Toward Group Optimization for the Practical Design of Building Systems

Lauren L. Stromberg, Alessandro Beghini, William F. Baker, Glaucio H. Paulino

July 26, 2011

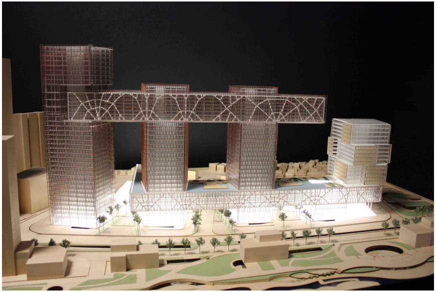
Architecture without evident engineering rationale

http://en.wikipedia.org/wiki/File:Hadid-Afragola.jpg

http://en.wikipedia.org/wiki/Beijing_National_Stadium

http://images.businessweek.com/ss/06/03/italy/source/3.htm

Combining Engineering and Architecture



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Combining Engineering and Architecture


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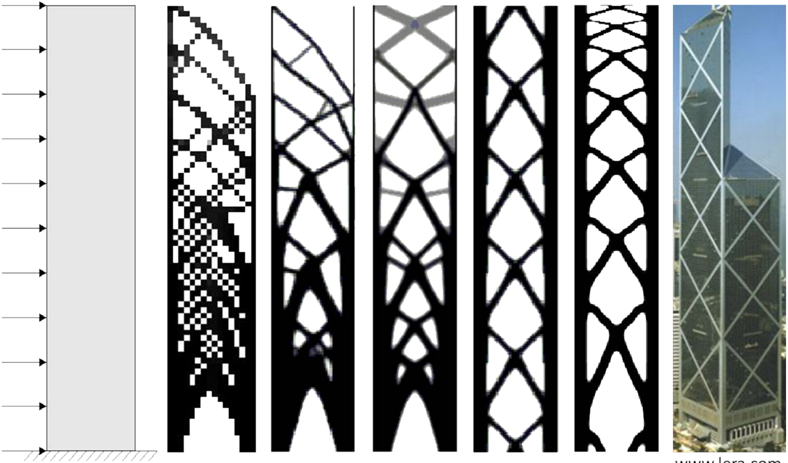
Background and motivation



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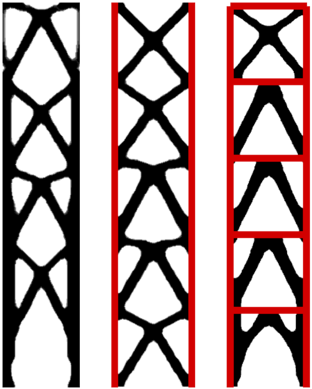
"The language of mathematics and rational engineering could not give form to architecture of substantive quality on its own, no more than could ungrounded aesthetic inclination. Rather, by conjoining creative energies and different perspectives, better innovative and responsive design solutions could be developed than either architect or engineer might conceive in isolation." Khan [2004].

Toward group optimization for the practical design of building systems

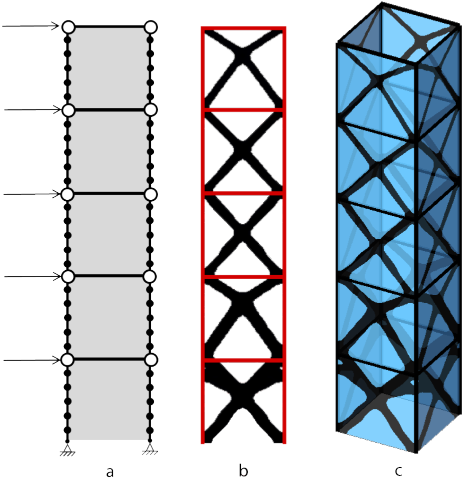


Stromberg et al. [2011]

Overall design process

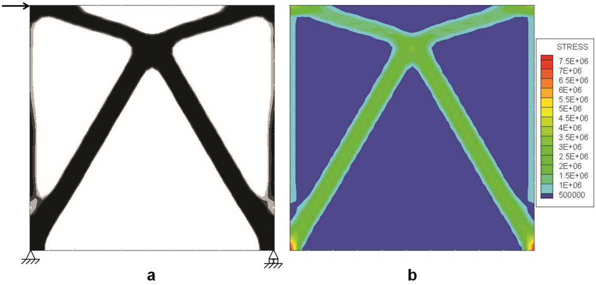
- size vertical line elements (columns) according to gravity load combinations (accounting for dead, superimposed dead and live loads) using technique in Baker [1992]
- run topology optimization on the continuum elements for lateral load combinations (accounting for wind and seismic loads)
- identify the optimal bracing layout based on results and create frame model
- optimize the member sizes using the virtual work methodology

Numerical example of a bracing system



Optimal Braced Frames - Constancy of Stress

Constant stress in optimized frame verified in the continuum- the Von-Mises stresses are nearly constant within each optimized member



Optimal Braced Frames - Constancy of Stress

In terms of the displacements u_i at each point of load application P_i, the compliance can be expressed as:

$$W_{ext} = \sum_i P_i u_i = \sum_j \frac{N_j^2 L_j}{EA_j} = W_{int}$$

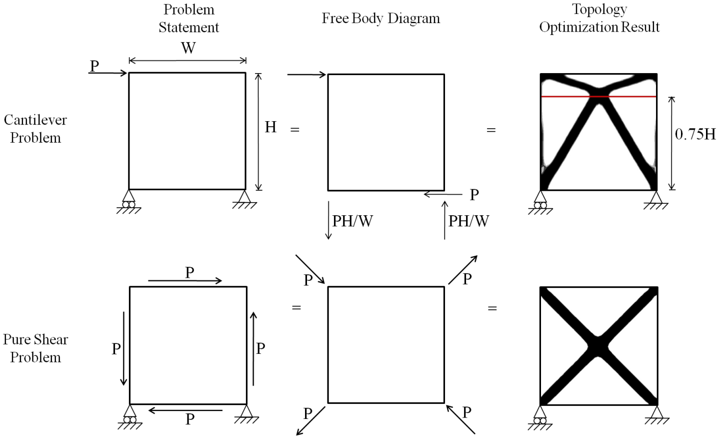
By introducing the Lagrangian multiplier constraint on the areas of the members,

$$W_{ext} = \sum_j \frac{N_j^2 L_j}{EA_j} + \lambda \left(\sum_j A_j L_j - V \right)$$

Differentiating with respect to the areas A_i and solving for the Lagrangian multiplier λ

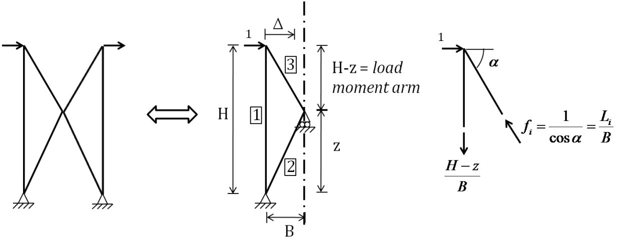
$$\lambda = \left(\frac{N_i}{A_i} \right)^2 \frac{1}{E} = \frac{\sigma^2}{E} = const$$

Optimal Single Module Bracing



Optimal Single Module Bracing

The optimal bracing geometry for a single module (even number of diagonals) is considered:



From the geometry, the forces in the members are

$$f_1 = \frac{H-z}{B}, \quad f_2 = \frac{\sqrt{B^2+z^2}}{B}, \text{ and } \quad f_3 = \frac{-\sqrt{B^2+(H-z)^2}}{B}$$

Optimal Single Module Bracing

Assuming each member to have a constant stress, $\sigma = \frac{F_i}{A_i}$,

$$\Delta = \sum_i \frac{f_i F_i L_i}{E A_i} = \frac{\sigma B}{E} \sum_i \frac{f_i L_i}{B}$$

The tip deflection of the frame is minimal when

$$\frac{\partial \Delta}{\partial z} = \frac{\sigma B}{E} \frac{\partial}{\partial z} \left(\sum_i \frac{f_i L_i}{B} \right) = 0$$

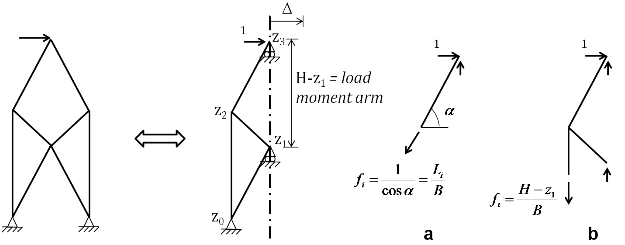
$$= \frac{\sigma B}{E} \frac{\partial}{\partial z} \left(H \left(\frac{H - z}{B^2} \right) + \frac{B^2 + z^2}{B^2} + \frac{B^2 + (H - z)^2}{B^2} \right) = 0$$

Thus, the brace work point height for minimal deflection is

$$\boxed{z = \frac{3}{4} H}$$

Optimal multiple module bracing for a point load

Now, adding one more set of diagonals (odd number of diagonals),



the forces in the diagonals are $f_i = \frac{L_i}{B}$ while the forces in the columns are given by $f_i = \frac{(H-z_i)}{B}$. The displacement at the top of the frame is

$$\Delta = \frac{\sigma B}{E} \sum_i \frac{f_i L_i}{B}$$

$$= \frac{\sigma B}{E} \left[\sum_i \left(\frac{L_i^2}{B^2} \right)_{braces} + \sum_j \left(\frac{(H - z_j) L_j}{B^2} \right)_{columns} \right]$$

Optimal multiple module bracing for a point load

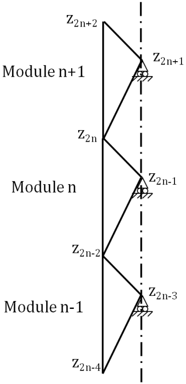
The frame with minimal top displacement is defined by the following equations:

$$\frac{\partial \Delta}{\partial z_1} = 0 \Rightarrow -3z_2 + 4z_1 = 0$$
$$\frac{\partial \Delta}{\partial z_2} = 0 \Rightarrow -H + 4z_2 - 3z_1 = 0$$

Therefore,

$$\boxed{\begin{aligned} z_1 &= \frac{3}{4} z_2 \\ z_2 &= \frac{4}{7} H \end{aligned}}$$

Optimal multiple module bracing for a point load



Generalizing the previous equations,

$$\frac{E\Delta}{\sigma B} = \sum_{n=1}^{N} \frac{\left(z_{2n} - z_{2n-1}\right)^2 + B^2}{B^2} + \frac{\left(z_{2n-1} - z_{2n-2}\right)^2 + B^2}{B^2} + \frac{\left(H - z_{2n-1}\right)^2}{B^2}\left(z_{2n} - z_{2n-2}\right)$$

Optimal multiple module bracing for a point load

Here N is the total number of modules and it is assumed that $z_{2n-2} < z_{2n-1} < z_{2n}$. By differentiating with respect to the nodal elevations z_{2n} (column work point) and z_{2n-1} (brace work point):

$$\frac{\partial}{\partial z_{2n-1}}\left(\frac{E\Delta}{\sigma B}\right) = 0 \Rightarrow -3z_{2n} + 4z_{2n-1} - z_{2n-2} = 0$$

$$\frac{\partial}{\partial z_{2n}}\left(\frac{E\Delta}{\sigma B}\right) = 0 \Rightarrow -z_{2n+1} + 4z_{2n} - 3z_{2n-1} = 0$$

These equations can be rewritten as follow:

$$z_{2n} = \frac{z_{2n-1} + z_{2n+1}}{2} - \frac{z_{2n+1} - z_{2n-1}}{4}$$

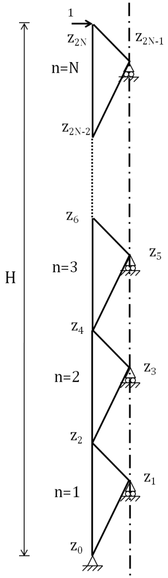
$$z_{2n-1} = \frac{z_{2n-2} + z_{2n}}{2} + \frac{z_{2n} - z_{2n-2}}{4}$$

Optimal multiple module bracing for a point load

From the previous expressions, two important geometric features of optimal braced frames are inferred:

1. The braced frame central work point z_{2n-1} is always located at 75% of the module height.
2. The module heights are all equal indicating that *patterns* are optimal

Optimal multiple module bracing for a point load



$$z_4 = \frac{8}{11} z_5$$

$$z_3 = \frac{7}{8} z_4 = \frac{7}{11} z_5$$

$$z_2 = \frac{4}{7} z_3 = \frac{1}{2} z_4 = \frac{4}{11} z_5$$

$$z_1 = \frac{3}{4} z_2 = \frac{3}{7} z_3 = \frac{3}{8} z_4 = \frac{3}{11} z_5$$

$$z_0 = 0$$

Optimal number of diagonals

By minimizing the volume of a frame,

$$V = \sum_i A_i L_i = \sum \frac{F_i}{\sigma} L_i = \frac{P}{\sigma} \sum_i f_i L_i = \frac{PB}{\sigma} \sum_i \frac{f_i L_i}{B}$$

or

$$V = \frac{PE}{\sigma^2} \Delta$$

For the first case ($m = 2$),

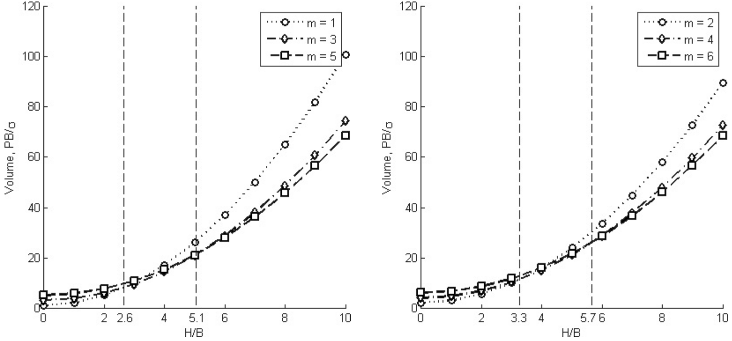
$$\begin{aligned} V &= \frac{PB}{\sigma} \sum_i \frac{f_i L_i}{B} \\ &= \frac{PB}{\sigma} \left(\frac{H^2}{4B^2} + \frac{B^2 + \frac{9H^2}{16}}{B^2} + \frac{\frac{H^2}{16} + B^2}{B^2} \right) \\ &= \frac{PB}{\sigma} \left(2 + \frac{7}{8} \left(\frac{H}{B} \right)^2 \right) \end{aligned}$$

Optimal number of diagonals

Number of Diagonals, m	Dimensionless Frame Volume, $V\sigma/(PB)$
1	$1 + \left(\frac{H}{B}\right)^2$
2	$2 + \frac{7}{8}\left(\frac{H}{B}\right)^2$
3	$3 + \frac{5}{7}\left(\frac{H}{B}\right)^2$
4	$4 + \frac{11}{16}\left(\frac{H}{B}\right)^2$
m (odd)	$m + \left[\frac{m+2}{2m+1}\right]\left(\frac{H}{B}\right)^2$
m (even)	$m + \left[\frac{1}{2} + \frac{3}{4m}\right]\left(\frac{H}{B}\right)^2$

Optimal number of diagonals

Dimensionless volume vs. height to width ratio, $\frac{H}{B}$

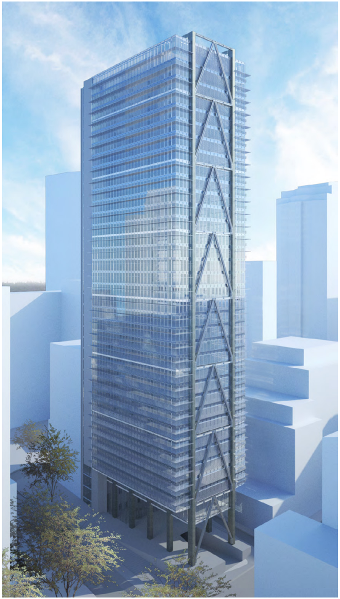


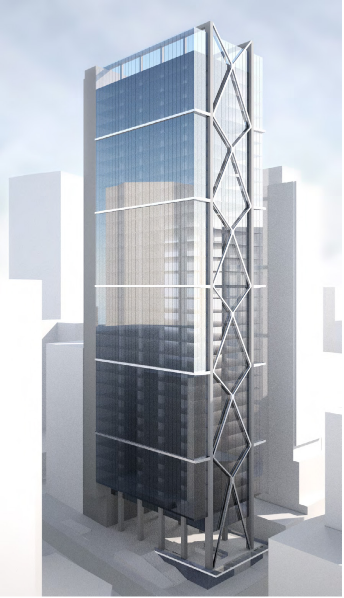
m odd   *m* even

Optimal Braced Frames

Image courtesy of Skidmore, Owings & Merrill, LLP

Optimal Braced Frames





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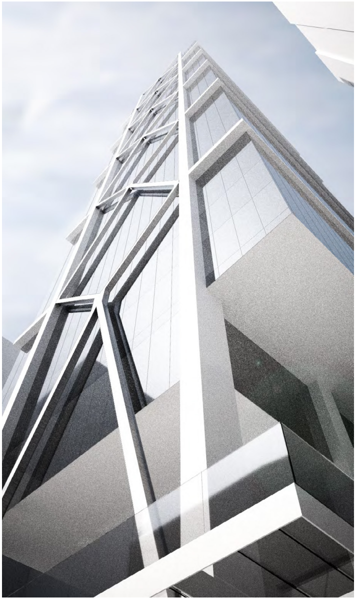
Optimal Braced Frames



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Conclusion

- Topology optimization, including group optimization, can and should be used for the practical design of buildings.

Conclusion

- Topology optimization, including group optimization, can and should be used for the practical design of buildings.

Thank you! Questions?

References I

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