11th US National Congress on Computational Mechanics
Minneapolis, Minnesota

Single-loop System Reliability-based Topology Optimization Accounting for Statistical Dependence between Limit-states

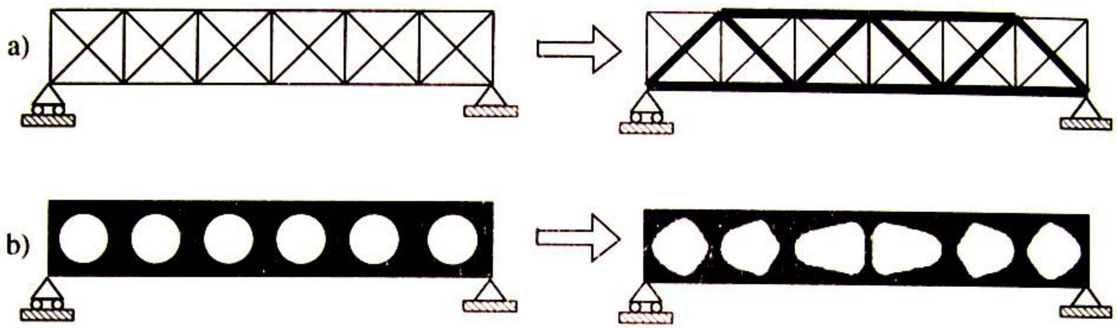
Tam Nguyen, Northeastern University
Junho Song, Glaucio Paulino, University of Illinois at Urbana-Champaign

07/25/2011

Department of Civil and Environmental Engineering
Northeastern University

Topology Optimization

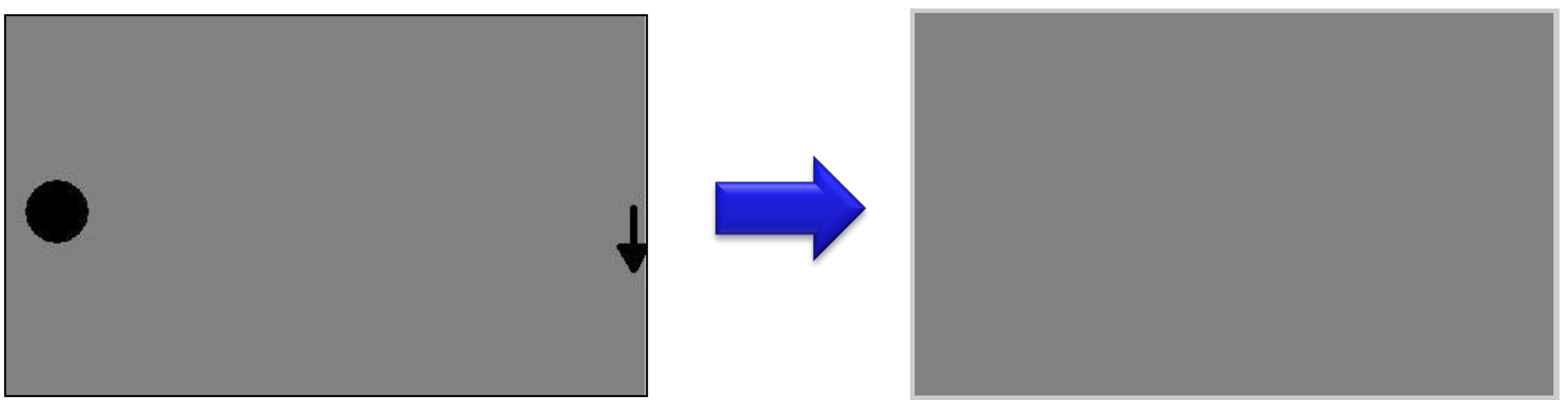
- **Classical structural design optimization: the optimal sizes or shapes for a given layout and connectivity**



size optimization

shape optimization

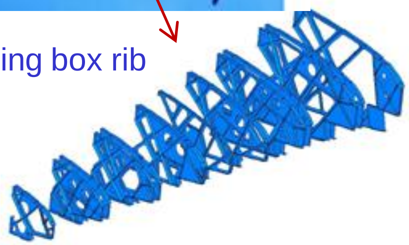
- **Topology optimization: free-form optimization**

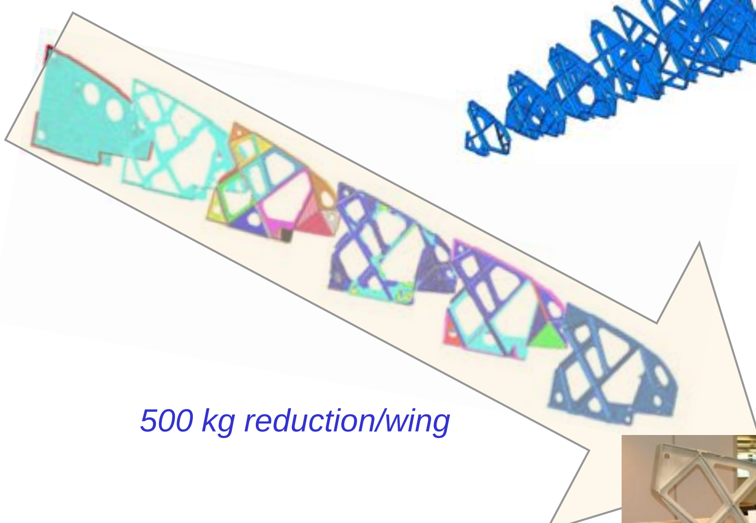


Topology Optimization Applications

Airbus

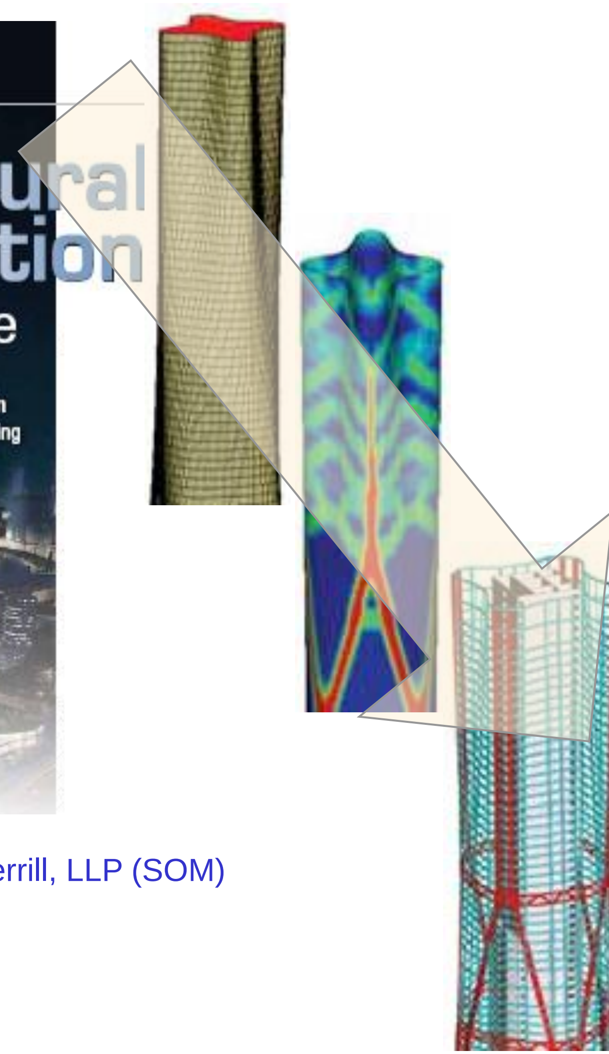
Wing box rib





500 kg reduction/wing

(www.altair.com)



Skidmore, Owings & Merrill, LLP (SOM)

Topology Optimization Procedure

Problem formulation

$$\min_{\rho} \quad C(\rho, \mathbf{u}_d) = \mathbf{f}^{\mathrm{T}} \mathbf{u}_d$$

$$s.t.: \quad \mathbf{K}(\rho)\mathbf{u}_d = \mathbf{f}$$

$$V(\rho) = \int_{\Omega} \rho(\psi) dV \le V_s$$

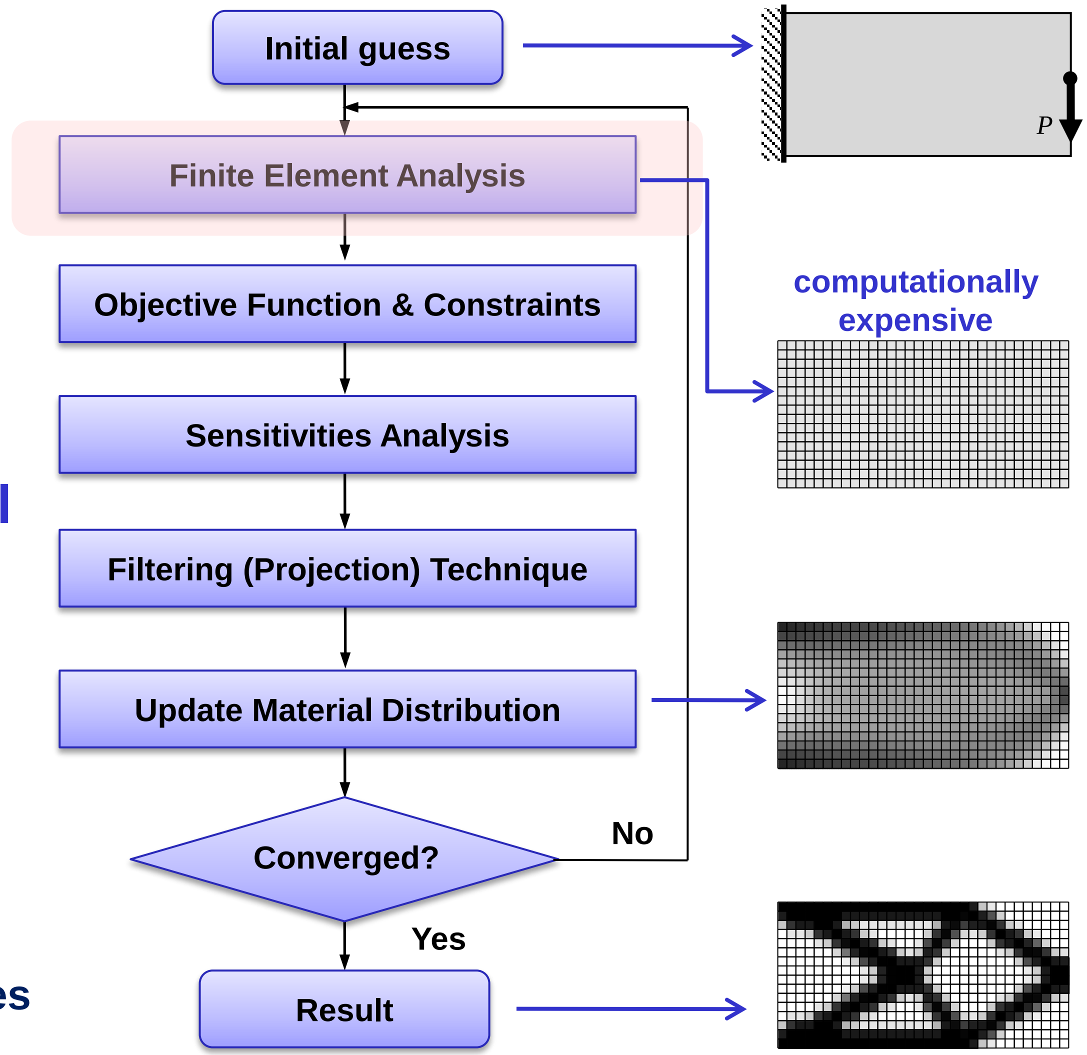
$$0 < \rho_{\min} \le \rho(\psi) \le 1$$

Solid and Isotropic Material with Penalization (SIMP)

$$E(\psi) = \rho(\psi)^p E^0$$

Optimizers

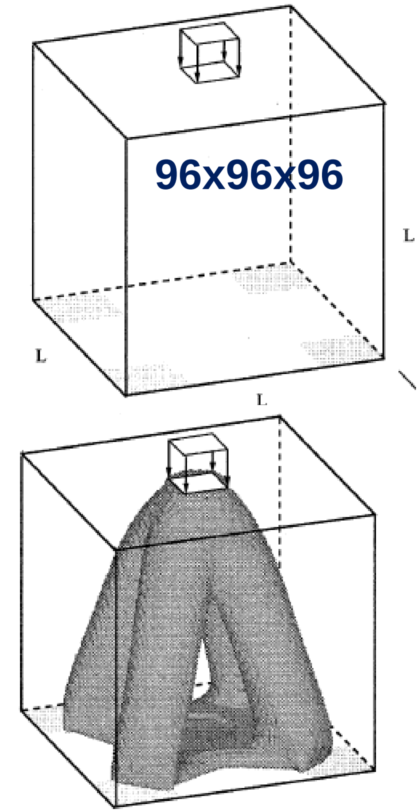
- Optimality Criteria (OC)
- Method of Moving Asymptotes (MMA)



High Resolution Topology Optimization

- **Large-scale (high resolution) TOP**
  - **Large number of finite elements**
  - **Computationally expensive: FEA cost**
- **Existing high resolution TOP**
  - **Parallel computing (Borrvall and Petersson, 2000)**
  - **Fast iterative solvers (Wang et al. 2007)**
  - **Approximate reanalysis (Amir et al. 2009)**
  - **Adaptive mesh refinement (de Stuler et al. 2008)**

Borrvall and Petersson, (2000), *IJNME*



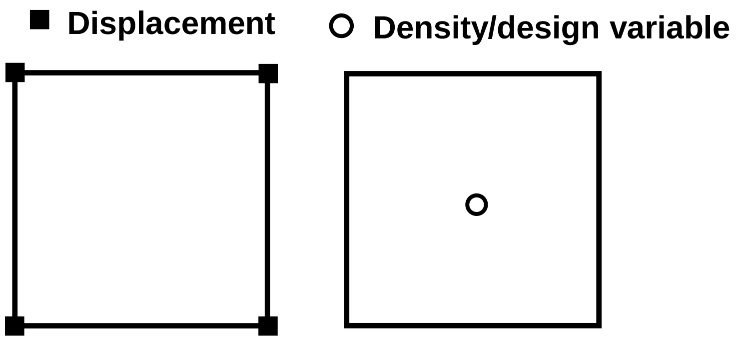
A stool (884,736 B8/U)

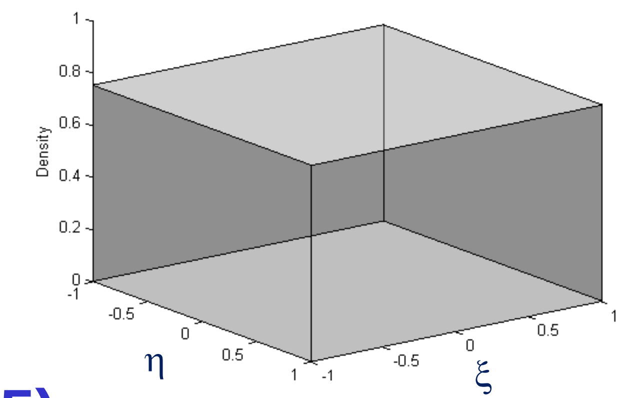
Same discretization for analysis and design

Multiresolution Topology Optimization (MTOP)

Conventional element-based approach (Q4/U)

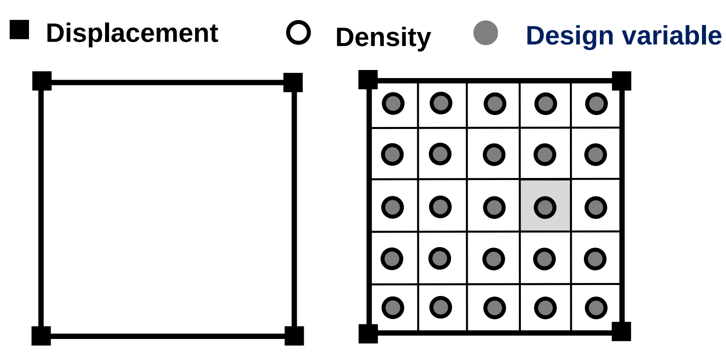
- Same discretization for displacement and density

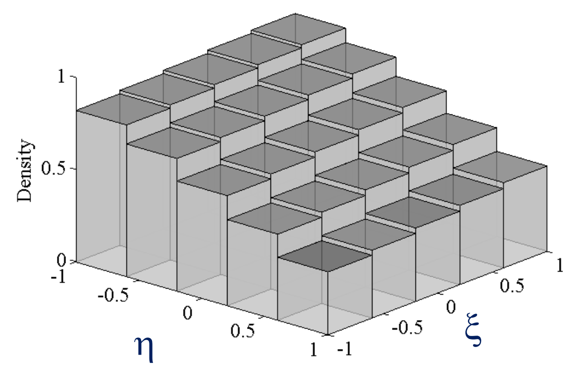




Proposed MTOP approach (Q4/n25)

- Different discretizations for displacement and density/design variables



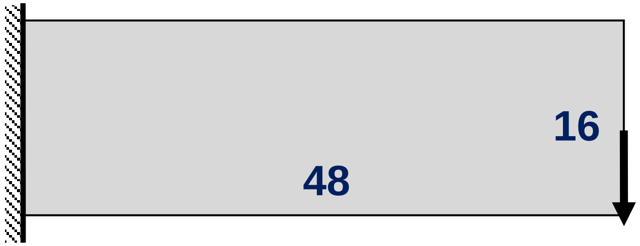


MTOP: 2D Cantilever Beam

Objective: minimum compliance

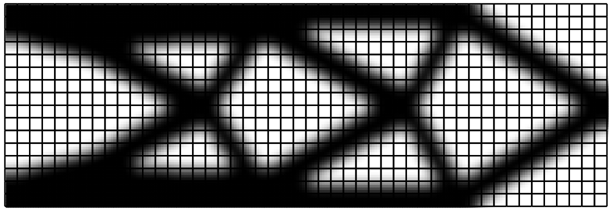
Constraint: *volfrac* = 0.5

Length scale: $r_{min} = 1.2$



Configuration

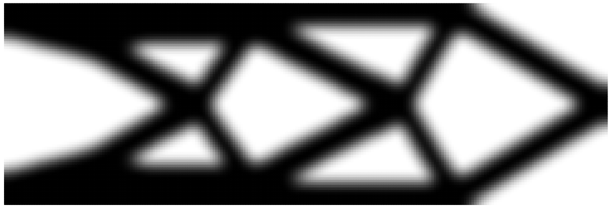
Coarse FE mesh



MTOP Q4/n25 FE mesh 48x16

(C=208.23)

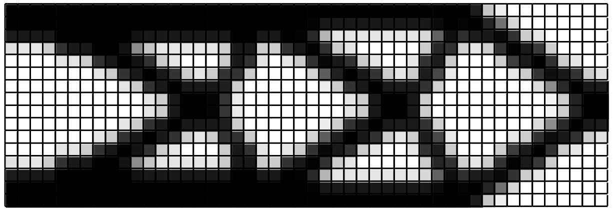
Fine FE mesh



Q4/U FE mesh 240x80

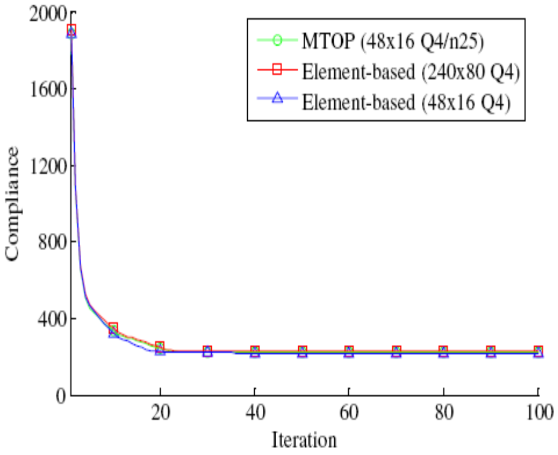
(C=210.68)

Coarse FE mesh



Q4/U FE mesh 48x16

(C=205.57)

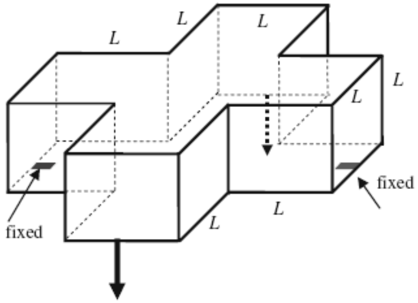


Convergence history

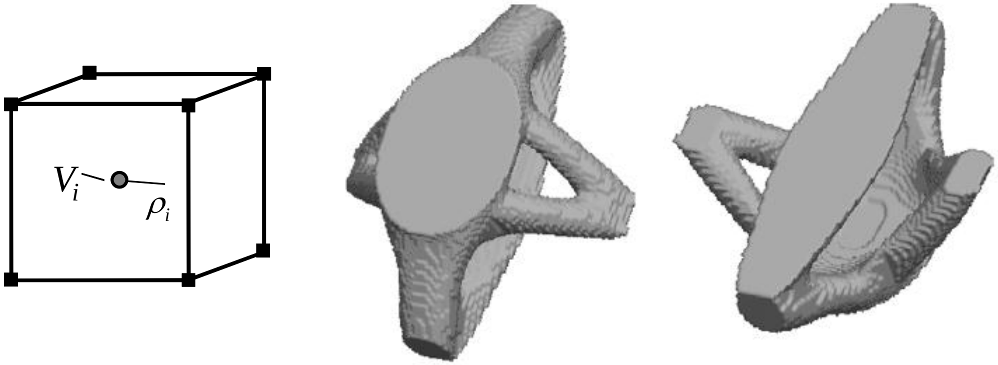
Nguyen *et al*. (2010), *JSMO*

MTOP: 3D Examples

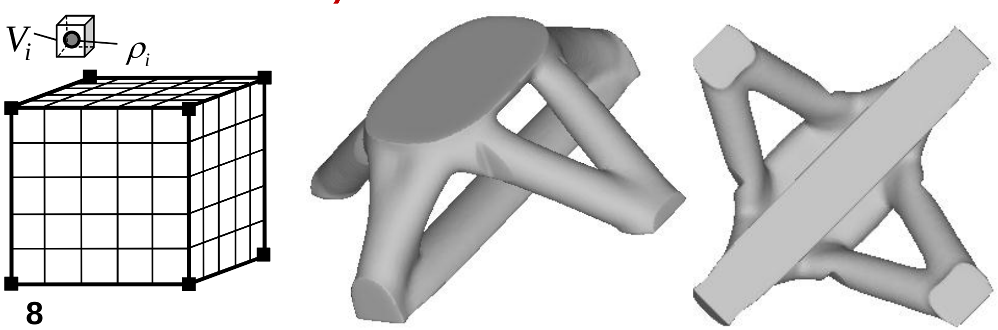
- 3D cross-shaped section



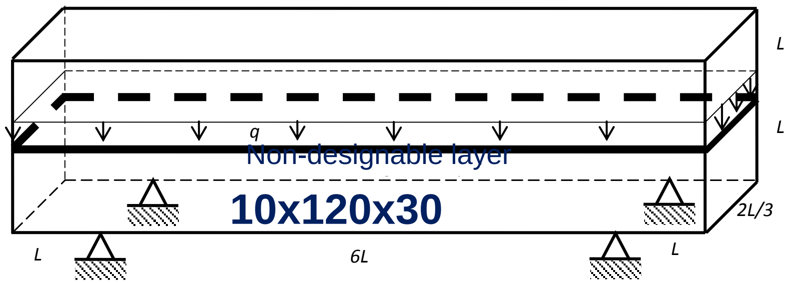
- Borrvall & Petersson 320,000 B8/U



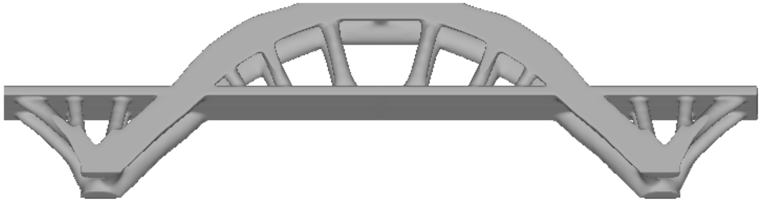
- MTOP: 5,000 B8/n125



- 3D bridge design



- MTOP: 36,000 B8/n125

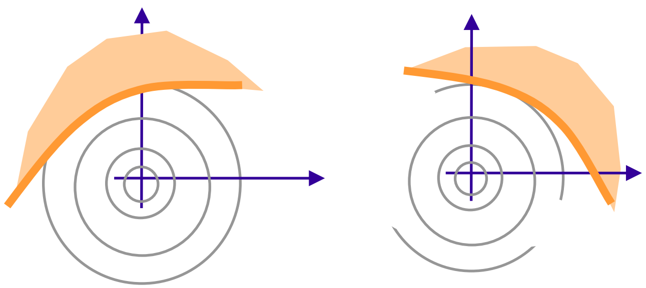


- An existing design

(http://www.sellwoodbridge.org)

System Reliability-Based Topology Optimization

Component RBTO

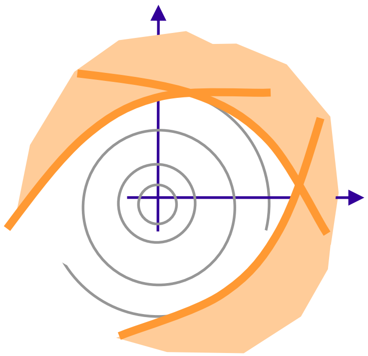


$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X}} \quad f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$

$$s.t. \quad P\left[g_i(\mathbf{d},\mathbf{X}) \le 0\right] \le P_i^t \quad i=1,\ldots,n$$

$$\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \qquad \boldsymbol{\mu}_\mathbf{X}^L \le \boldsymbol{\mu}_\mathbf{X} \le \boldsymbol{\mu}_\mathbf{X}^U$$

System RBTO



$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X}} \quad f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$

$$s.t. \quad P(E_{sys}) = P\left[\bigcup_k \bigcap_{i\in C_k} g_i(\mathbf{d},\mathbf{X}) \le 0\right] \le P_{sys}^t$$

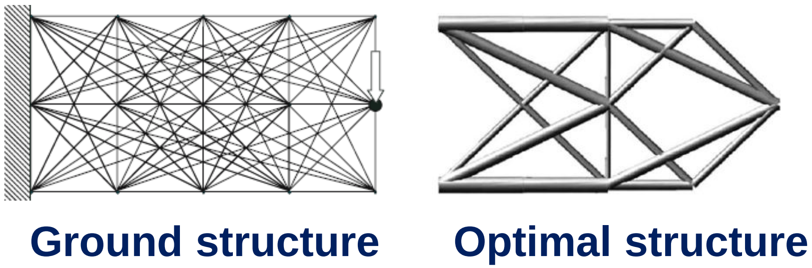
?

$$\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \qquad \boldsymbol{\mu}_\mathbf{X}^L \le \boldsymbol{\mu}_\mathbf{X} \le \boldsymbol{\mu}_\mathbf{X}^U$$

1. How to handle **dependent** limit-states in SRBTO?
2. How to compute **probability accurately** in RBTO?

Existing SRBTO Approaches

- **Discrete structures**
  - **Mogami *et al*. (2006)**
  - **Truss examples**

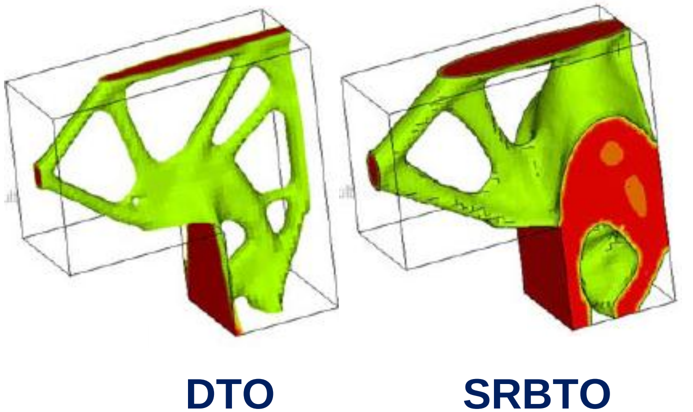


Mogami *et al*. (2006), *JSMO*

- **Continuum structures**
  - **Silvia *et al*. (2010)**
  - **Limit-states: statistically independent**

$$P(E_{sys}) = P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i) - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} P(E_i E_j) + \cdots + (-1)^{n-1} P(E_1 E_2 \cdots E_n)$$

$$P(E_1 E_2 E_3) = P(E_1) P(E_2) P(E_3)$$

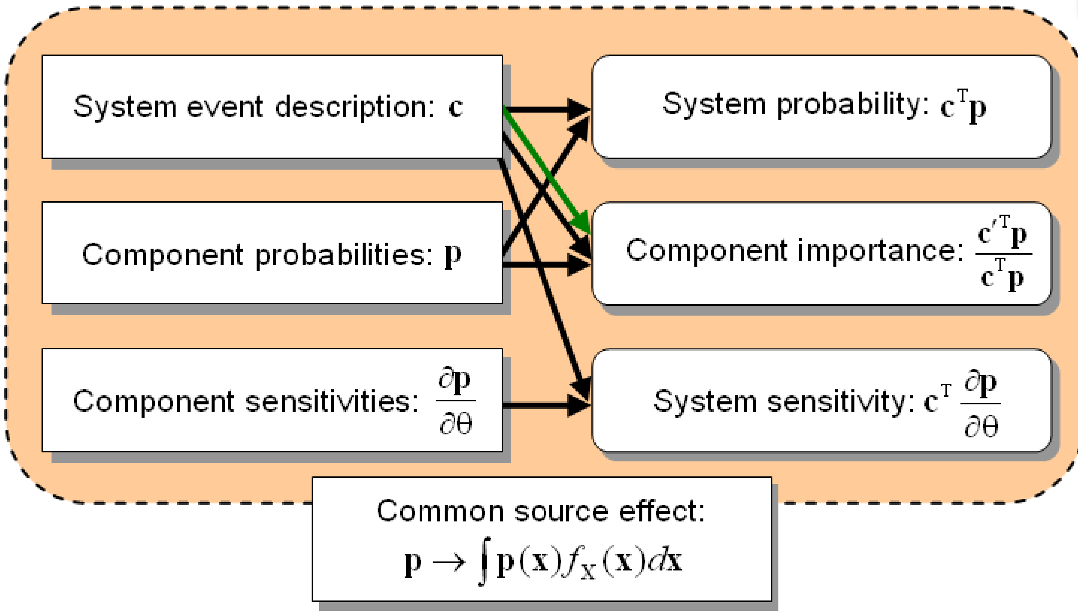


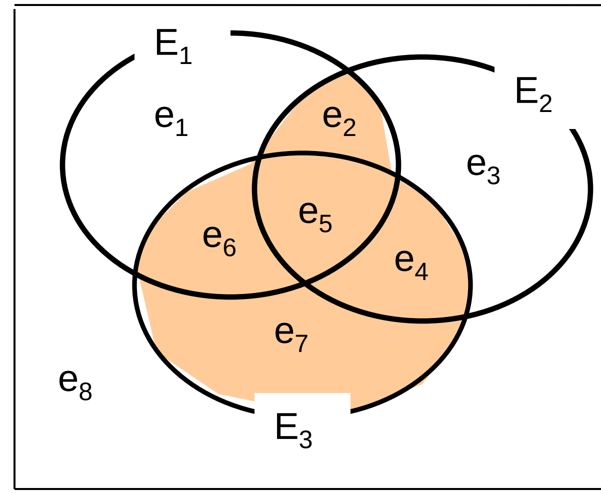
Silvia *et al*. (2010), JSMO

How to handle the dependence between limit-states?

Matrix-based System Reliability (MSR) Method

Song and Kang, (2009); *Structural Safety*





mutually exclusive and collectively exhaustive events (MECE)

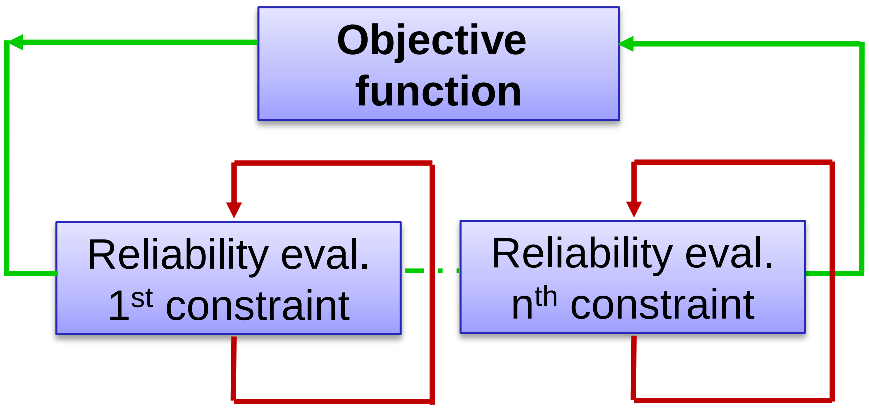
- **Convenient**: matrix-based procedures for c and p; easy SRA calculation (inner product)
- **General**: uniform application to series, parallel, and any general systems
- **Flexible**: inequality-type information; incomplete information (“LP bounds” method)
- **Efficient**: no need to re-compute “p”; replace “c” for SRA of a new event
- **Common Source Effect**: can account for statistical dependence between components
- **Decision Support**: parameter sensitivities, component importance measure; inferences

Proposed approach: SRBTO using MSR

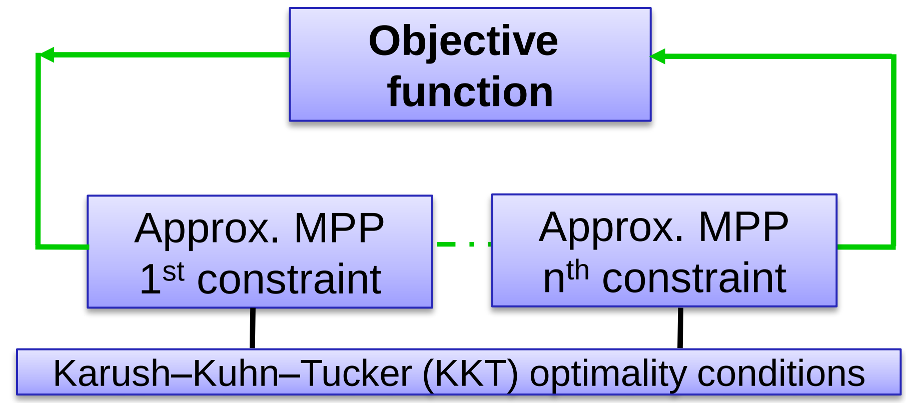
- **Adopt a single-loop RBTO**

Liang et al. (2007), McDonald & Mahadevan (2008)

  - **Double-loop RBTO**



  - **Single-loop RBTO**



- **Use MSR method to compute P_{sys} and its gradients**

$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X},P_1^t,\ldots,P_n^t} \quad f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$

$$s.t. \quad g_i(\mathbf{d},\mathbf{X}(\mathbf{U}_i^t)) \geq 0 \quad i=1,\ldots,n$$

Single-loop PMA

$$P_{sys} = \begin{cases} \int_\mathbf{s} \mathbf{c}^\mathrm{T}\mathbf{p}(\mathbf{s})\, f_\mathbf{S}(\mathbf{s})d\mathbf{s} \leq P_{sys}^t & \text{dependent} \\ \mathbf{c}^\mathrm{T}\mathbf{p} \leq P_{sys}^t & \text{indepdendent} \end{cases}$$

MSR method

SORM-based Single-loop CRBTO & SRBTO

Enhance the accuracy in Single-loop RBTO

- First-Order Reliability Method (FORM) → inaccurate for nonlinear limit-states
- Propose to use Second-Order Reliability Method (SORM) to improve the accuracy

SORM-based Single-loop CRBTO

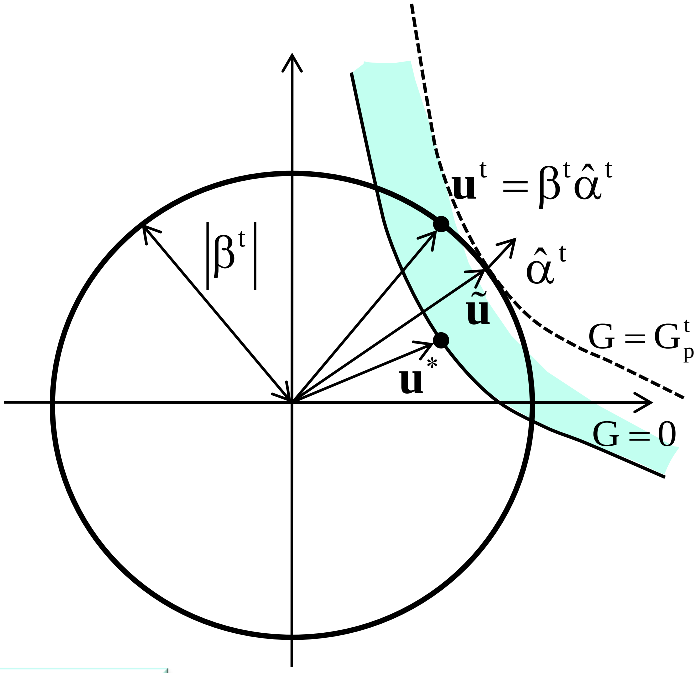
FORM-based

$$\beta_i^{t(k)} = \beta_i^t$$

At the k-th step

SORM-based

$$\beta_i^{t(k)} = \frac{\beta_i^t}{\beta_i^{t(k-1)(SORM)}} \times \beta_i^{t(k-1)}$$



SORM-based Single-loop SRBTO

FORM-based

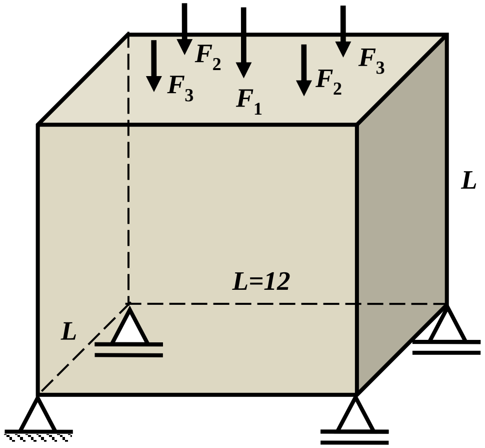
$$P(E_{sys};\mathbf{P}^t) = \begin{cases} \int_{\mathbf{s}} \mathbf{c}^T \mathbf{p}^t(\mathbf{s}) f_{\mathbf{s}}(\mathbf{s}) d\mathbf{s} \le P_{sys}^t \\ \mathbf{c}^T \mathbf{p}^t \le P_{sys}^t \end{cases}$$

At the k-th step

SORM-based

$$P_{sys} = P(E_{sys};\mathbf{P}^{t(SORM)}) = \begin{cases} \int_{\mathbf{s}} \mathbf{c}^T \mathbf{p}^{t(SORM)}(\mathbf{s}) f_{\mathbf{s}}(\mathbf{s}) d\mathbf{s} \le P_{sys}^t \\ \mathbf{c}^T \mathbf{p}^{t(SORM)} \le P_{sys}^t \end{cases}$$

SRBTO of a Cube

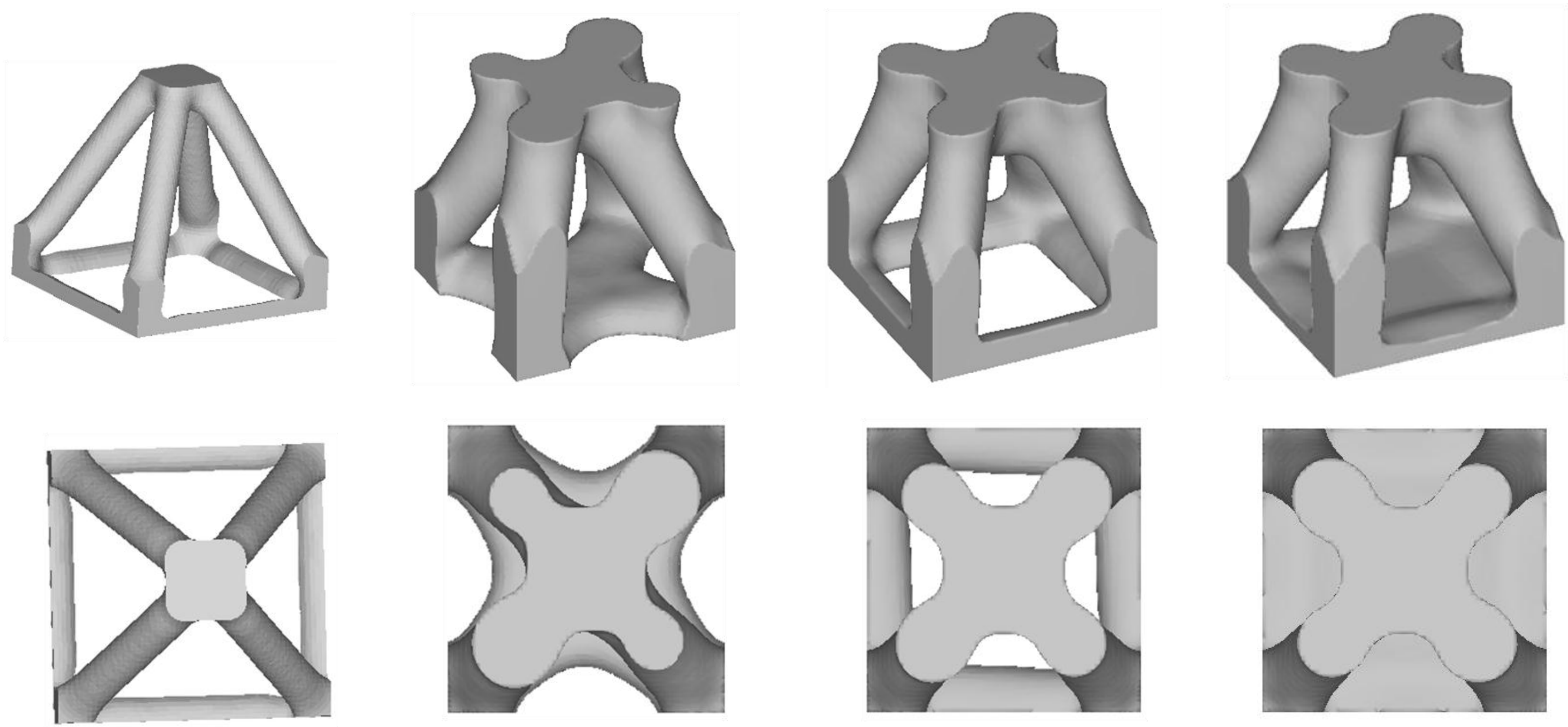


- **Objective: minimize volume $V(\rho)$**
- **Limit-states:** $g_i(\boldsymbol{\rho}, \bar{\mathbf{F}}_i) = 120 - C_i(\boldsymbol{\rho}, \bar{\mathbf{F}}_i), \quad i = 1, 2$
- **Random loads: : F ~ (F_1,F_2,F_3) ~**

  N(100,10), N(0,30), N(0,40)
- **Load cases:** $\bar{\mathbf{F}}_1 = (F_1, F_2), \quad \bar{\mathbf{F}}_2 = (F_1, F_3)$

- **Constraints**
  - **Deterministic TO (DTO):** $g_i(\boldsymbol{\rho}, \mathbf{f}) > 0, \quad i = 1, 2$
  - **Component RBTO (CRBTO):** $P(g_i(\boldsymbol{\rho}, \bar{\mathbf{F}}_i) \le 0) \le P_i^t, \quad i = 1, 2$
  - **System RBTO (SRBTO):** $P(\cup \cap g_i(\boldsymbol{\rho}, \bar{\mathbf{F}}_i) \le 0) \le P_{sys}^t$

Optimal Topologies



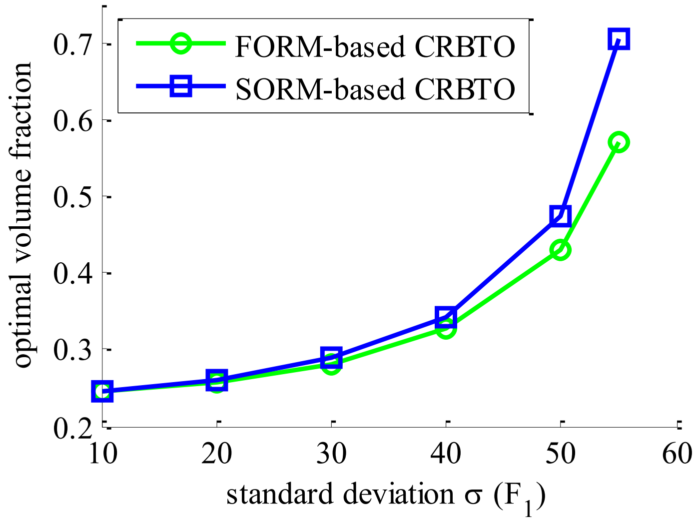
volfrac = 6.3%	*volfrac* = 24.4% (σ_F=10)	*volfrac* =22.3% (σ_{F1}=10)	*volfrac* =23.9% (σ_{F1}=20)
DTO	CRBTO	SRBTO	SRBTO

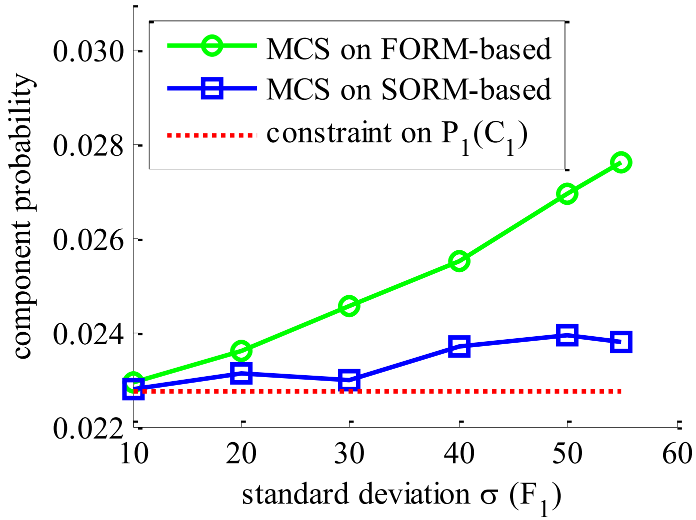
➢ **Different topologies were obtained**

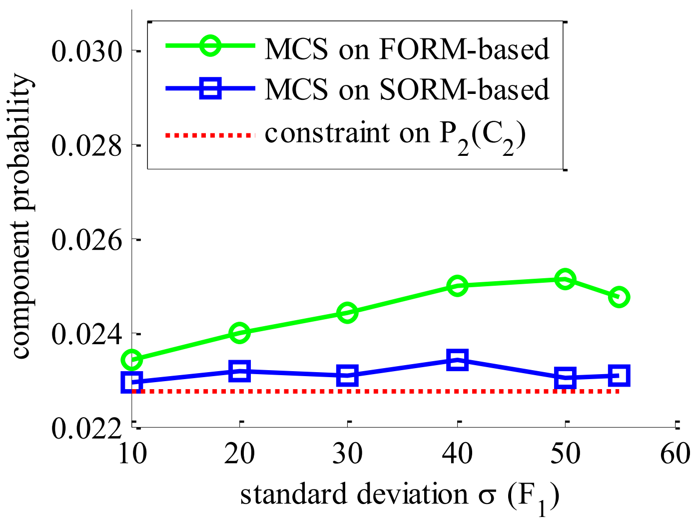
Improve Accuracy by Second-Order Reliability Method

Component RBTO

- **SORM-based provides more accurate results than FORM-based**



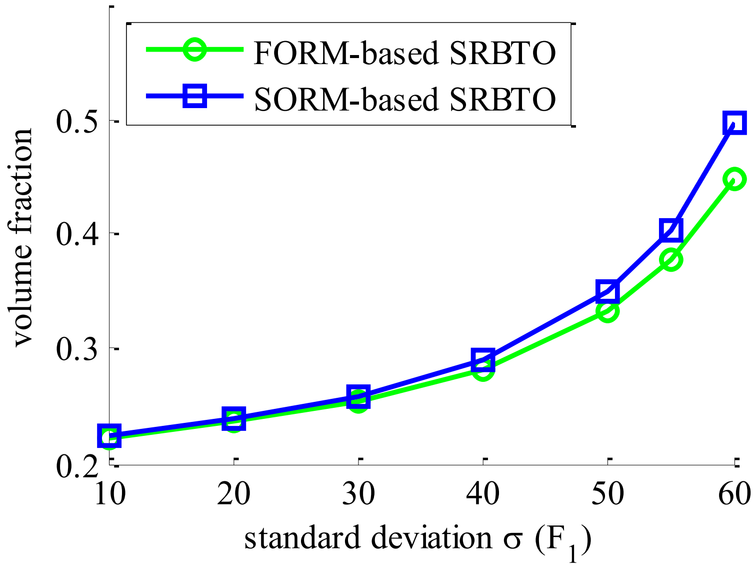


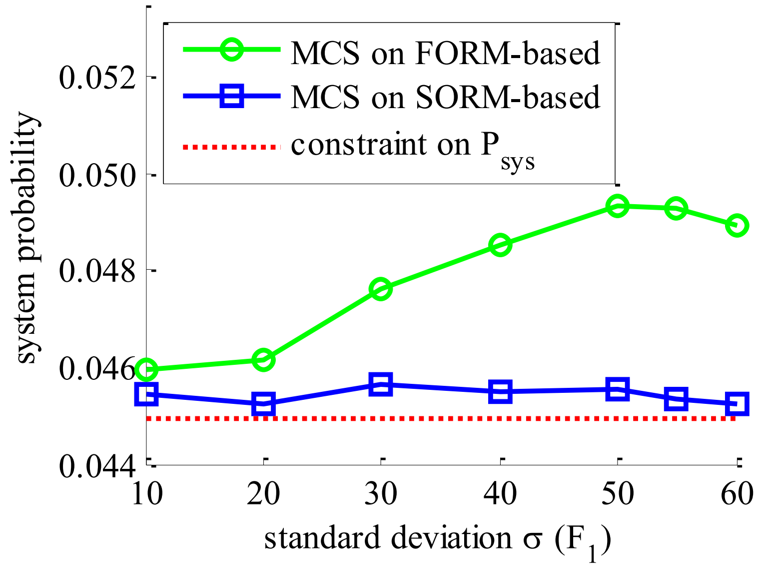


Improve Accuracy by Second-Order Reliability Method

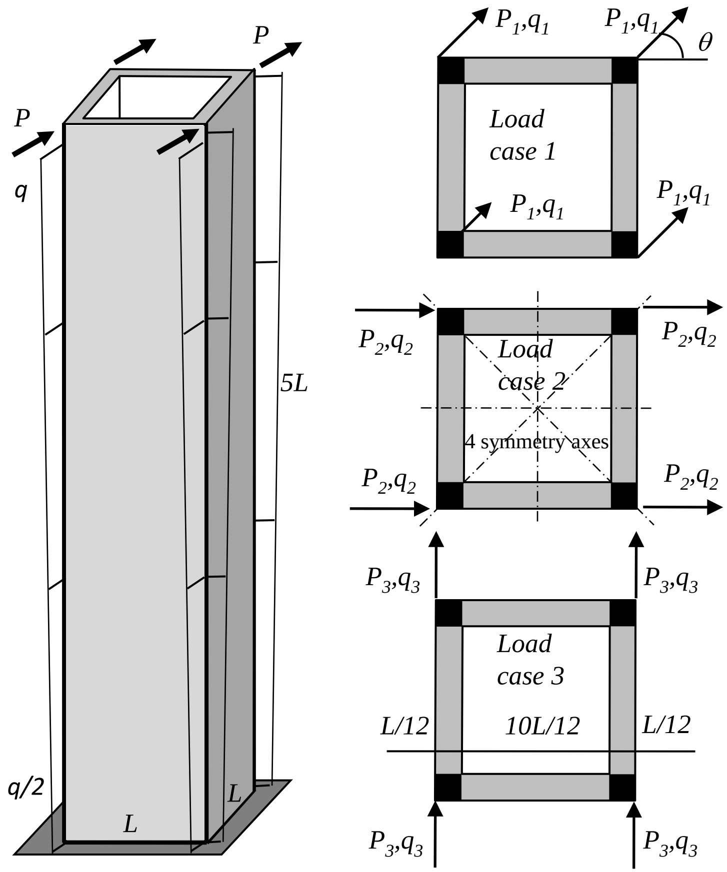
System RBTO

- **SORM-based provides more accurate results than FORM-based**





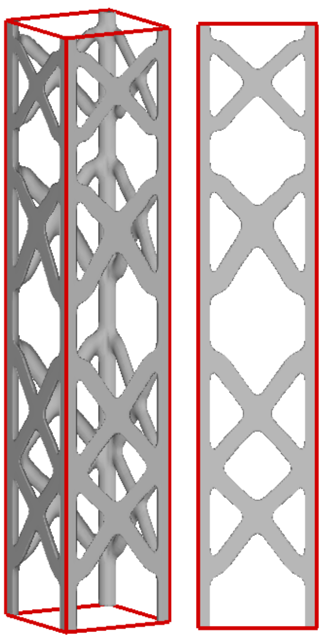
SRBTO of a Building Core



- **Objective: minimize volume $V(\rho)$**
- **Limit-states:** $g_i(\boldsymbol{\rho}, \bar{\mathbf{F}}_i) = C_i^0 - C_i(\boldsymbol{\rho}, \bar{\mathbf{F}}_i)$
- **Random loads: F ~ $(P_1, P_2, P_3, q_1, q_2, q_3)$**
- **Load cases:** $\bar{\mathbf{F}}_i = (P_i, q_i)$

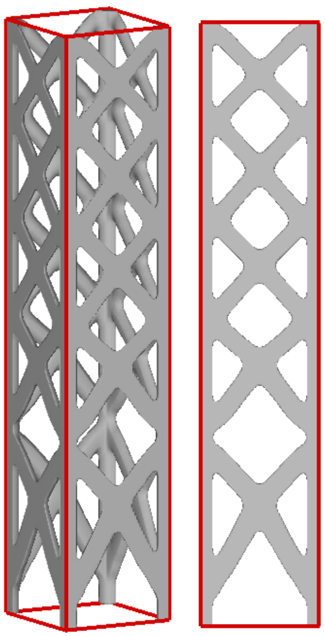
Load Cases	P		q (at top)		C_i^t
	mean	c.o.v	mean	c.o.v	
Case 1	70.71	0.30	2.82	0.15	250
Case 2	50.00	0.15	2.00	0.30	125
Case 3	50.00	0.20	2.00	0.15	125

Optimal Topologies of the Building Core



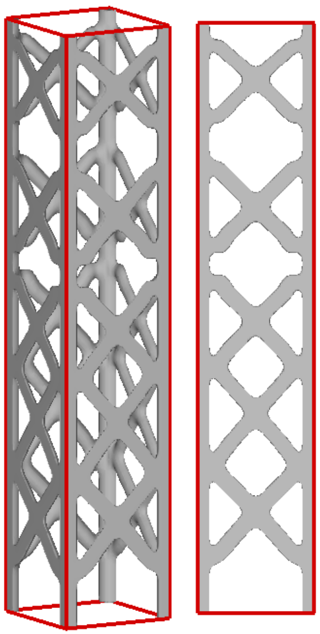
volfrac = **21.93%**

DTO



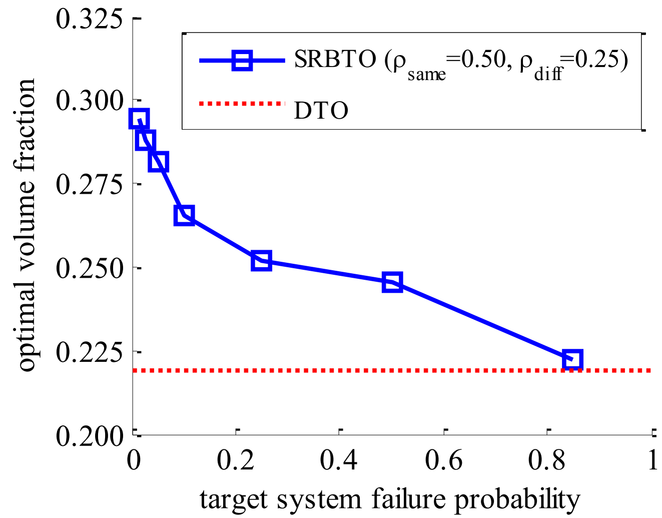
volfrac =**28.15%** (P_{sys}=0.05)

SRBTO



volfrac =**22.25%** (P_{sys}=0.85)

SRBTO



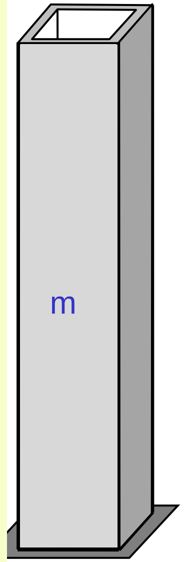
***volfrac* v.s P_{sys}**

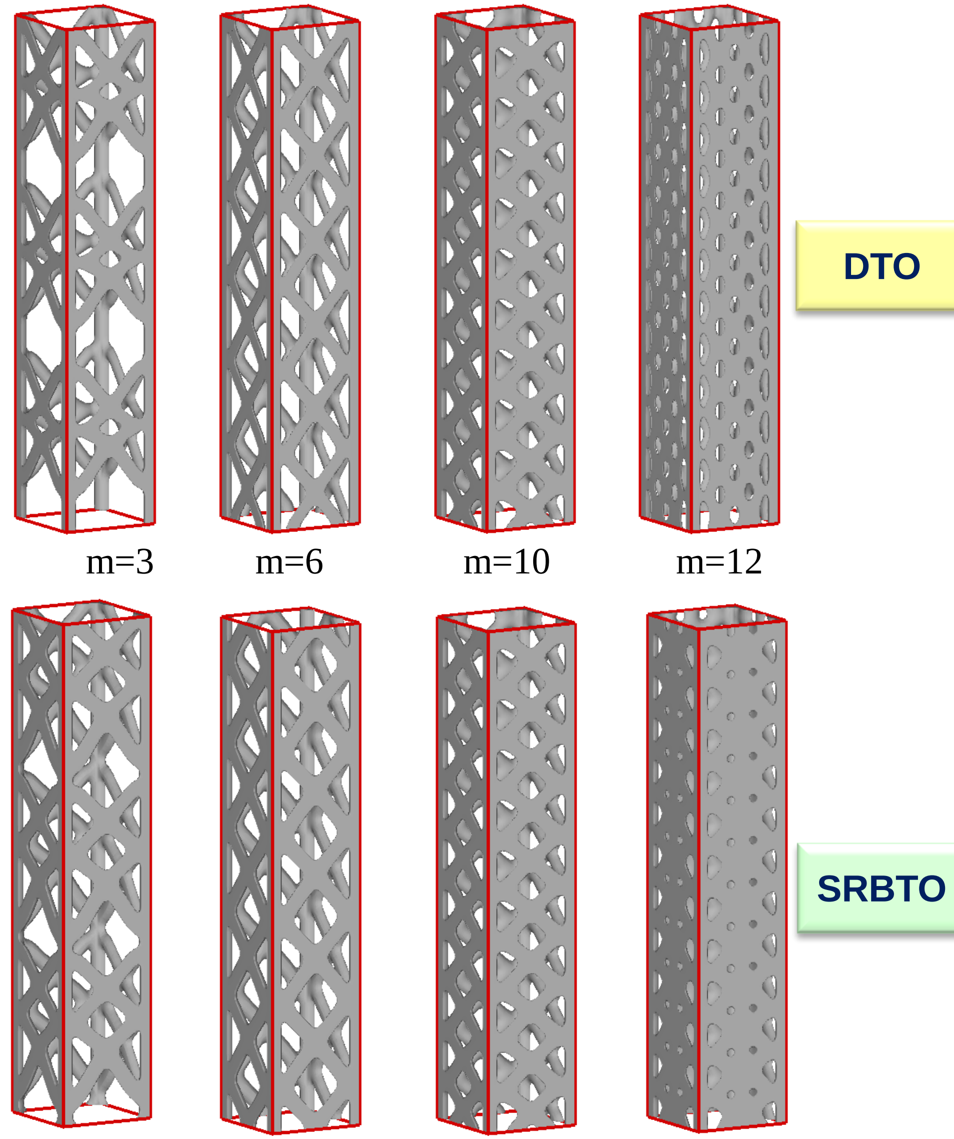
Probabilities: SRBTO/MSR v.s MCS

		P_1	P_2	P_3	P_{sys}
ρ_{same} = 0.5, ρ_{diff} = 0.25	SRBTO	0.02731	0.02088	0.00539	0.05000
	MCS	0.02747	0.02101	0.00542	0.05023
ρ_{same} = 0.5, ρ_{diff} = 0.25	SRBTO	0.26940	0.25973	0.20818	**0.50000**
	MCS	0.26977	0.26006	0.20800	0.50008
ρ_{same} = **0.9**, ρ_{diff} = **0.45**	SRBTO	0.02812	0.02227	0.00625	0.05000
	MCS	0.02816	0.02242	0.00638	0.05017

Building Core with Pattern Repetition

http://www.som.com





Summary & Conclusions

- **Multiresolution topology optimization (MTOP)**
  - Uses three distinct displacement, density, and design variable fields
  - Improves efficiency, apply to large-scale problems
- **System Reliability-based Topology Optimization (SRBTO/MSR)**
  - Uses Matrix-based system reliability (MSR) method
  - Enables uniform applications to general system events
  - Accounts for statistical dependence
- **SORM-based Single-loop CRBTO & SRBTO**
  - Employs second-order reliability method (SORM):
  - SORM-based Single-loop CRBTO & SRBTO
  - Improves accuracy
- **Numerical Examples**
  - Includes pattern repetition constraints
  - Compares with Monte Carlo simulations