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ON COMPUTATIONAL MECHANICS

On Fatigue Crack Growth Using Cohesive Zone Model

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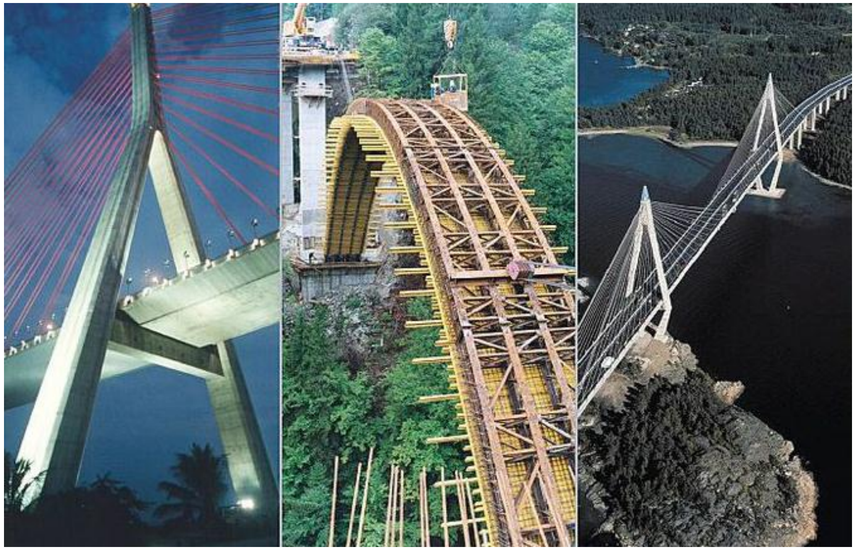
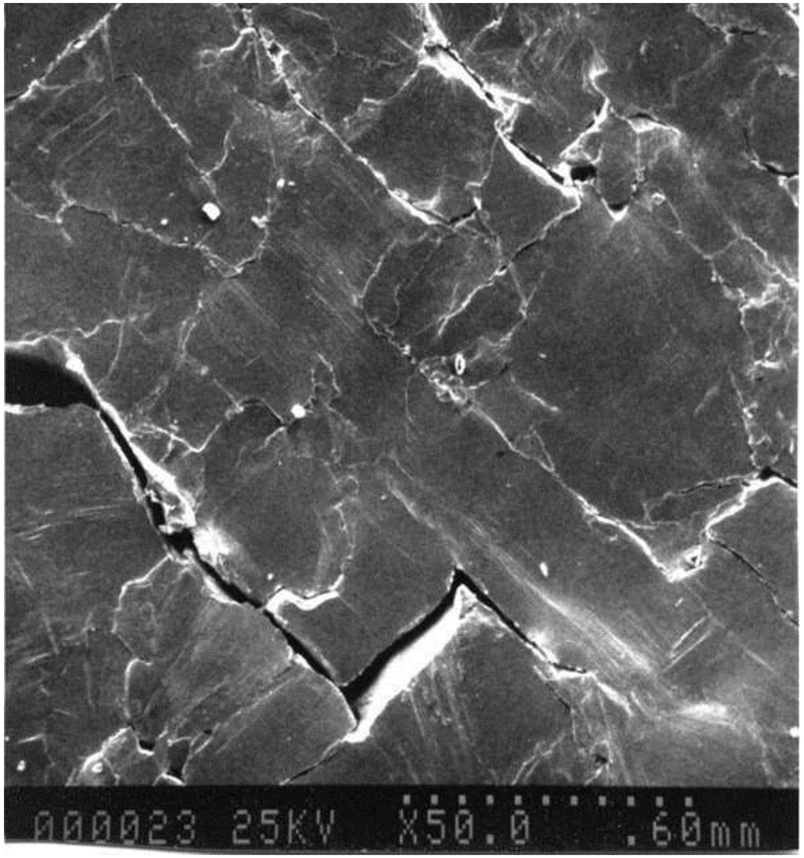
Motivation





Fatigue fracture along the interface between crystals in a metal
http://www.larrylawson.net/fatigue.htm

Previous Fatigue Crack Growth Models

Linear Elastic Fracture Mechanics

- Based on Paris type relations
- NASGRO, Southwest Research Institute (www.swri.org)
- AFGROW (www.afgrow.net), LexTech, Inc.

Nonlinear cohesive zone model

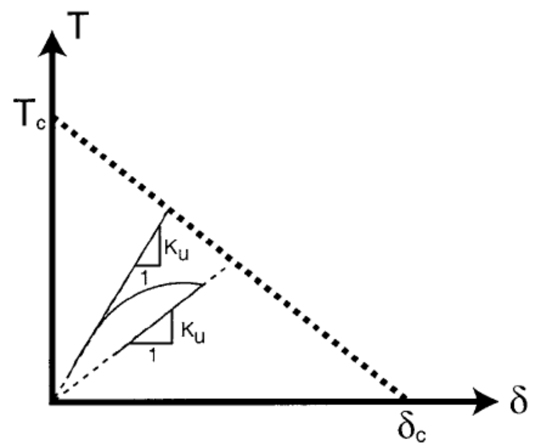
- Nguyen, Repetto, Ortiz, Radovitzky , 2001, A cohesive model of fatigue crack growth, IJF 110, 315-369
- Maiti, Geubelle, 2005, A cohesive model for fatigue failure of polymers , EFM 72, 691-708
- Roe, Siegmund, 2003, An irreversible cohesive zone model for interface fatigue crack growth simulation, EFM 70, 209-23
- Ural, Krishnan, Papoulia, 2009, A cohesive zone model for fatigue crack growth allowing for crack retardation, IJSS 46, 2453-2462

Model by Ngyen et al (2001)

Unloading/Reloading Relationship

$$\dot{T} = \begin{cases} K^{-}\dot{\Delta} & (\dot{\Delta} < 0) \\ K^{+}\dot{\Delta} & (\dot{\Delta} > 0) \end{cases}$$

- Unloading: $K^{-} = \dfrac{T_{\max}}{\Delta_{\max}}$
- Reloading: $\dot{K}^{+} = \begin{cases} -K^{+}\dot{\Delta} / \delta_f & (\dot{\Delta} > 0) \\ (K^{+} - K^{-})\dot{\Delta} / \delta_f & (\dot{\Delta} < 0) \end{cases}$
- When the softening starts?



Stiffness Decaying Factor, K^{+}

- Under uniform separation, Δ_0

$$K^{+}_{N} = e^{-N\Delta_0/\delta_f} K^{+}_{0}$$

O Nguyen, EA Repetto, M Ortiz and RA Radovitzky , 2001, A cohesive model of fatigue crack growth, IJF 110, 315-369

Model by Maiti and Geubelle (2005, 2006)

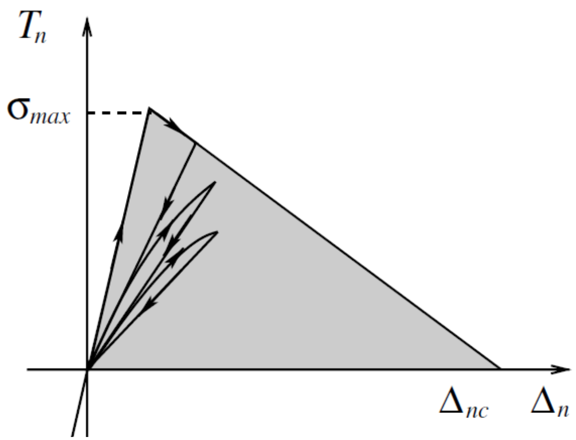
Reduction of Cohesive Stiffness

Maiti and Geubelle, 2005, A cohesive model for fatigue failure of polymers , EFM 72, 691-708

$$k_c = \frac{dT_n}{d\Delta_n} = F(N_f, T_n)$$

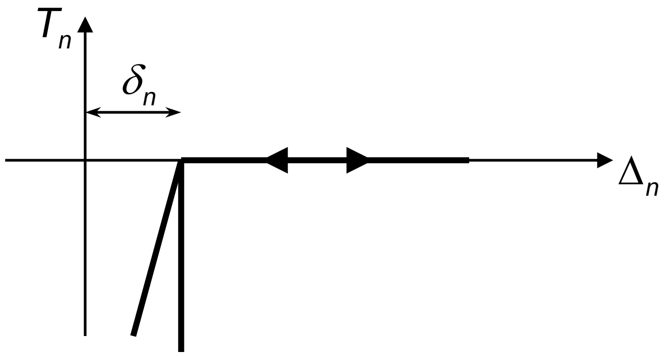
$$\dot{k}_c = -\frac{1}{\alpha} N_f^{-\beta} k_c \dot{\Delta}_n \qquad \dot{\Delta}_n \geq 0$$

$$k_c^{j+1} - k_c^{j} = -\frac{1}{\alpha}\left(N_f^{j}\right)^{-\beta} k_c^{j}\left(\Delta_n^{j+1} - \Delta_n^{j}\right)$$



- What if we change the load frequency or magnitude?

Crack Closure due to Wedge Effect



Maiti and Geubelle, 2006, Cohesive modeling of fatigue crack retardation in polymers: Crack closure effect, EFM 73, 22-41

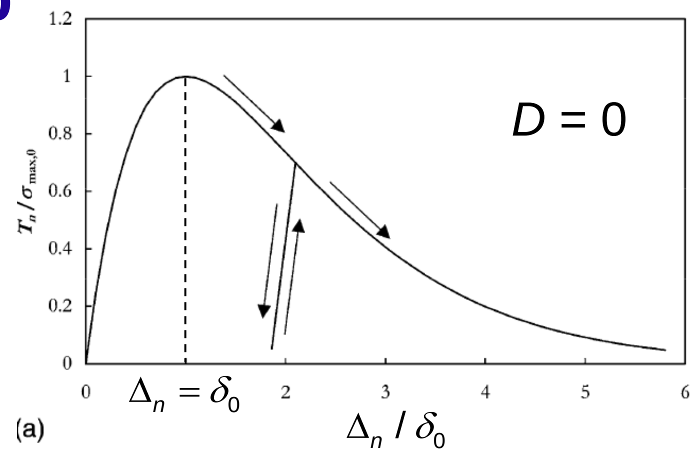
Model by Roe and Siegmund (2003)

◻ Unloading/Reloading Relationship

$$T_n = T_{n,\max} + k_n(\Delta_n - \Delta_{n,\max})$$

$$k_n = \boxed{\frac{\sigma_{\max} e}{\delta_0}}(1-D)$$

Initial slope in the exponential potential



- What if permanent deformation is negligible?
- What if the local cohesive traction does not reach the cohesive strength?

◻ Evolution of Damage Parameter

$$\dot{D} = \frac{|\dot{\Delta}|}{\delta_f}\left[\frac{T}{\sigma_{\max}(1-D)} - \frac{\sigma_f}{\sigma_{\max,0}}\right] H(\Delta - \delta_0) \qquad \dot{D} \geq 0$$

Roe and Siegmund, 2003, An irreversible cohesive zone model for interface fatigue crack growth simulation, EFM 70, 209-232

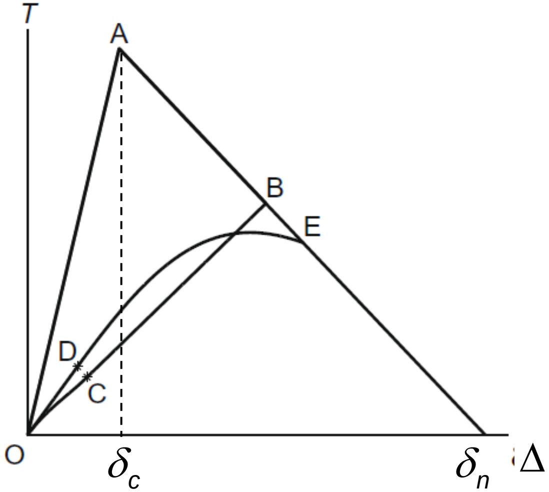
Model by Ural et al (2009)

Traction-Separation Relationship

$$T = F(k)\Delta \qquad F(k) = \frac{\sigma_c(1-k)}{k\delta_n + (1-k)\delta_c} \qquad C(k) = \sigma_c(1-k)$$

Damage Evolution

$$\dot{k} = \begin{cases} \alpha k(T-\beta C)\dot{\Delta} & T-\beta C > 0,\ \dot{\Delta} > 0 & DB \\ -\gamma k(T-\beta C)\dot{\Delta} & T-\beta C < 0,\ \dot{\Delta} < 0 & CO \\ 0 & (T-\beta C)\dot{\Delta} < 0 & OA, BC, OD \\ \dot{\lambda} & T = C,\ \dot{\Delta} > 0 & AB \end{cases}$$



No Damage Before Cohesive Strength

- Relatively lower cohesive strength is used in computational simulation

A Ural, VR Krishnan, KD Papoulia, 2009, A cohesive zone model for fatigue crack growth allowing for crack retardation, IJSS 46, 2453-2462.

Remarks

Previous Models

- Boundary between reloading and softening is not clear
- Model parameters should be free from the number of cycles
- Damage may occur before the cohesive traction reaches cohesive strength
- Little explanation on how the model parameters relate to the Paris "Law"

Goals

- Tackle the limitations in the previous models
- Provide a general fatigue crack growth model, which also captures Paris-type relations

Proposed Fatigue Crack Growth Model

Traction-Separation Relationship

$$T_n = \frac{\partial \varphi(\Delta_n, \Delta_t)}{\partial \Delta_n}(1 - D_T) \qquad \text{Softening condition}$$

$$T_n = \left(C_c + \frac{1 - C_c}{1 - C_o}\left(\frac{\Delta_n}{D_n \delta_n} - C_o \right) \right) \frac{\partial \varphi(D_n \delta_n, \Delta_t)}{\partial \Delta_n}(1 - D_T) \qquad \text{Unloading/Reloading}$$

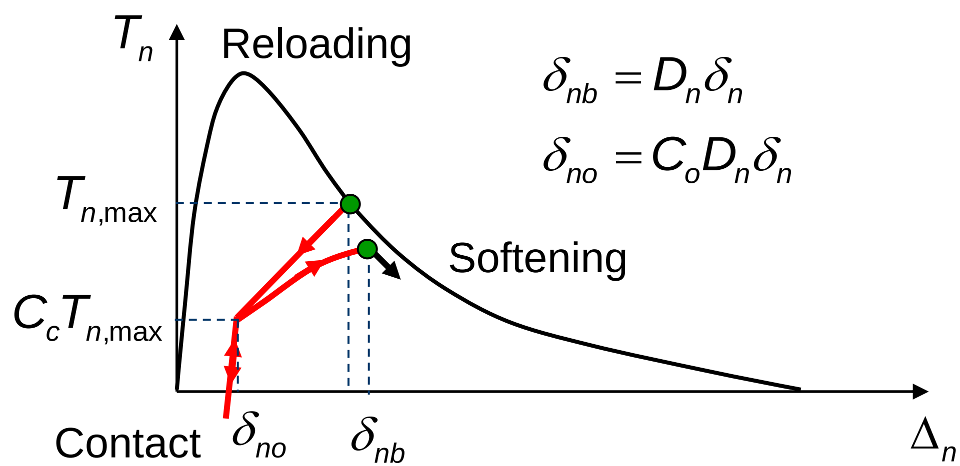
D_T: Damage associated with the rate of the cohesive traction

D_n: Damage associated with the rate of the cohesive separation

C_C: Crack closure effect (C_C=0.3)

C_O: Crack opening effect

$$\begin{aligned}
&\textit{Softening}: \delta_{nb} < \Delta_n < \delta_{nb} \\
&\textit{Reloading}: \delta_{no} < \Delta_n < \delta_{nb},\ \dot{\Delta}_n > 0 \\
&\textit{Unloading}: \delta_{no} < \Delta_n < \delta_{nb},\ \dot{\Delta}_n < 0 \\
&\textit{Contact}: \Delta_n < \delta_{no} \\
&\textit{Complete failure}: \Delta_n \geq \delta_n
\end{aligned}$$



Fatigue Crack Growth Model

Traction-Separation Relationship

$$T_n = \frac{\partial \varphi(\Delta_n, \Delta_t)}{\partial \Delta_n}(1 - D_T)$$ Softening condition

$$T_n = \left(C_c + \frac{1 - C_c}{1 - C_o}\left(\frac{\Delta_n}{D_n \delta_n} - C_o \right) \right) \frac{\partial \varphi(D_n \delta_n, \Delta_t)}{\partial \Delta_n}(1 - D_T)$$ Unloading/Reloading

D_T: Damage associated with the rate of the cohesive traction

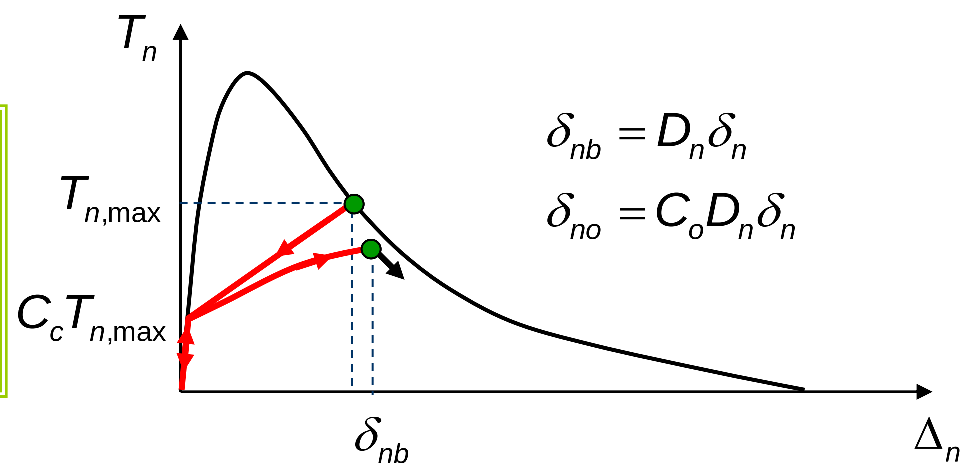
D_n: Damage associated with the rate of the cohesive separation

C_C: Crack closure effect (C_C=0.3)

C_O: Crack opening effect

$C_o = 0 \rightarrow \delta_{no} = 0$

$$T_n = \left(C_c + (1 - C_c)\frac{\Delta_n}{D_n \delta_n} \right) \frac{\partial \varphi(D_n \delta_n, \Delta_t)}{\partial \Delta_n}(1 - D_T)$$



Fatigue Crack Growth Model

Traction-Separation Relationship

$$T_n = \frac{\partial \varphi(\Delta_n, \Delta_t)}{\partial \Delta_n}(1 - D_T) \qquad \text{Softening condition}$$

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D_T: Damage associated with the rate of the cohesive traction

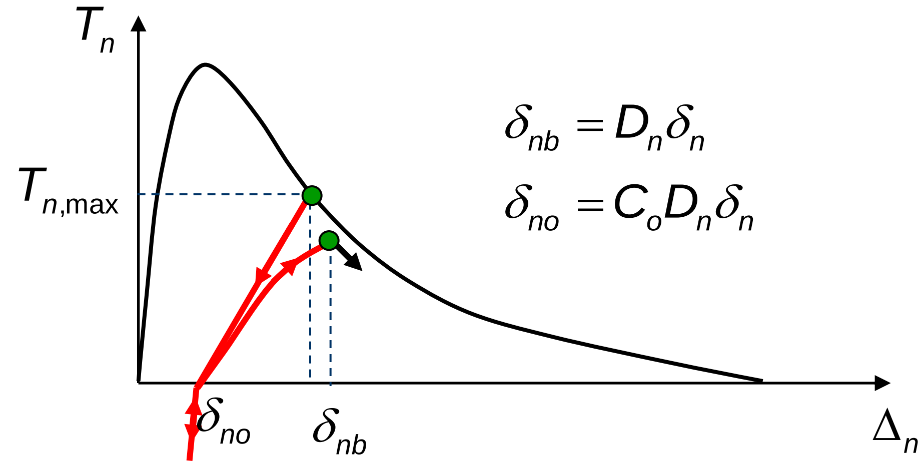
D_n: Damage associated with the rate of the cohesive separation

C_C: Crack closure effect (C_C=0.3)

C_O: Crack opening effect

$$C_c = 0$$

$$T_n = \frac{1}{1 - C_o}\left(\frac{\Delta_n}{D_n \delta_n} - C_o \right) \frac{\partial \varphi(D_n \delta_n, \Delta_t)}{\partial \Delta_n}(1 - D_T)$$



Fatigue Crack Growth Model

Traction-Separation Relationship

$$T_n = \frac{\partial \varphi(\Delta_n, \Delta_t)}{\partial \Delta_n}(1 - D_T) \qquad \text{Softening condition}$$

$$T_n = \left(C_c + \frac{1 - C_c}{1 - C_o}\left(\frac{\Delta_n}{D_n \delta_n} - C_o \right) \right) \frac{\partial \varphi(D_n \delta_n, \Delta_t)}{\partial \Delta_n}(1 - D_T) \qquad \text{Unloading/Reloading}$$

D_T: Damage associated with the rate of the cohesive traction

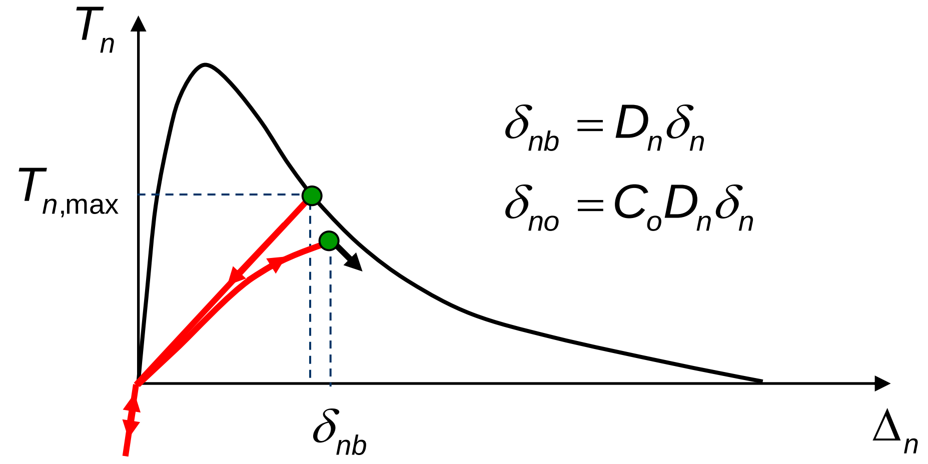
D_n: Damage associated with the rate of the cohesive separation

C_C: Crack closure effect (C_C=0.3)

C_O: Crack opening effect

$C_c = 0$, $C_o = 0$

$$T_n = \frac{\Delta_n}{D_n \delta_n} \frac{\partial \varphi(D_n \delta_n, \Delta_t)}{\partial \Delta_n}(1 - D_T)$$



Evolution of Damage Parameters

Damage Associated with Rate of Separation, D_n

$$\dot{D}_n = \begin{cases} \dot{\Delta}_n / \delta_n & \Delta_n \geq \delta_{nb} \text{ (Softening)} \\ \dot{\Delta}_n / \kappa_n \delta_n & \delta_{no} < \Delta_n < \delta_{nb},\ \dot{\Delta}_n > 0 \text{ (Reloading)} \\ 0 & \delta_{no} < \Delta_n < \delta_{nb},\ \dot{\Delta}_n < 0 \text{ (Unloading)} \\ 0 & \Delta_n < \delta_{no} \text{ (Contact)} \end{cases}$$

Damage Associated with Rate of Traction, D_T

$$\dot{D}_T = \begin{cases} \dot{T} / \kappa_t \sigma_{\max} & \text{(Reloading)} \\ 0 & \text{(Unloading / Softening)} \end{cases}$$

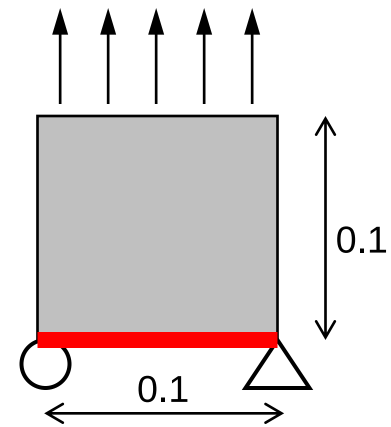
Material Parameters

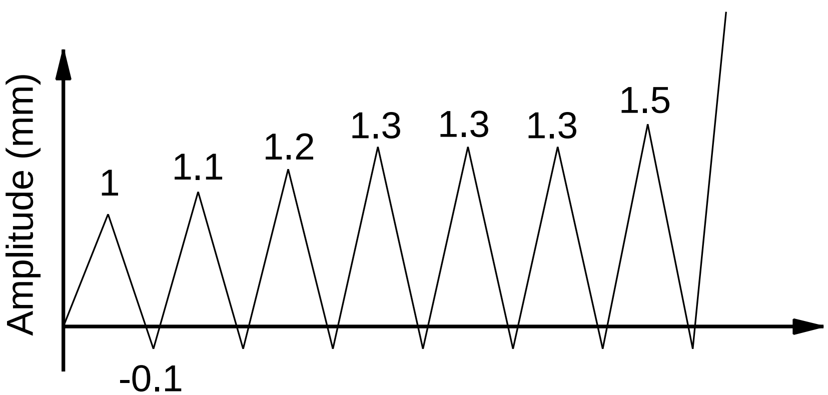
- Higher values of k_n and k_t (inversely proportional to damage), higher resistance in fatigue crack growth

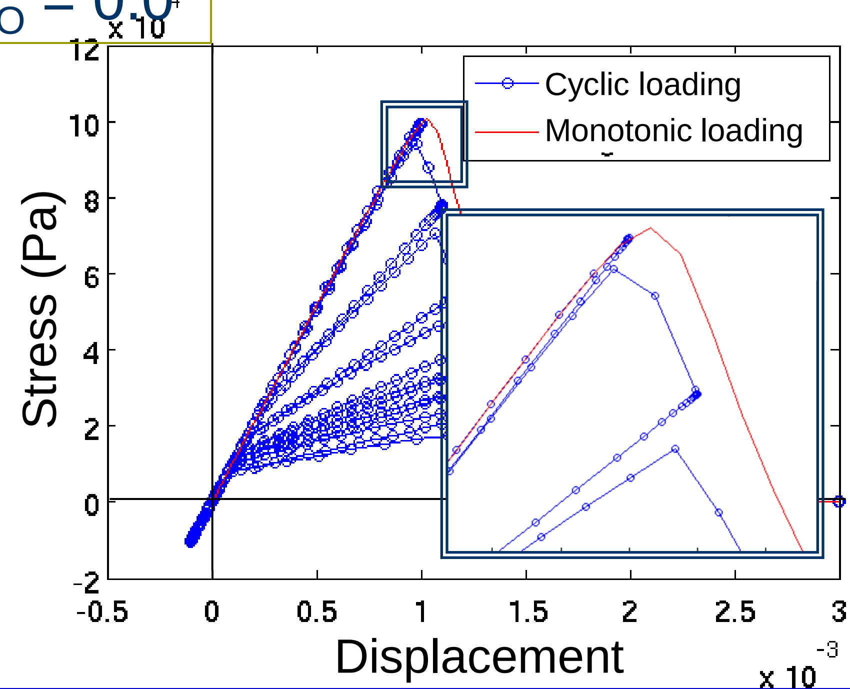
Simple Mode – I Problem

Problem Description

- Displacement Control
- E = 10.34 MPa, v = 0.3, t=0.04
- G_I = 98.1N/m, σ_{max} = 0.1MPa, α=0.3
- k_n = 10, k_t = 10, C_C = 0.3, C_O = 0.0

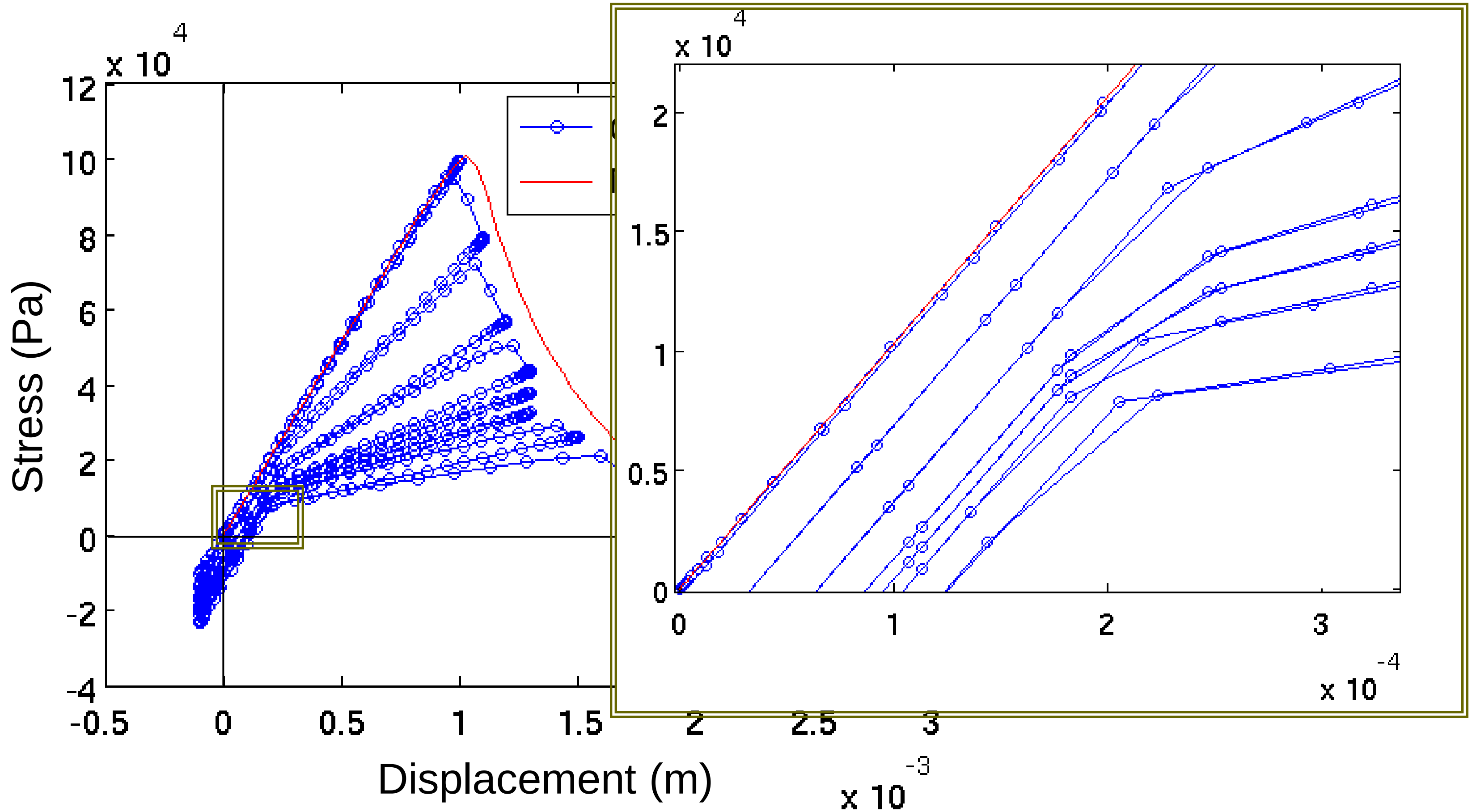






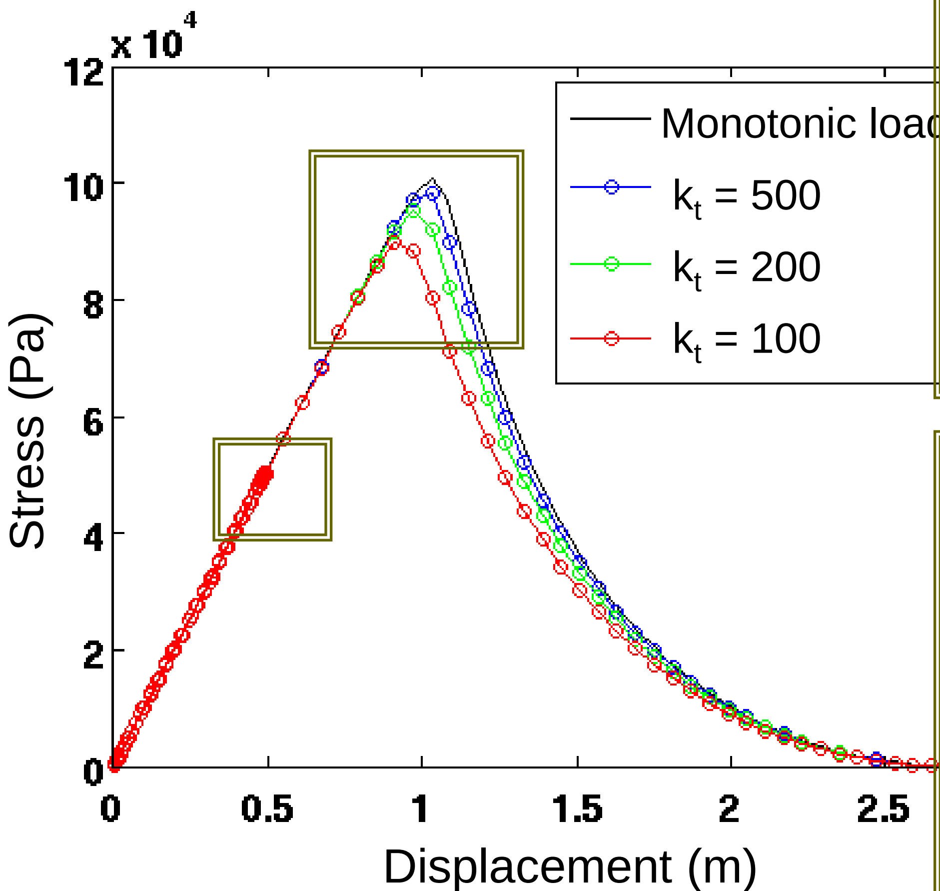
Effect of Crack Opening

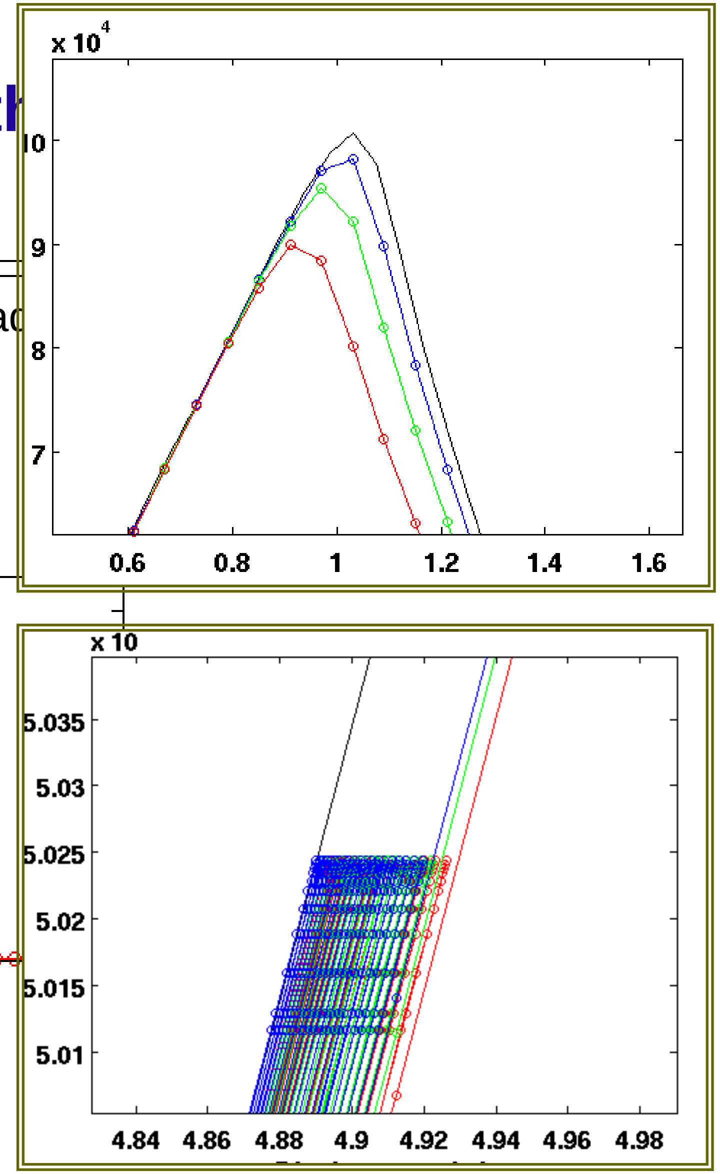
- **Parameters: k_n = 10, k_t = 10, C_C = 0.3, C_O = 0.1**



Constant Loading Amplitude

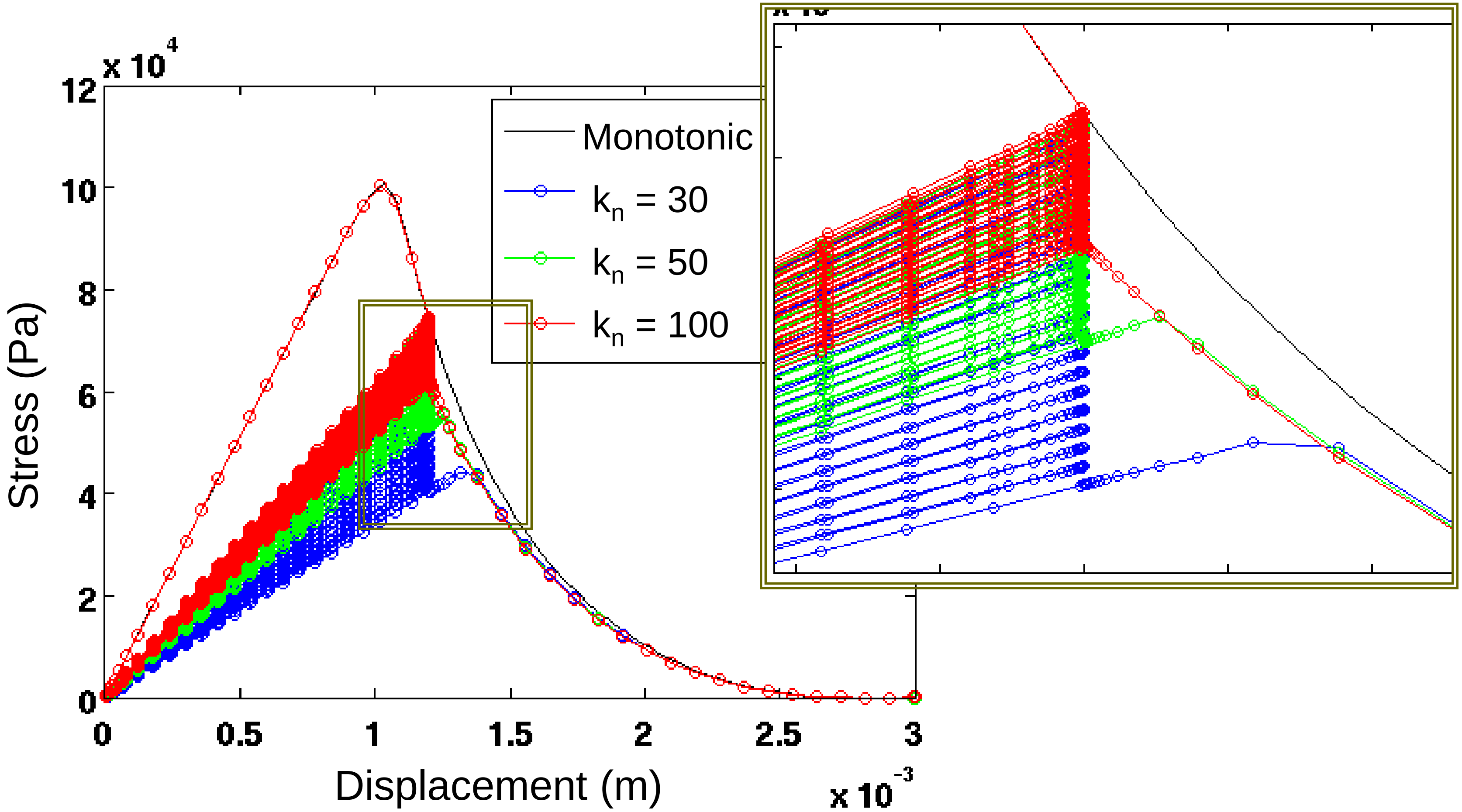
- **Effect of Damage Measure for th**





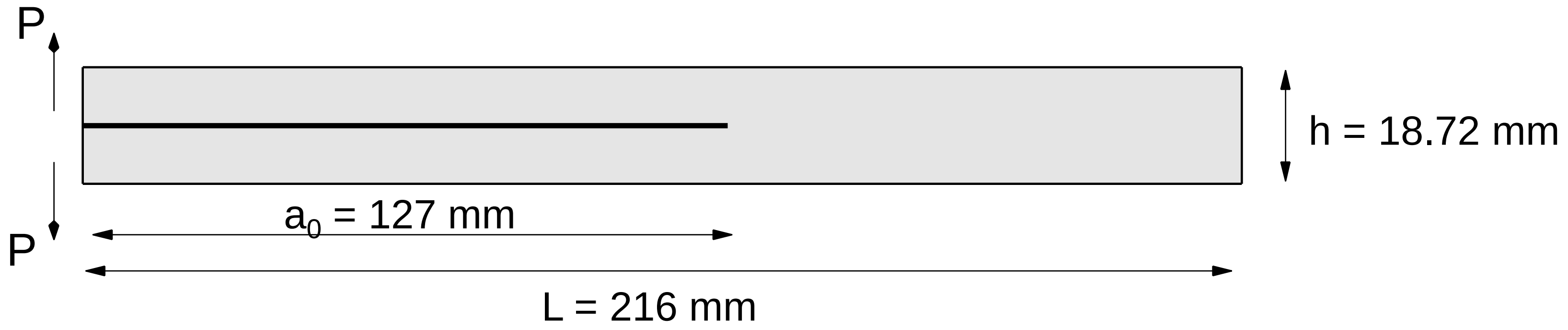
Constant Displacement Amplitude

Effect of Damage Measure for the Rate of Separation



Double Cantilever Beam

Problem Description

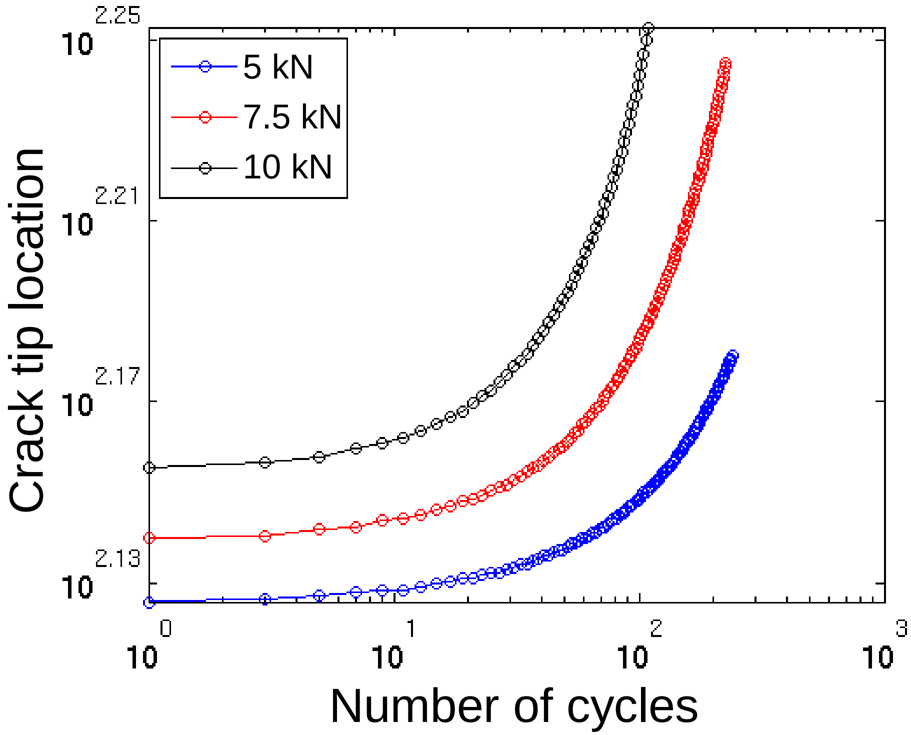


- Displacement Control
- E = 70 GPa, v = 0.33 , σ_{max} = 6.66MPa
- G_I = 675 N/m, α = 0.3
- k_n = 5/10/20, k_t = 40/80/∞, C_C = 0.3 , C_O = 0.0
- Constant loading amplitude (5, 7.5, 10 kN)

A Ural, VR Krishnan, KD Papoulia, 2009, A cohesive zone model for fatigue crack growth allowing for crack retardation, IJSS 46, 2453-2462.

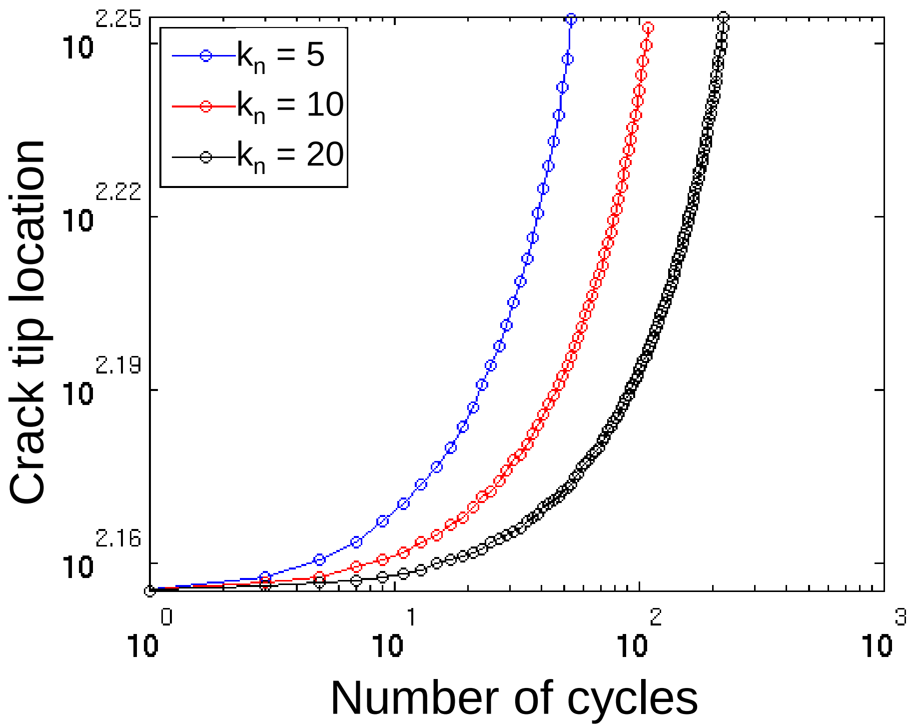
Computational Results

□ Load Magnitude



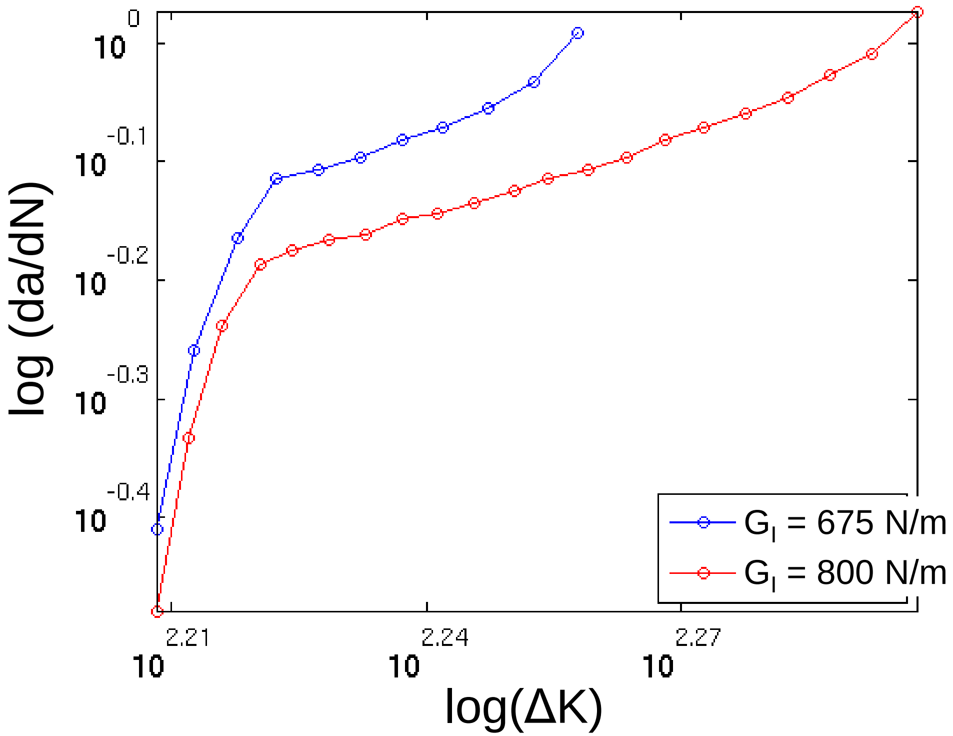
$k_n = 10, k_t = \infty$ (fixed)

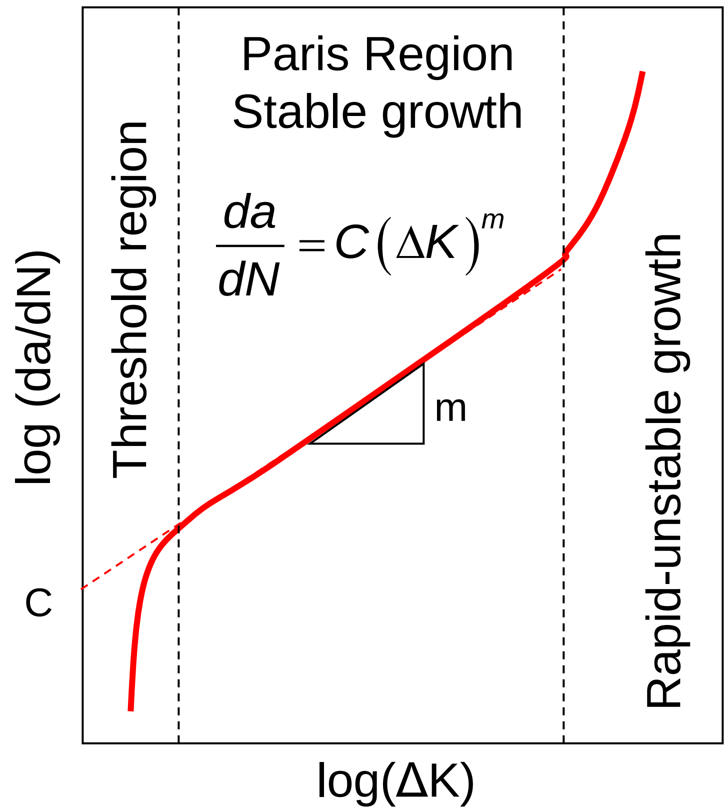
□ Rate of Separation



$P_{max} = 10$ kN, $k_t = \infty$ (fixed)

Effect of Fracture Energy

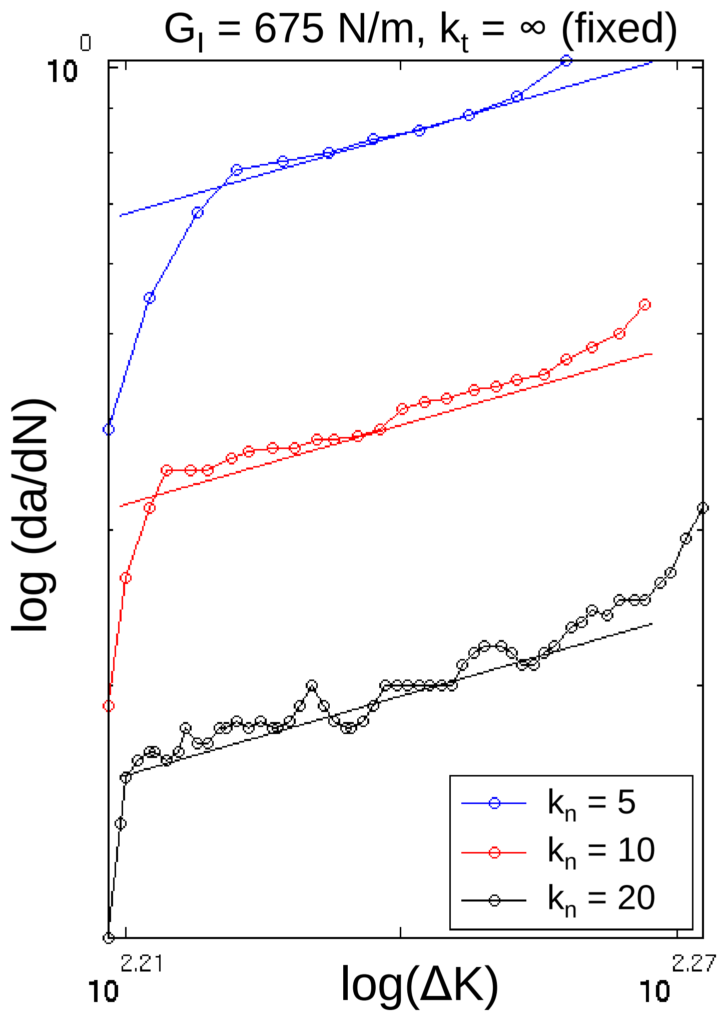




$$K = P\sqrt{\frac{12}{h}}\left(\frac{a}{h}+0.673\right)+\sqrt{\frac{2h}{\pi a}}-\left(0.815\left(\frac{a}{h}\right)^{0.619}+0.429\right)^{-1}$$

Foote and Buchwald, 1985, IJF 29, 125-134

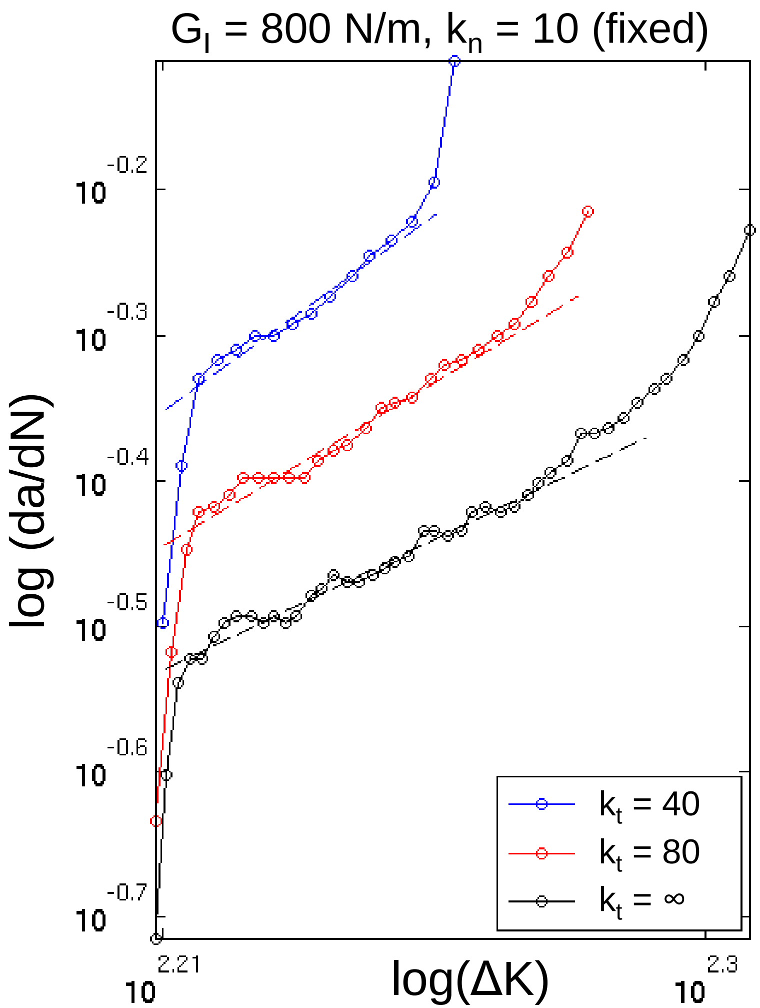
Comparison with Paris Equation



Paris' Parameters

- m = 3 / C = 1.6×10^{-7}, 0.75×10^{-7}, 0.37×10^{-7}

→ k_n is related to C



Paris' Parameters

- m = 3, 2.5, 2
- C = 1.04×10^{-7}, 10.7×10^{-7}, 112×10^{-7}

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Summary and Future Research

- Previous fatigue crack models show several limitations
- Proposed model is based on two damage measures
- Define softening and unloading/reloading conditions
- Fatigue damage occurs before the cohesive traction reaches the cohesive strength
- Extension to mixed-mode problems
- Further research needed on crack closure, bulk plasticity and temperature effects

Thank you for your attention !