Stable Topology Optimization with Arbitrary Polygons

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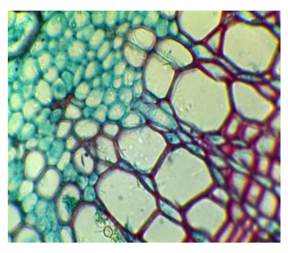
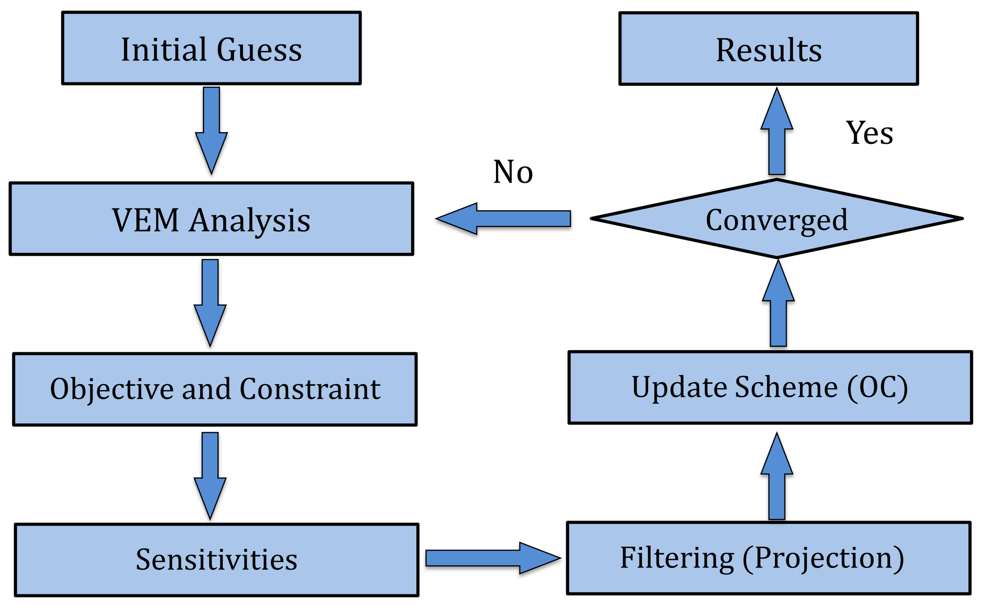
Motivation

- Polygonal elements are nature inspired
- Polygonal elements can alleviate mesh bias and produce optimal topologies
- To handle arbitrary/degenerate polygons, available techniques are less accurate and inefficient
- Virtual Element Method (VEM), provides stable and efficient alternative



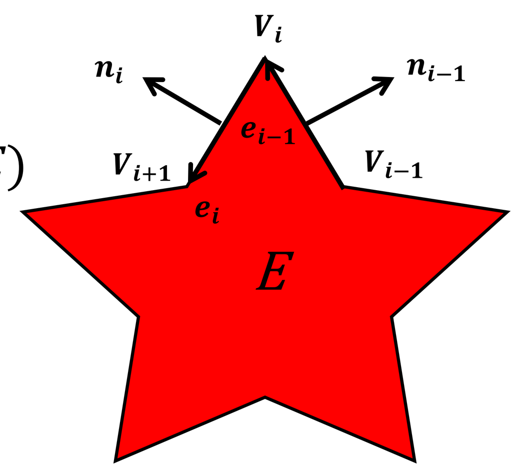
Optimization Formulation

$$\min \; \boldsymbol{f}^T \boldsymbol{u}$$
$$\text{s.t.} \;\; \boldsymbol{K}(E_e)\boldsymbol{u} = \boldsymbol{f}$$
$$\sum_{e=1}^{N} V_e \leq V_{max}$$
$$0 < \rho_{\min} \leq \rho < 1$$

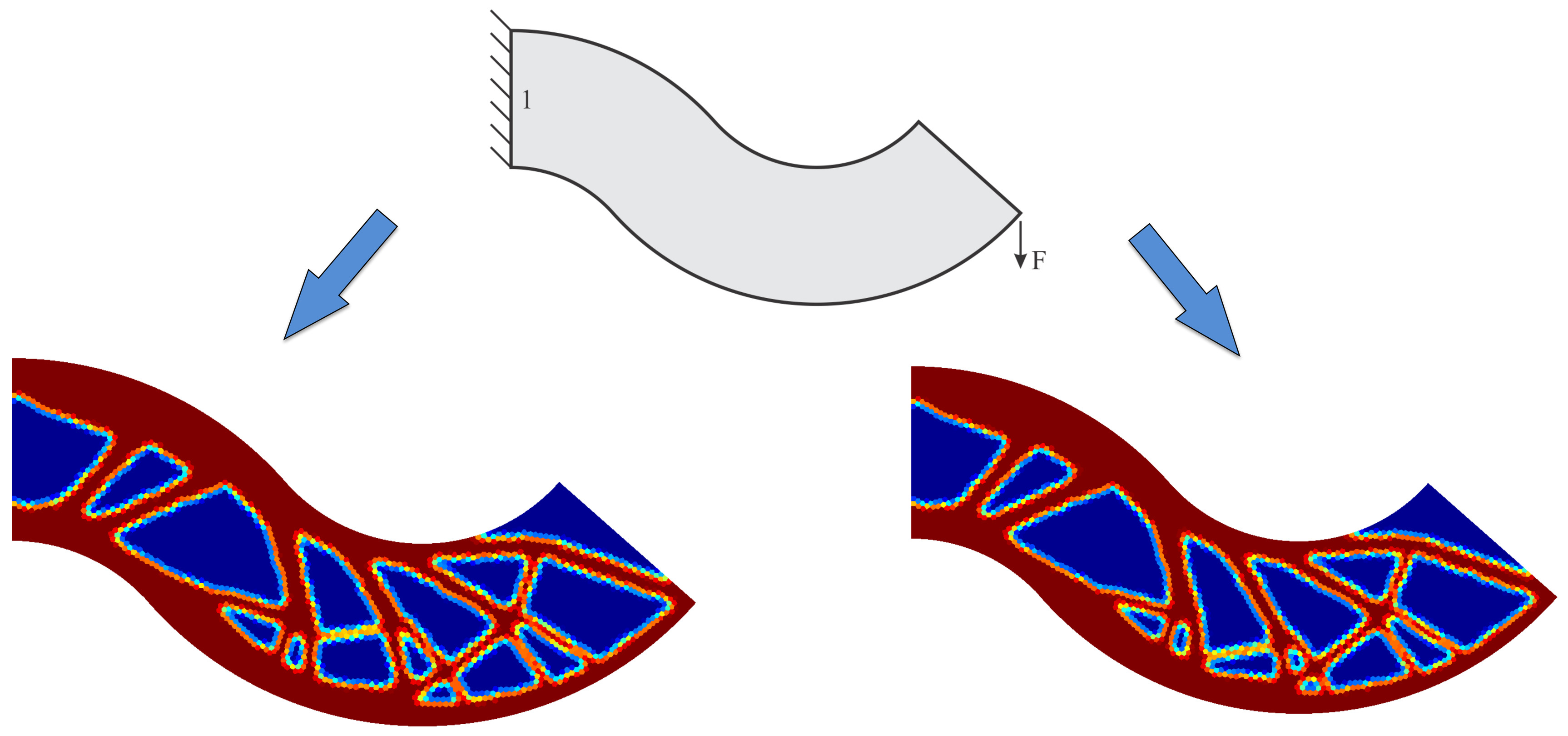


Virtual Element Method Formulation

- Two basic properties are required to be satisfied:
  1. Consistency: $a_h^E(\boldsymbol{p}_1, \boldsymbol{v}) = a^E(\boldsymbol{p}_1, \boldsymbol{v}) \; \forall E, \forall \boldsymbol{v} \in V^E, \forall \boldsymbol{p}_1 \in P_1(E)$
  2. Stability: $\exists \alpha^*, \alpha_* > 0, \ni \alpha_* a^E(\boldsymbol{v}, \boldsymbol{v}) \leq a_h^E(\boldsymbol{v}, \boldsymbol{v}) \leq \alpha^* a^E(\boldsymbol{v}, \boldsymbol{v}) \; \forall E, \forall \boldsymbol{v} \in V^E$
- Consistency can be ensured by exact integration:
  $$a_h^E(\boldsymbol{p}_1, \boldsymbol{v}) = \int_{\Omega^E} \nabla \boldsymbol{p}_1 : \boldsymbol{C} : \nabla \boldsymbol{v} \, d\Omega^E = \int_{\partial\Omega^E} (\nabla \boldsymbol{p}_1 : \boldsymbol{C} \cdot \boldsymbol{v}) \cdot \boldsymbol{n} \, d\partial\Omega^E$$
- A projection operator is defined: $\prod : V_h^E \rightarrow P_1(E)$:
  Such that: $a^E(\prod \boldsymbol{v}_h, \boldsymbol{q}) = a^E(\boldsymbol{v}_h, \boldsymbol{q})$ and $\prod \boldsymbol{q} = \boldsymbol{q} \; \forall \boldsymbol{q} \in P_1(E)$
- Finally, we get:
  $$a_h^E(\boldsymbol{u}_h, \boldsymbol{v}_h) = a^E(\prod \boldsymbol{u}_h, \prod \boldsymbol{v}_h) + (I - \prod) s^E(\boldsymbol{u}_h, \boldsymbol{v}_h)(I - \prod)$$



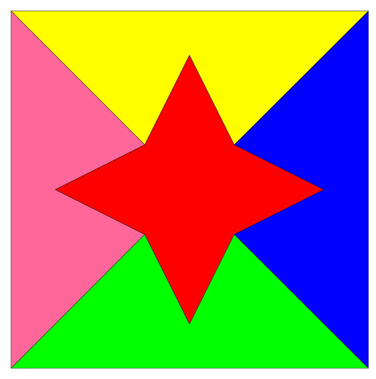
Topology Optimization with CVT Polygons



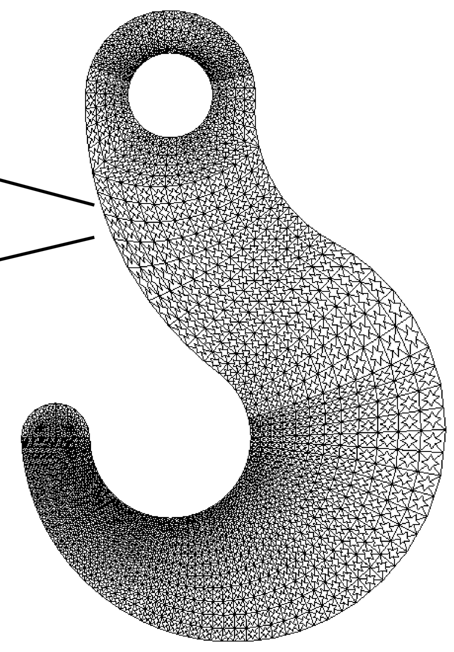
Optimization with VEM formulation
Objective:361.988

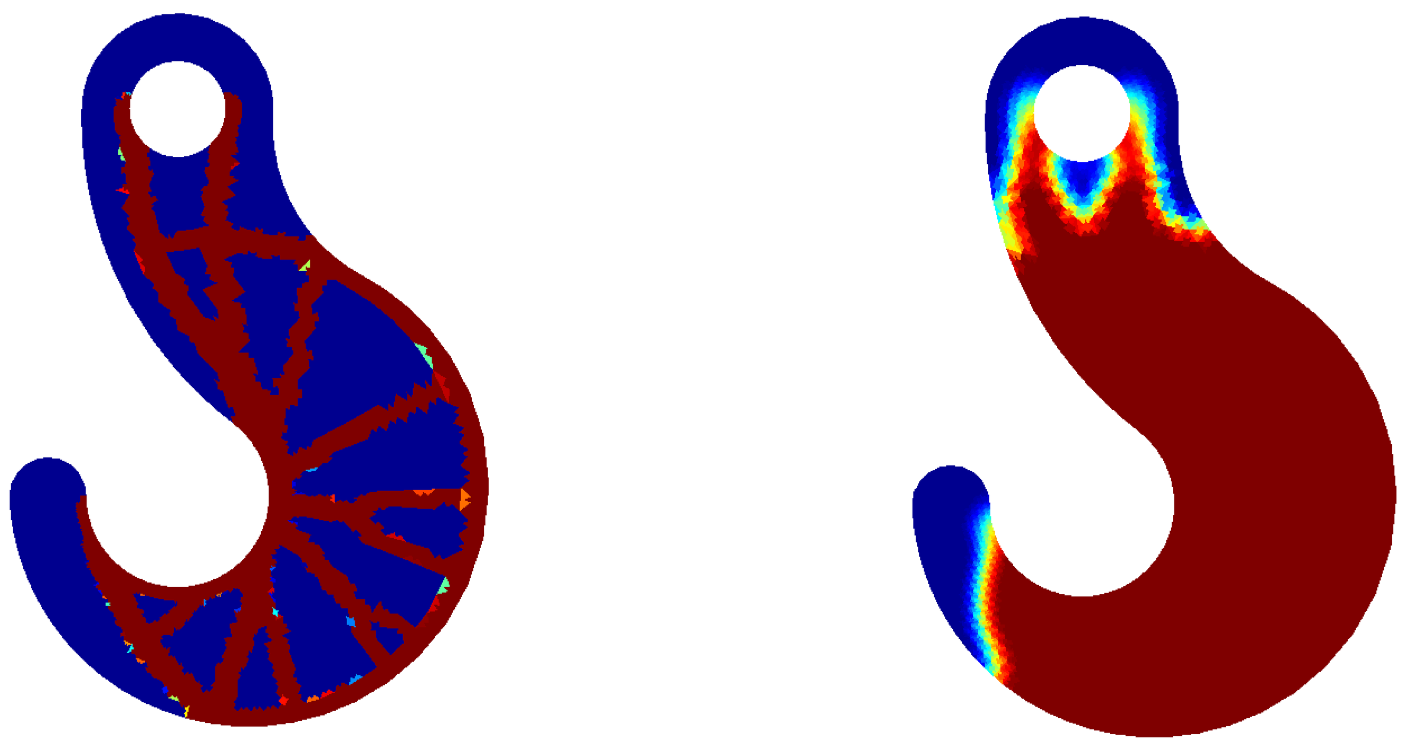
Optimization with PolyFEM formulation
Objective:362.023

Topology Optimization with Concave Polygons



One quad is divided into five concave polygons





Optimization with VEM formulation
Objective:2805.697

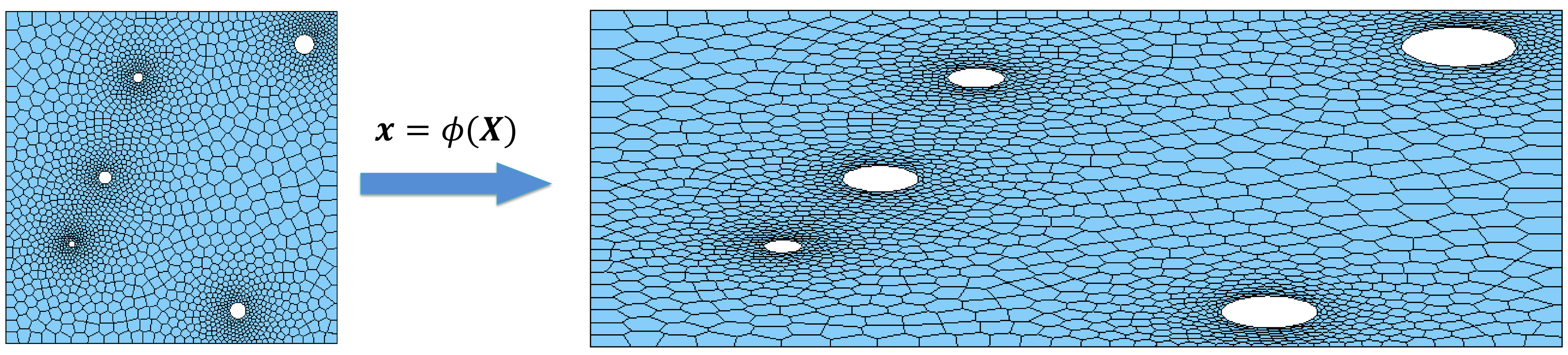
Optimization with PolyFEM formulation
Objective: 267223.208

Conclusion

- For regular polygonal mesh, PolyFEM and VEM formulation yields similar optimal design with very close compliance
- For concave/degenerate polygonal mesh, VEM result is stable while traditional PolyFEM results is numerically unstable
- VEM formulation allows more flexibility for the shape of elements in topology optimization

Future Work

- Topology optimization with geometrical and material nonlinearity with arbitrary polygons
- Nonlinear topology optimization for multi-physics and multi-scale problems



Reference

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