Topology Optimization of Functionally Graded Structures Using Polygonal Finite Element Interpolants

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Motivation

- Special element suitable for topology optimization of FGMs
- Checkerboard-free topology optimization solutions
- Favorable geometric configuration:

one-node vs. two-node connections

two lines vs. three lines of symmetry

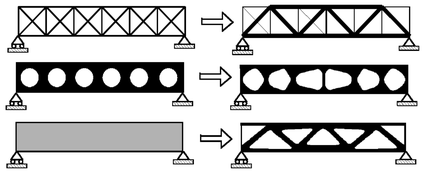
Introduction

- Different forms of structural optimization:

Size Optimization

Shape Optimization

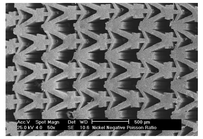
Topology Optimization



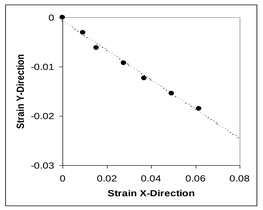
- Extensions to multiscale optimization problems:

Material with negative Poisson's ratio:

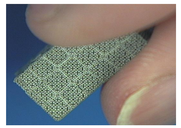
Nickel Structure

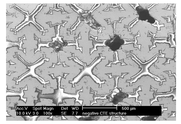


$$\upsilon = \frac{\varepsilon_y}{\varepsilon_x} = -0.3$$



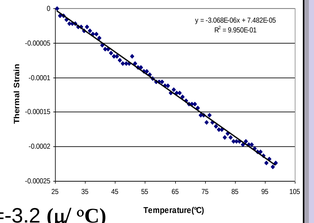
Material with negative coefficient of thermal expansion:





Ni/Fe=36/64

Ni/Fe=50/50



Predicted CTE=-3.2 (μ/ °C)

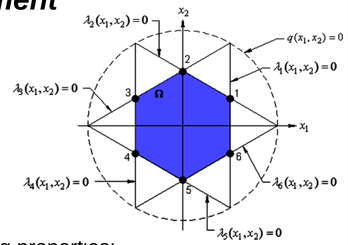
Courtesy of John Halloran, Department of Materials Science and Engineering, University of Michigan

Wachspress Hexagonal Element

- Wachspress shape functions:

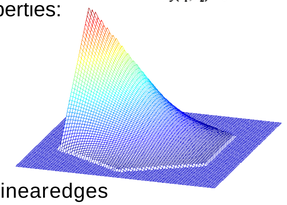
$$\phi_i(\mathbf{x}) = c_i \frac{\lambda_{i+2}\lambda_{i+3}\lambda_{i+4}\lambda_{i+5}}{q}$$

with $c_i = \dfrac{q(a_i)}{\lambda_{i+2}(a_i)\lambda_{i+3}(a_i)\lambda_{i+4}(a_i)\lambda_{i+5}(a_i)}$



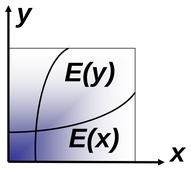
- These shape functions have the following properties:

1) $\sum_{i=1}^{6}\phi_i(\mathbf{x}) = 1, \quad 0 \le \phi_i(\mathbf{x}) \le 1$
2) $\phi_i(\mathbf{x}_i) = 1$ for all i and $\phi_i(\mathbf{x}_j) = 0$ for all $i \neq j$
3) $\sum_{i=1}^{6}\phi_i(\mathbf{x})\mathbf{x}_i = \mathbf{x}$
4) $\phi_i \in C^\infty$ within the polygon and ϕ_i piecewise linear at edges

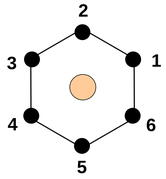


FGM-SIMP formulation

$$\mathbf{E}^H = \rho^p \mathbf{E}_0 e^{\alpha x + \beta y}$$



- Use of CAMD approach with nodal design variables
- Same interpolation functions are used to find density distribution within an element

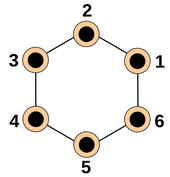


Constant material density

$$\rho(\mathbf{x}) = \sum_{i=1}^{6} \rho_i \phi_i(\mathbf{x})$$

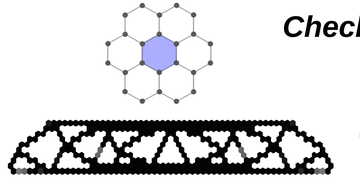
ϕ_i are the shape functions

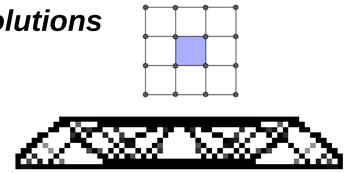
ρ_i are the nodal density

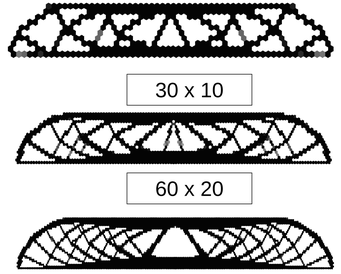


Continuously distributed material density

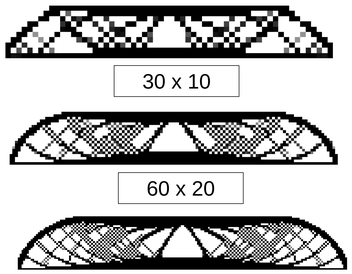
Checkerboard-free Solutions



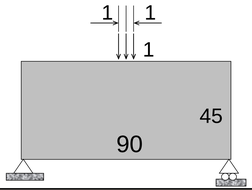


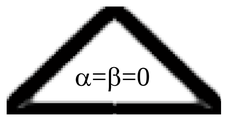


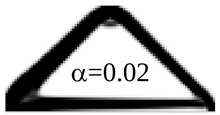
$$\min \mathbf{f}^T\mathbf{u}$$
$$\text{s.t.}\ \mathbf{K}(E_e)\mathbf{u} = \mathbf{f},$$
$$\sum_{e=1}^{N} V_e \le V$$
$$0 < \rho_{\min} \le \rho \le 1$$

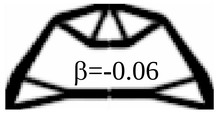


FGM-SIMP Results









Conclusions

- Topology optimization solutions using Wachspress hexagonal element are free from checkerboard patterns
- Wachspress interpolants provide potentially superior topology optimization solutions
- Topology optimization methods are powerful tools that can be used for the design of FGM structures

Acknowledgments