

# Geometrically and Topologically Unstructured Polygons for Dynamic Cohesive Fracture

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## Motivation

Brittle and quasi-brittle fracture problems appear across a wide range of fields, and cover a variety of applications

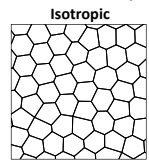
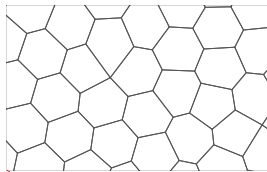


## Research Objectives

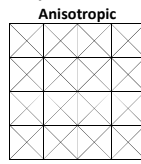
- Develop a computational framework that reduces the mesh induced bias which has historically been present in dynamic cohesive fracture simulations
- Employ polygonal finite elements, taking advantage of their adaptive features and ability to easily discretize any arbitrary problem domain
- Develop a series of topological operators, particularly adaptive refinement and adaptive element splitting, to make on-the-fly modifications to the mesh

## Polygonal Bulk Elements

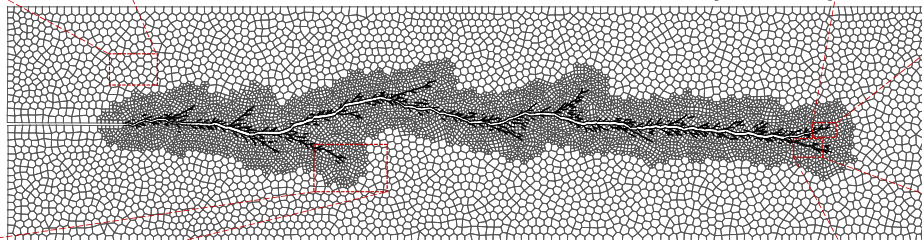
Polygonal finite elements provide an isotropic discretization of any arbitrary domain



Does not display bias to crack patterns

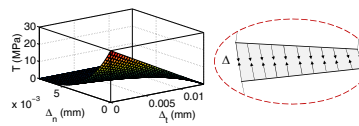


Displays bias to crack patterns

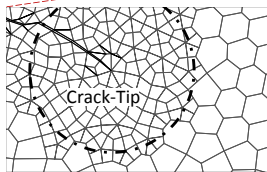
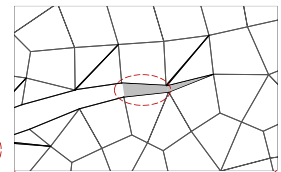


## Cohesive Elements for Fracture

The inelastic zone in front of the crack tip is described by a cohesive traction-separation relation

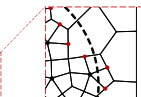
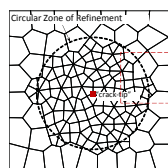


$$\Psi(\Delta_n, \Delta_t) = \min(\phi_n, \phi_t) + \left[ r_n \left( 1 - \frac{\Delta_n}{\delta_n} \right)^\alpha + (\phi_n - \phi_t) \right] \left[ r_t \left( 1 + \frac{|\Delta_t|}{\delta_t} \right)^\beta + (\phi_t - \phi_n) \right]$$



## Adaptive Refinement

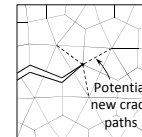
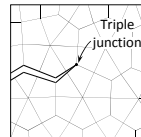
Every crack-tip is tracked and a region around each is refined with quadrilateral elements



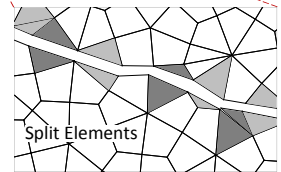
Hanging nodes are naturally accounted for by the element shape functions

## Adaptive Element Splitting

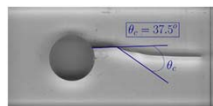
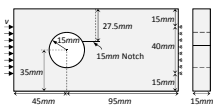
Cracks are allowed to split elements to help alleviate mesh induced restrictions.



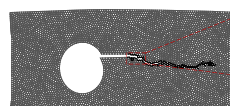
We developed an element splitting operator for arbitrary  $n$ -gons, but when combined with adaptive refinement the refined 4-gons will be permitted to be split.



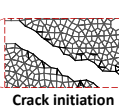
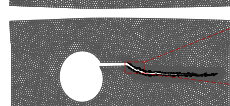
## Experimental Validation



Experimental result with crack initiation angle of  $\sim 37.5^\circ$



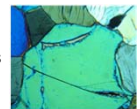
Crack initiation angle of  $\sim 16^\circ$



Crack initiation angle of  $\sim 35^\circ$

## Conclusions and Extensions

- Geometrically unstructured polygonal elements result in an isotropic discretization of the problem domain, but provide few paths for cracks to propagate along.
- By applying adaptive topological operators, such as, adaptive refinement and adaptive element splitting, the mesh induced bias can be alleviated.
- These techniques allow us to capture experimentally observed fracture patterns.
- A potential extension of this work, could be in the study of hierarchical materials. For example, at small scales, the polygonal elements could represent grains in metals and alloys while the adaptive refinement and splitting could be used to represent either intergranular or intragranular fracture.



## References

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