Prepublished Paper

This is a prepublished manuscript. The final manuscript is tentatively scheduled for V. 120, No. 6 and is subject to change.

The DOI for this paper is [insert DOI here] and will not change, but won’t be activated until the issue has been published.
Strut-and-Tie Models using Multi-material & Multi-Volume Topology Optimization: A Load Path Approach

Tuo Zhao\textsuperscript{a,1}, Ammar A. Alshanaq\textsuperscript{b,1}, David W. Scott\textsuperscript{c,2} and Glaucio H. Paulino\textsuperscript{a,2}

\textsuperscript{a} Department of Civil and Environmental Engineering, Princeton University, 54 Olden Street, Princeton, NJ 08544

\textsuperscript{b} Department of Civil Engineering, Yarmouk University, Irbid, Jordan 21163

\textsuperscript{c} Department of Civil Engineering & Construction, Georgia Southern University, Statesboro, GA 30460

\textsuperscript{1} T.Z. and A.A.A. contributed equally to this work.

\textsuperscript{2} To whom correspondence may be addressed. Email: dscott@georgiasouthern.edu or gpaulino@princeton.edu

ABSTRACT

The development of strut-and-tie models (STM) for the design of reinforced concrete deep beams considering a general multi-material and multi-volume topology optimization framework is presented. The general framework provides flexibility to control locations/inclination/length scale of the ties according to practical design requirements. Well-understood optimality conditions are applied to evaluate the performance of the optimized STM layouts. Specifically, the Michell number $Z$ (or load path) is used as a simple and effective criterion to quantify the STM. The experimental results verify that the STM layout with the lowest load path $Z$ achieves the highest ultimate load. Moreover, significantly reduced cracking is observed in the optimized STM layouts compared to the traditional STM layout. This observation implies that the optimized layouts may
require less crack control reinforcement, which would lower the total volume of steel required for
the deep beams.

1 INTRODUCTION

In 1904, Michell wrote the revolutionary paper “The Limits of Economy of Material in Frame-
structures,” which is a landmark in the field of optimization (in general) and topology optimization
(in particular). He derived the well-known Michell’s optimality conditions (Michell 1904), which
are the first ones that provide analytical ways to find optimal truss structures. The definition of the
optimal structures is the least-weight truss with given allowable stresses, which is also known as
the minimal total load path theory. The load path has been quantified using the Michell number,
$Z$, defined as follows,

$$Z = \sum_{e} |F_e|L_e = \sum_{e \in G^T} |F_e|L_e + \sum_{e \in G^C} |F_e|L_e$$ (1-1)

where $L_e$ and $F_e$ denote the length and internal axial force of the $e$th truss member in the structure,
respectively; $G^T$ and $G^C$ are the sets of tension and compression members, respectively. For any
statically determinate truss that is fully stressed (to the tensile stress limit $\sigma_T$ and compressive
stress limit $\sigma_C$), the volume of the truss can be calculated as follows,

$$V = \frac{\sum_{e \in G^T} |F_e|L_e}{\sigma_T} + \frac{\sum_{e \in G^C} |F_e|L_e}{\sigma_C} = \frac{(\sigma_C + \sigma_T)Z + (\sigma_C - \sigma_T)C}{2\sigma_C\sigma_T}$$ (1-2)

where $C = \sum_{e \in G^T} |F_e|L_e - \sum_{e \in G^C} |F_e|L_e$, which is known as the Maxwell number. Maxwell (1864)
states that $C$ is a constant value for given boundary and loading conditions, i.e., $C$ is independent
of structural layout. As a result, minimizing the load path $Z$ for a given design problem is
equivalent to minimizing the volume $V$ if the structure is fully stressed.

A pioneering work by Kumar (1978) applies the load path theory of truss frameworks to design
reinforced concrete deep beams by navigating optimal load transmission. Following and building
upon Kumar’s study, this work extends Michell’s optimality conditions to understand the optimal
load path for STM, and uses the load path $Z$ (or the Michell number) as a criterion to quantify the
efficiency of the STM. Compared to existing criteria (Schlaich, Schäfer, and Jennewein 1987; Xia,
Langelaar, and Hendriks 2020; He et al. 2020), the present criterion is simpler. The experimental
results in Section 5.2 verify that the STM layout with the lowest load path $Z$ (or Michell number)
achieves the highest ultimate load.

STM are powerful tools for analyzing and designing reinforced concrete structures. However,
traditional STM dramatically simplify the complex stress state found in deep concrete elements in
compression, which greatly limits their efficiency in many practical design applications. More
recently, topology optimization has been used to automatically generate STM, including the works
of Liang et al. (2000, 2001), Leu et al. (2006), Bruggi (2010), Mozaffari et al. (2020), and Zhou
and Wan (2021), which is just a small sample of references in the field. The optimized STM layouts
provide deeper insight into the load paths in reinforced concrete members and, ultimately, aid in
more efficient structural designs. However, most topology optimization formulations for STM use
only a single material assuming the struts and ties have the same linear behavior. Victoria et al.
(2011) extend the single material optimization using a bilinear material model with different
behaviors in compression and tension to represent the struts and ties, respectively. Gaynor et al.
(2012) and Jewett and Carstensen (2019) consider different materials for the struts and ties, but
most are typically restricted to a single volume constraint for both materials (i.e., each material
volume cannot be constrained separately). Thus, these models are limited in practical application.
In many real-world reinforced concrete structure design cases, restricting the location of
reinforcement (ties) to certain regions while controlling the allowable angle inclination or length
scale of ties according to design requirements is essential.
The above-mentioned limitations can be addressed by a general multi-material topology optimization approach, which efficiently accommodates an arbitrary number of materials and constraints (Zhang et al. 2018; Sanders et al. 2018). This general approach is applied to a novel STM framework using multi-material topology optimization with multiple volume constraints. The present framework allows the designer to adjust the ties' locations, inclinations, and scales based on practical design specifications.

2 RESEARCH SIGNIFICANCE

This work proposes a simple and efficient criterion (Michell number $Z$ in equation (1-1)) to quantify the efficiency of the topologically optimized STM. It is shown that the optimized STM layouts with lower $Z$ outperformed the traditional layout in terms of improving load-bearing capacity and ductility. The framework developed in this paper can form the benchmark of an efficient, general and practical STM design method for reinforced concrete structures.

3 MULTI-MATERIAL AND MULTI-VOLUME TOPOLOGY OPTIMIZATION FORMULATION FOR STRUT-AND-TIE MODELS

The design for an optimal STM layout consists of determining the cross-sectional areas of the truss members using the ground structure method (GSM) (e.g., Dorn et al 1964). In this method, the design domain is discretized using a set of nodes that are interconnected by truss members to form an initial ground (i.e., reference) structure (GS). Based on a tailored design update scheme, unnecessary members are gradually removed from the initial GS; the optimal STM design is then obtained. The proposed topology optimization formulation for STM using the GSM is given as:
\[
\begin{align*}
\min_{x_1, x_2} \ J(x_1, x_2) &= \min_{x_1, x_2} -\Pi(x_1, x_2, u(x_1, x_2)) \\
\text{s.t.} \quad \sum_{i \in G^j} L_i^T x_i - V_{max}^j \leq 0, \quad j = 1, \ldots, n, \text{and } i = 1, 2, \\
\text{with} \quad u(x_1, x_2) &= \arg\min_u \Pi(x_1, x_2, u).
\end{align*}
\] (3-1)

where \(x_1\) and \(x_2\) are the vectors of design variables (cross-sectional areas of the truss members) for struts (concrete) and ties (reinforcement), respectively, which can be constrained separately.

The objective function \(J\) is the negative total potential energy of the system in equilibrium, and \(u\) is the displacement vector (state variable) which is obtained as the minimizer of the potential energy \(\Pi\) - thus general nonlinear constitutive behavior can be incorporated (Sanders et al. 2020).

The formulation (3-1) considers a total of \(n\) independent volume constraints and denotes \(G^j\) as the set of material indices for the \(j\)th volume constraint. The term \(L_i^T x_i\) indicates the total volume associated with the design variable \(x_i\) with \(L_i\) being the length vector for the \(i\)th material, and \(V_{max}^j\) is the allowable volume for the \(j\)th volume constraint. The main feature of the formulation (3-1) is that it can efficiently handle a general setting of volume constraints. In particular, defining material sub-regions will allow the control of locations/inclination/length scale of the ties according to practical design requirements.

### 3.1 Design-variable update scheme to general volume constraints for STM

An essential component of any topology optimization framework is a reliable and efficient design-variable update scheme. Zhang et al. (2018) formulated a general design-variable update scheme tailored for the multi-material topology optimization formulation which does not require a pre-defined candidate material sequence, and can efficiently and effectively handle an arbitrary number of candidate materials and volume constraints. Inspired by this work, this work derives a design-variable update scheme for the present strut-and-tie optimization formulation (3-1).
The derivation of the design-variable update scheme is based on sequential explicit convex approximations. The objective function in the formulation (3-1) is approximated at optimization step \( k \) as a convex function constructed based on the objective function gradient (Christensen and Klarbring 2008; Groenwold and Etman 2008). Introducing a set of intervening variable vectors \( y_i(x_i) \), the approximation of the objective function at the \( k \)th optimization step is

\[
J^k(x_1, x_2) = J(x_1^k, x_2^k) + \sum_{i=1}^{2} \left[ \frac{\partial J}{\partial y_i}(x_1^k, x_2^k) \right]^T [y_i(x_i) - y_i(x_i^k)],
\]

(3-2)

where \( x_1^k, x_2^k \) are the values of the design variables at optimization step \( k \); \( \partial J/\partial y_i \) is the gradient of \( J \) with respect to the intervening variable \( y_i \), which depends on the gradient of \( J \) with respect to \( x_i \). In the following, to simplify notation, \( b_i \) denotes this gradient \( \partial J/\partial y_i \). Having defined the approximated objective \( J^k \), a sub-problem (by neglecting the constant terms in \( J^k \)) is formulated as,

\[
\min_{x_1, x_2} \sum_{i=1}^{2} b_i(x_1^k, x_2^k)^T y_i(x_i)
\]

s. t. \( \sum_{i \in G_j} L_i^T x_i - V_{max}^j \leq 0, j = 1, \ldots, nc \)

(3-3)

where \( x_{i,L}^{(e),k} \) and \( x_{i,U}^{(e),k} \) are the lower and upper bounds for the design variable \( x_i^{(e)} \) determined through a user-prescribed move limit. Introducing a set of Lagrange multipliers \( \lambda_j \) for each volume constraint, the Lagrangian of the above sub-problem is expressed as

\[
L(x_1, x_2, \lambda_1, \ldots, \lambda_{nc}) = \sum_{j=1}^{nc} \left\{ \sum_{i \in G_j} \left[ b_i(x_1^k, x_2^k)^T y_i(x_i) + \lambda_j L_i^T x_i \right] - \lambda_j V_{max}^j \right\}.
\]

(3-4)
The above Lagrangian function has a clearly separable structure with respect to each volume constraint. This means that the minimizer of the Lagrangian with respect to $x_i$, denoted as $x_i^*$, can be expressed in the following form:

$$x_i^{(e)*} = Q_i^{(e)}(x_1^k, x_2^k, \lambda_j), \quad \forall i \in G^j.$$  \hfill (3-5)

In other words, $x_i^*$ only depends on the Lagrange multiplier of the volume constraint associated with $x_i$. Plugging $x_i^*$ back into the Lagrangian gives the dual objective function

$$D(\lambda_1, \ldots, \lambda_{nc}) = L(x_1^*, x_2^*, \lambda_1, \ldots, \lambda_{nc}) = \sum_{j=1}^{nc} \left\{ \sum_{i \in G^j} \left[ [b_i(x_1^k, x_2^k)]^T y_i(x_i^*) + \lambda_j v_i^T x_i^* \right] \right\}. \quad (3-6)$$

Because $x_i^*$ only depends on $\lambda_j$ if $i \in G^j$, it concludes that the dual objective function $D$ also has a separable structure with respect to $\lambda_j$, namely, $D(\lambda_1, \ldots, \lambda_{nc}) = \sum_{j=1}^{nc} D^j(\lambda_j)$. As a result, the set of maximizing Lagrange multipliers $\lambda_1^*, \ldots, \lambda_{nc}^*$ can be computed by sequentially calculating the maximizing Lagrange multiplier $\lambda_j^*$ for each $D^j(\lambda_j)$. The general formula of the updated design variable is then obtained:

$$x_i^{(e),k+1} = Q_i^{(e)}(x_1^k, x_2^k, \lambda_j^*), \quad \forall i \in G^j.$$  \hfill (3-7)

Based on the above formula, because the update of the design variable depends only on the Lagrange multiplier of its corresponding volume constraint, the design variables associated with each volume constraint can be updated independently. The present design-variable update scheme has been applied to the STM design example shown next in Section 4.

4 NUMERICAL EXAMPLES

A practical computational tool for STM is developed to assist engineers in better understanding and designing reinforced concrete structures using the present multi-material topology optimization framework. The new STM framework will provide engineers with the flexibility to
specify the inclination/length scale of reinforcement and to control the tensile (tie) regions where
reinforcement needs to be placed depending on design requirements through the use of multiple
volume constraints.

A numerical study is conducted on the strut-and-tie models for the 2D reinforced concrete deep
beam shown in Figure 1(a). In this example, both struts and ties are modeled using truss elements
with bilinear material models, as shown in Figure 1(b). Five design scenarios are considered in
this numerical example. In the first scenario, two materials share the entire domain (see Figure
1(c)) and each material is associated with an individual volume constraint, i.e., \( V_{\text{max}}^j = 0.5V_{\text{max}} \),
\( j = 1,2 \). In the second and third scenarios, two materials share and split the domain shown in
Figure 1(e) and (g), and the tie region is constrained within two-thirds and half of the entire
domain, respectively. In the last two scenarios, struts and ties share and split the domain shown in
Figure 1(i) and (k), and the allowable angle inclination of ties is restricted to 90° and 45°,
respectively. The optimized results for the five scenarios are shown in Figure 1(d), (f), (h), (j), and
(l). From the comparison of the results, varying specified tie regions/inclinations can significantly
affect the STM and, in turn, the behavior of the resulting reinforced concrete beam.

Besides specifying the tie regions/inclinations, the present STM design framework allows
engineers to control the length scale of struts and ties either together or independently. Considering
the length scale of STM designs is important from a practical point of view since the difficulties
in the construction of the deep beam highly depend on the design of rebars in the STM. The length
control approach in the present STM design framework is demonstrated using the deep beam
example shown in Figure 1. Without any restrictions on the length scale, the layouts of the initial
ground structure for both struts and ties are shown in Figure 1(c). The corresponding optimized
STM design is shown in Figure 1(d) (which is the same plot as the one in Figure 2(a)). This design
can be simplified by restricting the length of members in the initial ground structures for both the struts and ties. For instance, assuming that the minimum length scale is $\sqrt{2}$, the optimized design is obtained, as shown in Figure 2(b), which has a simpler topology than the design shown in Figure 2(a). Two alternative designs in Figure 2(c) and 2(e) are obtained considering different minimum length scales as $\sqrt{5}$ and $\sqrt{10}$, respectively. Moreover, the length scale control can be applied to ties independently. For example, assuming that only ties are restricted to the minimum length scales, the optimized designs are achieved in Figure 2(d) and 2(f). These two designs have different topologies than the ones in Figure 2(c) and 2(e) accounting for length scale control for both struts and ties together. In summary, the flexible length control approach provides a variety of alternative STM configurations with different levels of complexity on the final topology.

The present STM framework generates a variety of optimized STM designs considering the specified tie regions, proper angles of inclination, and minimum length scale of the reinforcement. Furthermore, the efficiency of those alternative designs is investigated using the unified load path criterion in equation (1-1).

The design library in Figure 3 collects alternative STM designs for a deep beam structure. Among those designs, Figure 3(a) and 3(b) present conventional STM design layouts suggested by ACI code 318-19 (ACI Committee 318, 2019). The optimized designs with the present framework are shown in Figure 3(a) – 3(l). As all the designs in Figure 3 are statically determinant for the given boundary conditions, the internal forces in struts and ties can be easily calculated using static equilibrium conditions. Consequently, the load path $Z$ for each design is determined using equation (1-1). As the load path $Z$ decreases, the efficiency of the STM increases. For example, the standard ACI layout in Figure 3(a) has a larger load path $Z$ than the optimized layout in Figure 3(k). The testing results in section 5.2 demonstrate that the specimen with an optimized STM
layout (Figure 3(k)) can achieve a greater ultimate load than the specimen with the standard ACI layout for a given volume of tension reinforcement (Figure 3(a)).

5 EXPERIMENTAL EVALUATION OF OPTIMIZED STRUT-AND-TIE MODELS

The experimental research on RC deep beams presented in the literature focuses on the conventional strut-and-tie models suggested by the ACI code 318 (Schlaich, Schäfer, and Jennewein 1987; MacGregor et al. 1997; Breña and Roy 2009; Birrcher et al. 2009; Panjehpour et al. 2015; Ismail 2016; Ismail et al. 2017; Martinez et al. 2017; Rezaei et al. 2019; Kondalraj and Rao 2021). The present work selects two optimized strut-and-tie models (Figure 4(b) and 4(c)) and compares their behavior with the most common strut-and-tie model (Figure 4(a)) of ACI code 318-19. The designs employed contained one significant deviation from ACI 318 guidelines. That is, crack-control reinforcing (ACI 318-19, section 23.5) was omitted from the beam designs to allow an evaluation of the relative crack pattern development in the various beams. Five RC deep beam specimens were constructed: two for the standard ACI model, one for optimized layout I, and two for optimized layout II. All five specimens have the same geometry as shown in Figure 5.

Regarding the reinforcement arrangements of the specimens, a longitudinal reinforcement ratio \( \rho = 2\% \) was used for each specimen. To visualize the reinforcement layouts in a formwork, a 3D rendering of the reinforcement design was generated as shown in Figure 6(a) – (i), with more reinforcement details for the specimens given in Appendix A. Figure 6(h) and (i) show the designs involving multiple layers of rebars. Positioning those rebars inside formwork can be challenging from a construction perspective. To overcome this issue, customized rebar chairs were created using 3D printing. Using this method, the geometry of the rebar chairs can be specified for a given
reinforcement design. For example, the two customized rebar chairs in Figure 7 facilitated the accurate positioning of the reinforcing bars.

5.1 Experimental setup and procedure

The beams were cast with concrete having a compressive strength of 8.5 ksi (58.6 MPa) at the time of testing and reinforced with Grade-60 steel rebars (nominal yield strength of 60 ksi (414 MPa)). Rebar development was checked according to ACI 318-19; refer to Figure 5 for visualizations of beam and support locations and Appendix A for reinforcement layouts. The steel rebars of the five RC deep beams were instrumented with 350-Ω strain gauges prior to concrete casting. The locations of the strain gauges were selected to monitor the role of key reinforcing rebars in the optimized layouts; these locations are labeled in the three figures of Appendix A. Figures 8(a), 8(b), and 8(c) show the steel reinforcement used for the tested beams of the ACI layout, optimized layout I, and optimized layout II, respectively. Also, the lead wires for the strain gauges are presented in Figure 8. Moreover, two linear variable differential transformers (LVDT) were used to estimate the effective strain in the concrete struts. For the midspan deflection, a string potentiometer was used. An 890-kN (200-kip) load cell connected to a hydraulic load ram was used to record load values. The testing setup and instrumentation is shown in Figure 9.

The program of testing consisted of applying an increasing load while monitoring crack initiation on the beam. Once a crack was visually observed, the hydraulic jack valve was closed to hold the load constant and the cracks and their corresponding load values were highlighted on the beam. Additional load was then applied; this process was repeated until extensive cracks were observed which prevented further safe monitoring. At that point, continuous loading to complete failure was carried out and the failure load was recorded.

5.2 Load – deflection curves
An increasing load was applied to the specimen until it eventually failed. The load-deflection behavior of the specimens is shown in Figure 10 as solid/dashed curves. Each curve represents the applied load versus the midspan deflection of the deep beam. The ultimate loads (i.e., maximum loads measured in the specimens) considering the three different STM layouts are given in Table 1. It is observed that the optimized STM layout II (specimen #5), which has the lowest load path \( Z \), reaches the highest ultimate load; and the standard ACI STM layout (specimen #1 and #2), which has the largest load path \( Z \), achieves the lowest ultimate load. This indicates that the load path \( Z \) can serve as an effective criterion to evaluate the efficiency of the STM.

Moreover, the initial stiffness values are observed to decrease with the optimization in the reinforcement design. This can be attributed to the fact that the optimized designs have more reinforcement closer to the cross-section’s neutral axis which results in a smaller moment of area for transformed cross-sections when determining the overall transformed cross-sectional stiffness. Furthermore, deflection values are increased with the optimized layout I and II (specimen #3, #4, and #5) compared to the standard ACI layout (specimen #1 and #2). This demonstrates the improved behavior of the optimized designs in which more tension cracks in the central region and more steel yielding and, thus, more efficient load paths (\( Z \)) were observed.

### 5.3 Observed failure mode

The Failure mode of the specimens, considering the standard ACI STM layout (specimen #1 and #2) and the optimized layout I (specimen #3), was characterized by strut crushing. However, the ACI layout had larger crack widths compared to the optimized layout I, which can be attributed to the more efficient load path \( Z \) introduced by more inclined steel rebars to mitigate large crack widths. On the other hand, the specimens with optimized STM layout II (specimen #4 and #5) had a different failure mode characterized by bearing failure instead of strut failure as shown in Figure
When the optimized layout II was tested with a 6-inch bearing support plate (specimen #4), the failure mode was characterized by bearing failure showing the improved design obtained with more inclined steel rebars and a more optimal load path Z which mitigates the inclined strut failure mode of the tested specimen. To attempt to avoid an undesirable experimental failure mode in the optimized layout II specimens, the bearing plate width was increased from 6 to 8 inches (0.15 to 0.20 m) for the load test of specimen #5. A higher load capacity was observed using the 8-inch bearing plates, even though the section still failed in bearing. This shift of the controlling failure mode (for the optimized layout II) demonstrates the effectiveness of the novel STM layout. Future testing can incorporate more robust bearing layouts, which are expected to result in further optimization of the proposed STM.

Figure 11 shows how the total number of observed tension cracks (cracks in the midspan region of the beam) increased for the optimized layouts I (specimen #3) and II with 6-inch (specimen #4) and 8-inch (specimen #5) bearing schemes. Moreover, shear cracks were observed to reduce in the optimized layout I compared to the standard ACI design. No shear cracks were observed in the optimized layout II (both 6-inch and 8-inch bearing schemes). In terms of first crack loads in the struts, crack initiation was observed the ACI layout (specimen #1 and #2) at 66 kips (294 kN). Crack initiation for the optimized layout I (specimen #3) was observed at 75 kips (334 kN), and crack initiation for optimized layout II with a 6-inch bearing (specimen #4) was observed at 114 kips (507 kN). The strut crack initiation for optimized layout II with an 8-inch bearing (specimen #5) is not included in this discussion since the test was not halted for crack inspection after 94 kips (418 kN) for safety reasons; however, it can be reported that no cracks were observed up to the 94 kip (418 kN) load. These observations indicate a definable improvement in the load-carrying behavior for the optimized designs.
5.4 Observed strain gauge values

As discussed previously, strain gauges were used in specific locations on the internal reinforcement of the test specimens to attempt to compare the strain progression of the various reinforcing layouts under loading. Figure 12 shows the measured strain gauge values of the optimized layouts I (specimen #3) and II (specimen #5). As shown in Figure 12(a) for the optimized layout I, yielding was attained in the central region of the rebars (SG-2, 4, and 5), while SG-1 did not exhibit significant strain in the rebar because it was near the support. However, for the case of SG-3, it is interesting to note the sudden increase in strain values after the first tension crack and the increasing strain to eventual yielding with the progression of the strut crack. Figure 12(b) presents load-strain behavior for optimized layout II with an 8-inch bearing plate. Some key observations can be summarized as follows:

1) SG-1 and SG-5 located in the middle region demonstrated different behaviors due to the shape of the steel rebars they are attached to. Note that any gauges which appeared to fail early in the loading process are omitted from the reporting of data.

2) SG-3 and SG-4 were attached to the same steel rebar at two different inclinations; the steel in these regions has yielded, indicating that the inclined regions of the steel rebar participated in resisting transverse loading in the strut.

3) SG-6 behaved similarly to SG-3 and SG-4 but was on a different rebar, indicating similar behavior among all inclined rebars for optimized layout II.
Table 1 gives the ultimate experimental load attained for each specimen tested. As noted, the beams reinforced with optimized layout II (specimen #4 and #5) supported a larger applied load than the others. In addition, the controlling failure mode of the optimized II design shifted from compression to bearing; a more robust bearing detail would be expected to result in an even greater ultimate load for a beam with the optimized II layout. Thus, as expected, the STM layout has a substantial impact on the load-deflection behavior of deep beams. To provide a basis for comparison between the various beam layouts, the ultimate experimental loads are compared to a novel analysis procedure inspired by design guidelines in ACI 318-19. However, it is important to note that the analysis presented here does not directly follow the ACI design procedures (for optimized layouts). Rather, the analysis procedure is intended to give an insight into the effect of these layouts on the stress fields of the deep beams tested here. The failure modes checked are the capacity of the nodes, the capacity of the ties, and the capacity of the struts. Based on the failure modes observed in the testing, it has been assumed that the nominal strength of the strut controls the ultimate load of the beam for all layouts to simplify the comparison.

A sketch of the strut-and-tie models with the standard ACI layout, the optimized layout I, and the optimized layout II visualizing the force flow in the beam are shown in Figure 13(a), 13(b), and 13(c), respectively. The dashed lines, solid lines, and dimensionless round circles represent the compression elements, tension elements, and nodes (i.e., the intersection of struts and ties), respectively. The results of static analysis including the relative internal force magnitudes for ACI layout, optimized layout I, optimized layout II are shown in Table 2. This table also summarizes the assumed reinforcement inclination angles for each of three layouts.
For all the beams, the concrete compressive strength $f'_{c} = 8.5$ ksi (58.6 MPa) and the dimensions of the beams are as follows:

- Length of the truss $L = 60$ in. (1.52 m)
- Height of the truss $H = 15$ in. (0.38 m)
- Width of the beam $b_w = 9$ in. (0.23 m)
- Width of the bearing plates $b_1 = b_2 = 6.0$ in. (0.15 m). (Note that specimen #5 with the optimized layout II has a different bearing plate at supports, i.e., $b_1 = 8$ in. (0.2 m))
- Effective height of the tie $w_{T_1} = 4$ in. (0.1 m)
- Effective height of the node at the applied load $w_{T_2} = 2$ in. (0.05 m)

### 6.1 Standard ACI layout

For the standard ACI STM layout, the standard ACI analysis procedure is followed in a backward fashion in which the load is known and $\beta_s$ (the strut coefficient) is determined by calibrating the experimental results (i.e. the stress in the compression strut at the ultimate load) from testing to the analytical capacity of the strut in compression.

First, calculating the width of the strut at node #1 and at node #2 using equations (6-1) and (6-2), and then taking the lowest value corresponding to the highest stress (i.e. in this case $w_c^2$).

\[
\begin{align*}
    w_c^1 &= b_1 \sin \alpha_1 + w_{T_1} \cos \alpha_1 = 6.26 \text{ in. (0.16 m)} \\
    w_c^2 &= \frac{b_2}{2} \sin \alpha_1 + w_{T_2} \cos \alpha_1 = 3.13 \text{ in. (0.08 m)}
\end{align*}
\]

Then determining the compressive force in the strut from the known ultimate load $P = 71.1$ kips which is the average of specimen #1 and #2 (see Table 1):

\[
C_{2n} = 1.118P = 1.118 \times 71.1 \text{ kips} = 79.5 \text{ kips (354 kN)}
\]
The next step is the calculation of the effective compressive stress in the strut at node #2 using equation (6-4):

\[ f_{ce}^{s} = \frac{C_{2n}}{\frac{w_{e}^{2}}{2}b_{w}} = 2.82 \text{ ksi (19.4 MPa)} \]  
(6-4)

Using equation (6-5), the value of \( \beta_{s} \) is obtained, assuming that the strut and node confinement modification factor is equal to 1.0 (\( \beta_{c} = 1.0 \)):

\[ \beta_{s} = \frac{f_{ce}^{s}}{0.85\beta_{c}f'_{c}} = 0.39 \]  
(6-5)

It is noteworthy that this value is very similar to the value found in ACI 318-19 (\( \beta_{s} = 0.4 \)) for the case of no minimum reinforcement for crack control. The omission of minimum reinforcement was intended to allow a better evaluation for the performance of the novel layouts on the overall performance of the deep beam relative to one another. For this reason, significant transverse cracks were observed.

### 6.2 Optimized layout I

For the optimized layout I, the analysis procedure is the same as that given in Section 6.1 except that the resultant of the forces (\( C_{R} \)) acting at node #4 (see Figure 13(b)) and the corresponding \( \alpha_{R} \) are calculated with equations (6-6) and (6-7), respectively. This resolution of forces is intended to simplify the analytical procedure.

\[ C_{R} = \sqrt{(C_{2} \sin \alpha_{2} + C_{3} \sin \alpha_{3})^{2} + (C_{2} \cos \alpha_{2} + C_{3} \cos \alpha_{3})^{2}} = 1.118P \]  
(6-6)

\[ \alpha_{R} = \tan^{-1} \left( \frac{C_{2} \sin \alpha_{2} + C_{3} \sin \alpha_{3}}{C_{2} \cos \alpha_{2} + C_{3} \cos \alpha_{3}} \right) = 0.464 \]  
(6-7)

Then the width of the strut at node #1 and at node #4 is calculated as:

\[ w_{e}^{1} = b_{1} \sin \alpha_{1} + w_{T1} \cos \alpha_{1} = 7.16 \text{ in. (0.18 m)} \]  
(6-8)
\[
\frac{w_c^4}{2} = \frac{b_2}{2} \sin \alpha_R + w_{T2} \cos \alpha_R = 3.13 \text{ in. (0.08 m)} \tag{6-9}
\]

As \( w_c^4 < w_c^1 \), then \( w_c^4 \) is used for the calculation of the effective compressive stress in the strut at node #4. Given the ultimate load \( P = 114 \text{ kips} \) reached during the corresponding experimental test (see Table 1), the effective compressive stress is obtained as:

\[
f_{ce}^s = \frac{C_R}{w_c^4 b_w} = \frac{1.118P}{w_c^4 b_w} = 4.52 \text{ ksi (31.2 MPa)} \tag{6-10}
\]

In turn, the value of \( \beta_s \) is calculated as, assuming that the strut and node confinement modification factor is equal to 1.0 (\( \beta_c = 1.0 \)):

\[
\beta_s = \frac{f_{ce}^s}{0.85\beta_c f_c'} = 0.625 \tag{6-11}
\]

The larger value of \( \beta_s = 0.625 \) calculated for optimized layout I compared to that calculated for the standard ACI layout indicates a more efficient load path in the optimized layout. It should be noted that the \( \beta_s \) value calculated for optimized layout I is less than the value \( \beta_s = 0.75 \) which assumes the inclusion of minimum distributed reinforcement for crack control.

### 6.3 Optimized layout II

Similar to the procedure shown in Section 6.2, the resultant of the forces acting at node #13 (see Figure 13(c)) is determined \( (C_R) \) for optimized layout II with its corresponding \( \alpha_R \) as shown in equations (6-12) and (6-13).

\[
C_{Rx} = C_2 \sin \alpha_{11} + C_3 \sin \alpha_7 + C_4 \sin \alpha_2 + C_5 \sin \alpha_3 = 0.5P
\]

\[
C_{Ry} = C_2 \cos \alpha_{11} + C_3 \cos \alpha_7 + C_4 \cos \alpha_2 + C_5 \cos \alpha_3 = P
\]

\[
C_R = \sqrt{(C_{Rx})^2 + (C_{Ry})^2} = 1.118P \tag{6-12}
\]
\[ \alpha_R = \tan^{-1}\left( \frac{C_{Rx}}{C_{Ry}} \right) = 0.464 \]  

(6-13)

For the optimized layout II with 6-inch bearing plate (i.e., \( b_1 = 6 \) in. \((0.15 \text{ m})\)), the width of the strut at node \#1 and at node \#13 is calculated as:

\[ w_c^1 = b_1 \sin \alpha_1 + w_{T1} \cos \alpha_1 = 7.16 \text{ in. } (0.18 \text{ m}) \]  

(6-14)

\[ w_c^{13} = \frac{b_2}{2} \sin \alpha_R + w_{T2} \cos \alpha_R = 3.13 \text{ in. } (0.08 \text{ m}) \]  

(6-15)

where the smaller value \( w_c^{13} \) is selected for calculating the effective compressive stress in the strut at node \#13. It is also known that the ultimate load \( P = 131 \) kips from the experiment (see Table 1). Following the same procedure presented in Section 6.2, the value of \( \beta_s \) is calculated as, assuming that the strut and node confinement modification factor is equal to 1.0 (\( \beta_c = 1.0 \)):

\[ \beta_s = \frac{f_{ce}}{0.85 \beta_c f'_{c_e}} = \left( \frac{C_R}{w_c^{13} b_w} \right) / (0.85 \beta_c f'_{c_e}) = \left( \frac{1.118P}{w_c^{13} b_w} \right) / (0.85 \beta_c f'_{c_e}) = 0.719 \]  

(6-16)

For the optimized layout II with 8-inch bearing plate (i.e., \( b_1 = 8 \) in. \((0.2 \text{ m})\)), the width of the strut at node \#1 and at node \#13 is obtained as:

\[ w_c^1 = b_1 \sin \alpha_1 + w_{T1} \cos \alpha_1 = 8.94 \text{ in. } (0.23 \text{ m}) \]  

(6-17)

\[ w_c^{13} = \frac{b_2}{2} \sin \alpha_R + w_{T2} \cos \alpha_R = 3.13 \text{ in. } (0.08 \text{ m}) \]  

(6-18)

Given the ultimate load \( P = 189 \) kips from the experiment (see Table 1), the value of \( \beta_s \) is obtained as, assuming that the strut and node confinement modification factor is equal to 1.0 (\( \beta_c = 1.0 \)):

\[ \beta_s = \left( \frac{1.118P}{w_c^{13} b_w} \right) / (0.85 \beta_c f'_{c_e}) = 1.038 \]  

(6-19)

All the results are summarized in Table 3. The resulting \( \beta_s \) for the optimized layout II with 6-inch bearing plate (specimen \#4) shows a slightly improved performance because of the premature
bearing failure, but with even more improvement in the ultimate load. However, $\beta_s$ for optimized layout II with an 8-inch bearing plate (specimen #5) shows that a more desirable load path is utilized when premature bearing failure is suppressed. It is important to note that the calculations for both optimized layout II with 6-inch and 8-inch are based on the assumption that the nominal strength of the strut controls the ultimate failure. Nonetheless, for the specific case of optimized layout II, a bearing failure was observed rather than a strut failure, so the actual values of $\beta_s$ will be higher than the calculated value herein.

7 CONCLUDING REMARKS

This work presents a multi-material/multi-volume topology optimization framework to design practical strut-and-tie model (STM) layouts for reinforced concrete structures. Inspired by the Michell’s optimality conditions, the efficiency of optimized STM layouts is quantified by the load path $Z$ (or the Michell number), which serves as a simple and efficient criterion for evaluating any STM. An experimental testing program indicated that the optimized STM layouts demonstrated significantly improved behavior than the traditional layout using the same overall reinforcement ratio. As such, it is expected that designs with optimized layouts could result in a demonstrably smaller total volume of reinforcing steel needed to resist a given set of design loads, resulting in a more economical design. It is acknowledged that at present the more complex reinforcement layouts may increase fabrication costs. However, given advancements in reinforcing layout construction using computer-aided reinforcing fabrication, it is anticipated that reducing the total volume of steel needed in a given deep beam design will eventually result in a more efficient and cost-effective design. Given the significantly reduced cracking observed in the optimized layouts compared to the traditional STM layout, it is possible that the optimized layouts would require less
crack control reinforcement, potentially further reducing the total volume of steel needed in the deep beams.

In addition, to extend the present framework as a practical reinforced concrete structures design tool, further research could be conducted in the following aspects: 1) 3D design domain with complex stress state; 2) incorporating realistic plasticity material models in the optimization framework for concrete and steel; 3) achieving higher structural ductility through transverse reinforcement optimization; 4) application of the design and analysis procedures presented herein to a database of known reliable experimental results for deep beams available in the established literature.

APPENDIX A

This appendix details the reinforcement design including the location of strain gauges for the standard ACI STM layout (Figure 14), optimized layout I (Figure 15), and optimized layout II (Figure 16), respectively.

APPENDIX B

Appendix B includes an example to show the calculation of the Michell number $Z$ for the STM layout in Figure 3(d). Since this truss system is statically determinate, the internal axial force of the members can be calculated using the equilibrium conditions, i.e., $N_{12} = N_{67} = -0.56P$, $N_{24} = N_{46} = -0.61P$, $N_{34} = N_{45} = -0.53P$, $N_{13} = N_{57} = 0.25P$, $N_{23} = N_{56} = 0.47P$, and $N_{35} = P$ (see labeled node numbers in Figure 17). Moreover, the length of each truss member is given as $L_{12} = L_{67} = 0.56H$, $L_{24} = L_{46} = 1.82H$, $L_{34} = L_{45} = 1.6H$, $L_{13} = L_{57} = 0.75H$, $L_{23} = L_{56} = 0.71H$, and $L_{35} = 2.5H$. Therefore, the Michell number $Z$ can be obtained using equation (1-1) as

$$Z = \sum |F_e|L_e = 2|F_{12}|L_{12} + 2|F_{24}|L_{24} + 2|F_{34}|L_{34} + 2|F_{13}|L_{13} + 2|F_{23}|L_{23} + |F_{35}|L_{35} = 8.08PH.$$
AUTHOR BIOGRAPHIES

Tuo Zhao is a Postdoctoral Research Associate in the Department of Civil and Environmental Engineering at Princeton University. His research interests include multi-material topology optimization and nonlinear analysis of reinforced concrete structures. He was the winner of the ACI Georgia Chapter LaGrit F. "Sam" Morris Student Scholarship.

Ammar A. Alshannaq is an Assistant Professor in the Department of Civil Engineering at Yarmouk University. His research interests include repairing and strengthening of existing RC structures, characterization and testing of FRP materials, and optimization and novel design of deep RC beams.

David W. Scott is a Professor in the Department of Civil Engineering and Construction at Georgia Southern University, GA, USA. He is a member of ACI Committees 364, Rehabilitation; 440, Fiber-Reinforced Polymer Reinforcement; 546, Repair of Concrete; and E706, Concrete Repair Education. His research interests include novel materials & systems for civil infrastructure, evaluation of highway safety structures, and repair of concrete structures.

Glaucio H. Paulino is the Margareta Engman Augustine Professor of Civil and Environmental Engineering at Princeton University, NJ, USA. His research interests include the development of methodologies to characterize the deformation and fracture behavior of existing and emerging materials and structural systems, and topology optimization for multiscale/multiphysics problems.

REFERENCES

ACI Committee 318. 2019. 'Building Code Requirements for Structural Concrete and Commentary', American Concrete Institute.


Ismail, Kamaran Sulaiman. 2016. 'Shear behaviour of reinforced concrete deep beams', University of Sheffield.


TABLES AND FIGURES

List of Tables:

Table 1 – Comparison of load path $Z$ and the ultimate load for three strut-and-tie layouts.

Table 2 – Relative magnitudes of internal forces (as a fraction of applied load $P$) and angles (in radians) for the ACI layout, optimized layout I, and optimized layout II as shown in Figure 13(a), Figure 13(b) and Figure 13(c), respectively.

Table 3 – Summary of analysis results for all layouts (see Figure 13 for node numbering).

List of Figures:

Figure 1 – Optimized strut-and-tie models for a reinforced deep beam considering various design scenarios. (a) Geometry of the beam with highlighted design domain (i.e., grey region). (b) Simplified bilinear material model for both struts and ties. (c) Scenario 1: strut (red) and tie (blue) regions share the design domain; (e)&(g) Scenario 2&3: the tie region can only occupy two thirds and half of the entire domain, respectively. (i)&(k) Scenario 4&5: the allowable angle inclination of the tie is $90^\circ$ and $45^\circ$, respectively. (d)(f)(h)(j)&(l) Corresponding optimized STM designs for scenarios 1 – 5.

Figure 2 – Flexible length control for the optimized STM (assuming that the dimension of the unit square in the background grid for each design is 1). (a) An optimized design without length constraints for both struts and ties. (b) (c) & (e) Optimized designs with length constraints for both struts and ties, where the upper bound of the length is defined as $\sqrt{2}$, $\sqrt{5}$, and $\sqrt{10}$, respectively. (d) & (f) Optimized designs with length constraints on ties only, with the allowable length defined as $\sqrt{5}$ and $\sqrt{10}$, respectively.

Figure 3 – Deep beam STM design library. The load path $Z$ (i.e., equation (1-1)) indicates the efficiency of the alternative designs in this library. As the load path $Z$ decreases through (a) to (l),
the efficiency of the corresponding design improves. The three highlighted layouts (i.e., (a), (d) & (k)) are selected for the experimental validation in Section 5.

Figure 4 – Three layouts of the strut-and-tie models for experimental validation. (a) conventional model in the ACI code 318-19. (b) and (c) two optimized strut-and-tie models.

Figure 5 – Deep beam specimen geometry for evaluating the optimized strut-and-tie models.

Figure 6 – 3D representations of the STM reinforcement layouts. (a)(b)(c) Three selected STM layouts designated as standard ACI layout, optimized layout I, and optimized layout II, respectively; (d)(e)(f) Front view of reinforcement designs; (g)(h)(i) Perspective view of reinforcement designs.

Figure 7 – Positioning reinforcement using 3D printer rebar chairs (PRUSA® Fused Filament Fabrication (https://www.prusa3d.com/), with PLA plastic material).

Figure 8 – Steel reinforcement cages used for (a) the ACI layout, (b) optimized layout I, and (c) optimized layout II.

Figure 9 – Experimental setup and testing station.

Figure 10 – Testing results of the deep beams considering different STM layouts. Note: 1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

Figure 11 – Failure modes of the deep beams considering standard ACI with 6-inch bearing plate, optimized I layout with 6-inch bearing plate, optimized II layout with 6-inch bearing plate, and optimized II layout with 8-inch bearing plate, respectively.

Figure 12 – Strain gauge values with loading for (a) Optimized layout I, and (b) Optimized layout II (with 8-inch bearing plate).

Figure 13 – Deep beam strut-and-tie models. (a) Standard ACI STM layout. (b)(c) Optimized STM layouts I &II, respectively.
Figure 14 – Reinforcement details of the specimen with the standard ACI STM layout. Note: the labeled dimensions are inches and 1.0 in. = 25.4 mm.

Figure 15 – Reinforcement details of the specimen with the optimized STM layout I. Note: the labeled dimensions are inches and 1.0 in. = 25.4 mm.

Figure 16 – Reinforcement details of the specimen with the optimized STM layout II. Note: the labeled dimensions are inches and 1.0 in. = 25.4 mm.

Figure 17 – Labeled node numbers for the truss system in Figure 3(d).

Table 1 – Comparison of load path $Z$ and the ultimate load for three strut-and-tie layouts.

<table>
<thead>
<tr>
<th>STM layout</th>
<th>Load path (equation (1-1))</th>
<th>Specimen</th>
<th>Ultimate load, kips (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard ACI</td>
<td>$9.00 PH$</td>
<td>#1 (6&quot; bearing plate)</td>
<td>73.5 (327)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#2 (6&quot; bearing plate)</td>
<td>68.7 (305)</td>
</tr>
<tr>
<td>Optimized I</td>
<td>$8.08 PH$</td>
<td>#3 (6&quot; bearing plate)</td>
<td>114 (506)</td>
</tr>
<tr>
<td>Optimized II</td>
<td>$7.78 PH$</td>
<td>#4 (6&quot; bearing plate)</td>
<td>131 (583)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>#5 (8&quot; bearing plate)</td>
<td>189 (839)</td>
</tr>
</tbody>
</table>

Table 2 – Relative magnitudes of internal forces (as a fraction of applied load $P$) and angles (in radians) for the ACI layout, optimized layout I, and optimized layout II as shown in Figure 13(a), Figure 13(b) and Figure 13(c), respectively.

<table>
<thead>
<tr>
<th></th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI</td>
<td></td>
<td></td>
<td></td>
<td>1.118</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimized I</td>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
<td>$\alpha_3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.2500</td>
<td>0.4714</td>
<td>0.5590</td>
<td>0.6067</td>
<td>0.5336</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tan^{-1}(2)$</td>
<td>$\tan^{-1}(2/7)$</td>
<td>$\tan^{-1}(4/5)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimized II</td>
<td>$T_1$</td>
<td>$C_1$</td>
<td>$C_2$</td>
<td>$C_3$</td>
<td>$C_4$</td>
<td>$C_5$</td>
</tr>
<tr>
<td></td>
<td>0.5590</td>
<td>0.6622</td>
<td>0.1169</td>
<td>0.3030</td>
<td>0.0958</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
<td>$\alpha_3$</td>
<td>$\alpha_7$</td>
<td>$\alpha_{11}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tan^{-1}(2)$</td>
<td>$\tan^{-1}(1)$</td>
<td>$\tan^{-1}(2)$</td>
<td>$\tan^{-1}(3/4)$</td>
<td>$\tan^{-1}(1/5)$</td>
<td></td>
</tr>
</tbody>
</table>
Table 3 – Summary of analysis results for all layouts (see Figure 13 for node numbering).

<table>
<thead>
<tr>
<th>Layout</th>
<th>(w_1^c)</th>
<th>(w_2^c)</th>
<th>(C_{2n})</th>
<th>(f_{ce}^s)</th>
<th>(\beta_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard ACI layout</td>
<td>6.26 in</td>
<td>3.13 in</td>
<td>79.5 kips</td>
<td>2.82 ksi</td>
<td>0.390</td>
</tr>
<tr>
<td>Optimized layout I</td>
<td>7.16 in</td>
<td>3.13 in</td>
<td>128 kips</td>
<td>4.52 ksi</td>
<td>0.625</td>
</tr>
<tr>
<td>Optimized layout II with 6-inch bearing plate</td>
<td>7.16 in</td>
<td>3.13 in</td>
<td>146 kips</td>
<td>5.20 ksi</td>
<td>0.719</td>
</tr>
<tr>
<td>Optimized layout II with 8-inch bearing plate</td>
<td>8.94 in</td>
<td>3.13 in</td>
<td>211 kips</td>
<td>7.49 ksi</td>
<td>1.038</td>
</tr>
</tbody>
</table>

* Note: 1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN, and 1.0 ksi = 6.89 MPa.
Figure 1 – Optimized strut-and-tie models for a reinforced deep beam considering various design scenarios. (a) Geometry of the beam with highlighted design domain (i.e., grey region). (b) Simplified bilinear material model for both struts and ties. (c) Scenario 1: strut (red) and tie (blue) regions share the design domain; (e)&(g) Scenario 2&3: the tie region can only occupy two thirds
and half of the entire domain, respectively. (i)&(k) Scenario 4&5: the allowable angle inclination of the tie is $90^\circ$ and $45^\circ$, respectively. (d)(f)(h)(j)&(l) Corresponding optimized STM designs for scenarios 1 – 5.

Figure 2 – Flexible length control for the optimized STM (assuming that the dimension of the unit square in the background grid for each design is 1). (a) An optimized design without length constraints for both struts and ties. (b) (c) & (e) Optimized designs with length constraints for both struts and ties, where the upper bound of the length is defined as $\sqrt{2}$, $\sqrt{5}$, and $\sqrt{10}$, respectively. (d) & (f) Optimized designs with length constraints on ties only, with the allowable length defined as $\sqrt{5}$ and $\sqrt{10}$, respectively.
Figure 3 – Deep beam STM design library. The load path $Z$ (i.e., equation (1-1)) indicates the efficiency of the alternative designs in this library. As the load path $Z$ decreases through (a) to (l), the efficiency of the corresponding design improves. The three highlighted layouts (i.e., (a), (d) & (k)) are selected for the experimental validation in Section 5.
Figure 4 – Three layouts of the strut-and-tie models for experimental evaluation. (a) conventional model in the ACI code 318-19. (b) and (c) two optimized strut-and-tie models.

Figure 5 – Deep beam specimen geometry for evaluating the optimized strut-and-tie models.
Figure 6 – 3D representations of the STM reinforcement layouts. (a)(b)(c) Three selected STM layouts designated as standard ACI layout, optimized layout I, and optimized layout II, respectively; (d)(e)(f) Front view of reinforcement designs; (g)(h)(i) Perspective view of reinforcement designs.

Figure 7 – Positioning reinforcement using 3D printer rebar chairs (PRUSA® Fused Filament Fabrication (https://www.prusa3d.com/), with PLA plastic material)
Figure 8 – Steel reinforcement cages used for (a) the ACI layout, (b) optimized layout I, and (c) optimized layout II.
Figure 9 – Experimental setup and testing station.

Figure 10 – Testing results of the deep beams considering different STM layouts. Note: 1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.
Figure 11 – Failure modes of the deep beams considering standard ACI with 6-inch bearing plate, optimized I layout with 6-inch bearing plate, optimized II layout with 6-inch bearing plate, and optimized II layout with 8-inch bearing plate, respectively.
Figure 12 – Strain gauge values with loading for (a) Optimized layout I, and (b) Optimized layout II (with 8-inch bearing plate).
Figure 13 – Deep beam strut-and-tie models. (a) Standard ACI STM layout. (b)(c) Optimized STM layouts I & II, respectively.
Figure 14 – Reinforcement details of the specimen with the standard ACI STM layout. Note: the labeled dimensions are inches and 1.0 in. = 25.4 mm.
Figure 15 – Reinforcement details of the specimen with the optimized STM layout I. Note: the labeled dimensions are inches and 1.0 in. = 25.4 mm.
Figure 16 – Reinforcement details of the specimen with the optimized STM layout II. Note: the labeled dimensions are inches and 1.0 in. = 25.4 mm.
Figure 17 – Labeled node numbers for the truss system in Figure 3(d).