## Interfacial cracking in a graded coating/substrate system loaded by a frictional sliding flat punch

## - Electronic supplementary material -(Figure 7-13)

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Figure 7. Variations of punch-edge stress intensity factors (a)  $K_T/K_{0T}$  and (b)  $K_L/K_{0L}$  versus punch location e/c for different values of interlayer thickness  $h_2/h$  and shear modulus ratio  $\mu_1/\mu_3$  (h/c=1.0,  $\delta/c=0.5$ ,  $\mu_p=0.5$ ,  $K_{0T}=\sigma_0(2\delta)^{-\omega}$ ,  $K_{0L}=\sigma_0(2\delta)^{-\chi}$ ,  $\sigma_0=P/2\delta$ ).



Figure 8. Variations of crack-tip stress intensity factors (a)  $K_{\rm I}(-c)/K_0$ , (b)  $K_{\rm I}(+c)/K_0$ , (c)  $K_{\rm II}(-c)/K_0$ , and (d)  $K_{\rm II}(+c)/K_0$  versus punch location e/c for different values of coating thickness  $h_1/c$  ( $\mu_1/\mu_3=5.0$ ,  $\mu_f=0.5$ ,  $h_2/c=0.5$ ,  $\delta/c=0.5$ ,  $K_0=\sigma_0c^{1/2}$ ,  $\sigma_0=P/2\delta$ ).



Figure 9. Variations of punch-edge stress intensity factors (a)  $K_T/K_{0T}$  and (b)  $K_L/K_{0L}$  versus punch location e/c for different values of coating thickness  $h_1/c$  ( $\mu_1/\mu_3=5.0$ ,  $\mu_7=0.5$ ,  $h_2/c=0.5$ ,  $\delta/c=0.5$ ,  $K_{0T}=\sigma_0(2\delta)^{-\omega}$ ,  $K_{0L}=\sigma_0(2\delta)^{-\chi}$ ,  $\sigma_0=P/2\delta$ ).



Figure 10. Variations of crack-tip stress intensity factors (a)  $K_{\rm I}(-c)/K_0$ , (b)  $K_{\rm I}(+c)/K_0$ , (c)  $K_{\rm II}(-c)/K_0$ , and (d)  $K_{\rm II}(+c)/K_0$  versus punch location e/c for different values of punch width  $\delta/c$  ( $\mu_1/\mu_3=5.0$ ,  $\mu_f=0.5$ ,  $h_1/h=h_2/h=0.5$ , h/c=1.0,  $K_0=\sigma_0c^{1/2}$ ,  $\sigma_0=P/2\delta$ ).



Figure 11. Variations of punch-edge stress intensity factors (a)  $K_T/K_{0T}$  and (b)  $K_L/K_{0L}$  versus punch location e/c for different values of punch width  $\delta/c$  ( $\mu_1/\mu_3=5.0$ ,  $\mu_f=0.5$ ,  $h_1/h=h_2/h=0.5$ , h/c=1.0,  $K_{0T}=\sigma_0(2\delta)^{-\omega}$ ,  $K_{0L}=\sigma_0(2\delta)^{-\chi}$ ,  $\sigma_0=P/2\delta$ ).



Figure 12. Variations of crack-tip stress intensity factors (a)  $K_{\rm I}(-c)/K_0$ , (b)  $K_{\rm I}(+c)/K_0$ , (c)  $K_{\rm II}(-c)/K_0$ , and (d)  $K_{\rm II}(+c)/K_0$  versus punch location e/c for different values of friction coefficient  $\mu_f (\mu_1/\mu_3=5.0, \delta/c=0.5, h_1/h=h_2/h=0.5, h/c=1.0, K_0=\sigma_0 c^{1/2}, \sigma_0=P/2\delta)$ .



Figure 13. Variations of punch-edge stress intensity factors (a)  $K_T/K_{0T}$  and (b)  $K_L/K_{0L}$  versus punch location e/c for different values of friction coefficient  $\mu_f (\mu_1/\mu_3=5.0, \ \delta/c=0.5, \ h_1/h=h_2/h=0.5, \ h/c=1.0, \ K_{0T}=\sigma_0(2\ \delta)^{-\omega}, \ K_{0L}=\sigma_0(2\ \delta)^{-\chi}, \ \sigma_0=P/2\ \delta).$