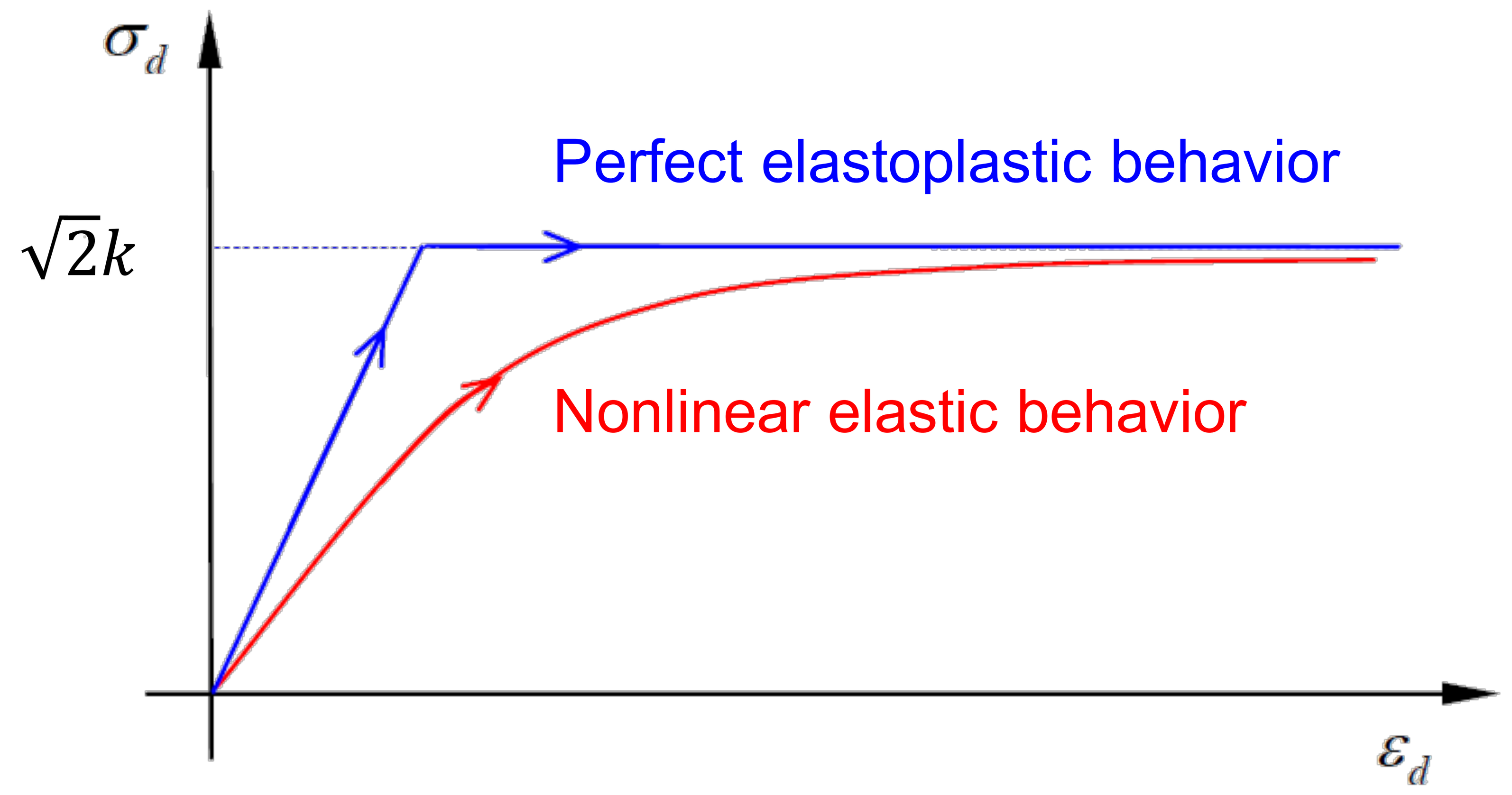
# Topology Optimization with Nonlinear Constitutive Model Governed by von Mises Criteria


Tuo Zhao[a], Adeildo S. Ramos Jr.[b], Glaucio H. Paulino[a]

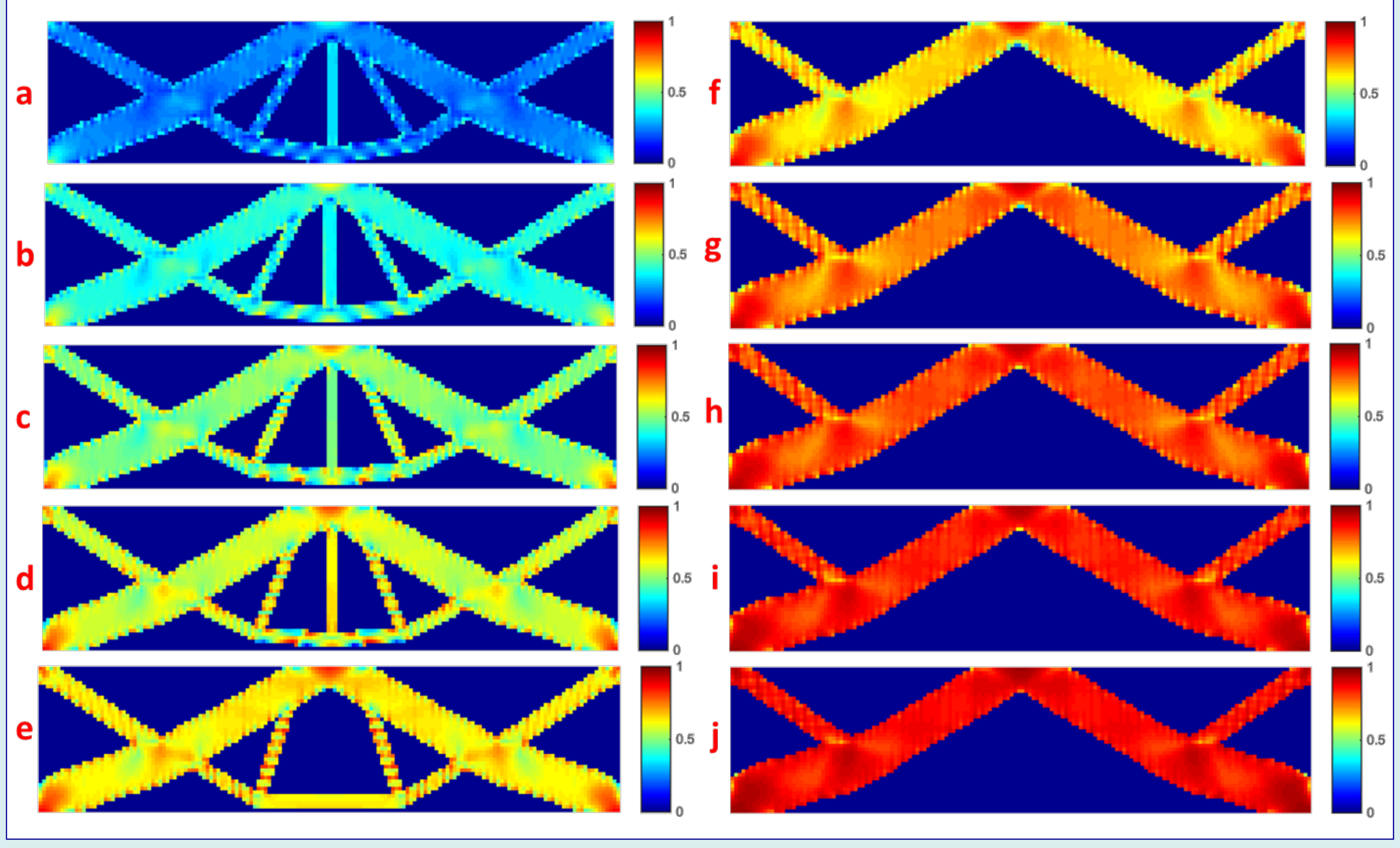
$a$: Georgia Institute of Technology, Atlanta, U.S. $b$: Federal University of Alagoas, Maceió, AL, Brazil


## Motivation

- ❖ von Mises yield criterion $f(\boldsymbol{\sigma}) = \sigma_d - \sqrt{2}k \leq 0$
- ❖ Nonlinear elastic constitutive model $\boldsymbol{\sigma} = K\varepsilon_v \boldsymbol{I} + 2\mu(\varepsilon_d)\boldsymbol{\varepsilon_d}$




## von Mises Stress Distribution




## Formulation and Sensitivity

$$\max_{\boldsymbol{x}} \quad U(\boldsymbol{x}, \boldsymbol{u}(\boldsymbol{x}))$$
$$s.t. \quad V \leq V_0$$

with

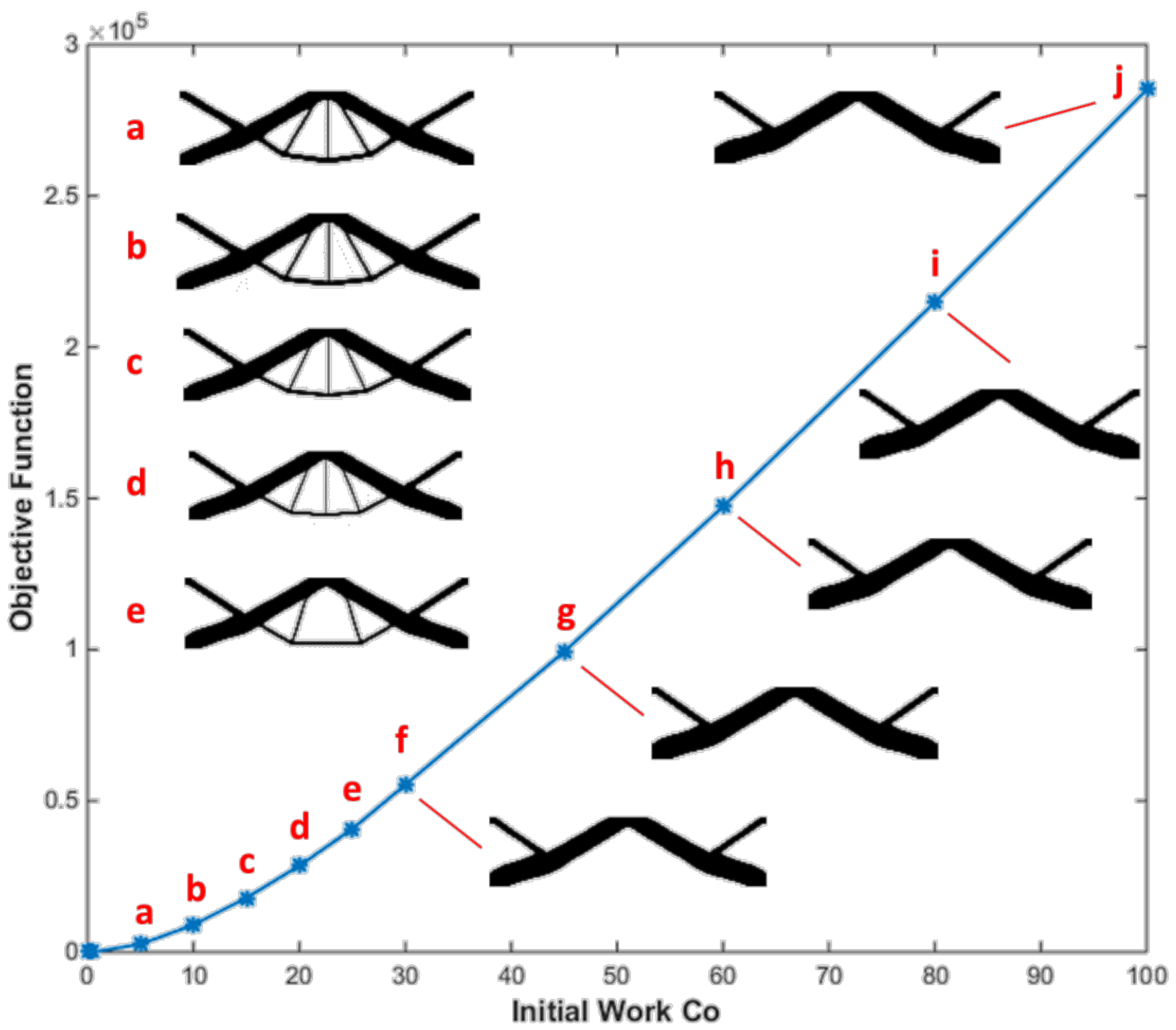
$$\begin{cases} \boldsymbol{T}(\boldsymbol{x}, \boldsymbol{u}(\boldsymbol{x})) = \lambda \boldsymbol{f_0} \\ \boldsymbol{f}_0^T \boldsymbol{u} = 2C_0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \min_{\boldsymbol{u}} \quad U(\boldsymbol{u}) \\ s.t. \quad \boldsymbol{f}_0^T \boldsymbol{u} = 2C_0 \end{cases}$$

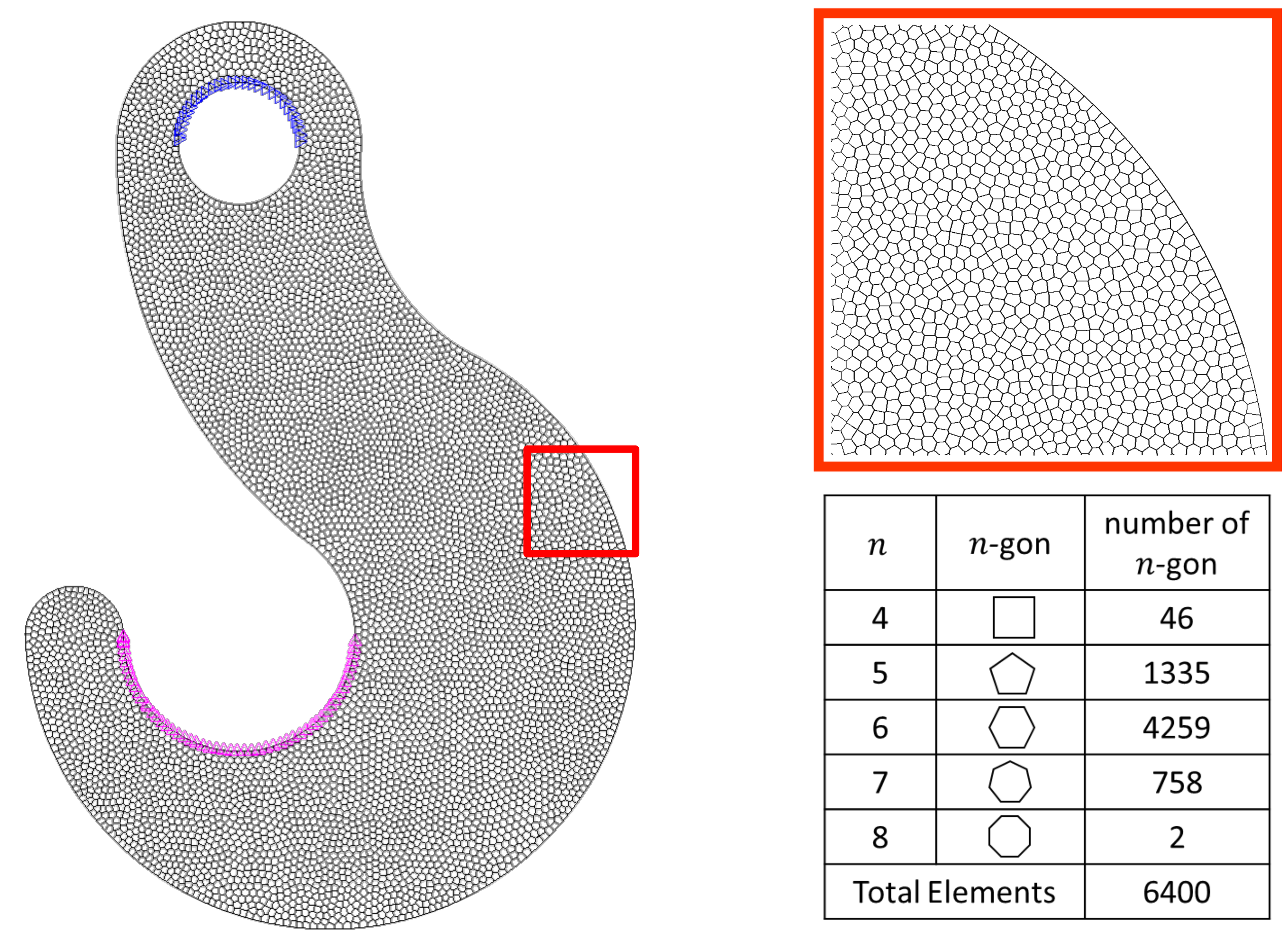
$$\frac{\partial U(\boldsymbol{x}, \boldsymbol{u}(\boldsymbol{x}))}{\partial x_e} = \frac{\partial U}{\partial x_e} + \left(\frac{\partial U}{\partial \boldsymbol{u}}\right)^T \frac{\partial \boldsymbol{u}}{\partial x_e} = \frac{\partial U}{\partial x_e} + \lambda \boldsymbol{f_0}^T \frac{\partial \boldsymbol{u}}{\partial x_e}$$

$$\boxed{\frac{\partial U(\boldsymbol{x}, \boldsymbol{u}(\boldsymbol{x}))}{\partial x_e} = \frac{\partial U}{\partial x_e}}$$

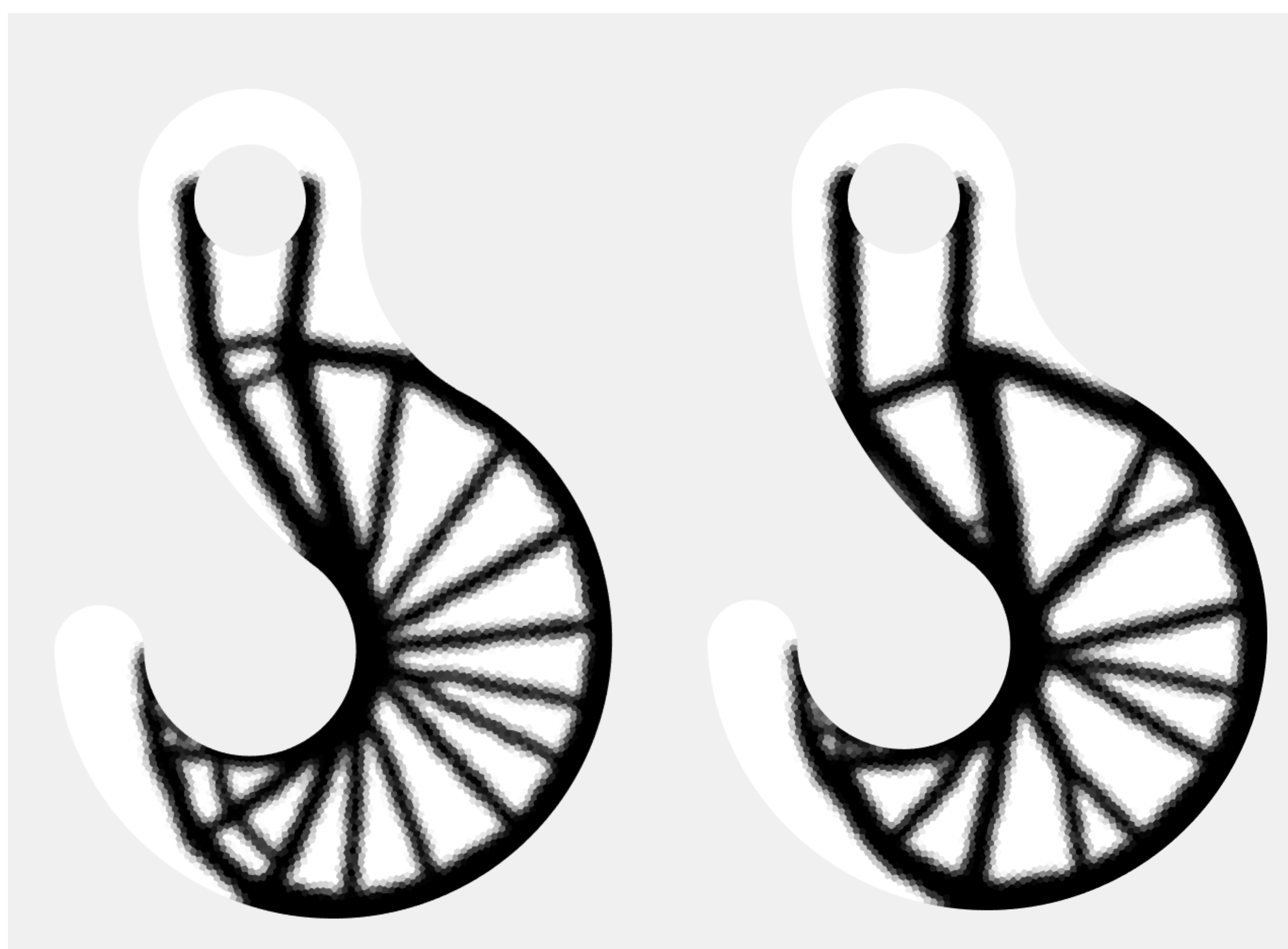
## Optimal Solutions




## Polygonal Elements



| $n$ | $n$-gon | number of $n$-gon |
|---|---|---|
| 4 | | 46 |
| 5 | | 1335 |
| 6 | | 4259 |
| 7 | | 758 |
| 8 | | 2 |
| Total Elements | | 6400 |



Linear optimized topology    Nonlinear optimized topology Initial work $C_0 = 5100$

## Conclusions

- Obtained convergent results which leads to optimal structures in the plastic range.
- Solved the nonlinear topology optimization problems in arbitrary domains by using polygonal FE meshes.

## References


[1] Pasquali, P. (2008) Análise Limite de Estruturas Através de uma Formulação em Elasticidade Não-Linear. *Master Thesis.*

[2] Klarbring, A., and Strömberg, N. (2013) Topology optimization of hyperelastic bodies including non-zero prescribed displacements. *Structural and Multidisciplinary Optimization.*

[3] Talischi, C., Paulino, G.H., Pereira, A., Menezes, I.F.M. (2012) PolyTop: a Matlab implementation of a general topology optimization framework using unstructured polygonal finite element meshes. *Structural and Multidisciplinary Optimization.*