Topology optimization with polygonal finite elements

C. Talischi, G.H. Paulino, A. Pereira, I. Menezes

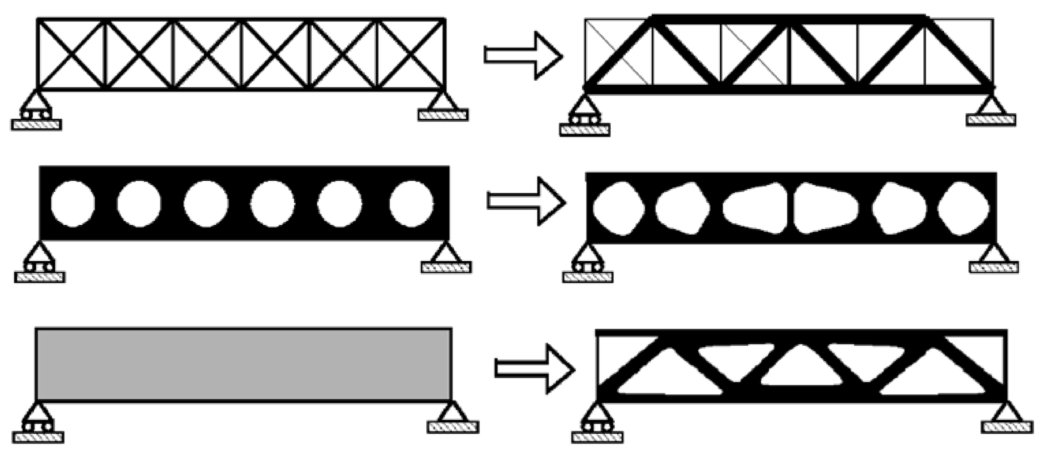
Department of Civil and Environmental Engineering
University of Illinois at Urbana-Champaign

Introduction

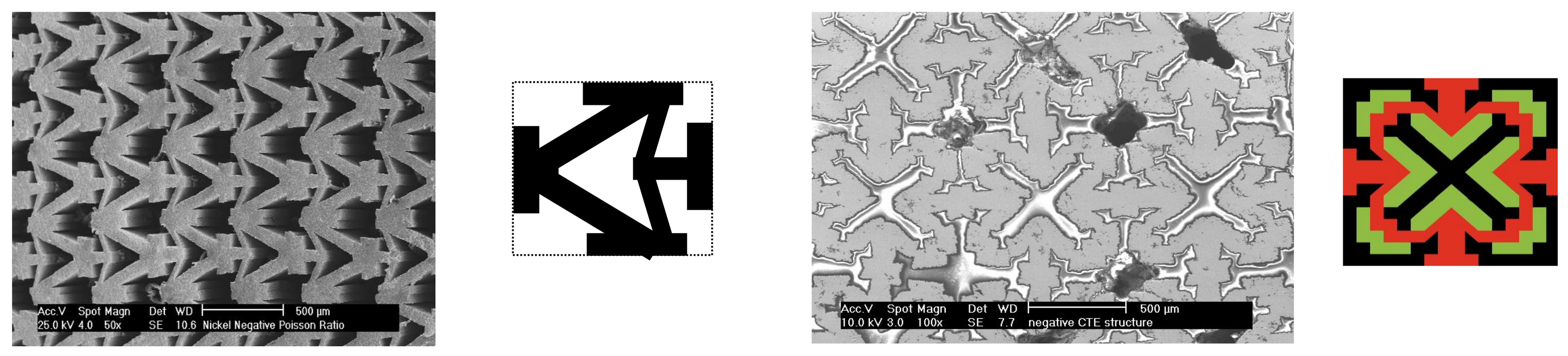
Sizing Optimization

Shape Optimization

Topology Optimization



❖ Extensions to multiscale, multiphysics problems:



Negative Poisson's Ratio *Negative thermal expansion*

Courtesy of Prof. John Halloran, Material Science and Engineering, University of Michigan

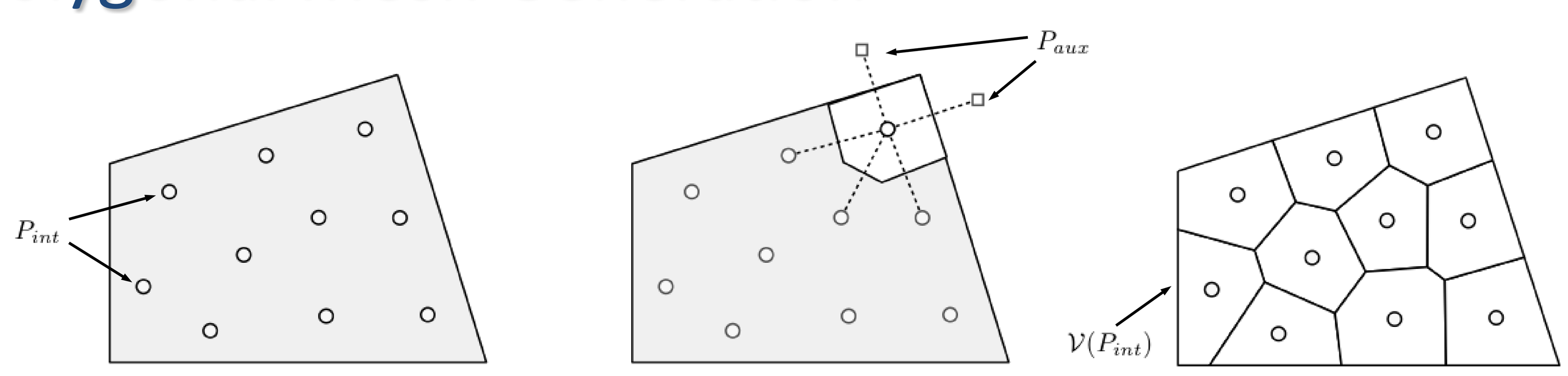
Motivation

❖ Uniform grids are traditionally used to parameterize and analyze design

❖ In addition to numerical instabilities, the constrained geometry of these meshes can bias the orientation of members in optimal design

❖ This work examines the use of polygonal finite element in topology optimization to address these issues

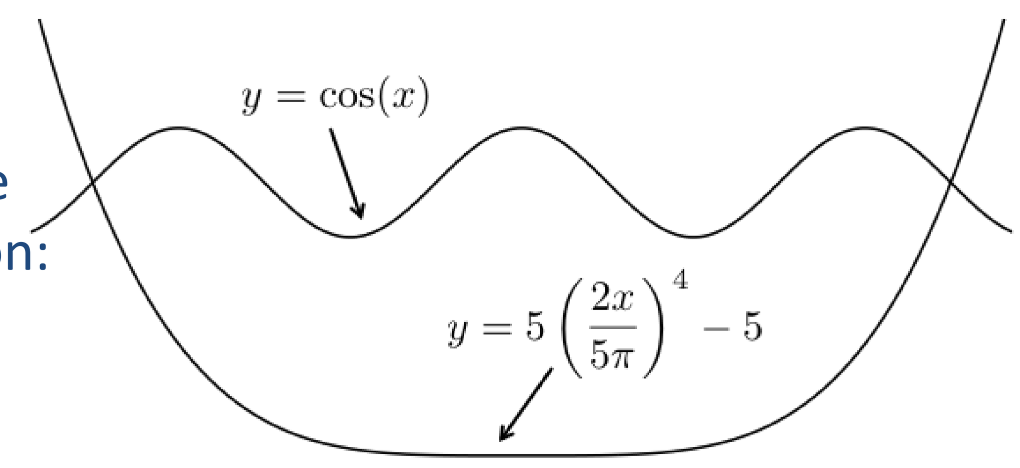
Polygonal Mesh Generation



The use of auxiliary points guarantee that the resulting Voronoi diagram includes an approximation to the boundary

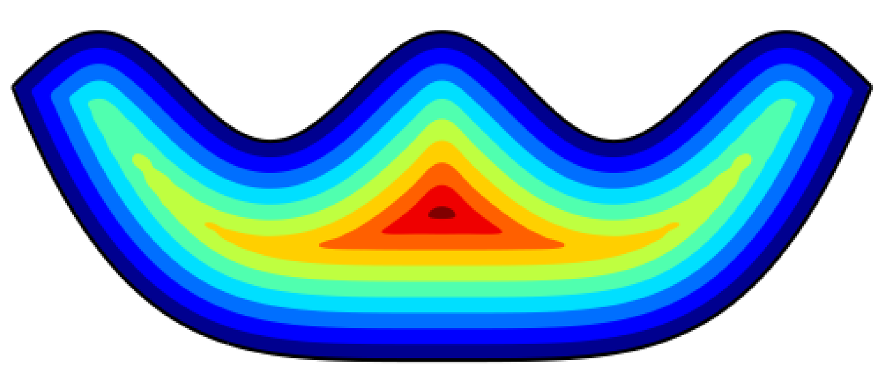
The domain is described by the zero level set of a given function:

$$f(\boldsymbol{x}) = 0$$

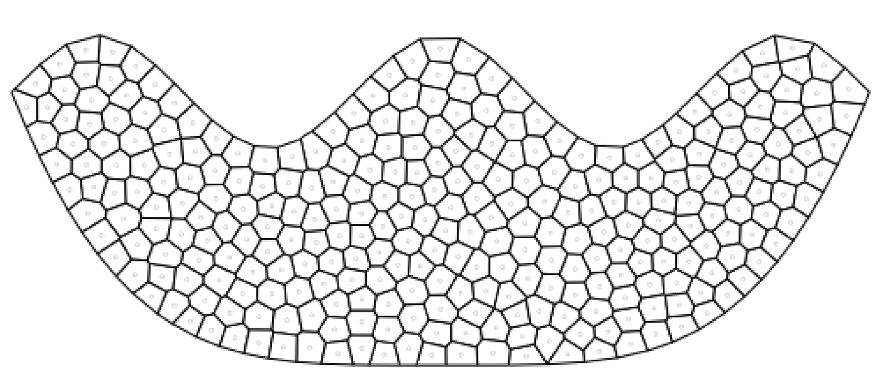


Placement and reflection of seeds can be carried out generically using a signed distance function:

$$\boldsymbol{x}_R = \boldsymbol{x} - 2d(\boldsymbol{x})\frac{\nabla d(\boldsymbol{x})}{|\nabla d(\boldsymbol{x})|}$$



The resulting mesh using the Centroidal Voronoi Tesselation of the point set:

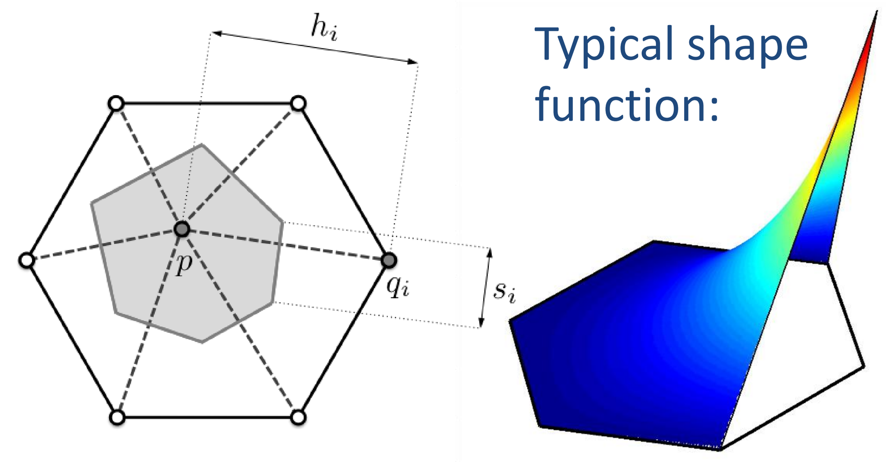


Finite element formulation

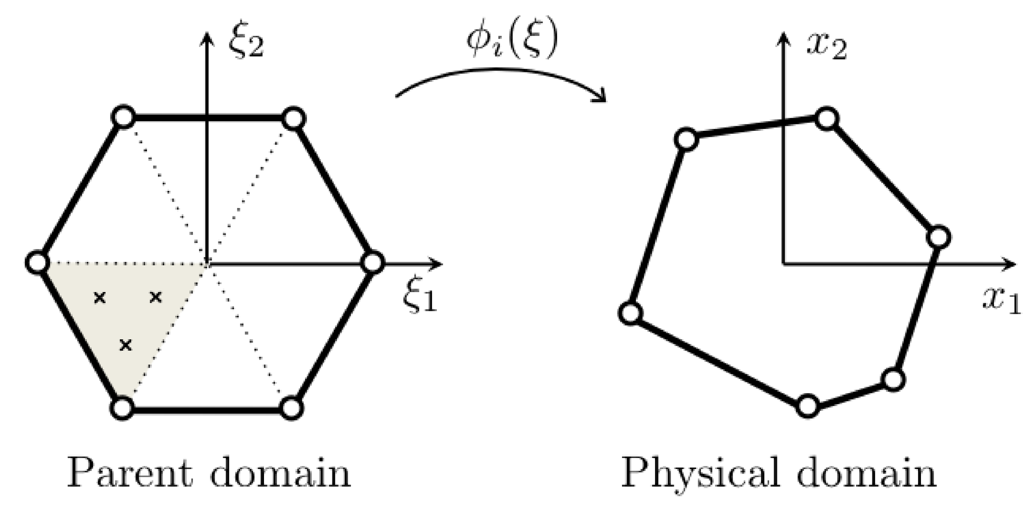
For a convex polygon, the Laplace interpolant is defined as:

$$\phi_i(\boldsymbol{x}) = \frac{w_i(\boldsymbol{x})}{\sum_{j=1}^{n} w_j(\boldsymbol{x})}$$

where $w_i(\boldsymbol{x}) = \dfrac{s_i(\boldsymbol{x})}{h_i(\boldsymbol{x})}$



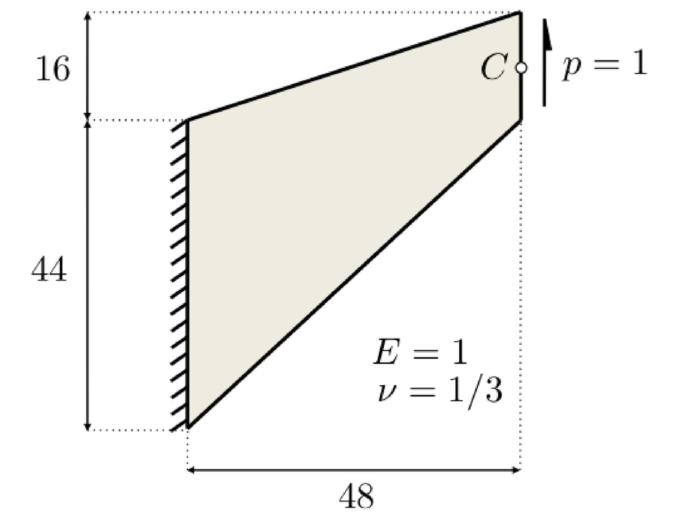
An isoparametric mapping from regular n-gons to any convex polygon is constructed using these shape functions

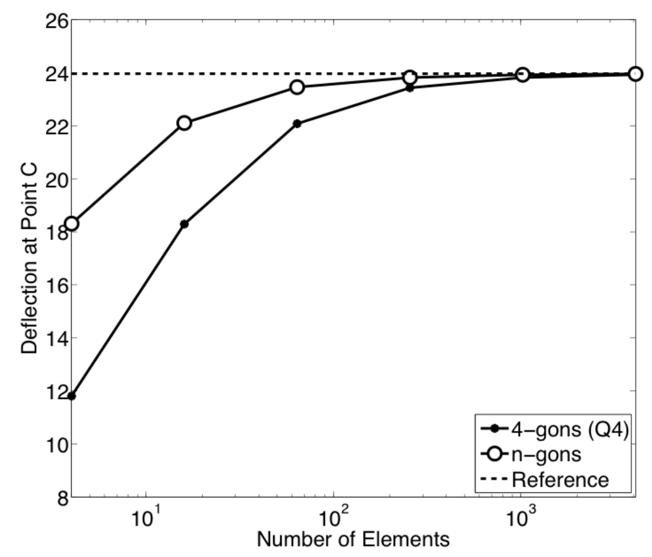


Weak form integrals are evaluated by triangulating the parent element and using the usual quadrature rules

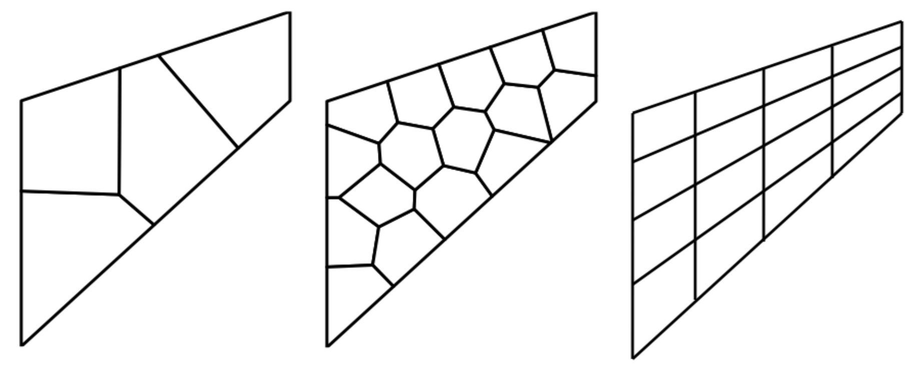
Numerical performance

Cook's problem consisting of a tapered panel subjected to uniform shear loading:





Polygonal elements are not as stiff as the quad elements

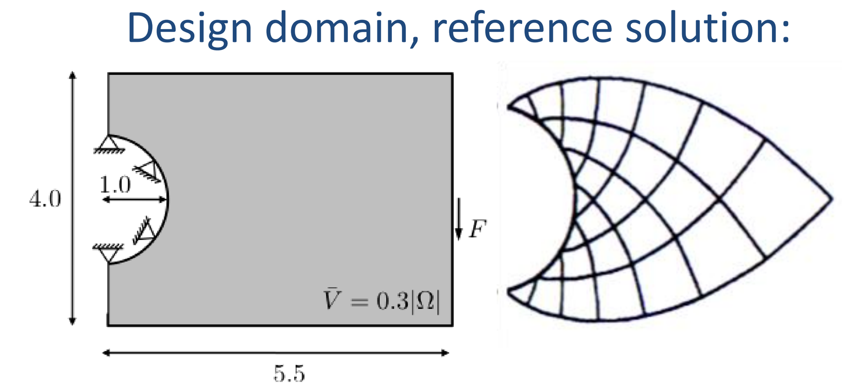


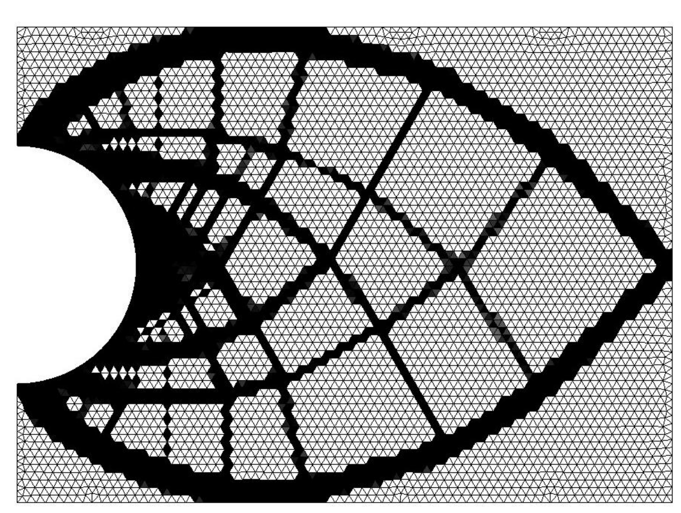
Meshes used: note the progressive refinement for quads and independent refinement for polygons

Minimum compliance design:

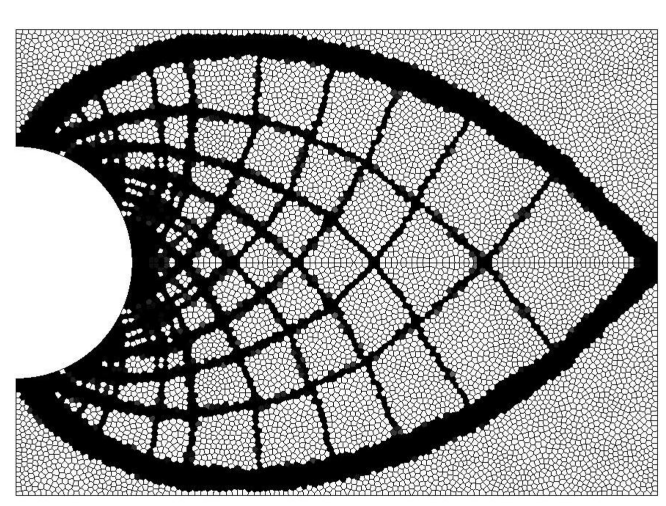
The T6 mesh suffers from the limitation of its geometry while the CVT meshes have the flexibility to represent the optimal layout:

Design domain, reference solution:





~9000 T6 elements



10000 Polygonal elements

Conclusions

❖ Solutions of discrete topology optimization problems with fixed mesh representation include a form of mesh dependency that stems from the geometric features of the spatial discretization

❖ To address this problem, we employ fully unstructured meshes to reduce the influence of simplex geometry on optimization solutions

Reference: Talischi C, Paulino GH, Pereira A, Menezes IFM (2009). Polygonal finite elements for topology optimization: A unifying paradigm. *Int J Numer Meth Engng* , (Submitted)