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A Bar and Hinge Model for Scalable Structural Analysis of Origami

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Motivation

Origami is popular in science and engineering because of:

self-assembly • **deployment** • **compact stowage** • **adaptability**

Applications can range in scale from metamaterials and micro-robotics to aerospace systems and deployable architecture

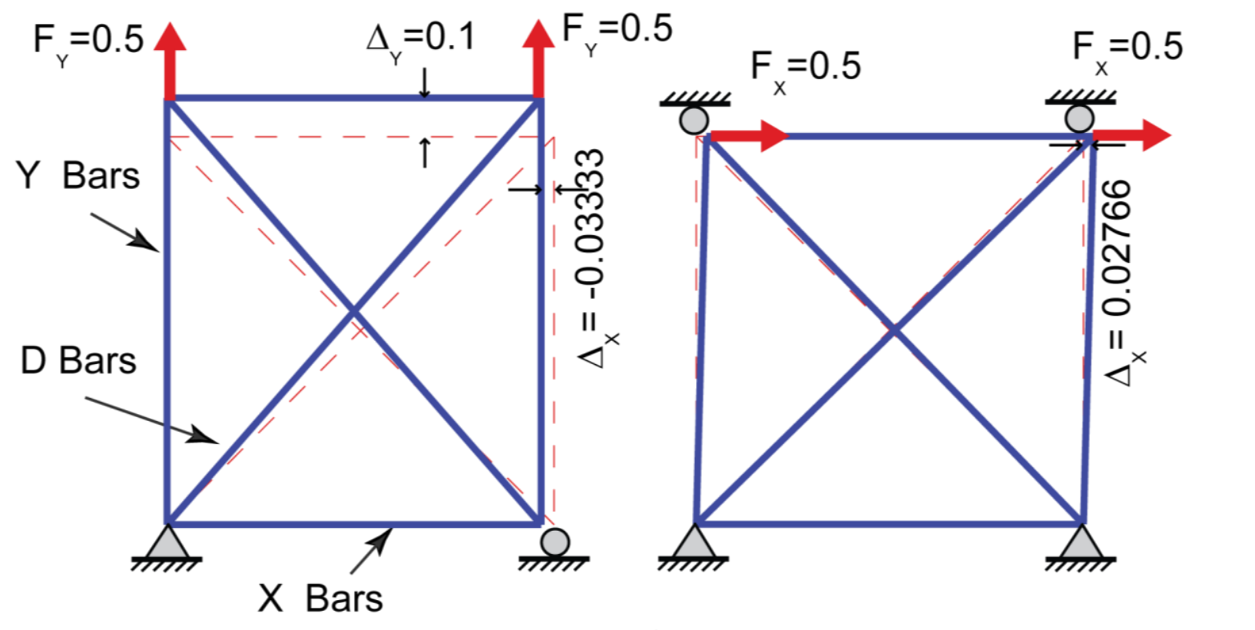
In this research we improve a bar and hinge model to enable *scalable*, *efficient* and *simplified* structural analysis of origami

Elastic Behaviors of Origami

Stretching and Shear of Panels

An indeterminate frame of bar elements is used to capture in-plane isotropic behavior of the panels

Young's modulus: E
Thickness: t
Poisson's ratio: ν

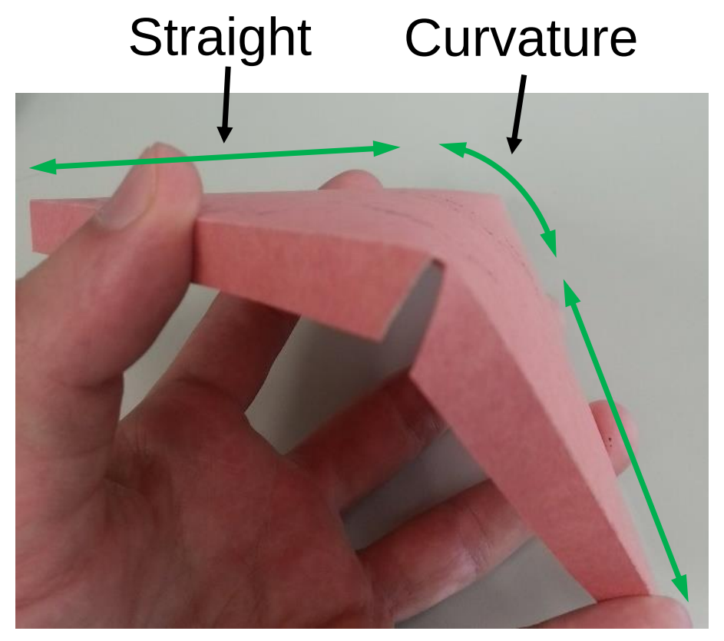


Tensile patch test **Shear patch test**

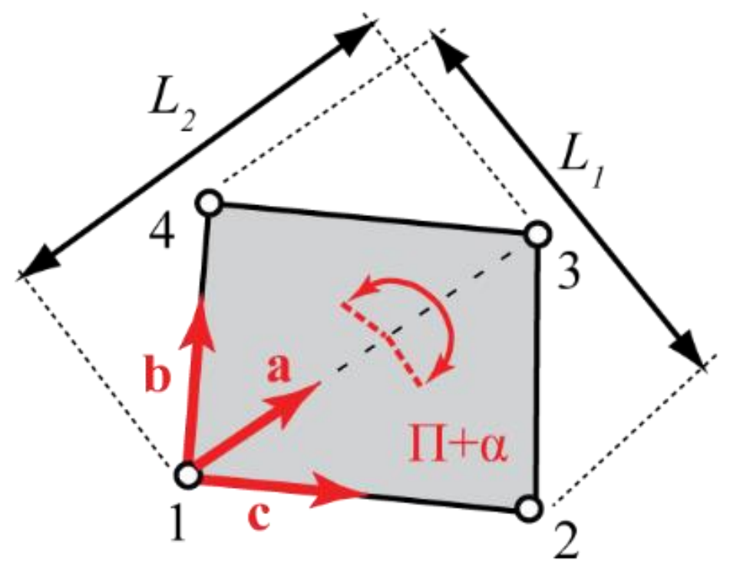
Bar stiffness definition: $A_{X/Y} = t\frac{W^2 - \nu H^2}{2W(1-\nu^2)}$ $A_D = t\frac{\nu(H^2 - W^2)^{3/2}}{2HW\ (1-\nu^2)}$ $\boxed{K_S = \frac{EA_{Bars}}{L}}$

Bending of Panels

Out-of-plane bending curvature localizes along the shorter diagonal of the panels. Rotational hinges simulate the stiffness



Bending of panel



Rotational hinge model

Panel stiffness scaling:

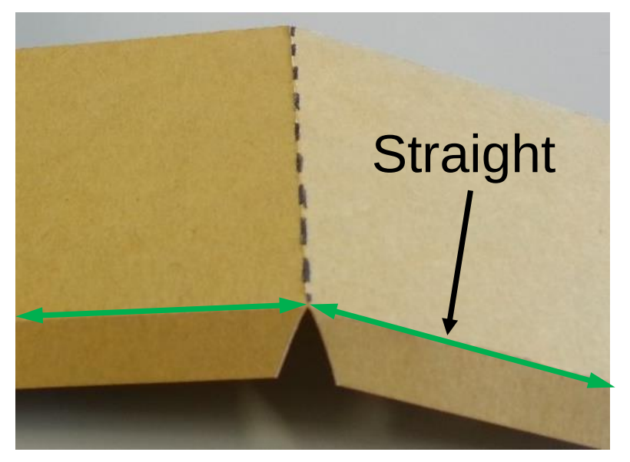
$E_T \sim k\left(\frac{L_2}{t}\right)^{1/3}$ k, the bending modulus is: $k = \frac{Et^3}{12(1-\nu^2)}$

Stiffness definition:

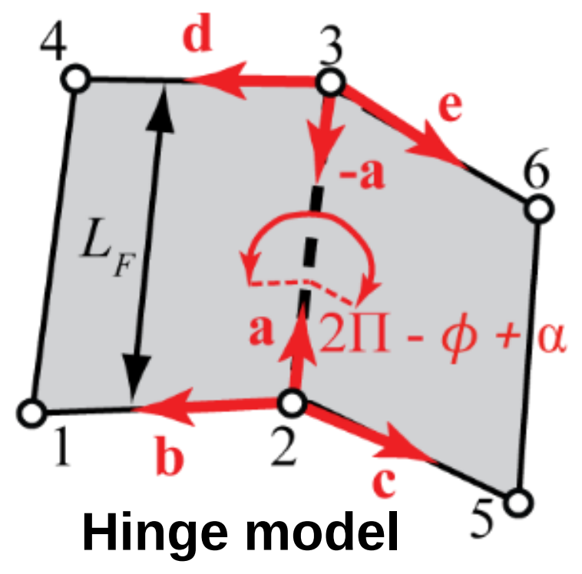
$$\boxed{K_B = C_B\frac{Et^3}{12(1-\nu^2)}\left(\frac{L_2}{t}\right)^{1/3}}$$

Bending of Fold Lines

Bending curvature is concentrated on the fold. Ratio R_{FP} relates bending stiffness of fold to panel, and depends on material and physical properties



Bending at prescribed fold line

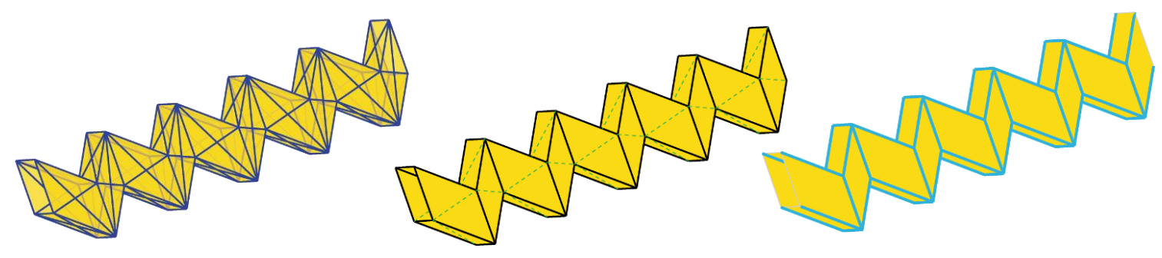


Hinge model

$$\boxed{K_F = R_{FP}C_B\frac{L_F}{2}\frac{Et^3}{12(1-\nu^2)}\left(\frac{1}{t}\right)^{1/3}}$$

Assembly of Bar and Hinge Model

- ➢ Length scale dependent
- ➢ Isotropic
- ➢ Includes material properties E, t, and ν



$$\mathbf{K} = \begin{bmatrix} C \\ J_B \\ J_F \end{bmatrix}^T \begin{bmatrix} \boxed{\mathbf{K}_S} & 0 & 0 \\ 0 & \boxed{\mathbf{K}_B} & 0 \\ 0 & 0 & \boxed{\mathbf{K}_F} \end{bmatrix} \begin{bmatrix} C \\ J_B \\ J_F \end{bmatrix}$$

Eigenvalue Analyses

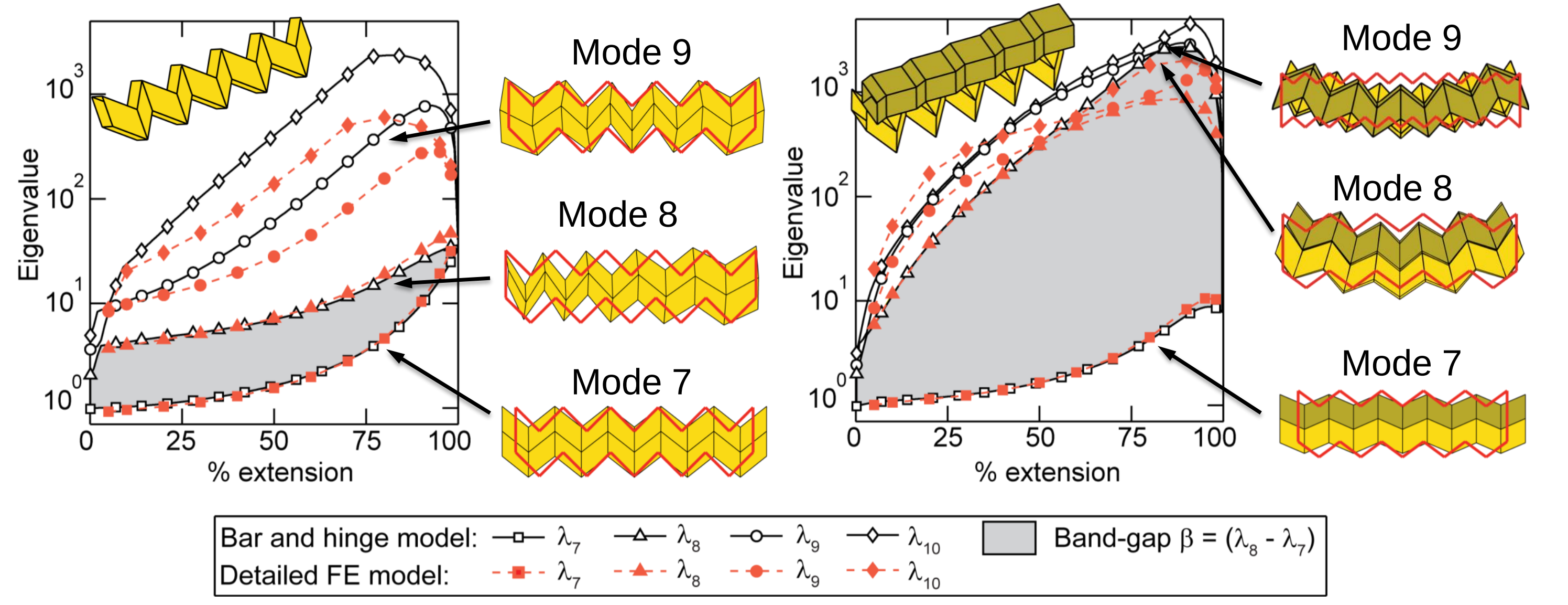
$\mathbf{K}$ = Stiffness matrix

$\mathbf{M}$ = Mass matrix

$$\mathbf{K}\mathbf{v}_i = \lambda_i\mathbf{M}\mathbf{v}_i \qquad i = 1, \ldots, N_{dof}$$

Eigenvalue Eigenmode

Analysis of single tube **Analysis of zipper tubes**



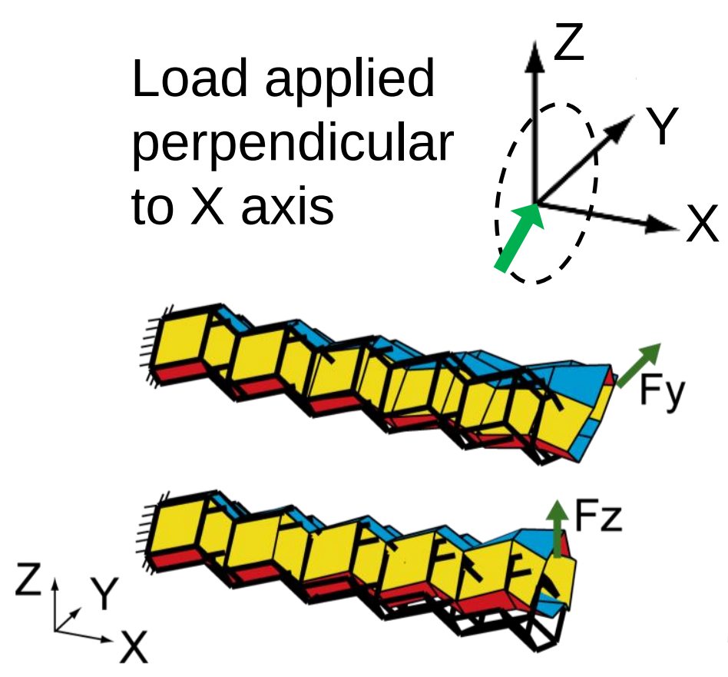
- Eigenvalue analyses can be used to predict the kinematic motion (Mode 7) and other deformation modes of the structure
- Increasing the band-gap makes a stiff, yet deployable origami
- The model approximates modes with panel and fold bending well, but underestimates the stiffness of stretching and shearing

Static Stiffness Analyses

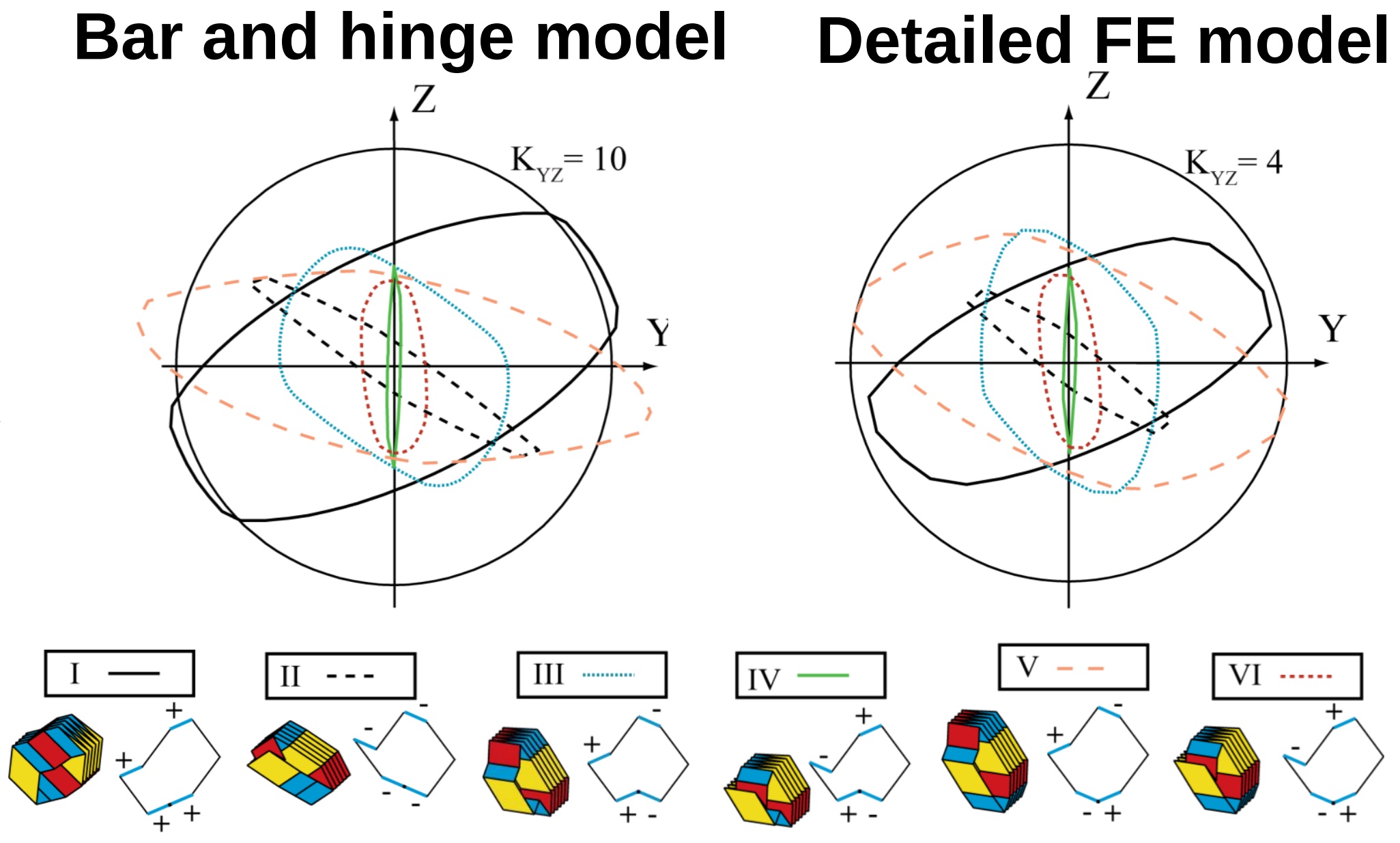
$$\mathbf{F} = \mathbf{K}\Delta$$

Force Displacement

Load applied perpendicular to X axis



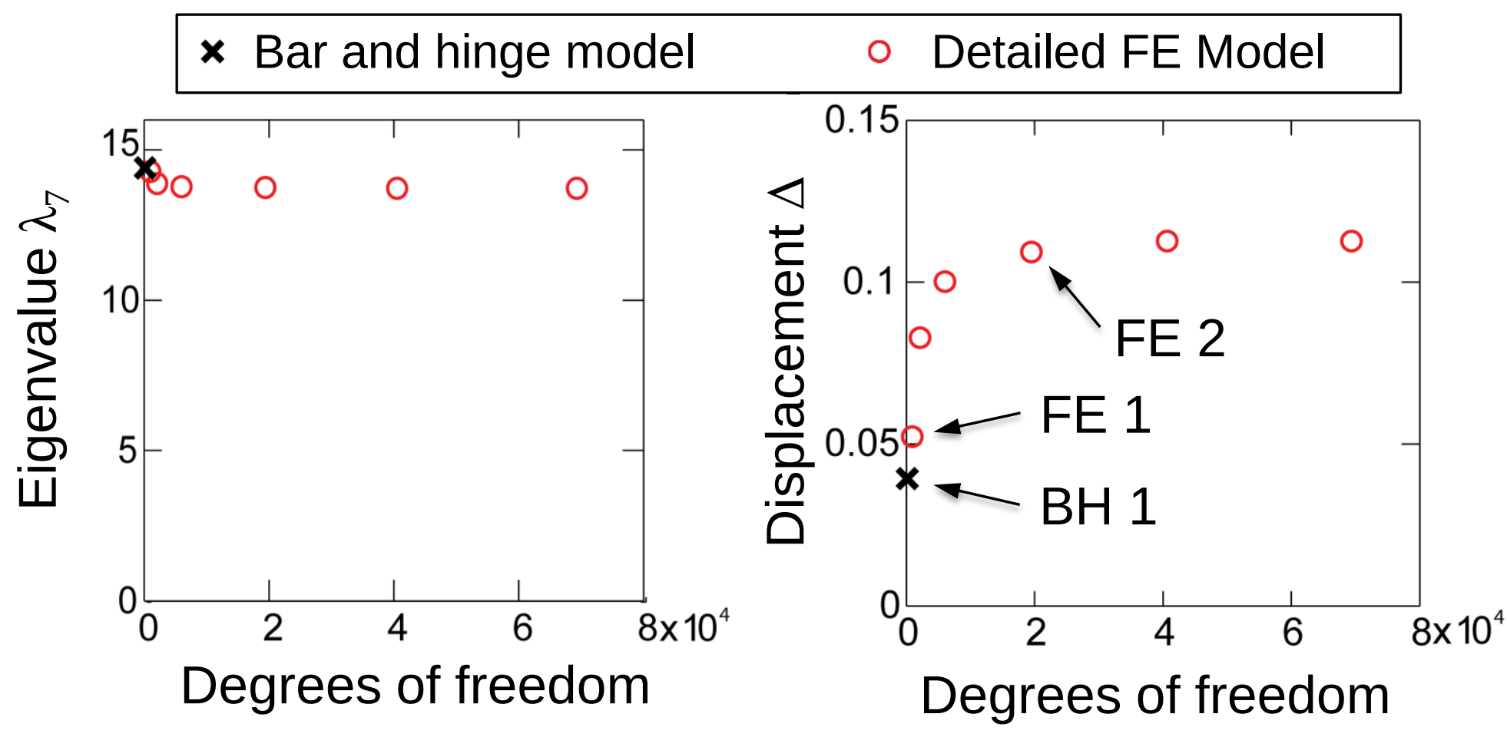
Bar and hinge model **Detailed FE model**



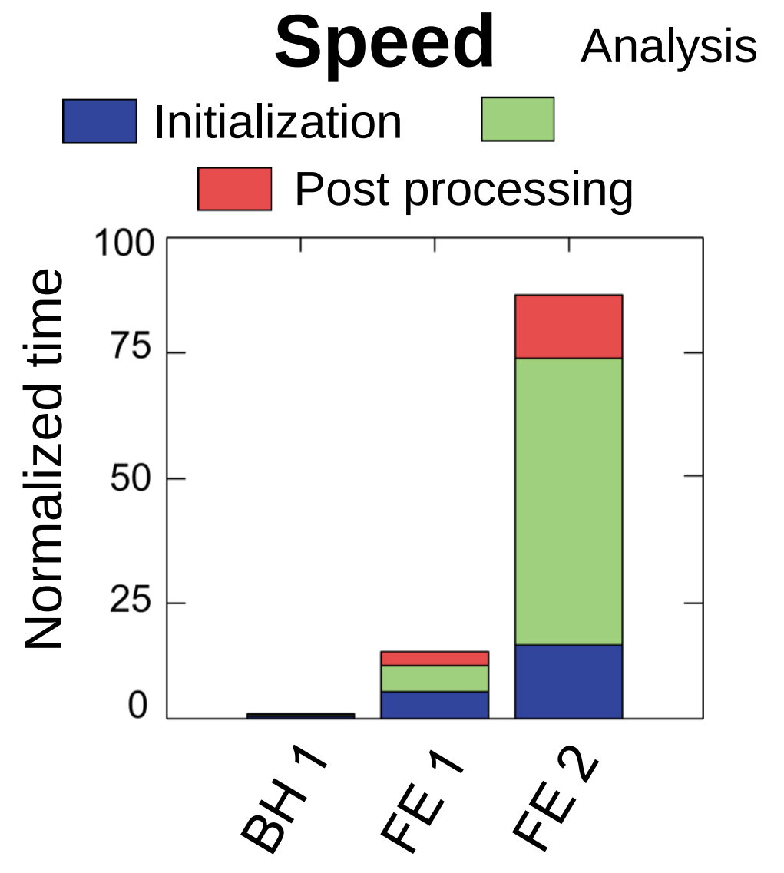
- The bar and hinge model captures the anisotropy for reconfiguring polygonal tubes, but overestimates stiffness

Model Accuracy and Efficiency

Convergence



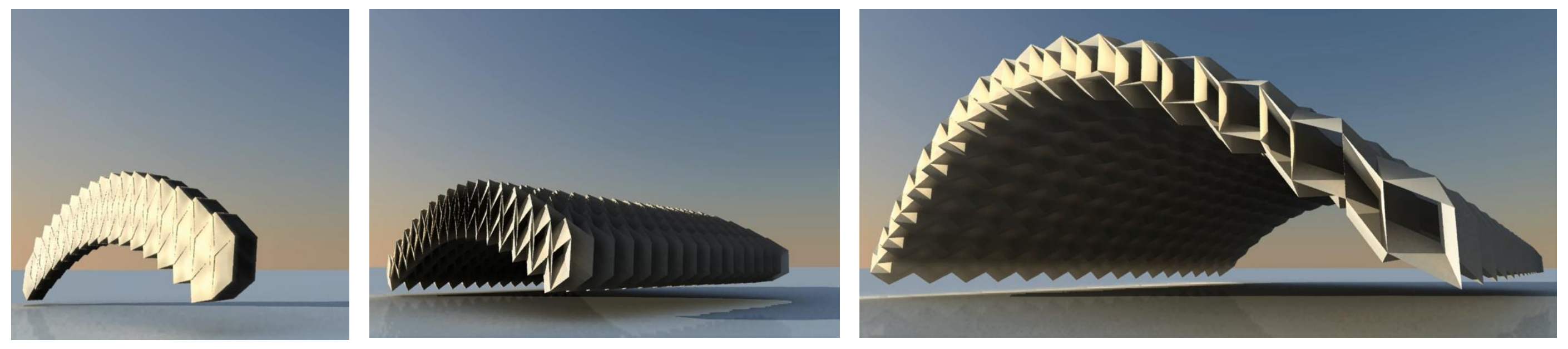
Speed



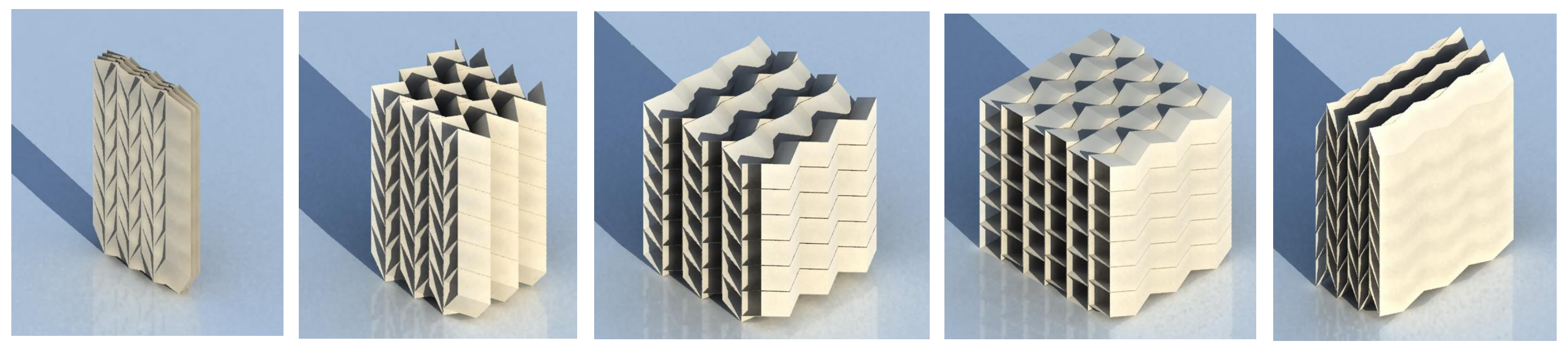
- The efficient bar and hinge model is provides an accuracy comparable to detailed FE models for a same number of degrees of freedom

Benefits and Extensions of Model

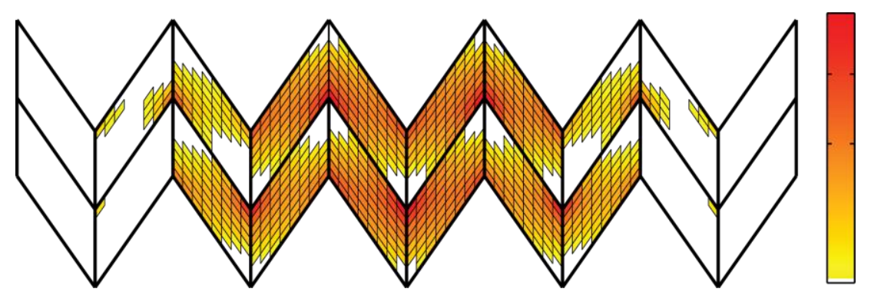
- Model is simple to understand, implement, modify and use. This makes it valuable to the growing community of origami researchers and enthusiasts
- The bar and hinge model is scalable, isotropic and incorporates material properties t, E, and ν
- It provides sufficient accuracy for global structural analysis of origami



- The model is efficient in comparison to detailed FE analyses making it suitable for extensions such as:
  - ➢ Large displacement simulations
  - ➢ Modeling elasto-plastic hinge elements
  - ➢ Parametric variations for geometric design
  - ➢ Optimization of cellular origami type structures



Limitations



Localized panel bending energy from detailed FE analysis

- The bar and hinge model cannot capture localized effects accurately
- Stiffness for stretching and shearing of the panels is overestimated in comparison to the bending deformations
- The factor relating fold to panel stiffness (R_{FP}) and the factor defining the panel bending (C_B) stiffness have not been thoroughly investigated

References

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