Structural Topology Optimization employing the Allen-Cahn Evolution Equation on Unstructured Polygonal Meshes

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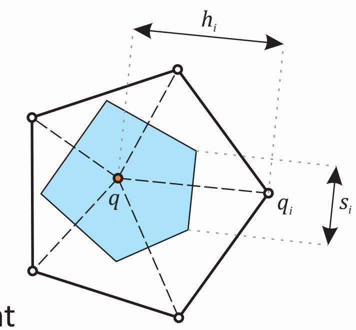
Motivation

- Implicit function methods such as level-set method, although attractive, require periodic reinitialization, for example, to a signed distance function for numerical convergence.
- Polygonal/Polyhedral elements circumvent the mesh bias observed in standard finite element meshes (triangles/tetrahedra or quads/bricks).
- Venture into the non-Cartesian domains for topology optimization.

Polygonal Finite Elements

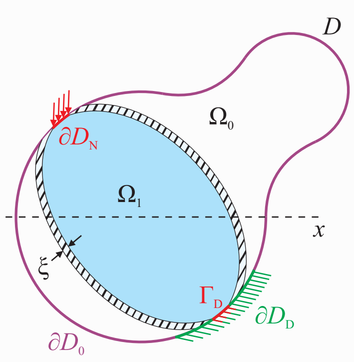
Unstructured polygonal/polyhedral meshes provide convenience and flexibility in discretizing complicated design domains.

Natural neighbor scheme based Laplace interpolants are used to construct a finite element space of polygonal elements.



$$N_i(\boldsymbol{x}) = \frac{\alpha_i(\boldsymbol{x})}{\sum_{\mathcal{P}} \alpha_j(\boldsymbol{x})}, \qquad \alpha_i(\boldsymbol{x}) = \frac{s_i(\boldsymbol{x})}{h_i(\boldsymbol{x})}, \qquad \boldsymbol{x} \in \mathbf{R}^2$$

Phase-Field Method



$$\begin{cases} \phi = 1 & \boldsymbol{x} \in \Omega_1, \\ 0 < \phi < 1 & \boldsymbol{x} \in \xi, \quad \text{Diffuse interface} \\ \phi = 0 & \boldsymbol{x} \in \Omega_0. \end{cases}$$

Allen-Cahn Equation

$$\frac{\partial \phi}{\partial t} = \kappa \nabla^2 \phi + \phi(1-\phi)\left[\phi - \frac{1}{2} - 30\eta \frac{\bar{J}'(\phi_{t1})}{\|\bar{J}'(\phi_{t1})\|}\phi(1-\phi)\right]$$

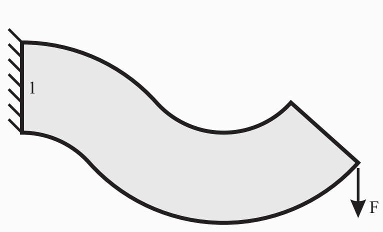
CVT-based Finite Volume Method

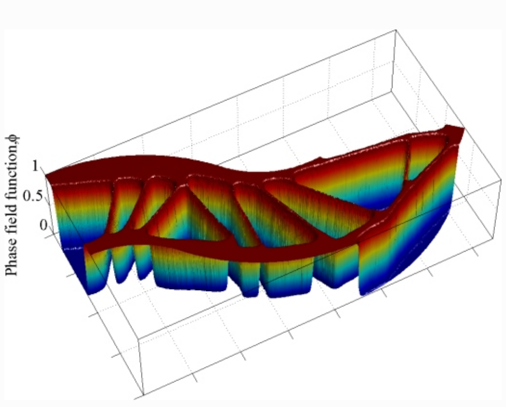
$$\int_{t,D_p} \frac{\partial \phi}{\partial t} dt dD = \int_{t,\Gamma_p} \kappa \boldsymbol{\nabla}\phi \cdot \boldsymbol{n} dt d\Gamma - \int_{t,D_p} f'(\phi)\, dt dD$$

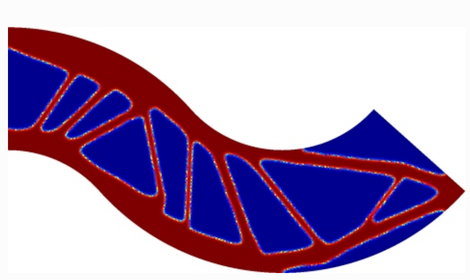
$$\phi_p^{n+1} = \begin{cases} \dfrac{V_p \phi_p^n + P_3}{V_p\left(1 - \left(1-\phi_p^n\right) r\left(\phi_p^n\right)\Delta t\right)} & \text{for } r\left(\phi_p^n\right) \le 0 \\ \dfrac{V_p \phi_p^n \left(1 + r\left(\phi_p^n\right)\Delta t\right) + P_3}{V_p\left(1 + \phi_p^n r\left(\phi_p^n\right)\Delta t\right)} & \text{for } r\left(\phi_p^n\right) > 0 \end{cases}$$

Minimum Compliance Design

Accurately discretizing such a domain using Cartesian grids is challenging.



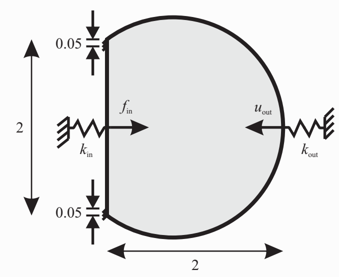


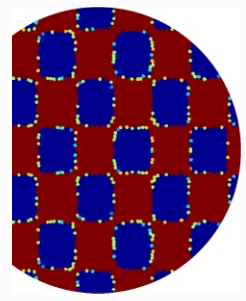


Converged Topology

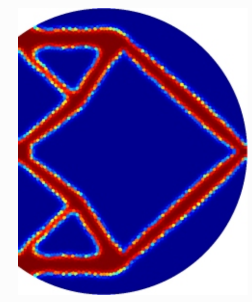
Compliant Mechanism Design

Phase-field method using polygonal elements can be used to design compliant mechanisms.





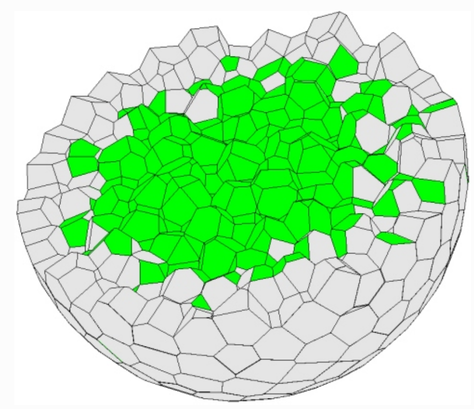
Initial Topology



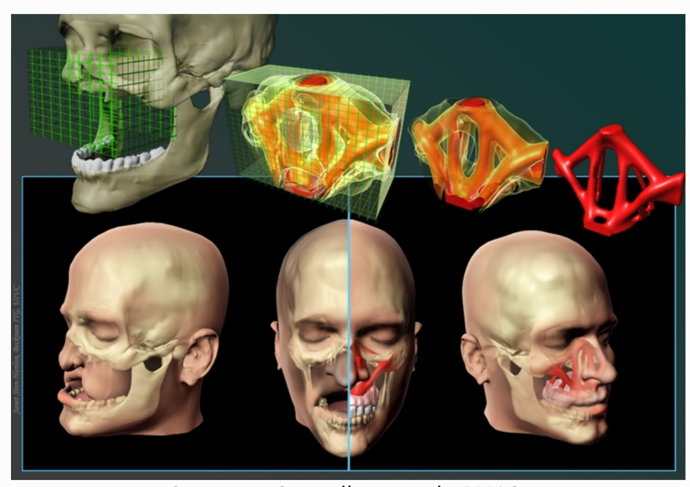
Converged Topology

Extensions

Natural extension to three-dimensions using polyhedral meshes to study real world problems.



Courtesy: Leonardo et. al.



Courtesy: Sutradhar et. al., PNAS

Phase-field method using polyhedral meshes paves the way for medical engineering applications including craniofacial segmental bone replacement.

Conclusions

- Phase-field based topology optimization with polygonal elements offers a general framework for topology optimization on arbitrary domains.
- Meshes based on simplex geometry such as quads/bricks or triangles/tetrahedra introduce intrinsic bias in standard FEM, but polygonal/polyhedral meshes do not.
- Polygonal/polyhedral meshes based on Voronoi tessellation not only provide greater flexibility in discretizing non-Cartesian design domains but also remove numerical anomalies such as one-node connections and checkerboard pattern in density based methods.

References

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