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Connecting architecture and engineering through structural topology optimization

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ABSTRACT

One of the prevalent issues facing the construction industry in today's world is the balance between engineering and architecture: traditionally, the goal of the architect has focused more on the aesthetics, or "form" of a structure, while the goal of the engineer has been focused on stability and efficiency, or its "function". In this work, we discuss the importance of a close collaboration between these disciplines, and offer an alternative approach to generate new, integrated design ideas by means of a tailored structural topology optimization framework, which can potentially be of benefit to both the architectural and structural engineering communities. Several practical case studies, from actual collaborative design projects, are given to illustrate the successes and limitations of such techniques.

1. Introduction

Design professionals (such as architects and engineers) strive for a balance between different and sometimes conflicting goals for any particular project. Traditionally (at least in recent tradition) we can perhaps generalize that the goal of the architect has been leaning towards aesthetics and the goal of the engineer has been focused on stability and efficiency. In the more distant past (say, in medieval times during which great cathedrals were being built) the specialization of *architecture* and *engineering* that exists today did not.

In many instances there is a chasm between the vision of the architect and the sensibility of the engineer, between the aesthetics or appearance of a structure and its corresponding skeleton. We can argue that the distinction is between form and function – the form being the domain of the architect and the function of the engineer, but often the architect is as much concerned with "function" as the engineer, perhaps in a very different sense, and the engineer is as concerned with "form" as the architect, but perhaps differently than the architect.

The architect might speak of the building in ethereal terms and dealing with how people may experience the building and the philosophy of the design. The engineer might speak in more explicit and quantitative terms. They, of course, talk about the same building, yet not they only have different ways of describing it, but different ideas about what it should be. Since both architects and engineers are critical in the design of a building, the result can be (at worst) a compromise (neither the architect nor the engineer is completely happy) or (ideally) a synergistic result (where both are happy and proud, and the result is a sum even greater than the contributions of both participants).

Vitruvius (a Roman architect of the 1st century AD) wrote that a good building should satisfy the three principles of strength, utility and beauty (*firmitas*, *utilitas*, *venustas*). A building designed with aesthetics but without enough engineering to stand is unacceptable. A building designed only to stand but without regard for how it will be used or how people will respond to it is equally unacceptable.

Just as cathedrals "pushed the envelope" of design and technology, we are continuing to stretch limits with what we are designing. Innovations in design tools and philosophies about design, as well as innovations in fabrication and construction, are enabling designs to be realized which recently would not have been able to be built. In some instances, an architect is able to design something which would have been impossible to an engineer before. In an (unfortunate, we think) environment where an architect will envision a building without any regard or sense for engineering principles but can instruct the engineer to "make it work" more things are now possible. In a more collaborative environment architects and engineers work together to envision and realize incredible structures. Super-tall skyscrapers are one example of buildings requiring such close collaboration.

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Architects and engineers specialize in their disciplines, and even people within a discipline may specialize in a particular aspect of it. But, the process of design is extremely collaborative from the very start of a project. This reduces the problem of going too far in a design direction without considering several aspects. Architects inspire engineers and engineers inspire architects in all of our designs (even if it may be difficult to pin-point the origin of a particular idea, and even if some might be reluctant to admit it).

Historically, there are architects whose visions of aesthetics produce designs with very strong structural sensibility and innovative ideas. Such buildings have influenced the fields of architecture and engineering tremendously. Examples of these architects include: *Antonio Gaudi*, who used physical models to calculate sophisticated structures; *Buckminster Fuller*, whose philosophical ideas about holistic design, synergetics and geometry led to innovative structures such as the geodesic dome; *Felix Candela*, creating thin-shell concrete structures which are efficient and beautiful; and others (refer to Fig. 1).

The same issue that exists for architects and structural engineers also exists between architects and other types of engineers. An environmental engineer will consider as part of the *function* of the building its cost and efficiency to operate, the comfort of its occupants, and its sustainability. The collaborative efforts between architects and environmental engineers is similar in nature to that between architects and structural engineers, not to mention possible trade-offs in the design of a project due to perhaps divergent goals of structural and environmental engineers. One recent example of a similar multi-disciplinary design optimization can be seen in the flexible workflow framework for engineering design optimization presented by Crick et al. [4]. This example illustrates how a process with conflicting requirements of the different disciplines attempts to converge upon a description that represents an acceptable compromise in the design space.

On this note, we reflect on the innovative work of a well-known structural engineer, *Fazlur Khan*, who was influenced by the collaboration with the architect, *Bruce Graham*, which changed the idea of modern building architecture. Sabina Khan [5] described that *Bruce Graham* "inspired Khan to strive for structural systems that were not only structurally efficient but also worthy of becoming the core idea on which architectural design could center".

1.1. Motivation for structural topology optimization

As a possible avenue to achieve balance between the form and function, the authors strive to introduce a new, modified topology optimization framework, specifically for the design industry. Topology optimization can be used as a means to minimize the material consumption in a structure, while at the same time providing a tool to generate design alternatives of benefit to both the engineering and architectural communities, where the architecture works closely with the structural engineering in these proposed designs. This tool can be an initial step towards the creation of efficient designs and provides an interactive rational process for a project where architects and engineers can more effectively incorporate each other's ideas, rather than simply "making it work". In such a situation, the architecture might not "sacrifice" design for efficiency. Furthermore, the question of whether function follows form or vice-versa will no longer be of concern because through the use of structural topology optimization, the architecture and engineering are integrated together.

1.2. Paper organization

The remainder of this paper is organized as follows: in the next section, we review existing topology optimization techniques in the literature and corresponding numerical implementations. Then, in Section 3 the topology optimization framework for buildings and other structural applications is discussed. In Section 4, several case studies are presented for a variety of high-rise buildings and other architectural problems to illustrate the aesthetic value of topology optimization in this context. Finally, we conclude with some remarks about the application of these ideas.

2. Existing techniques in literature

Researchers have previously developed many computational optimization tools, in which the goal is to reduce the cost or material usage in a structure while satisfying specific design criteria. Among these tools, there are the cases of size optimization, shape optimization, genetic algorithms, topology optimization and others. The existing state of the art technologies are discussed next.

2.1. Background information

Size optimization is commonly used for finding the optimal cross-sectional area of beam elements in a frame or calculating the optimal thicknesses of plate elements while satisfying design criteria. In this method, the shape or connectivity of members may not change, but they may be removed during the process ([6]).

An alternative technique, *shape optimization*, looks at the shape of the initial material layout in a design domain and morphs the shape boundaries to obtain an optimal solution. In this case, the optimization can reshape the material inside the domain, but retains its topological properties such as number of holes ([7,8]).

Optimization tools commonly used in the industry are based on genetic algorithms, where principles from nature and natural selection can be used to identify the ideal design for a specific criteria in a certain design domain ([9]). Though this technique works on a wide range of problems (including size and shape optimization) and does not require the use of potentially complicated derivatives, it often requires more function evaluations and is not necessarily convergent, even to local minima ([10]). For a review of these techniques, the reader can refer to the paper by Suzuki and Kikuchi [11].

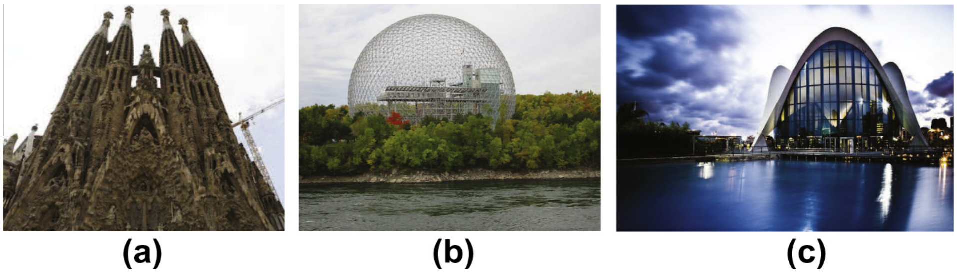

Fig. 1. Examples of structures by architects with strong and innovative engineering concepts: (a) Antonio Gaudi ([1]), (b) Buckminster Fuller ([2]), (c) Felix Candela ([3]).

To overcome some of the limitations present in the above techniques, topology optimization is introduced. Topology optimization is a mathematical, usually (but not always) gradient-based design tool which determines the location in a design domain to place material based on the loads and boundary conditions for a specific objective (i.e. a target deflection, compliance, etc.). The feasible solutions can have any shape, size or connectivity. In this technique, the finite element method (FEM) is applied by splitting a design domain into several small pieces, known as finite elements. In a topology optimization solution, each element is used to represent the conceptual design in the same fashion as a pixel of an image by containing a density that is either solid (black) or void (white).

2.2. Numerical schemes for topology optimization

A major innovation in topology optimization originated with the introduction of a technique for the least-weight layout of trusses in Michell [12]. From there, the work was extended to discrete structures by the works of Rozvany and Prager [13], Rozvany [14], Prager [15,16], which mainly focused on the optimal layout geometry of discretized cantilever beams and trusses with finite numbers of joints and members.

Concerning numerical schemes and the exploration of topology optimization for continuum structures, the paper by Bendsoe and Kikuchi [17] introduced the homogenization method, which was later used extensively by Allaire et al. [18]. With further maturation, came the Solid Isotropic Material with Penalization (SIMP) model ([19–22]), evolutionary techniques such as Evolutionary Structural Optimization (ESO) ([23,24]), level-set methods ([25–28]), methods based predominately on the topological derivative ([29,25,30]) and phase-field methods ([31]). While the latter three appear promising in the more recent literature, they also seem to be in their early phases of development. Additionally, according to Zhou and Rozvany [32], Rozvany et al. [33], the ESO methods often produce non-optimal solutions. The use of SIMP became common practice and showed tremendous promise for new research techniques with the introduction of the 99-line code for Matlab® available in Sigmund [34]. Thus, the tool proposed in this work to generate alternative designs where the engineering is integrated with the architecture is based on a SIMP approach using a Matlab® code.

2.3. Topology optimization for buildings and other structural engineering applications

Though topology optimization has been used in other fields, with applications spanning from mechanical to aerospace ([6,35–38]), the ideas presented in this paper attempt to transition the technology towards more applications in the civil engineering industry.

In recent years, few attempts have been made to use optimization techniques for such structural applications. The paper by Tomás and Martí [39] discusses techniques to improve the structural behavior and resistance to bending moments in concrete shells. In Walls and Elvin [40], an approach is introduced to efficiently group discrete frame members so that the optimal sizes produce a structure with the least weight. Additionally, Wang [41] optimized the shape of frame structures by introducing criteria to minimize the maximum bending moment. Furthermore, in the work of Geyer [42] an overview of a component-oriented design process for the multidisciplinary design of buildings is described.

In regards to topology optimization, several approaches have been proposed. For example, in Mijar et al. [43] a framework was introduced using Reuss and Voigt constitutive mixing rules for the effective stiffness for topology optimization of braced frames. The work of Neves et al. [44] tailors the topology optimization problem for stability design, where the critical buckling load serves at the objective function. Similarly, in [45] structural frames are studied with natural frequency as the objective. A method combining sizing, shape, and topology optimization was developed by Lagaros et al. [46] to design steel structures with web openings. Additionally, Liang et al. [47] and Liang [48] proposed a technique for the optimization of multi-story steel building frames using a performance index as element removal criteria. More recently, the work by Allahdadian and Boroomand [49] discusses the design and retrofitting of braced frame structures subject to dynamic loads. In this paper, topology optimization is taken one step further: it is employed to assist in the overall layout optimization and creation of innovative designs for the entire design process from architecture to engineering.

2.4. Selection of objective functions

In the design of any structure, it is important to select the right objective function to suit the problem. Minimum compliance, or maximum stiffness, is one objective, which can be used in its own merit and also as a surrogate to explore other metrics, such as ductility, natural frequencies, eigenmodes, P-Δ effects, buckling and stability, depending on the problem being explored.

In the conceptual design phase of a high-rise building, the majority of concerns are usually related to the overall stiffness/drift requirements under lateral loads [50]; therefore, many of the decisions made during this process are related to defining the lateral system, or providing stiffness/drift control. The minimization of compliance subjected to volume constraint is one example where topology optimization can be explored and has been shown advantageous, as demonstrated in the examples that follow. Other structural objective functions, such as buckling or those listed above, may be too specialized for the initial design phase, but can also provide valuable insight later on in the design process.

Through the selection of the objection function and other metrics that might fit the problem being studied, the engineer can then present the architect with a spectrum of solutions based on these parametric studies. This selection process has been shown to provide new ways to look at designs, which in turn inspires the overall design of the structure.

2.5. Necessity for layout and manufacturing constraints in building design

To ensure the topology optimization results, which often consist of non-practical material layouts and components, are capable of being built, we discuss the need for additional constraints to the topology optimization problem. The following gives a summary of some of the layout and manufacturing constraints we have incorporated, among many possible options, into the topology optimization design problem.

To eliminate undesirable results (i.e. those with very thin members, checkerboarding, etc.), several sensitivity and density filters have been developed in the context of manufacturing constraints. Bourdin [51] proposed a filtering technique by replacing point-wise element densities with a regularized density field using a convolution operator. The work of Borrvall and Petersson [52] used the idea of regularized density control to develop a density filter. Similarly, a density filter was proposed by Sigmund [53] by using morphology-based restriction schemes with a fixed-length scale to eliminate the gray regions between solid and void elements. Sensitivity filters have been studied extensively by Sigmund [34,54]. The interested reader can refer to the review paper by Sigmund and Petersson [55] for further discussion on these techniques. As an alternative to filtering techniques, constraint methods, such as perimeter control (Ambrosio and Buttazzo [56], Haber et al. [57]) could also be imposed to alleviate numerical instabilities and results of poor quality.

In the context of building design, a manufacturing constraint may be necessary on the size of members used in a structure, for instance. In the case of steel structures, the minimum and maximum member sizes might be constrained in accordance with the available shapes in the American Institute of Steel Construction (AISC) specifications ([58]). Other practical limitations include "pick-weight" and other construction limitations. The work of Guest et al. [59] and Guest [60] demonstrated methods to limit the minimum and maximum member sizes using projection techniques with a fixed-length scale. These techniques project the neighboring design variables on the element densities which also eliminated mesh dependent solutions (different solutions for different levels of mesh refinement) and numerical instabilities, such as checkerboarding (alternating patches of solid and void material) in the results.

In addition, there may be the need to incorporate mechanical, electrical and plumbing design constraints with the structural design, e.g. to allow for a hole to run a pipe through a beam. In this case, a constraint on the size of the hole could be applied. For example, in Almeida et al. [61] the topology optimization problem is modified using an inverse projection technique to control the size of such a void.

Patterns and symmetry constraints can provide an aesthetic advantage (as we address later) in addition to reducing costs on the construction. For steel structures, these constraints allow costly connection geometries to be reused throughout the height of a building, while improving the quality control. On the other hand, concrete structures with patterns and symmetry constraints allow formwork to be reused throughout to increase the efficiency of the construction process. With constraints on panel sizes of the glass curtain walls, the necessity for costly special glass shapes can be eliminated and panels can be reused throughout the height. These techniques can be incorporated into the optimization by adding mapping schemes for the design variables, such as the ones proposed by Almeida et al. [62] or by Huang and Xie [63] in the context of evolutionary structural optimization (ESO) for periodic structures.

Under typical loading conditions, the columns of a building will be larger in size at the base and smaller at the top. *Pattern gradation* constraints provide a means to smoothly transition the material layout design along the height of a structure. Moreover, at the top of a building in diagrid structures, shear behavior dominates whereas the base is typically controlled by the overturning moment giving rise to optimal bracing angles around 45° at the top and more vertical near the base. This was studied in [64] through additional mapping schemes and stress trajectories.

Other constraints for building components that could be incorporated into the structural topology optimization problem include casting ([65]), extrusion ([66,67]), and machining ([68]).

The methodology presented in Stromberg et al. [64] represents an initial attempt at identifying optimal bracing angles. However, it presents some limitations, such as high concentrations of material towards the edges of the domain, consistent with the flange vs. web behavior, described in Section 4 of [64]. Such concentrations cause confusion in identifying the working point at the column to the diagonal intersection. In addition, the columns created using a continuum are so wide that they possess high flexural stiffness. In practice, this is not realistic because the columns are narrower. Moreover, since the continuum topology optimization problem has a constraint on the volume fraction and a large amount of material forms the column members, a relatively low volume is available for the diagonals. As a result, there is an "incomplete" diagonalization in the frame (i.e. missing diagonals at the base of the frame). Thus, a methodology using a combination of discrete members and continuum quadrilateral members to overcome this issue was introduced in [69]. The advantages of this new technique illustrate a complete diagonalization along the height of the building, where each diagonal is clearly identified as part of the proposed overall design process.

3. Topology optimization framework

Using topology optimization, a computational framework that can provide architects and engineers with ample freedom to explore novel designs while still satisfying principles from structural engineering and mechanics is introduced in this section. This software platform, as described in Stromberg et al. [64], can be the basis for a tool that provides designs that identify with both engineering and architectural communities, encouraging integrated designs. It also has the theoretical capabilities (associated with topology optimization) to be used in other fields such as automotive, aerospace structures, and microelectromechanical systems (MEMS).

3.1. Theoretical background

In topology optimization, we seek the optimal layout of material for a given design domain in terms of an objective function. The generalized problem statement for the topology optimization problem can be written as follows:

$$\begin{aligned} &\min_{\mathbf{d}} f(\mathbf{d},\mathbf{u}) \\ &\quad s.t.\ g_i(\mathbf{d},\mathbf{u}) = 0 \quad \text{for } i = 1,\ \ldots,\ k \\ &\quad s.t.\ g_i(\mathbf{d},\mathbf{u}) \leq 0 \quad \text{for } i = k+1,\ \ldots,\ m \end{aligned} \tag{1}$$

where $\mathbf{d}$ is the design field, $\mathbf{u}$ is the response, and they are related through the constraint functions g_i. A canonical example is the minimum compliance problem:

$$\begin{aligned} &\min_{\mathbf{d}} f(\mathbf{d},\mathbf{u}) = \mathbf{p}^T\mathbf{u} \\ &\quad s.t.\ g_1(\mathbf{d},\mathbf{u}) = \mathbf{K}(\mathbf{d})\mathbf{u} - \mathbf{p} \\ &\qquad\quad g_2(\mathbf{d}) = V(\mathbf{d}) - \overline{V} \end{aligned} \tag{2}$$

where g_1 represents the equilibrium constraint, while g_2 is the constraint on the allowable volume of material, $\overline{V}$. The general framework described by Eq. (1) can be used for both gradient and non-gradient based optimization methods, where the response $\mathbf{u}$ could be natural frequency, stress levels, ductility, eigenmodes, $P - \Delta$ effects, and so on.

By means of relaxation, the well known ill-posedness of the topology optimization problem, or lack of a solution in the continuum setting ([70–73]), can be overcome. Thus, a continuous variation of density in the range $[0, 1]$ is applied rather than restricting each density to an integer value of 0 or 1 guaranteeing the existence of a solution in this setting. To avoid singularities in the global stiffness matrix, $\mathbf{K}(\mathbf{d})$, a small parameter greater than zero, d_{min}, is specified.

The Solid Isotropic Material with Penalization (SIMP) ([19–22]) model is commonly used to solve topology optimization problems. In this formulation, the stiffness and element density are related through a power-law relation of the form:

$$\mathbf{E}(\mathbf{x}) = \mathbf{E}_{min} + \mathbf{d}(\mathbf{x})^p(\mathbf{E}_0 - \mathbf{E}_{min}) \tag{3}$$

where E_0 is the Young's modulus of the solid phase of material and p is a penalization parameter to eliminate intermediate densities with $p \geqslant 1$. The SIMP model ensures that material properties continuously depend on the material density at each point. The penalization parameter, p, forces the material density towards 0 or 1 (void or solid respectively) by penalizing regions of intermediate densities (gray zones) where $\mathbf{d}$ assumes values in the range of 0 to 1.

Additionally, by using continuation, the penalization parameter, p, is increased over the range of 1–4 in increments of 0.5 until convergence at each value is achieved. This technique further penalizes the intermediate densities throughout the process.

3.2. Work flow

While this topology optimization framework is based on engineering theory, it has advantages for both engineers and architects. From the engineering standpoint, a finite element analysis of the structure is performed during each iteration to ensure the design is structurally sound. On the other hand, it also includes the rendering capabilities of final results for architects to use in generating ideas for potential designs. The optimization is done by the engineers in Matlab®; the result is interpreted and transferred to CAD or rendering software through input/output (text) files.

Though topology optimization results are guided by engineering judgment, several options can be changed in the structural context to explore different outcomes. For example, a different design space or various combinations of loads and boundary conditions can be explored. Then, this message is conveyed to the architect via an interpretation or rendering of the topology optimization results by the structural engineer into a frame representing the gravity and/or lateral system of the structure. These variations can be used to give architects several logical options, from which they can choose the most aesthetically pleasing or applicable design. The architects can then integrate the structural components with other building components (mechanical, electrical, façade, plumbing, glass work, cladding, elevators, etc.).

In some cases the structural system of a project is evident and expressed, in others the structural system is covered with the façade. In the first case, the structure *is* the architecture, such as the cases of a bridge, viaduct, long span road structure or some high-rise buildings (John Hancock Center, London's Broadgate Exchange House). These structures emphasize pure engineering to satisfy structural principles, while the second category uses the architecture or external features of the building to cover the engineering components rather than incorporating them into the design.

The intent of topology optimization is to enable architects and engineers to work together to express the structure together with the architecture. For example, in the design of a building, the criteria of the structural engineer may focus on the tip deflection limits, the lateral load resisting systems (braced frames or concrete core), the sizing and placement of the structural members (i.e. beam and columns) and the ability to simplify the design by using symmetry and patterns, among others. On the other hand, the architect may consider a different range of criteria regarding the aesthetics of the building, such as the value of views, cladding (e.g. glass facade), incorporation of landscaping (green areas), symmetrical appearance and patterns. An example of an integrated design is shown in Fig. 2, where topology optimization was explored as a means to incorporate the structural criteria. Incorporation of the architectural criteria might be further implemented through a variety of approaches ([74]).

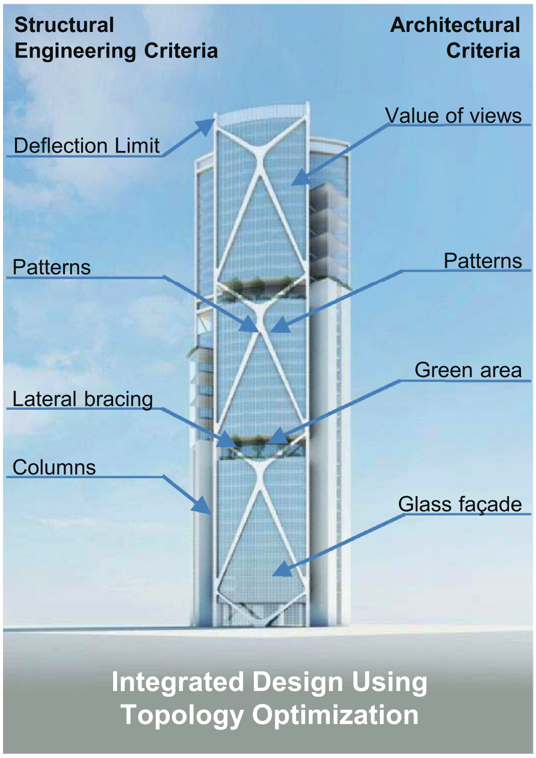

Fig. 2. Optimized building illustrating the concept of integrated design.

4. Case studies

In this section, key concepts and case studies are presented to provide the reader with several examples of the advantages and limitations of the proposed framework for design projects. The case studies employ the aforementioned concepts and integrate them within an interdisciplinary framework.

4.1. Design and parametric modeling

Parametric modeling is a key concept in modern design, as it is very commonly used to provide architects and engineers with a common ground to communicate and exchange ideas. Variations on the parameters produce variations on the design, which in turn has architectural and structural implications.

4.1.1. World Trade Center Tower One spire

As an example, in the design of the World Trade Center Tower One, the spire containing broadcasting equipment was designed in a process using a flexible parametric model, as shown in Fig. 3. The design was the result of a close collaboration between engineers and architects. The parametric model allowed the designers to explore and analyze variations considering size (spire diameter at different elevations and overall height), proportion (ratio between the height and maximum diameter), number of panels, perforation patterns and structural soundness. In addition to the spire, the structural diagrid system of an earlier proposal for the tower itself was designed according to the parametric model illustrated in Fig. 4.

4.1.2. Bridges connecting building towers

A parametric approach similar to that used for the World Trade Center was adapted to our framework for the design of the Zendai competition (China). The aim was to create a unique and innovative design for the upper "bridge" structure spanning between several towers ([75]). This space was approximated as a beam, discretized with several four node quadrilateral (Q4) elements. The gravity load on the mesh was applied as a series of equal point loads at nodal locations. The mesh was constrained with pin supports at the nodes corresponding to the locations where the towers would support the "bridge". At the first stage in the design process, a parametric study was performed to present the architects with several feasible options for the design, using different combinations of layout constraints (variations on symmetry, patterns, minimum member size, etc.). As an example, for the design shown in Fig. 5, each section of the design space between supports of the beam was constrained to have a similar pattern, a technique known as pattern gradation.

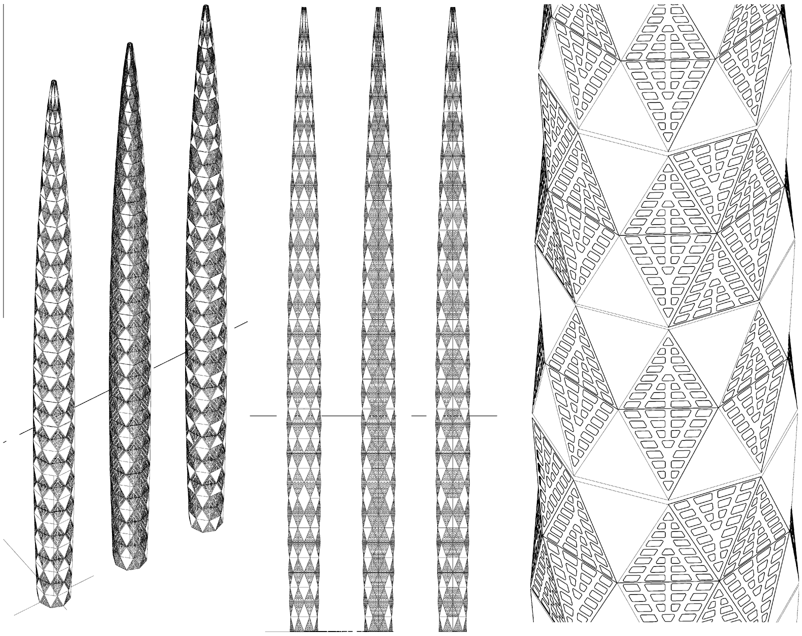

Fig. 3. Parametric studies for the design for the spire for World Trade Center Tower One.

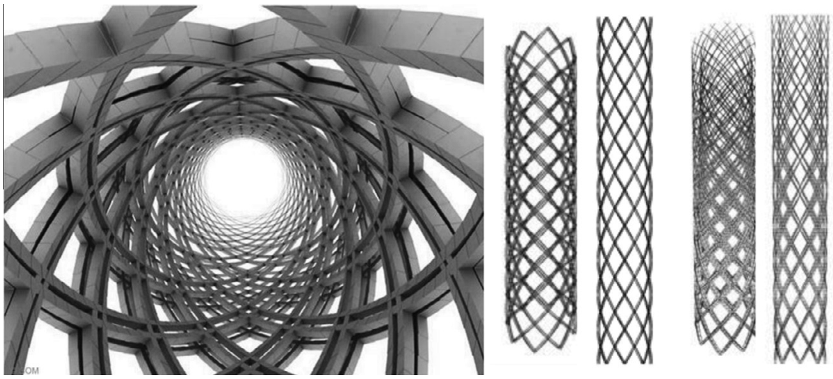

Fig. 4. Renderings of the parametric model for the conceptual design of the World Trade Center Tower One, NY.

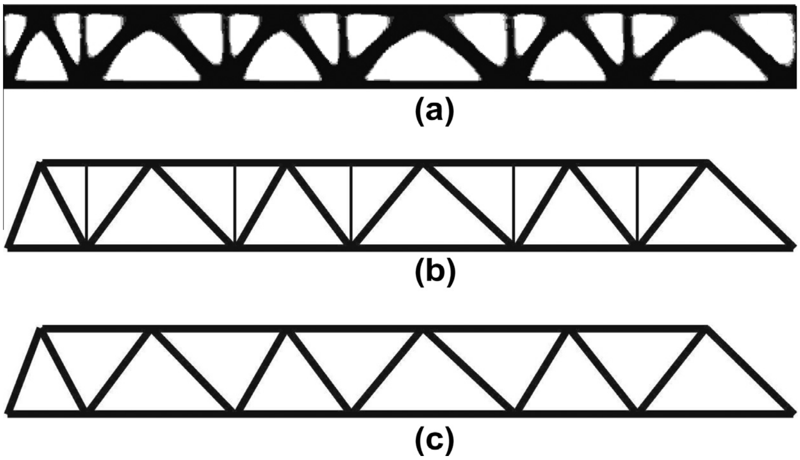

Fig. 5. Illustration of the concept design for the structural system of the Zendai competition: (a) topology optimization results with pattern constraints, (b) discretized structure as interpreted by engineers, (c) Warren truss.

This result proved quite similar to a Warren truss, which was deemed too common and ordinary from an architectural standpoint.

The design that was later selected by the architects was the alternative which was produced using topology optimization without any layout constraints (illustrating the iterative design

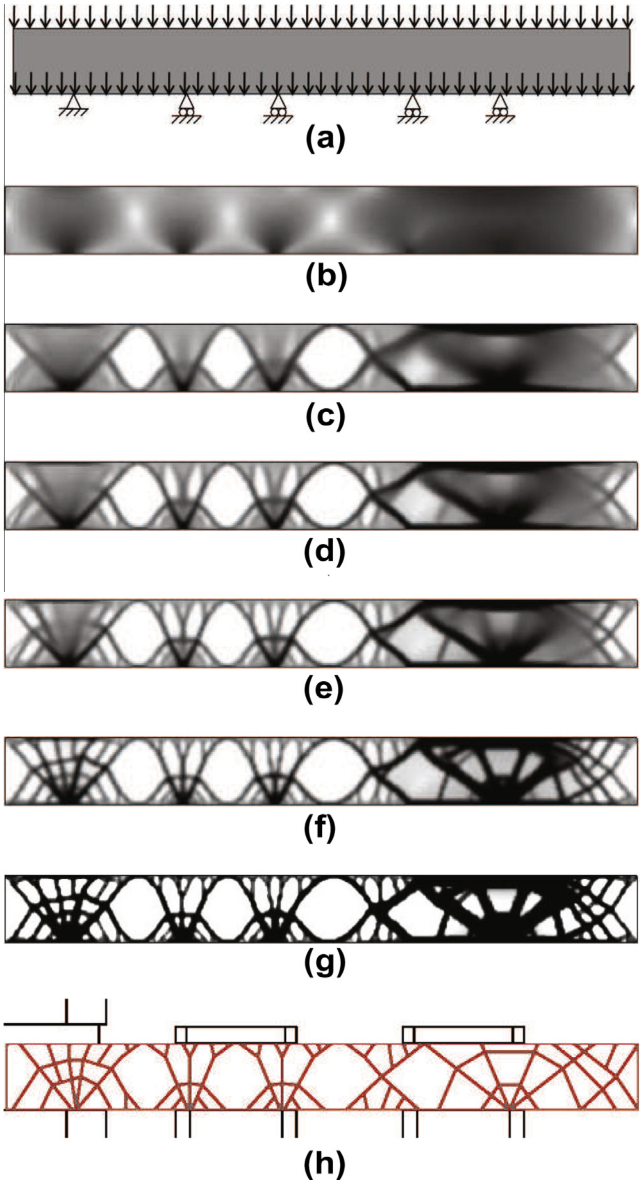

Fig. 6. Topology optimization for design of the upper "beam" spanning several towers for the Zendai competition: Iteration (a) 0, (b) 1, (c) 10, (d) 15, (e) 20, (f) 40, (g) 100 (final design). (h) Resulting engineering interpretation.

process). The new design (shown in Fig. 6), shared some elements of "biomimetic architecture", which was thus deemed more exciting and organic from an aesthetic point of view, hence it was embraced in the project. This new design was compared to the idea of "spider webs", something more abstract and unique in nature, but still with engineering rationale behind the structure. For example, in nature a spider web is extremely efficient: very lightweight, but also very strong – an analogy for the resulting topology optimization design. The final images of the architectural physical model of the project are following in Fig. 7. The results show that the left-most and right-most supports give way to the development of a bounded Michell-like truss.

4.2. High-rise buildings

Several examples of the application of topology optimization for the conceptual design of the bracing systems or skin of high-rise building structures are given in Figs. 8–10. Together they emphasize a variety of design and analysis considerations.

Fig. 7. Picture of the physical model for the concept design of the Zendai competition using topology optimization results.

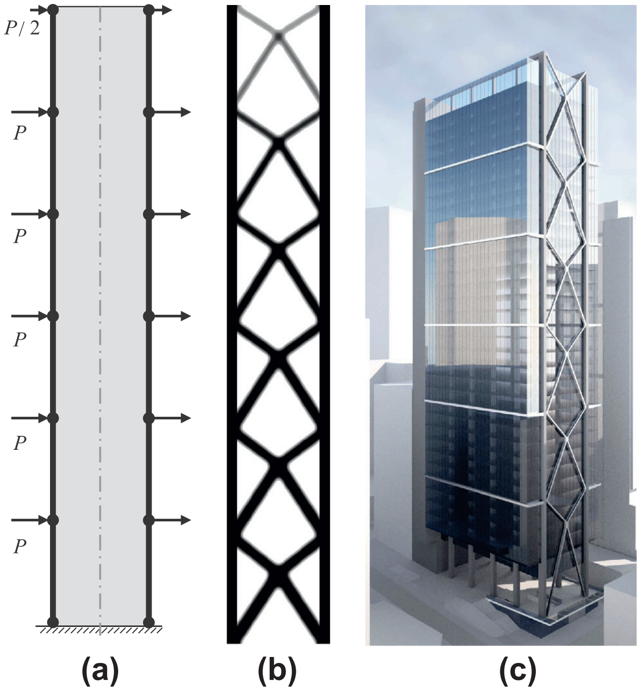

Fig. 8. Illustration for the concept design of a 288 m tall high-rise in Australia, which shows the engineering and architecture expressed together: (a) problem statement, (b) results of the topology optimization, (c) renderings of the design.

4.2.1. A 288m tall building

Fig. 8 shows the conceptual design for a 288m tall high-rise building in Australia that was inspired by the topology optimization of the bracing system. The topology optimization was performed using a combined element technique ([69]), where the behavior is modeled using both 2D continuum (Q4) elements and

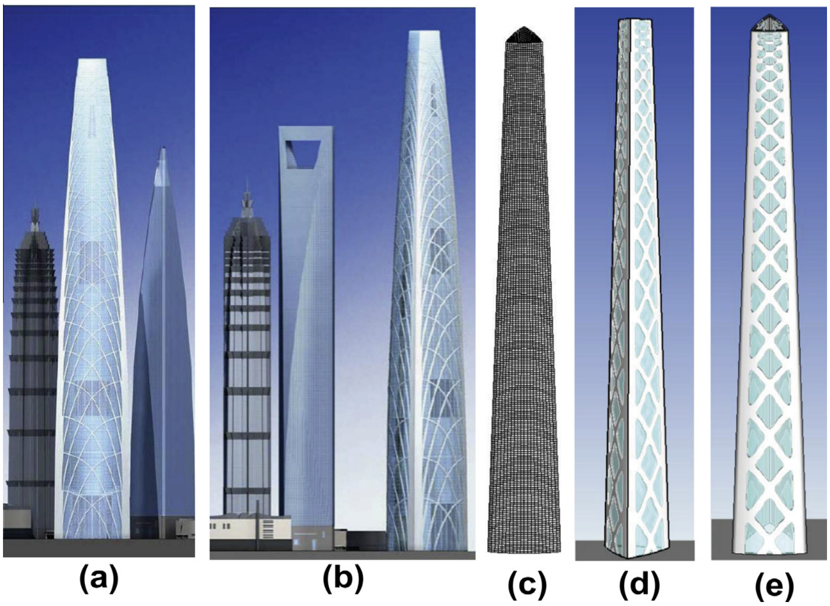

Fig. 9. Illustration of the concept design for the Z3 competition in Shanghai, China: (a and b) renderings of the elevations of the building in context to surroundings, (c) finite element mesh, (d and e) results of the topology optimization.

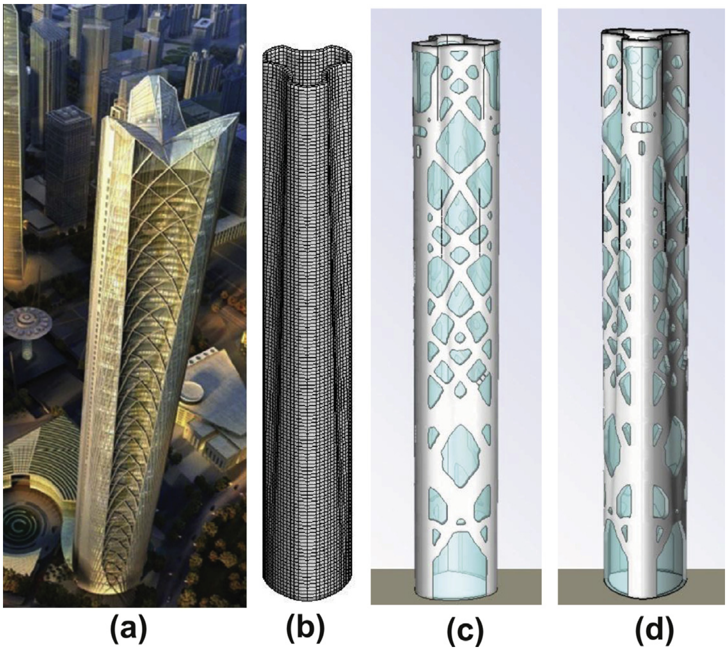

Fig. 10. Illustration of the concept design for the Wuhan competition: (a) architectural rendering of the final design, (b) finite element mesh, (c and d) topology optimization results.

beam/column elements. The final results show a bracing system in which the densities increase as the load increases throughout the height of the structure where the patterns emerge naturally (i.e. no layout constraints were applied in this study), which provide some aesthetic value to the design.

4.2.2. A 580m tall building

In Fig. 9, topology optimization with pattern constraints was used for a competition entry for a 580m tall high-rise building in Shanghai, China. The building is triangular in plan; each side of the triangle is convex and is linearly tapered from the bottom to the top. The mesh was constructed using 28,800 eight node brick elements (B8) with the same tapering. Additionally, the base of the structure was fixed and a lateral wind load was applied to the building in the form of point loads at the top. The constraints imposed here include three-way symmetry with pattern gradation. The resulting bracing system is similar to the principle stress trajectories of the structure subject to the wind load, which were used to create the original design. For a more in-depth discussion of this technique, the reader can refer to reference ([64]).

4.2.3. An 84-story mixed-use building

Next, a competition entry for an 84-story mixed-use building in Wuhan, China has been analyzed. Similarly to the previous example, the building in plan is triangular with convex sides, however, it is not tapered along its height. An additional challenge from a

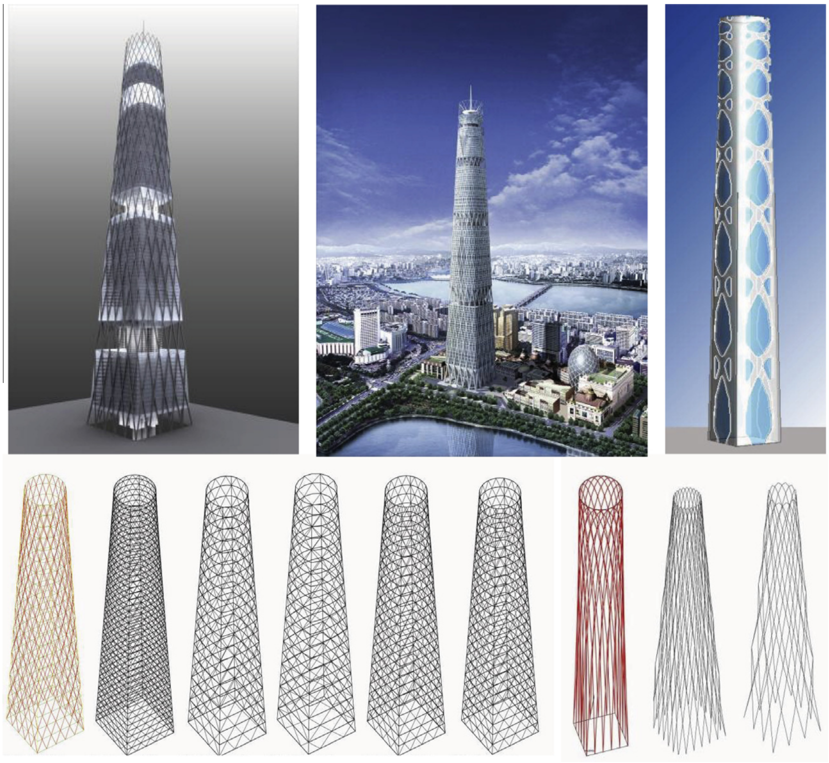

Fig. 11. Illustration of the concept design for Lotte Tower: (top left and center) renderings of the final design, (top, right) topology optimization result ([64]) and (bottom) parametric studies.

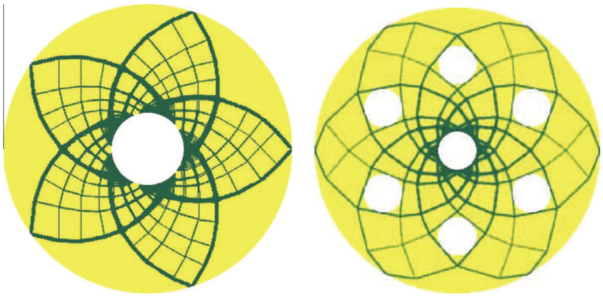

Fig. 12. "Creation of a star" (as designed by structural engineers for a holiday card (left)) and variation on the original design (right).

meshing standpoint is a progressively increasing circular arc cut out from the sides from the base to the top of the building. This building was meshed with 9,000 B8 elements. The loads and boundary conditions are the same as the previous example. The constraints include three-way symmetry, but no pattern gradation due to the uniqueness of the geometry on the surface. The architectural renderings are shown together with the topology optimization results in Fig. 10. Similar to the previous example, the structural bracing system in the competition entry was inspired by the principle stress trajectories under wind load for a high-rise.

4.2.4. Conceptual design of the Lotte Tower

The design of the Lotte Tower in Seoul (see Fig. 11) used an exterior diagrid structure transformed in shape from a square at the base to a circle at the top and represented another close collaboration between engineers and architects similar to the one for the World Trade Center Tower One. After the tower design was completed, the building was revisited using topology optimization with pattern constraints (top, right of Fig. 11). The mesh for the optimization was created with 12,800 B8 elements. Loading and boundary conditions were similar to the previous examples.

The resulting design emphasizes the concept of principle stress trajectories of the structure, which are traced in a cascading pattern in the renderings of the competition entry. The fundamentals

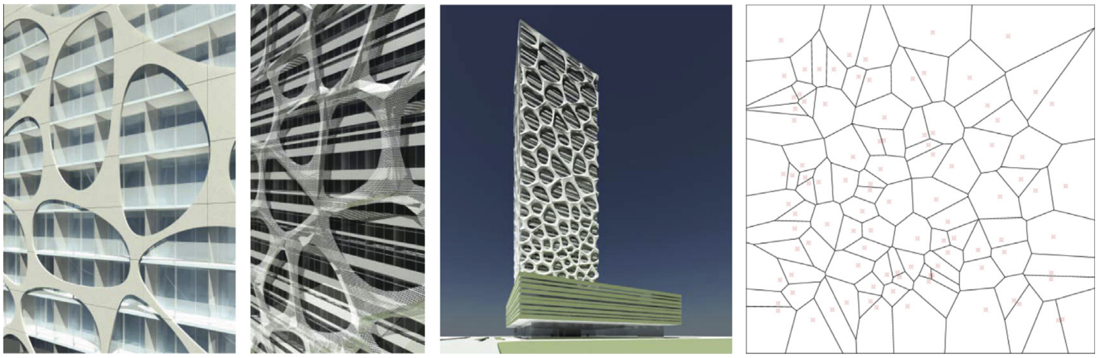

Fig. 13. The concept of Voronoi diagrams used in architectural design for a tower in Tianjin, China.

behind the structural engineering are evident in the respect that the column sizes in all the examples increase from the top to the base due to gravity loads and the diagonals of the cross braces show larger angles at the base of the structure than at the top due to the moment-shear interaction. These designs also illustrate the engineering and the architecture expressed together in the outer skin of the structure.

4.3. Geometrical/architectural patterns for design

Geometry is one aspect of design which straddles both architecture and engineering. (Indeed geometry can be found to relate to many fields, from art to music and dance, and is as prevalent in natural and human creations.) Patterns, as implemented in our framework, are one expression of geometry in a language which can be appreciated and used as a tool by both architects who are conveying in the design an aesthetic and abstract idea and engineers who are optimizing and stabilizing the design.

Part of the excitement in collaborations among architects and engineers (who, it sometimes seems, speak in different languages) is when they try to explain their points-of-view to each other. In this sense, one viewpoint is that architects work in an abstract language, and engineers in a more concrete language. On the other end, another viewpoint is that architects deal with space and materials that will be directly experienced in the building, and engineers deal with entities, like stress and strain, that cannot be seen. When these abstract structural concepts are visualized (often to explain to the architect a concept or analysis result that could be conveyed to another engineer with some words or mathematical expression), interesting possibilities arise. Stress patterns (sometimes these are incredibly beautiful) become architectural design ideas. These design ideas may satisfy the engineering requirements for the building.

4.3.1. Exploring Voronoi diagrams

Recently, the educational software “PolyMesher” [76] (general purpose polygonal mesh generator) and “PolyTop” [77] (general topology optimization framework) were used to design a holiday card (see Fig. 12). The software uses the concept of Voronoi diagrams and polygonal finite elements to generate novel designs, illustrating how an engineering framework can also be used to create works of art. Thus by providing engineers and/or architects in the industry with such a tool, a new area of design might emerge based on geometrical architectural patterns.

Voronoi diagrams, in addition to their use as an alternative for standard finite element meshes in topology optimization, can also be used as a concept in other areas of architectural design as well. The images shown in Fig. 13 illustrate an elevation pattern for a competition for a tower in Tianjin, China. While visually interesting (although aesthetics can always be argued), there are more objective properties that we can identify, and perhaps these are the same properties that make them useful in analysis.

There is a continuity of edges and vertices. While not a triangulated structure, an edge will always terminate with two other edges. We also have some control of the distribution of edges/members in structures modeled this way. Notice, for example, in the image showing the full tower, “cells” closer to the bottom of the tower are smaller than cells near the top of the tower. While the initial sets of points which generated the Voronoi diagram is random, we can control the distribution to create this property.

5. Conclusions

In this paper, the tailored topology optimization approach can be used as an approach that might lead to a better integration of engineering and architecture in the design of buildings. This framework eliminates the question of whether form follows function or vice versa, and resulting designs embrace the structural engineering together with the architecture to create innovative aesthetically pleasing structures with evident structural engineering components.

Similar to the findings of Crick et al. ([4]), we note also here that even though each group has its own vocabulary and models, the shared parametric model in our case (or conversely algorithm in [4]) was used to establish a common ground for communication with one another where topology optimization was used as the common language. This resulted in final engineering designs that were a compromise on the design decisions negotiated by the specialists themselves.

To conclude this work, we reflect on the views of Fazlur Khan on the integration of engineering and architecture:

> *“The language of mathematics and rational engineering, Khan maintained, could not give form to architecture of substantive quality on its own, no more than could ungrounded aesthetic inclination. Rather, by conjoining creative energies and different perspectives, better innovative and responsive design solutions could be developed than either architect or engineer might conceive in isolation.” [5].*

This quote illustrates the advantages that a multidisciplinary tool, such as the topology optimization framework proposed in this paper, can present to the design industry.

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