Analysis of Origami-Inspired Structures

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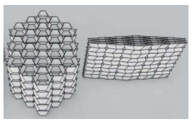
Motivation and Research Objective

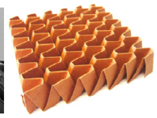
- Wide range of application of origami including but not limited to:

Deployable structures
Kuribayashi and You (2006)



Metamaterials
Schenk and Guest (2013)

Adaptive structures
http://www.arup.com/



Folded cores
Heimbs (2007)

- Investigating the capability of the pin-jointed bar framework approach for the analysis of folded shell structures subjected to external loads by stiffness analysis and calculation of mechanical properties of the known folded sheets (Miura-ori and Egg-box) computationally.

Modeling and Analysis of Folded Shell Structures

- **Modeling using constrained pin-jointed bar framework**

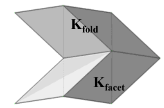
Model using rigid bars and triangulate each facet using additional bar, and impose constraint F to model folding.

$$F = \sin(\phi) = \sin(\phi(P)) \qquad d\phi = J\,d = \frac{1}{\cos(\phi)} \sum \frac{\partial F}{\partial p_i} dp_i$$

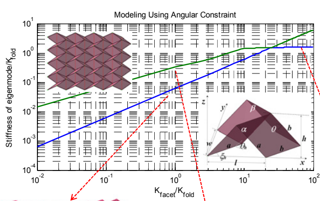
ϕ : dihedral fold angle between two adjoining facets; **P**: vector of nodal coordinates of the facets; C: the compatibility matrix.

$$\bar{C}d = \begin{bmatrix} e \\ d\phi \end{bmatrix} \qquad \bar{C} = \begin{bmatrix} C \\ J_{facet} \end{bmatrix}$$

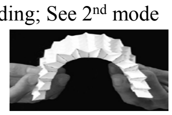
$$K = \begin{bmatrix} C \\ J_{facet} \\ J_{fold} \end{bmatrix}^T \begin{bmatrix} G_{bar} & 0 & 0 \\ 0 & G_{facet} & 0 \\ 0 & 0 & G_{fold} \end{bmatrix} \begin{bmatrix} C \\ J_{facet} \\ J_{fold} \end{bmatrix} \qquad Kd = f$$



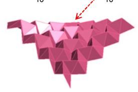
Stiffness Analysis of Miura-ori Sheet



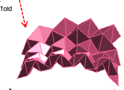
Saddle shape under bending: Positive Poisson's ratio under bending; See 2nd mode



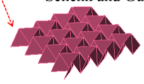
Schenk and Guest (2011)



1st mode at K_{facet}/K_{fold} =1
Twisting

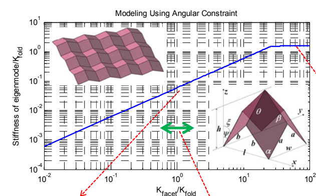


2nd mode at K_{facet}/K_{fold} =1, 10
Saddle

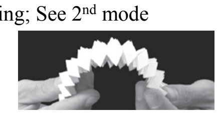


1st mode at K_{facet}/K_{fold} =80
Rigid Origami Behavior

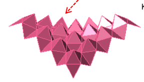
Stiffness Analysis of Egg-box Sheet



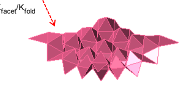
Spherical shape under bending: Negative Poisson's ratio under bending; See 2nd mode



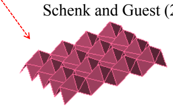
Schenk and Guest (2011)



1st mode at K_{facet}/K_{fold} =1
Twisting



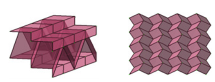
2nd mode at K_{facet}/K_{fold} =1
Spherical



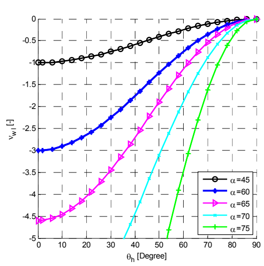
1st mode at K_{facet}/K_{fold} =80
Rigid Origami Behavior

In-plane Poisson's Ratio of Miura-ori: Numerical Calculation

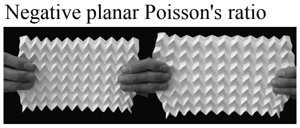
- Choose the material properties (K_{bar} and K_{facet}/K_{fold}) to ensure rigid origami behavior.
- The system is singular. Apply appropriate constraint to remove rigid body modes, without impeding the planar motion.
- The first eigen-modes show rigid origami behavior



Examples used in this part

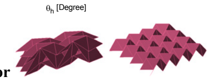


Negative planar Poisson's ratio



Schenk and Guest (2011)

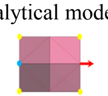
1st eigen-mode showing rigid origami behavior

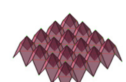


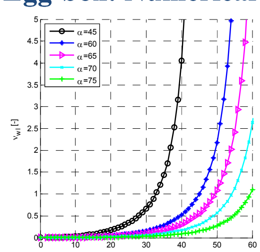
In-plane Poisson's Ratio of Egg-box: Numerical Calculation

- Similar approach explained for Miura-ori is used for Egg-box as well. Numerical results match well with the analytical model.

Examples used in this part

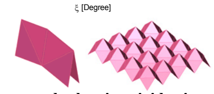






Positive planar Poisson's ratio

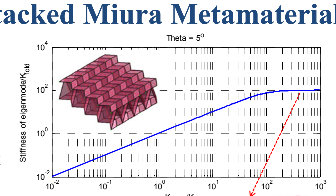
Schenk and Guest (2011)



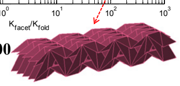
1st eigen-mode showing rigid origami behavior

Stiffness Analysis of Stacked Miura Metamaterial

- The example includes three layers of Miura with special geometry attached to each other. First mode at large values of K_{facet}/K_{fold} shows that the material folds uniformly: A meta-material property of stacked Miura



1st mode at K_{facet}/K_{fold} =800
Rigid Origami Behavior



Conclusions and Future Research

- Rigid origami behavior is captured very well in the analysis.
- Numerical verification of the mechanical properties of the patterns used here match well with the experimental and analytical results.
- Current code can be used for analysis of other patterns and for ground structure topology optimization of origami-inspired structures.

References

- Eidini M., Paulino, G. H., "Analysis and mechanical properties of origami-inspired structures, to be submitted.
- Schenk M. and Guest S., "Origami folding: A structural engineering approach," *5OSME*, 2011.
- Schenk M. and Guest S., "Geometry of Miura-folded metamaterials," *Proceedings of the National Academy of Sciences*, vol. 110, no. 9, pp. 3276-3281, 2013.

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