Cohesive Zone Modeling of Dynamic Crack Propagation in Homogeneous and Functionally Graded Materials

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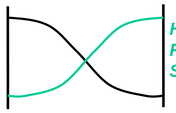
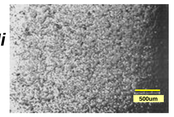
❖ Background

Ideal Behavior of Material Properties in a Ceramic-Metal FGM

Fracture Toughness
Thermal Conductivity



High Temperature Resistance Compressive Strength



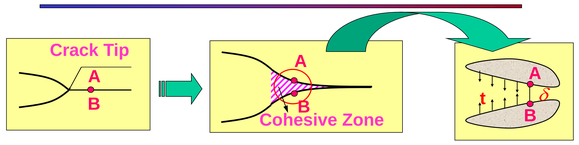
Metal Rich CrNi Alloy

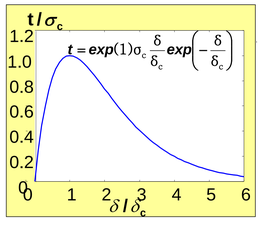
Ceramic Rich PSZ

(Ilschner, 1996)

FGMs Offer a Composite's Efficiency w/o Stress Concentrations at Sharp Material Interfaces

Cohesive Fracture Model





t: Cohesive Traction
δ: Surface Displacement Jump
ϕ: Free Energy Density Function per Unit Area

$$\phi = e\sigma_c\delta_c\left[1-\left(1+\frac{\delta}{\delta_c}\right)\exp\left(-\frac{\delta}{\delta_c}\right)\right]$$

$$t = \frac{\partial\phi}{\partial\delta}$$

X. Xu and A. Needleman, 1995, Numerical simulations of dynamic crack growth along an interface, *International Journal of Fracture,* Vol 74, pp.289-324

❖ Numerical Scheme

Explicit Updating Scheme

PVW: $\int_v(\mathbf{div}\sigma-\rho\ddot{\mathbf{u}})\delta\mathbf{u}dv+\int_s(\mathbf{T}-\sigma\mathbf{n})\delta\mathbf{u}ds=0$

Divergence theorem

$$\int_v(\boldsymbol{\sigma}:\delta\mathbf{E}+\rho\ddot{\mathbf{u}}\cdot\delta\mathbf{u})dv-\int_s\mathbf{T}\cdot\delta\Delta\mathbf{u}ds=\int_s\mathbf{T}\cdot\delta\mathbf{u}ds$$

Finite Element Discretization

Cohesive surface

$$\mathbf{M}\ddot{\mathbf{u}}=\mathbf{F}-\mathbf{R}_{\text{int}}+\mathbf{R}_{\text{cohesive}}$$

F: external load vector
R_{int}: internal force vector
$R_{cohesive}$: cohesive force vector

Explicit Updating Scheme:

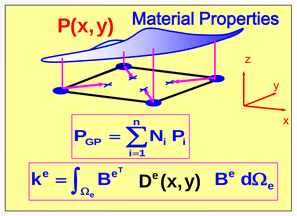
$$\mathbf{u}_{n+1}=\mathbf{u}_n+\dot{\mathbf{u}}_n\Delta t+\frac{1}{2}\ddot{\mathbf{u}}_n\Delta t^2$$

$$\ddot{\mathbf{u}}_{n+1}=\mathbf{M}^{-1}\left(\mathbf{F}-\mathbf{R}_{\text{int}(n+1)}+\mathbf{R}_{\text{coh}(n+1)}\right)$$

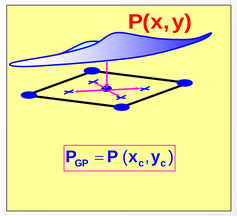
$$\dot{\mathbf{u}}_{n+1}=\dot{\mathbf{u}}_n+\frac{\Delta t}{2}(\ddot{\mathbf{u}}_n+\ddot{\mathbf{u}}_{n+1})$$

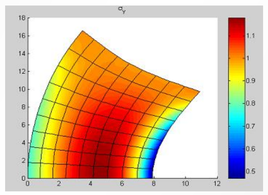
Material Gradation

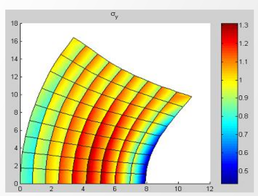
Graded element-- Generalized isoparametric Formulation



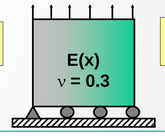
Homogeneous Elements







Unaveraged nodal stress



Unaveraged nodal stress

❖ CZM for FGM

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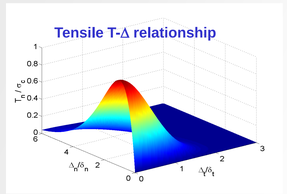
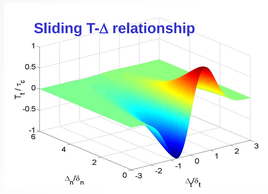
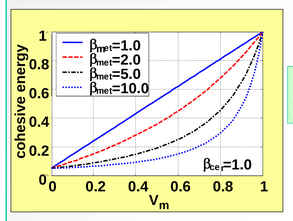




$$\mathbf{T}_{\text{fgm}}(x)=\left(\frac{V_m(x)}{V_m(x)+\beta_m[1-V_m(x)]}\right)\mathbf{T}_{\text{met}}+\left(\frac{1-V_m(x)}{1-V_m(x)+\beta_c V_m(x)}\right)\mathbf{T}_{\text{cer}}$$

$V_m(x)$: Volume Fraction of Metal
β_m, β_c: Material Parameters Calibrated with Experiments



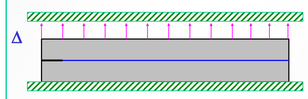
Calibration of β_m and β_c

Fracture Test (CT specimen, etc.) compare P-Δa curve FEM simul., Vary β_m, β_c

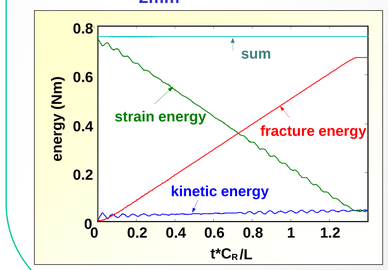
calibrate β_m and β_c

Z.-H. Jin, G. H. Paulino and R. H. Dodds Jr., 2001, Finite element investigation of quasi-static crack growth in functionally graded materials using a novel cohesive zone fracture model, *ASME Journal of Applied Mechanics*, Vol. 69, pp.370-379

Verification



- Mesh convergence
- Crack tip speed
- Energy balance



Energy dissipated :

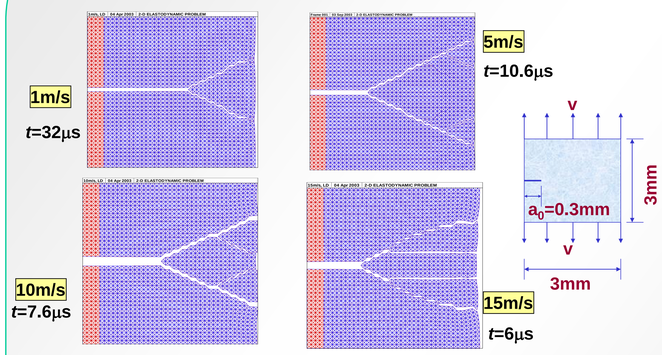
$$E_{fracture}=G_I A=0.66935\,N\cdot m$$

FEM result:

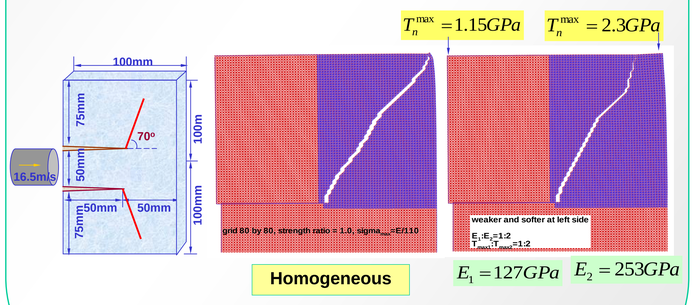
$$E_{fracture}=0.66944\,N\cdot m$$

❖ Examples

Example 1: Branching



Example 2: Mixed-mode Crack



Homogeneous

$E_1=127GPa$ $E_2=253GPa$

Z. Zhang and G.H.Paulino, 2004, Cohesive Zone Modeling of Dynamic Failure in Homogeneous and Functionally Graded Materials, *International Journal of Plasticity (in press).*

❖ Future Work

Intrinsic vs. Extrinsic CZM:

Intrinsic CZM	*Extrinsic CZM*
Initial elastic: artificial compliance introduced	*Initial Rigid*: no artificial compliance introduced
Implementation: cohesive elements inserted before simulation starts	*Implementation*: cohesive elements inserted adaptively

Topology-based Data Structure

Insert new surfaces→topology changes

Balance between
-- efficient retrieval of adjacency information
-- memory storage

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