Unraveling Soft Material Behavior through Finite Deformations

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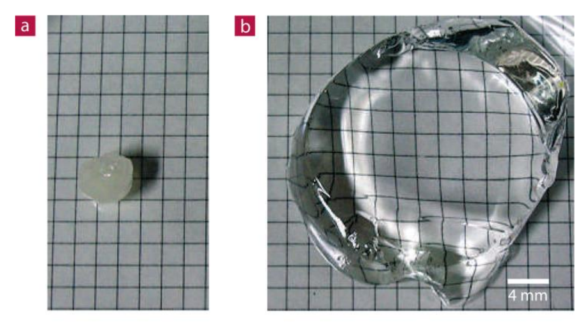
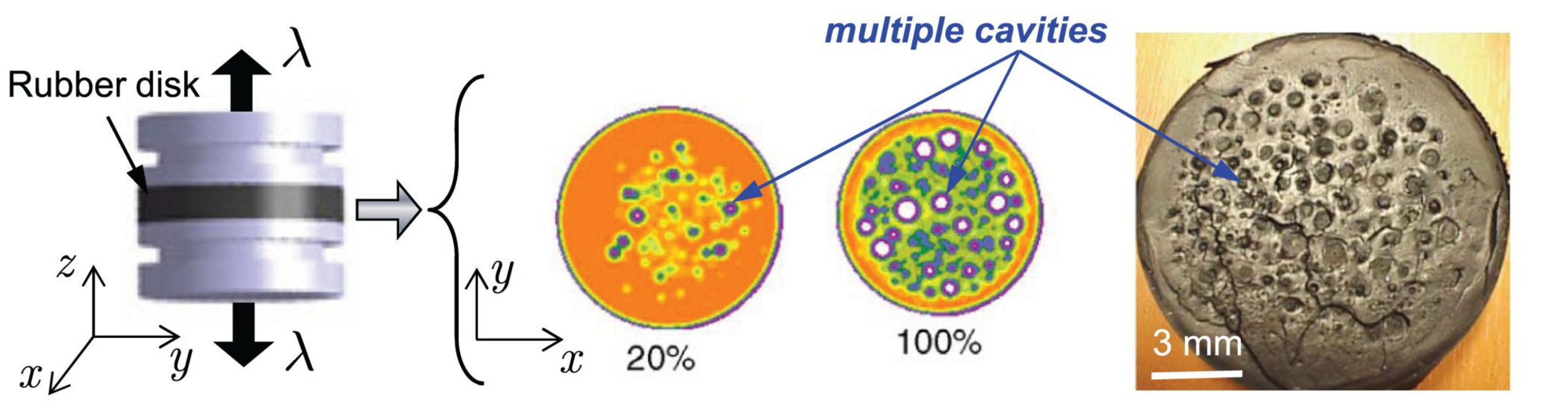
Motivation

- Soft organic materials, such as electro- and magneto-active elastomers and gels; which are elastic by nature, hold tremendous potential for new high-end technologies, e.g. next generation sensors and actuators.
- Soft materials often possess complex microstructures, with underlying local deformations which are typically larger than macroscopic ones. This makes the modeling of soft materials challenging.



Swelling of lipophilic polyelectrolyte gel



“Poker chip” test

In-situ X-ray images at various applied stretches λ

Image after a stretch of 100%

Two-field mixed variational principle

- Find $(\mathbf{u}^*, p^*)$ such that:

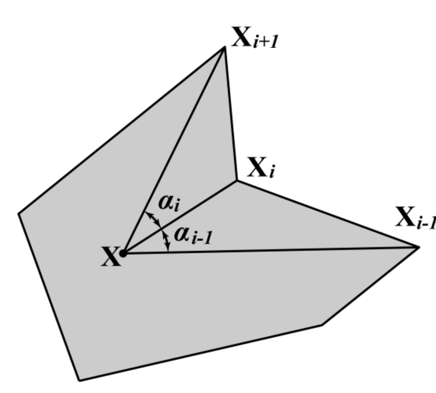
$$\Pi(\mathbf{u}^*, p^*) = \inf_{\mathbf{u}} \sup_{p} \Pi(\mathbf{u}, p)$$

where:

$$\Pi(\mathbf{u}, p) = \int_{\Omega_0} [-W_C(\mathbf{X}, \mathbf{F}(\mathbf{u}), p) + p(J(\mathbf{u}) - 1)] \mathrm{d}\Omega_0 - \int_{\Omega_0} \mathbf{f}_0 \cdot \mathbf{u} \mathrm{d}\Omega_0 - \int_{\partial\Omega_0} \mathbf{t}_0 \cdot \mathbf{u} \, \mathrm{d}\,\partial\Omega_0$$

$$W_C(\mathbf{X}, \mathbf{F}, p) = \sup_{J} \, [p(J-1) - W(\mathbf{X}, \mathbf{F}, J)]$$

Polygonal finite element

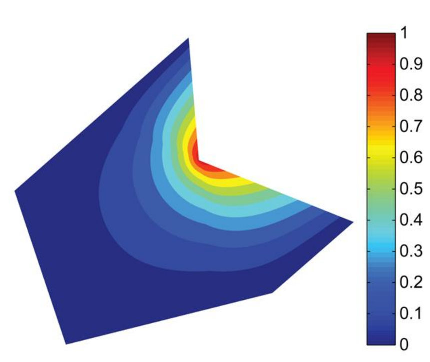


- Displacement field is approximated by Mean Value coordinates:

$$\varphi_i(\mathbf{X}) = \frac{w_i(\mathbf{X})}{\sum_{j=1}^{n} w_j(\mathbf{X})}$$

where w_i is given by:

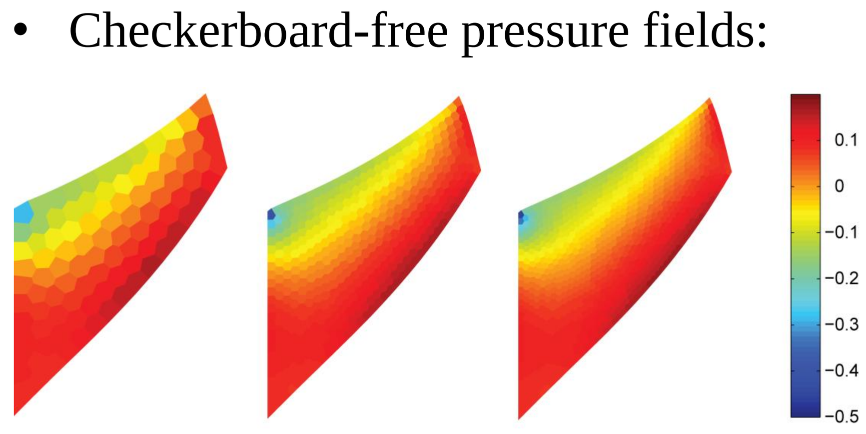
$$w_i(\mathbf{X}) = \frac{\tan\left(\frac{\alpha_{i-1}}{2}\right) + \tan\left(\frac{\alpha_i}{2}\right)}{||\mathbf{X} - \mathbf{X}_i||}$$



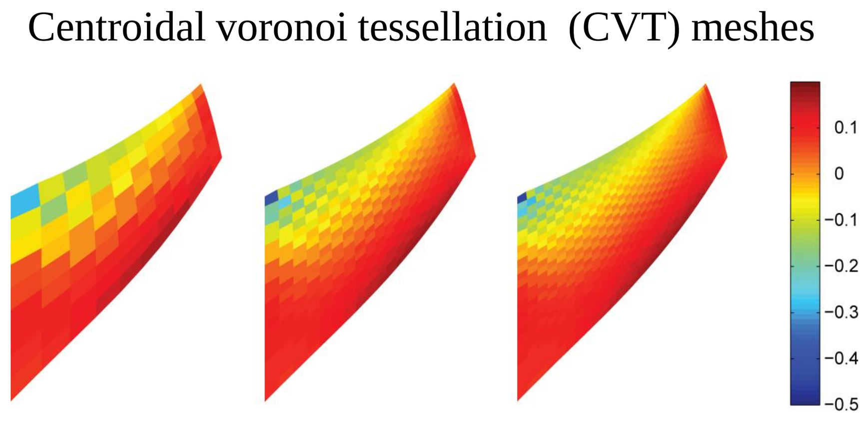
- Pressure field is interpolated by piece-wise constant functions.

Numerical stability and accuracy

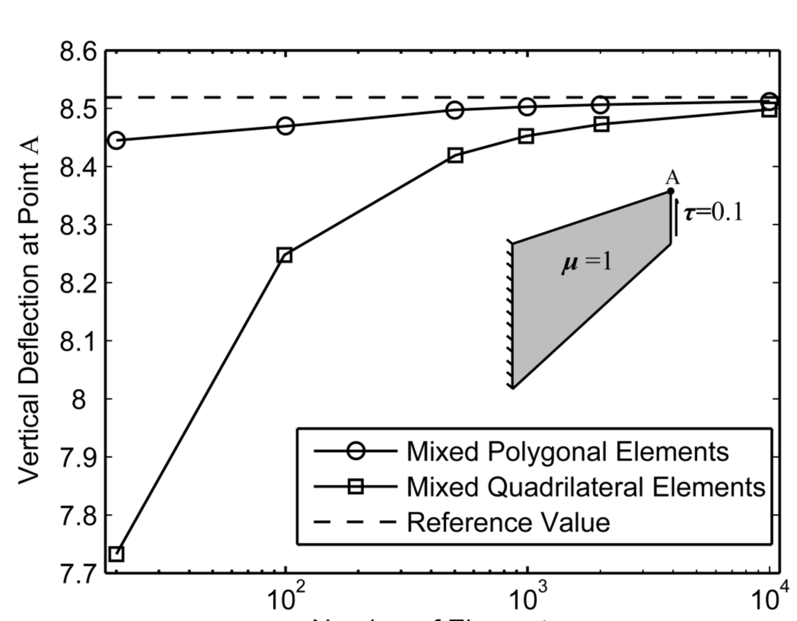
- Checkerboard-free pressure fields:



Centroidal voronoi tessellation (CVT) meshes

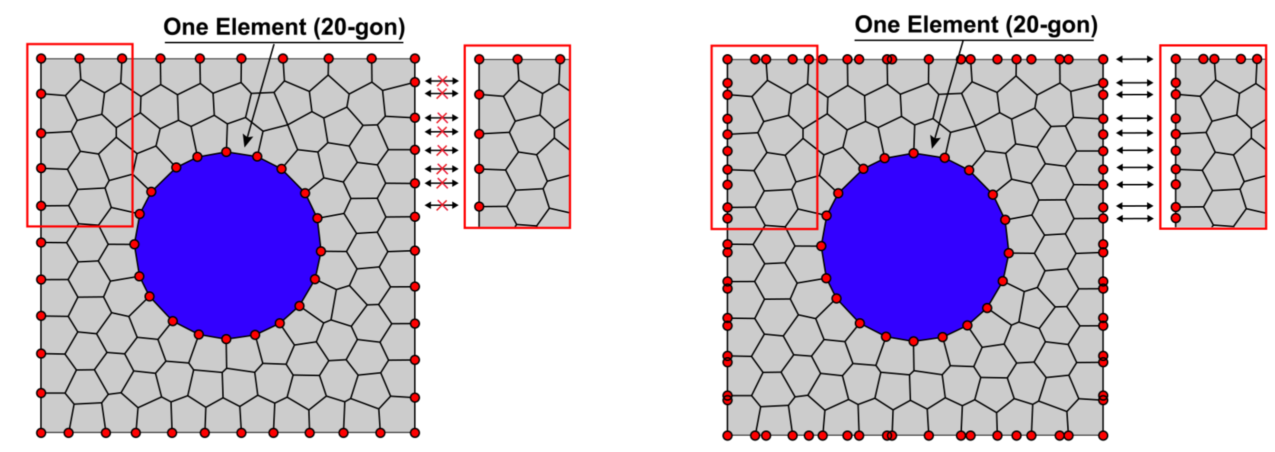


Linear quadrilateral meshes

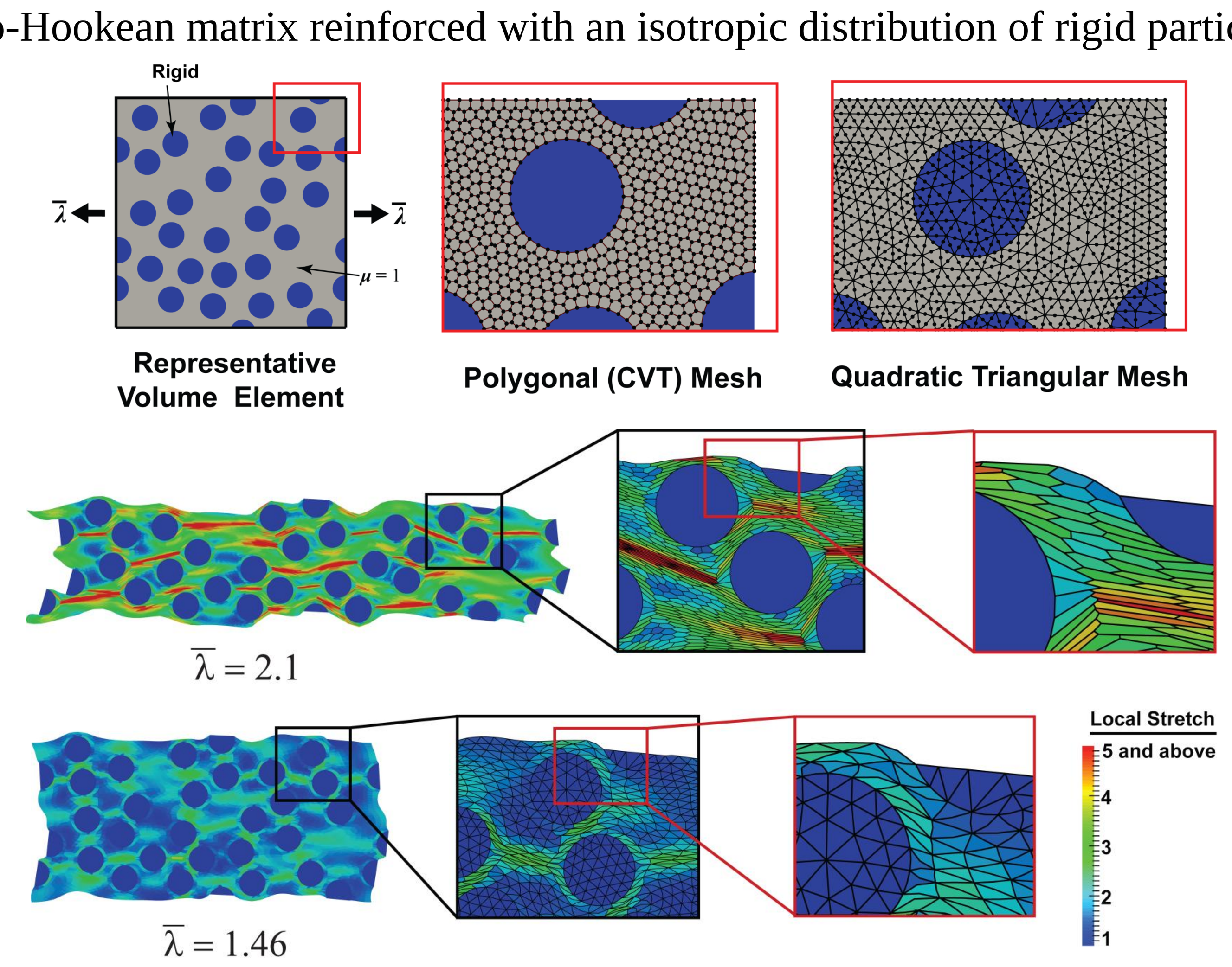


Filled elastomers

- Geometrical advantages to model inclusions and periodic boundary conditions:

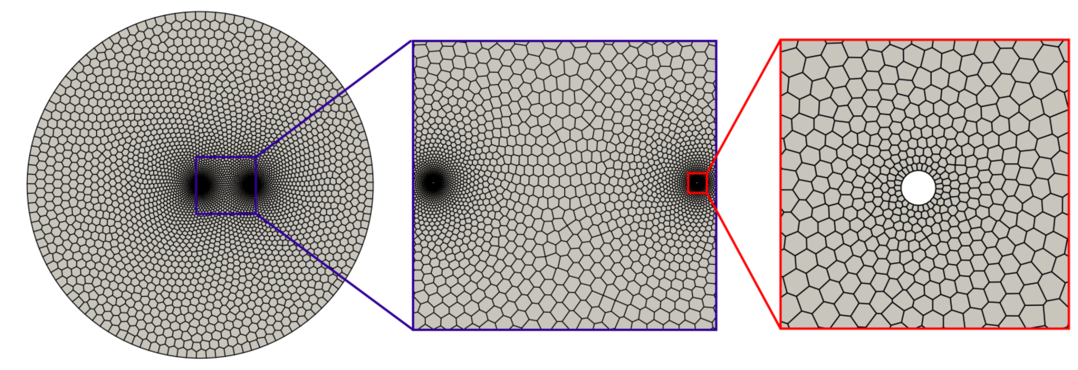


- Neo-Hookean matrix reinforced with an isotropic distribution of rigid particles:

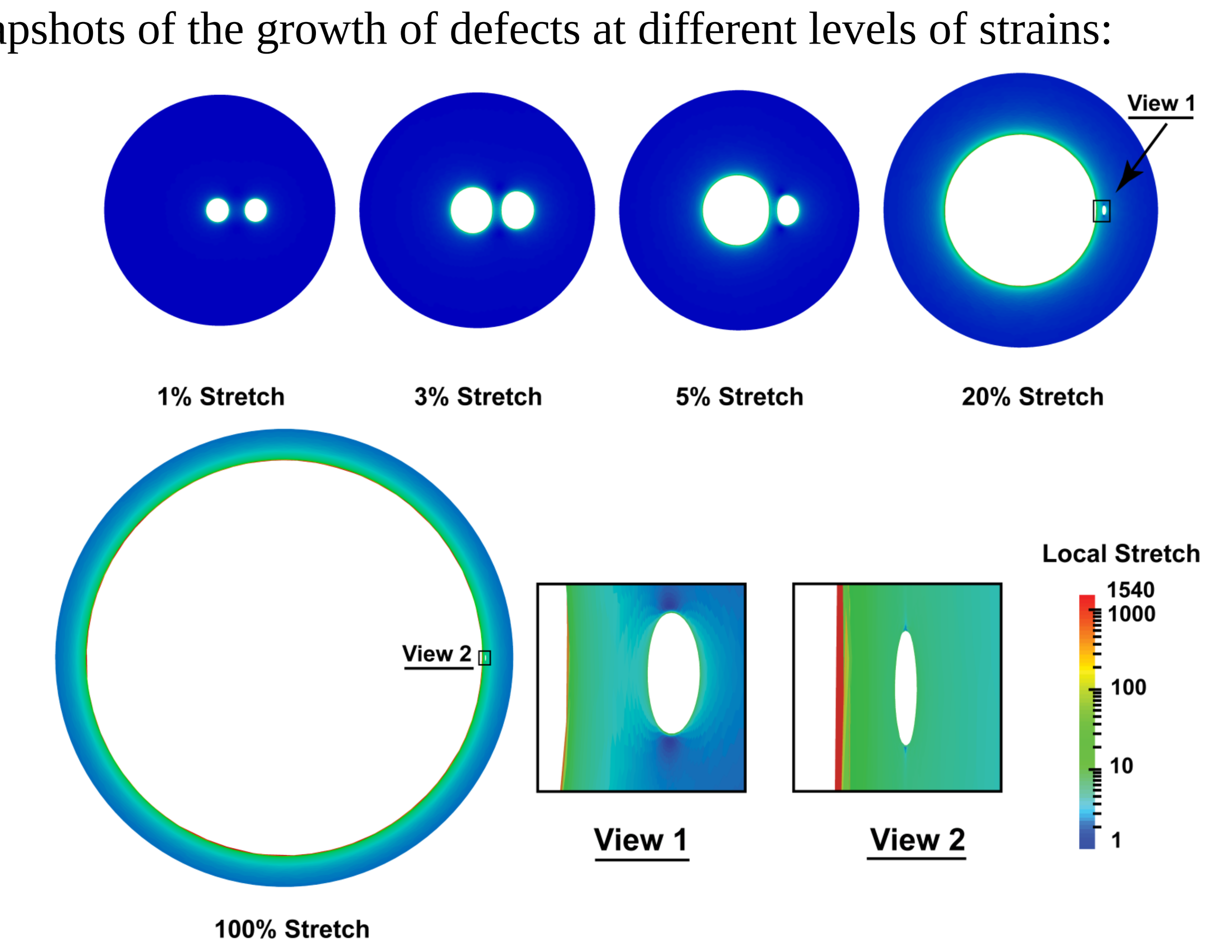


Cavitation

- Graded polygonal mesh bridging two length scales:



- Snapshots of the growth of defects at different levels of strains:



Conclusions

- Polygonal elements are numerically stable on Voronoi-type meshes without any additional treatments.
- Polygonal elements are more geometrically favorable in modeling inclusions with arbitrary geometry, incorporating periodic boundary conditions and bridging different length scales.
- Polygonal elements appear to be more tolerant to large local deformations than classic triangular and quadrilateral elements.

References

- Chi H, Talischi C, Paulino GH, Lopez-Pamies O, “Polygonal finite element for finite elasticity”, IJNME, In preparation
- Talischi C, Paulino GH, Pereira A, Menezes IMF, “Polymesher: A general-purpose mesh generator for polygonal elements written in Matlab.”, JSMO, vol.45, No3, pp.309-328, 2012

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