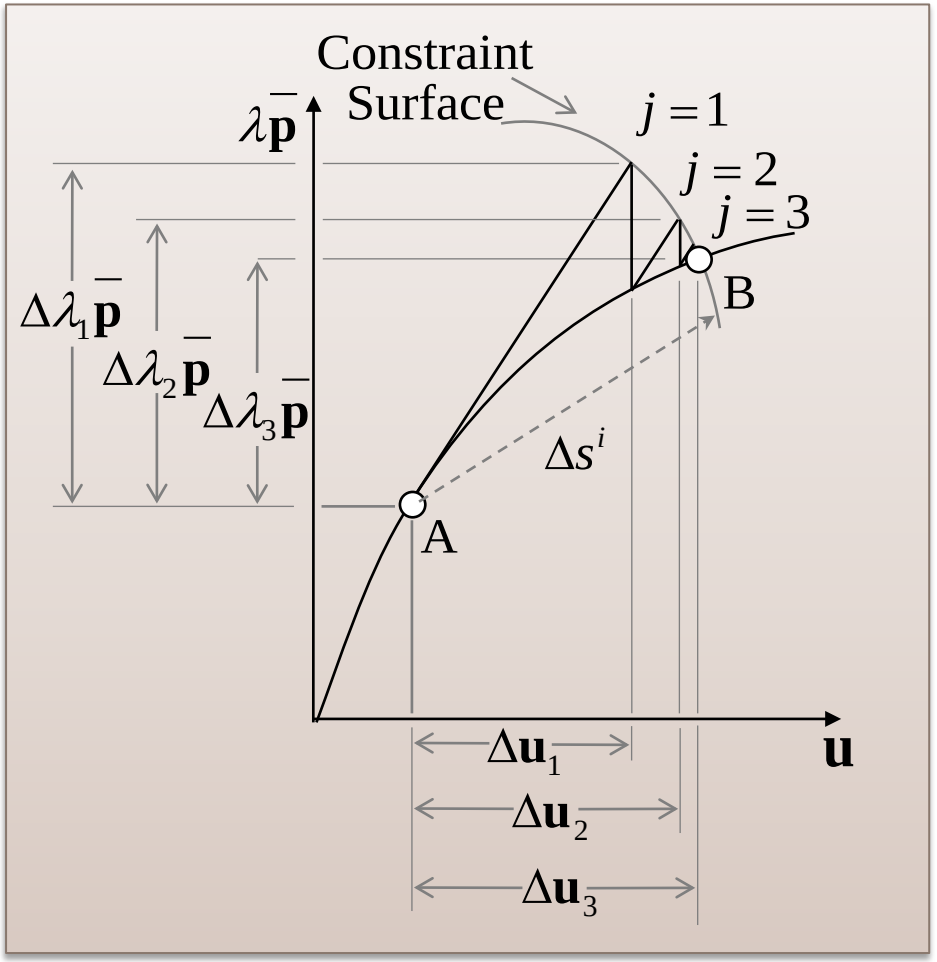
Crisfield MA. *Non-linear Finite Element Analysis of Solids and Structures. Volume 1: Essentials*. 1991. | Ramm E. *Nonlinear Finite Element Analysis in Structural Mechanics*. 1981. | Bergan PG, et al. *IJNME*. 1978. | Riks E. *International Journal of Solids and Structures*. 1979. | Wempner GA. *International Journal of Solids and Structures*. 1971. | Yang Y-B, McGuire W. *Proceedings of the 1985 International Conference on Numerical Methods in Engineering: Theory and Applications*. | Yang, Y.B. and Sheih, M.S. *AIAA Journal* 1990. |Krenk S. *IJNME* 1995. | Kouhia R. *CMAME* 2008.

Common nonlinear solution schemes

Load control method



Arc-length control method

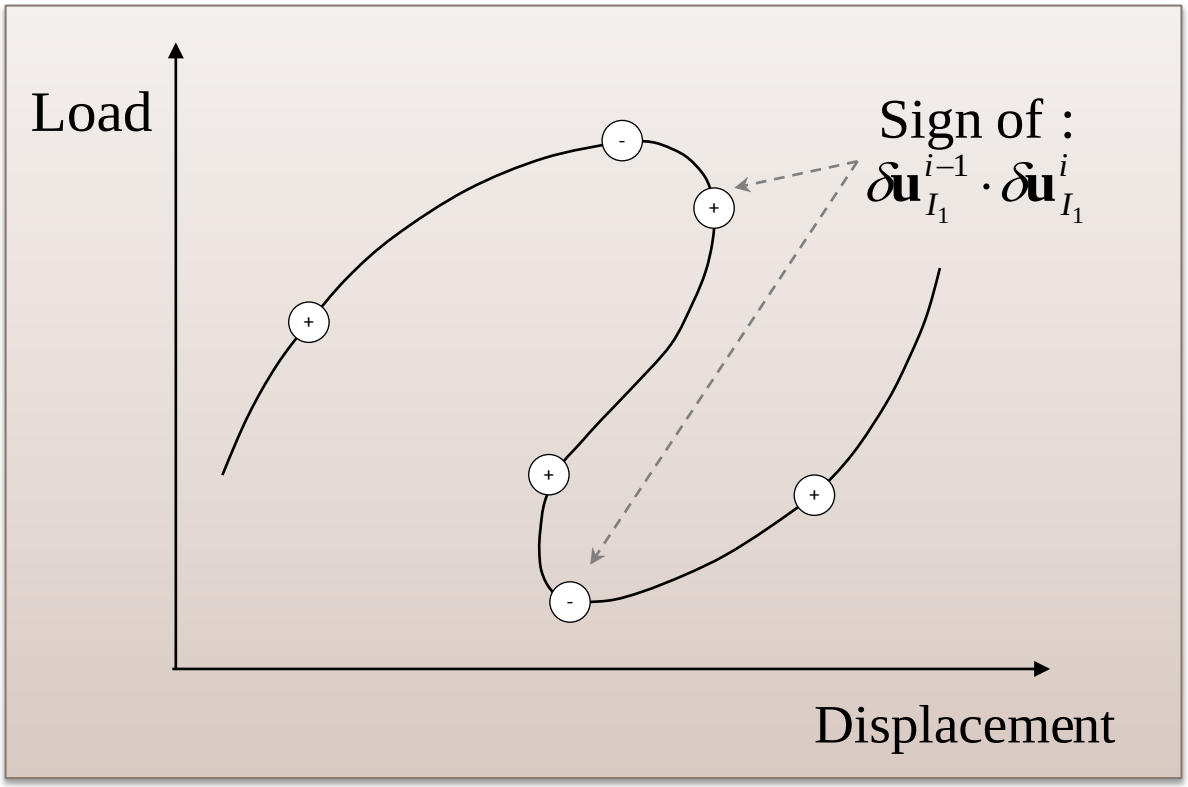


Generalized displacement control method

- Load parameter is based on a physical quantity called the Generalized Stiffness Parameter (GSP)

$$\text{GSP} = \frac{\delta \mathbf{u}_{1_I}^{1} \cdot \delta \mathbf{u}_{1_I}^{1}}{\delta \mathbf{u}_{1}^{i-1}{}_{I} \cdot \delta \mathbf{u}_{j_I}^{i}}$$

- The sign of the GSP changes only at load limit points



Yang, Y.B. and Sheih, M.S. *AIAA Journal* 1990.

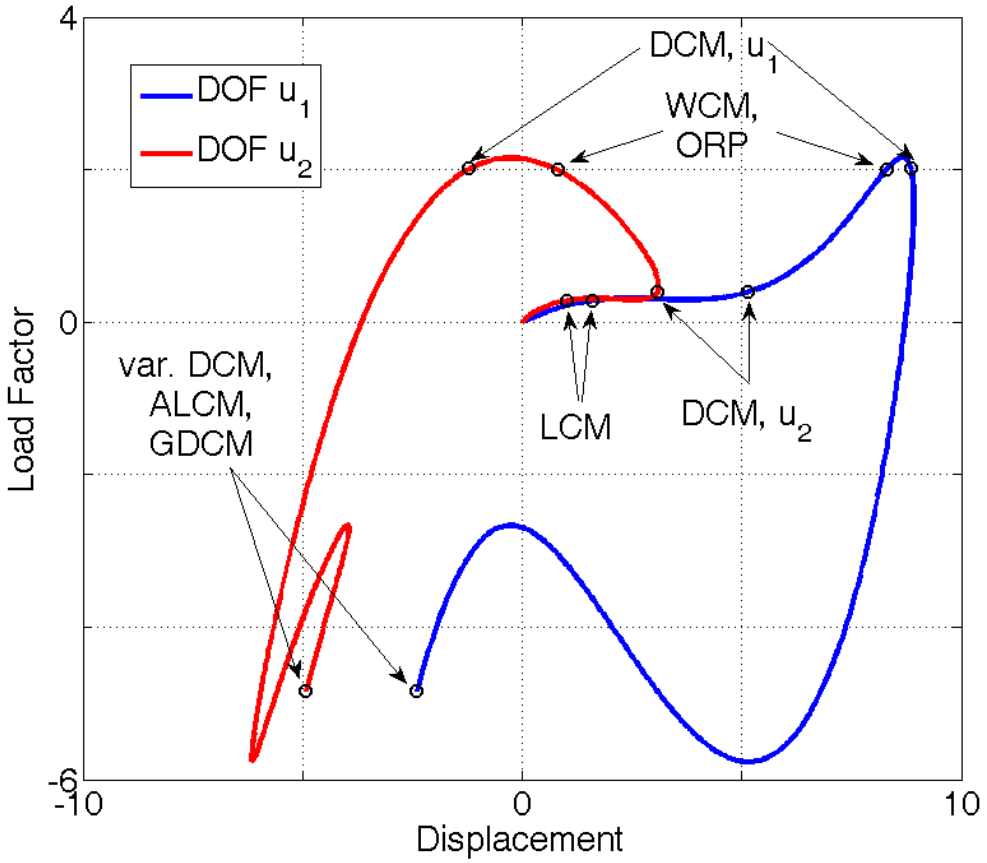
Two-dimensional function

- Internal force given by

$$q(u) = \begin{pmatrix} 10u_1 + 0.4u_2^3 - 5u_2^2 \\ 0.4u_1^3 - 3u_1^2 + 10u_2 \end{pmatrix}$$

- Tangent matrix

$$K(u) = \begin{bmatrix} 10 & 1.2u_2^2 - 10u_2 \\ 1.2u_1^2 - 6u_1 & 10 \end{bmatrix}$$



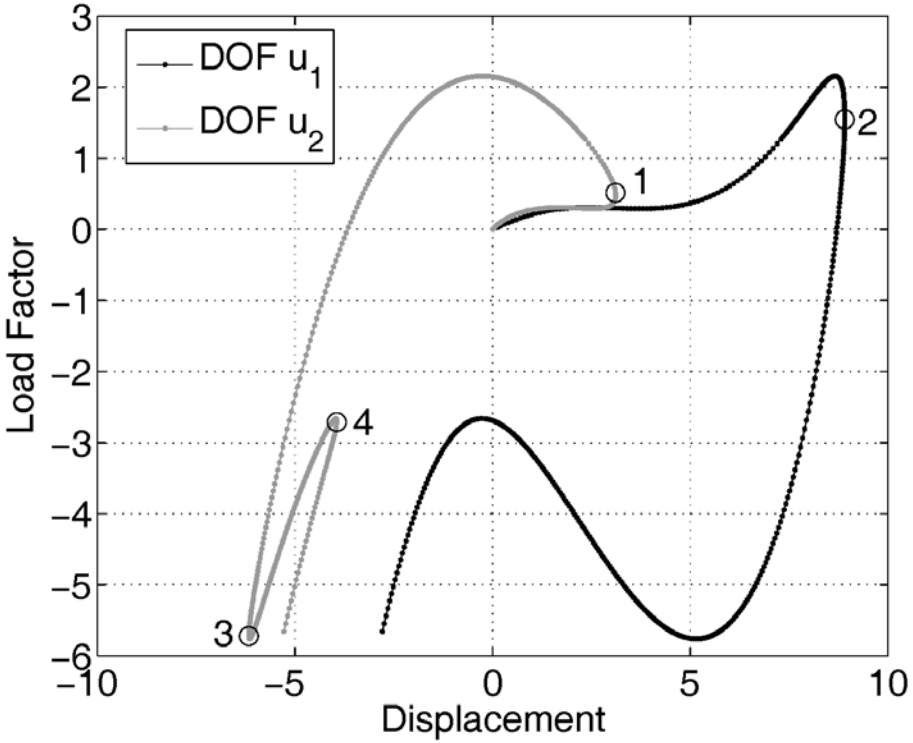
LCM = load control method
DCM = displacement control method
ALCM = arc-length control method
WCM = work control method
GDCM = generalized displacement control method
ORP = orthogonal residual procedure
var = variable
u1,u2 = control degrees of freedom

Ilinca Stanciulescu. Personal communications. 2009.

Two-dimensional function and the variable DCM

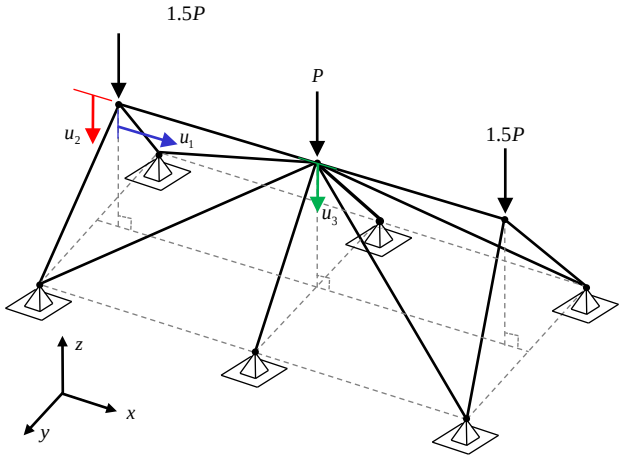
- Systematically select the best control parameter

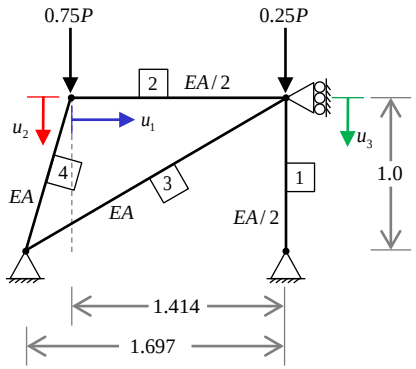
Variable displacement control method		
Step	Control	Snap back
1-73	u_1	Step 58 in u_2, Label 1
74-216	u_2	Step 158 in u_1, Label 2
217-445	u_1	Step 266 in u_2, Label 3
		Step 398 in u_2, Label 4



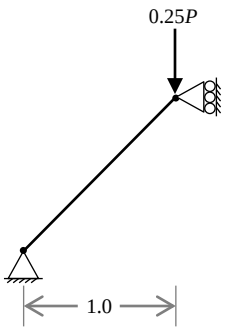
Fujii F, et al. Computers & Structures. 1992.

Twelve bar truss

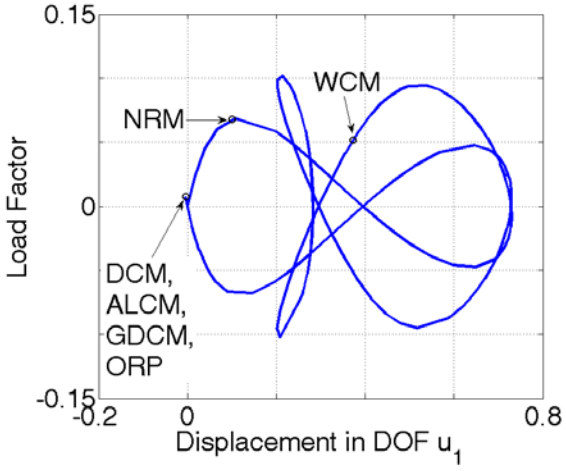


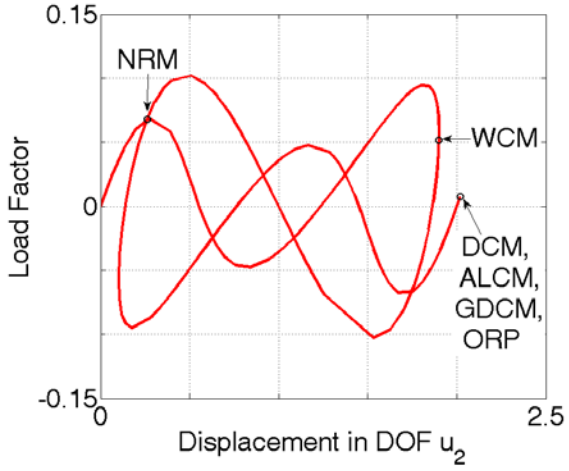


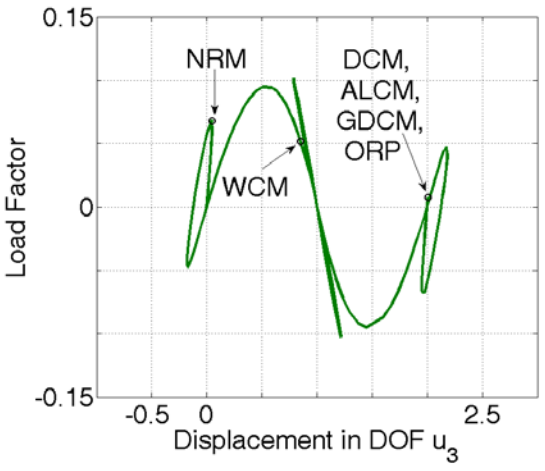
x-z view



y-z view



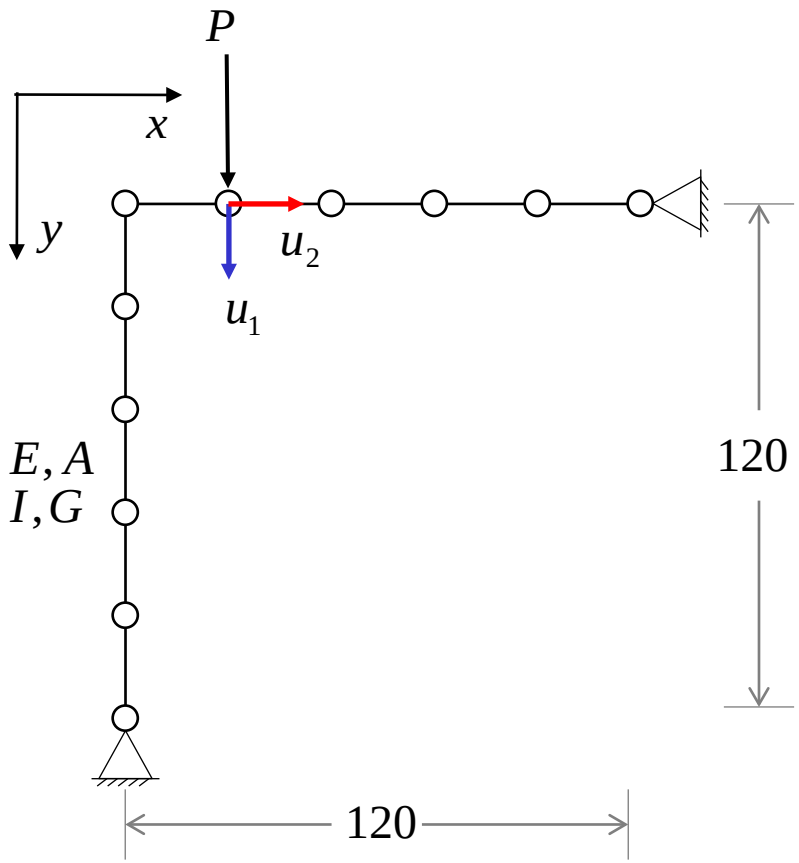


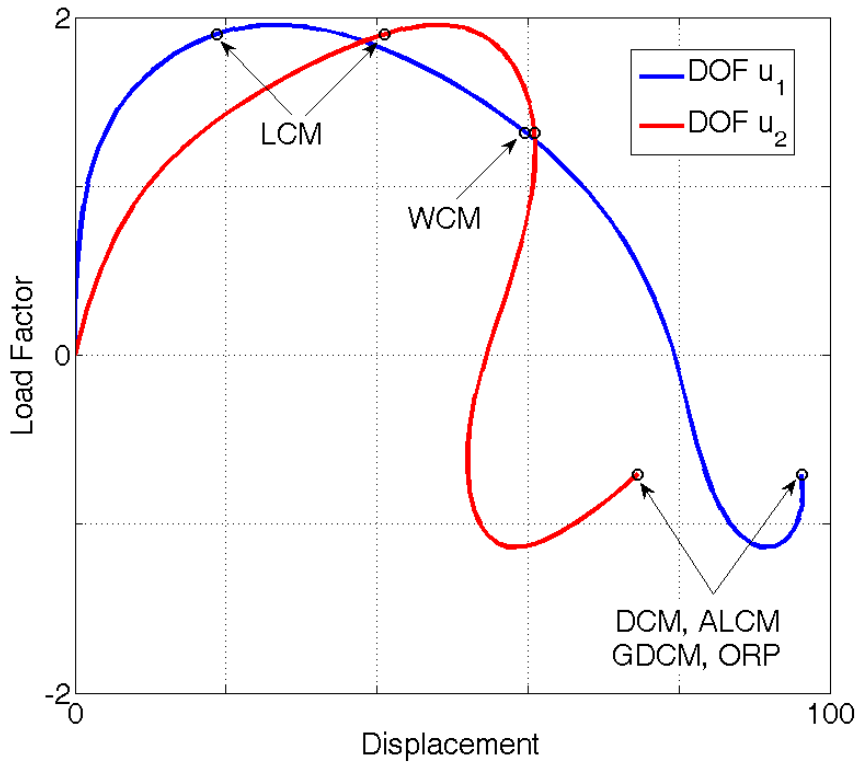


Yang Y-B and Leu LJ. *CMAME* 1991.
Krenk S and Hededal O. *CMAME* 1995.

NRM = newton raphson method, DCM = displacement control method, ALCM = arc-length control method, WCM = work control method, GDCM = generalized displacement control method, ORP = orthogonal residual procedure

Lee frame





Lee SL et al. *ASCE Journal of Engineering Mechanics*. 1968.
Schweizerhof KH and Wriggers P. *CMAME* 1986.
Parente E and Vaz LE. *IJNME* 2001

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Evaluation of solution schemes

- In general the most robust and powerful solution schemes in the context of the N+1 space are
  - Variable displacement control method
  - Arc-length control method
  - Generalized displacement control method

- Standard displacement control method and work control method have predictable difficulties at displacement limit points

- Orthogonal residual procedure is sensitive to input parameters and behavior is sometimes unpredictable near limit points

Conclusions and future work

- NLS++ is a powerful computational framework that provides seamless implementation of solution schemes of interest
  - Gives user flexibility to select the most appropriate scheme for a particular problem
  - Portable, effective and extendable

- Plan to integrate NLS++ with the new finite element code based on topological data representation, called TopFEM

- Details and code available in M.S. thesis

www.ideals.illinois.edu

www.sofieleon.com