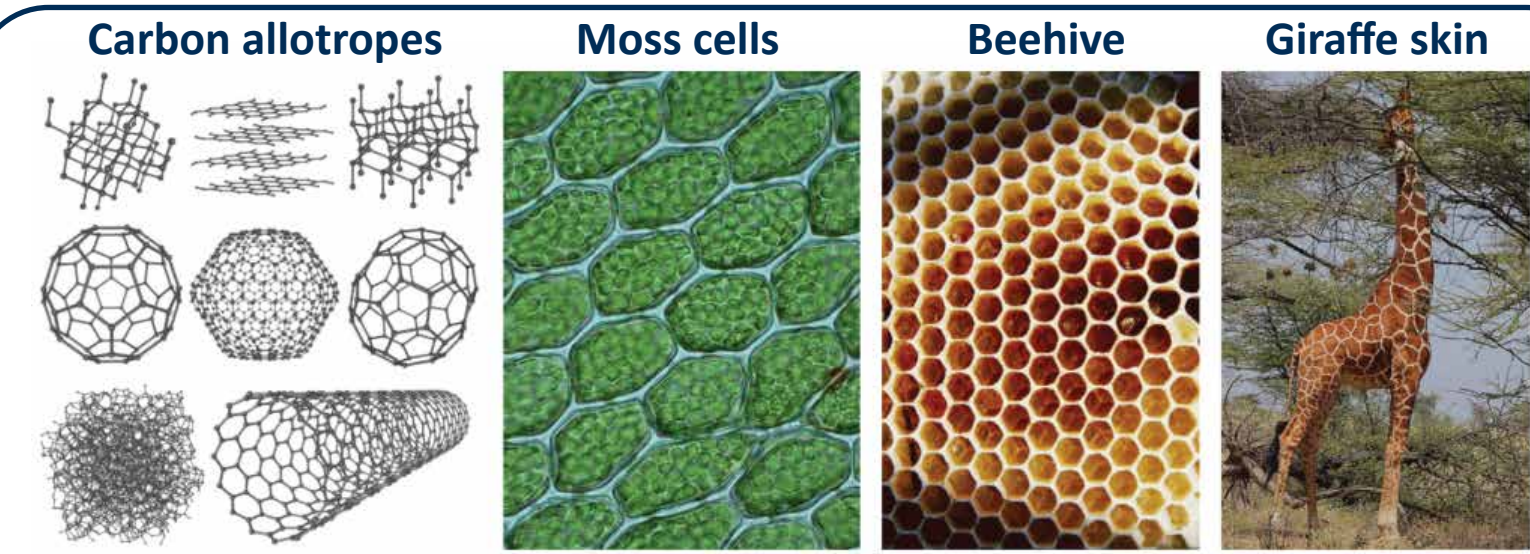


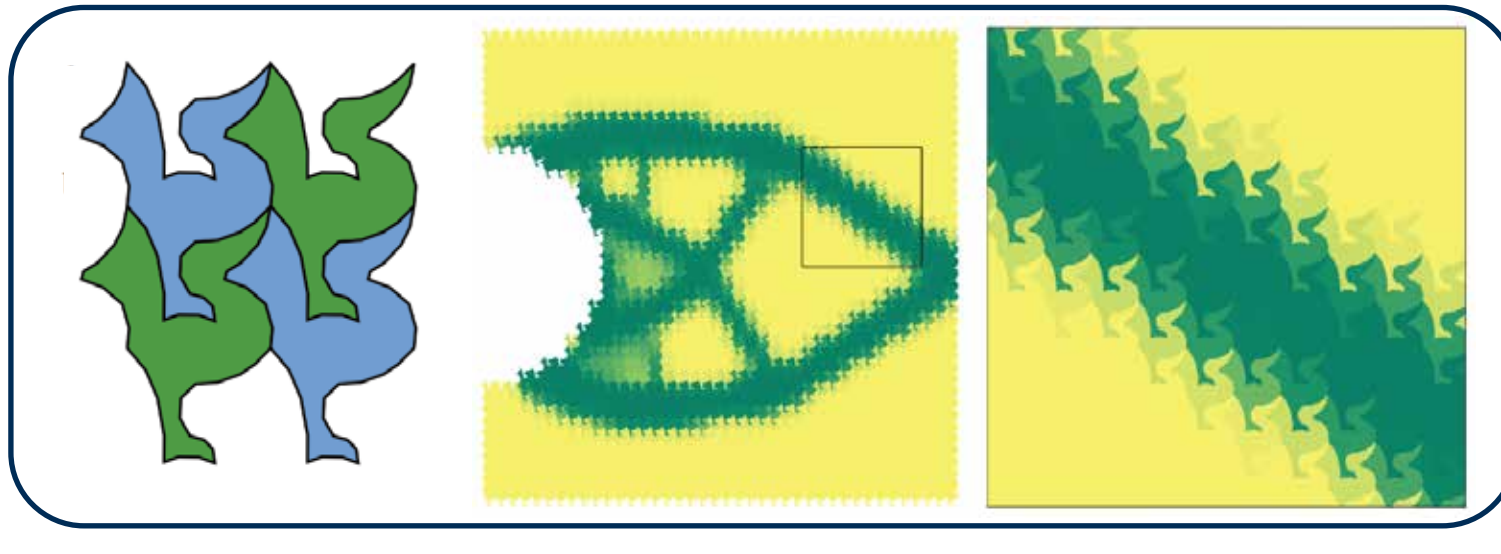
Continuum topology optimization

Polygonal finite elements (PolyMesher & PolyTop)

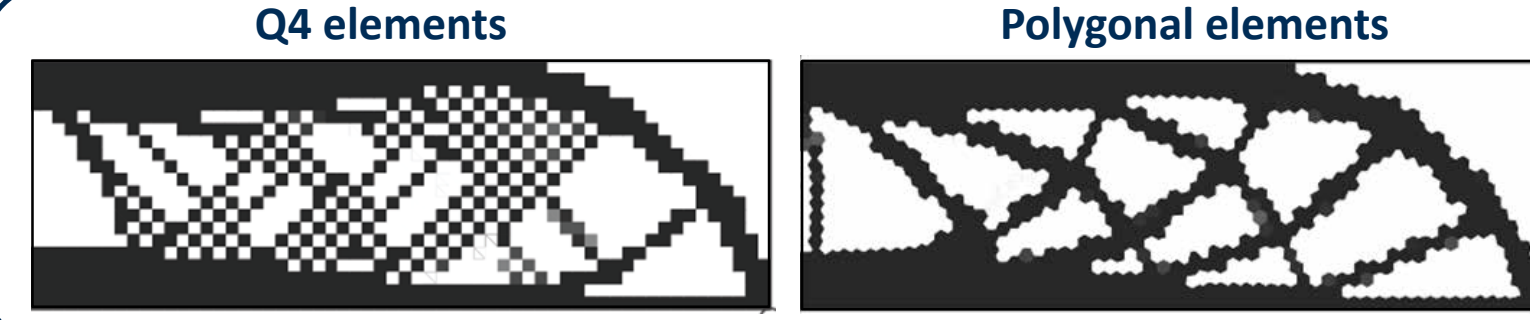
Polygons in nature



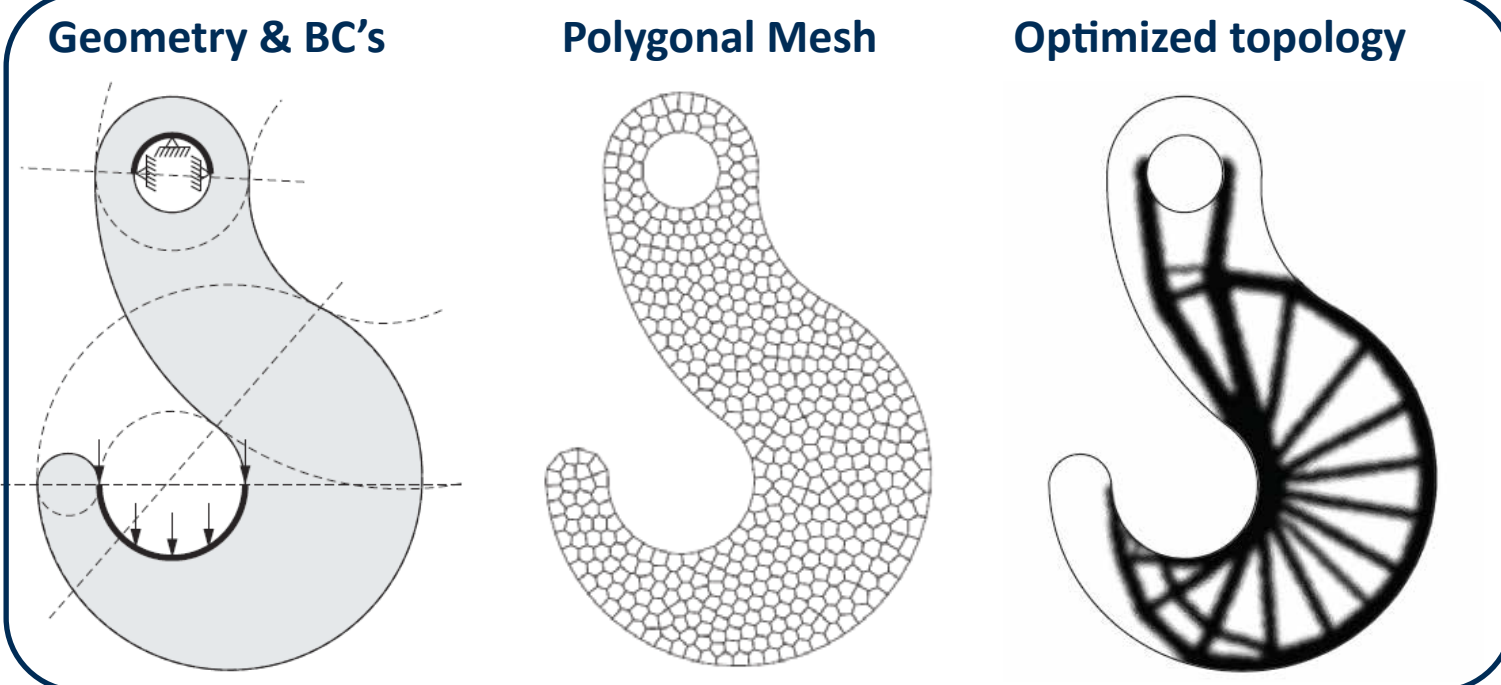
M.C. Escher-inspired elements



Naturally avoid checkerboard patterns



Optimization on arbitrary domains



Paulino and Gain. "Bridging art and engineering using Escher-based virtual elements." *Journal of Structural and Multidisciplinary Optimization*. 51:867-883. 2015.

Talisch, Paulino, Pereira, and Menezes. "PolyMesher: a general-purpose mesh generator for polygonal elements written in Matlab." *Journal of Structural and Multidisciplinary Optimization*. 45:309-328. 2012.

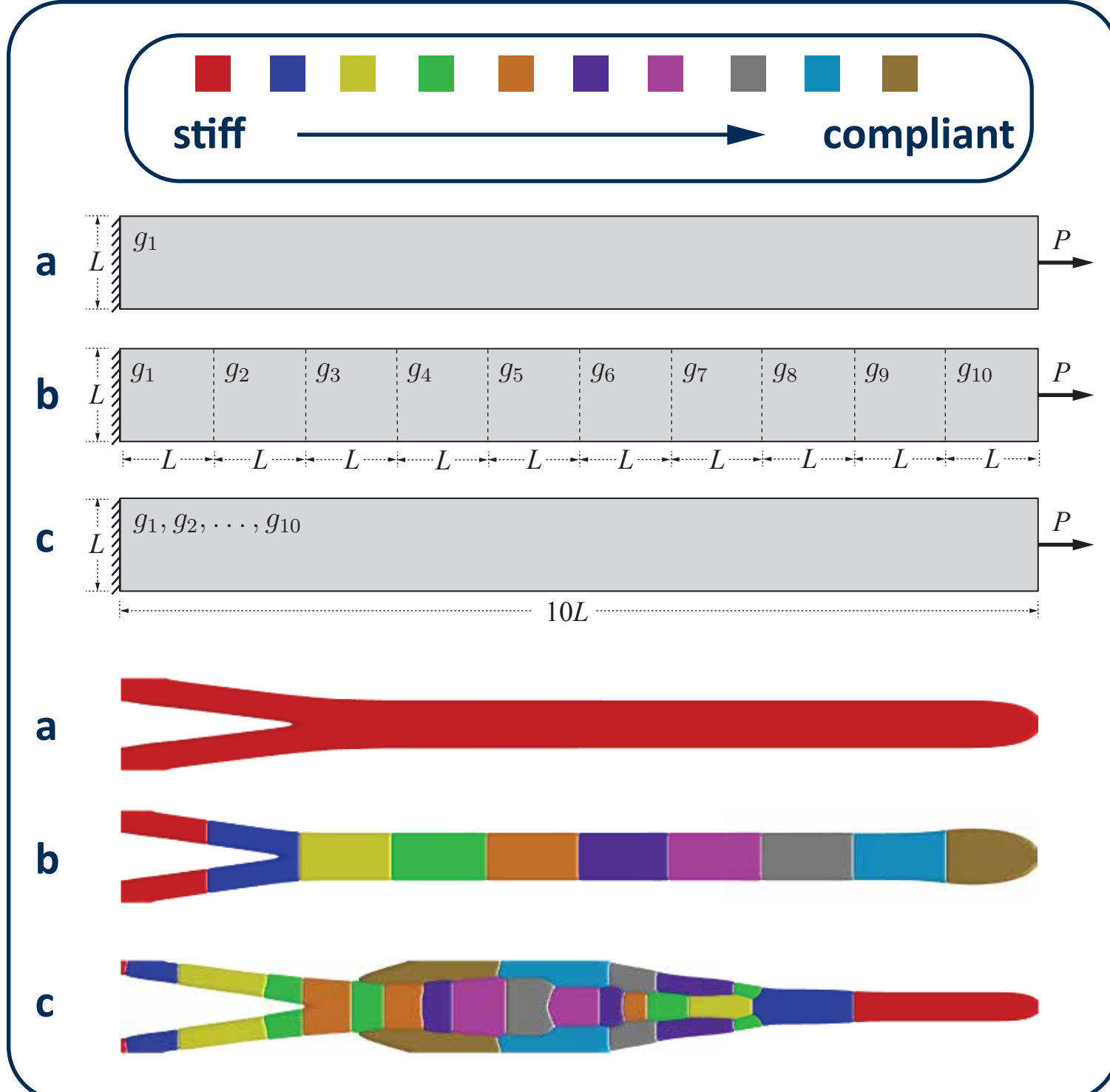
Talisch, Paulino, Pereira, and Menezes. "PolyTop: a Matlab implementation of a general topology optimization framework using unstructured polygonal finite element meshes." *Journal of Structural and Multidisciplinary Optimization*. 45:329-357. 2012.

Many materials and arbitrary volume constraints

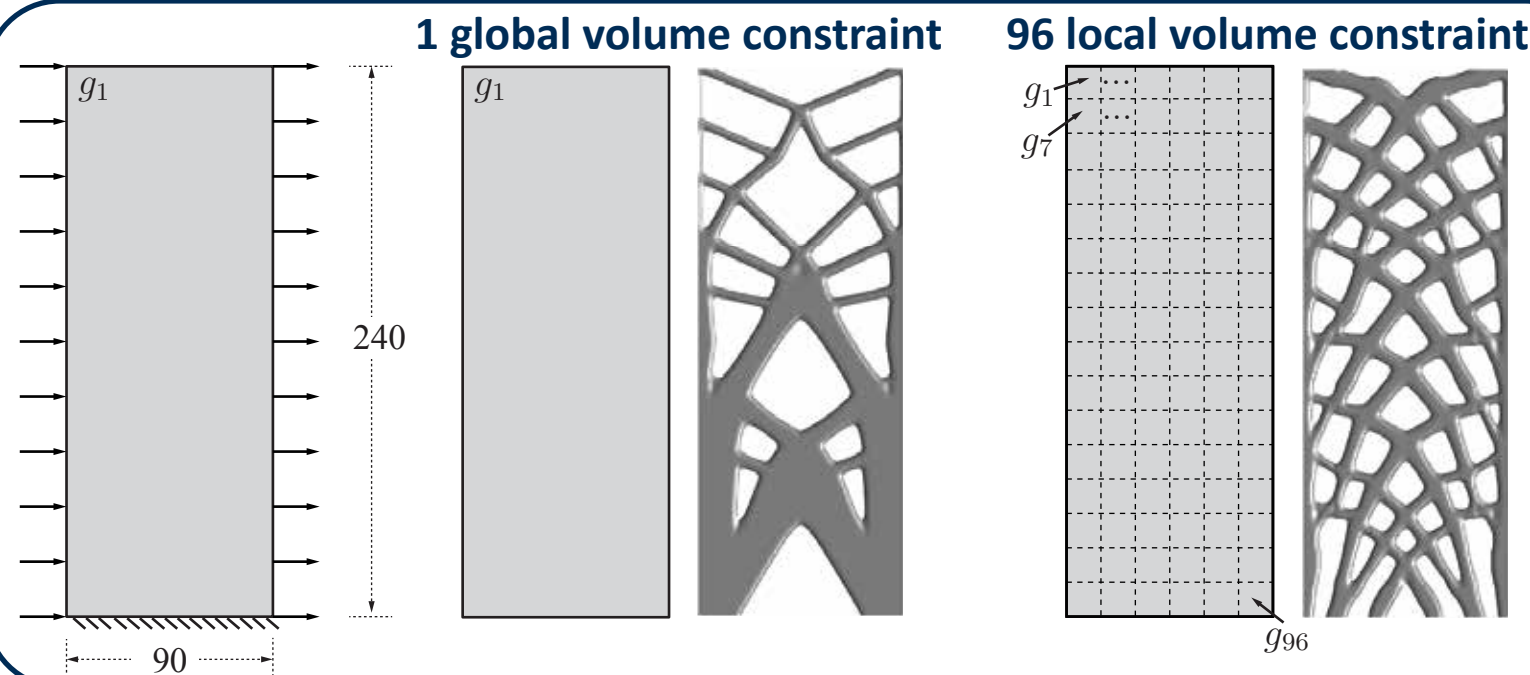


$$\begin{aligned} \min_{\mathbf{x}_1, \dots, \mathbf{x}_m} \quad & J = \mathbf{f}^T \mathbf{u}(\hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_m) \\ \text{s.t.} \quad & g_j = \sum_{i \in \mathcal{G}_j} \sum_{e \in \mathcal{E}_j} \hat{x}_i^e V^e \leq V_j^{\max} \\ & j = 1, \dots, N_c \\ & 0 \leq x_i^e \leq 1 \\ \text{with} \quad & \mathbf{K}(\hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_m) \mathbf{u}(\hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_m) = \mathbf{f} \end{aligned}$$

10-material designs



Hundred(s) of constraints



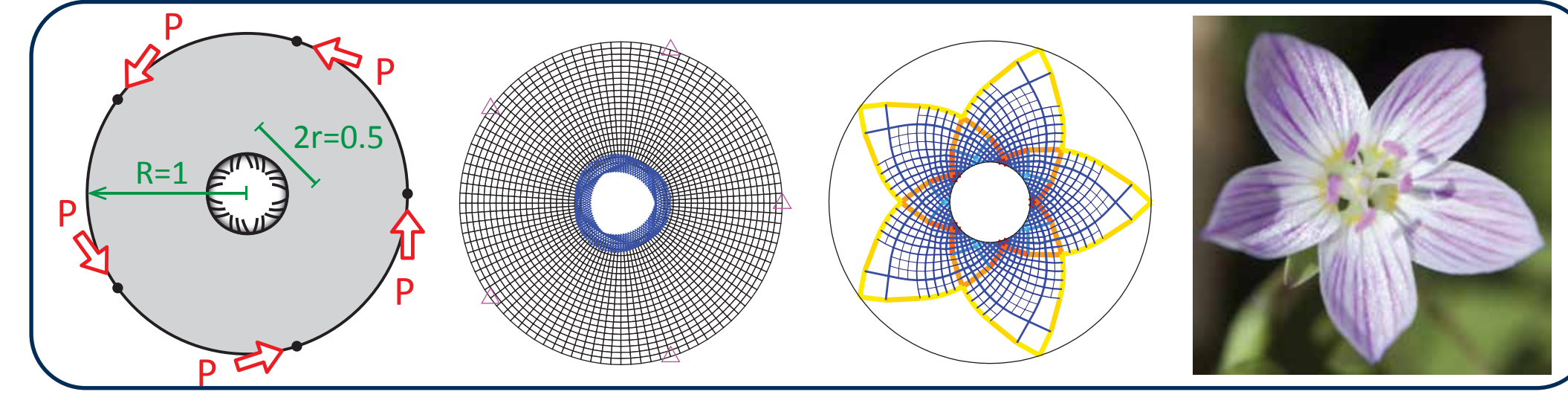
Zhang, Paulino, and Ramos Jr. "Multi-material topology optimization with multiple volume constraints: A general approach applied to ground structures with material nonlinearity." *Journal of Structural and Multidisciplinary Optimization*. Accepted.

Sanders, Aguiló, and Paulino. "Multi-material continuum topology optimization with arbitrary volume and mass constraints." Under review.

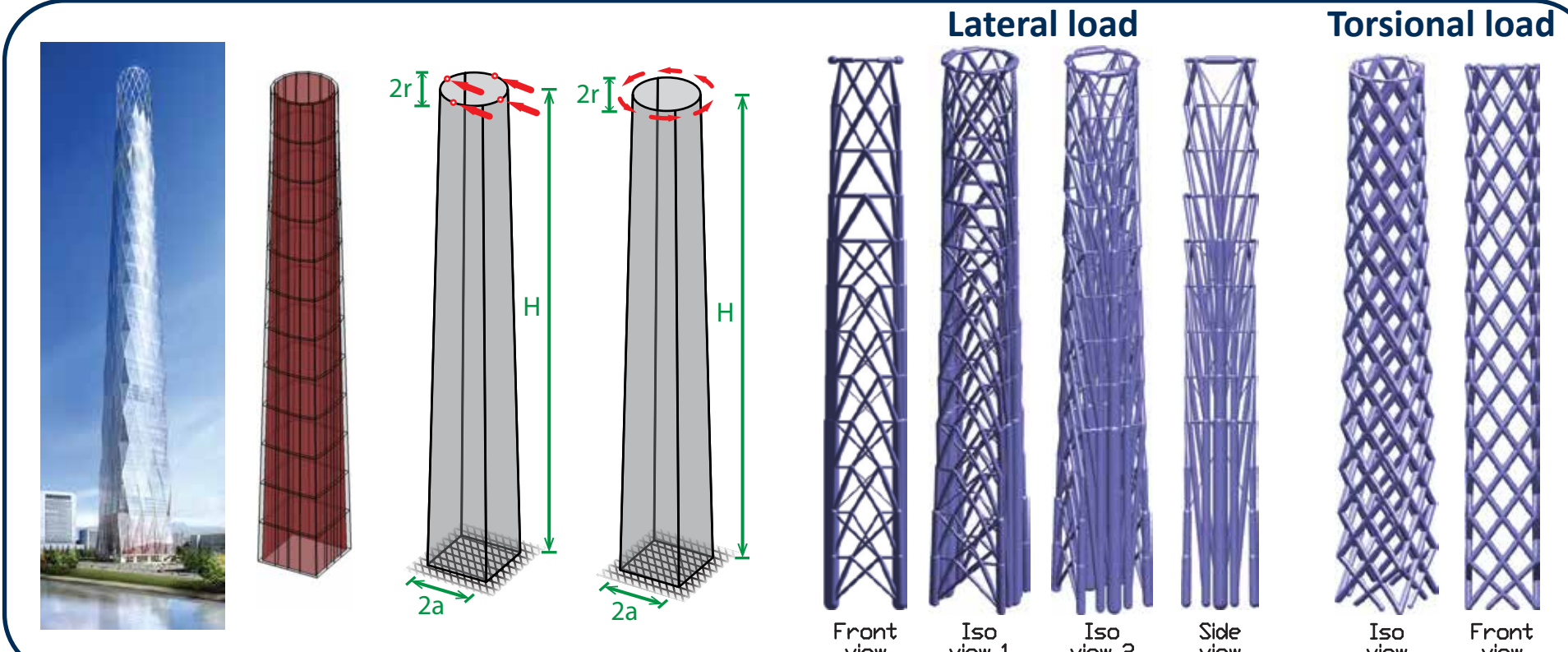
Discrete topology optimization

Plastic formulation (GRAND & GRAND3)

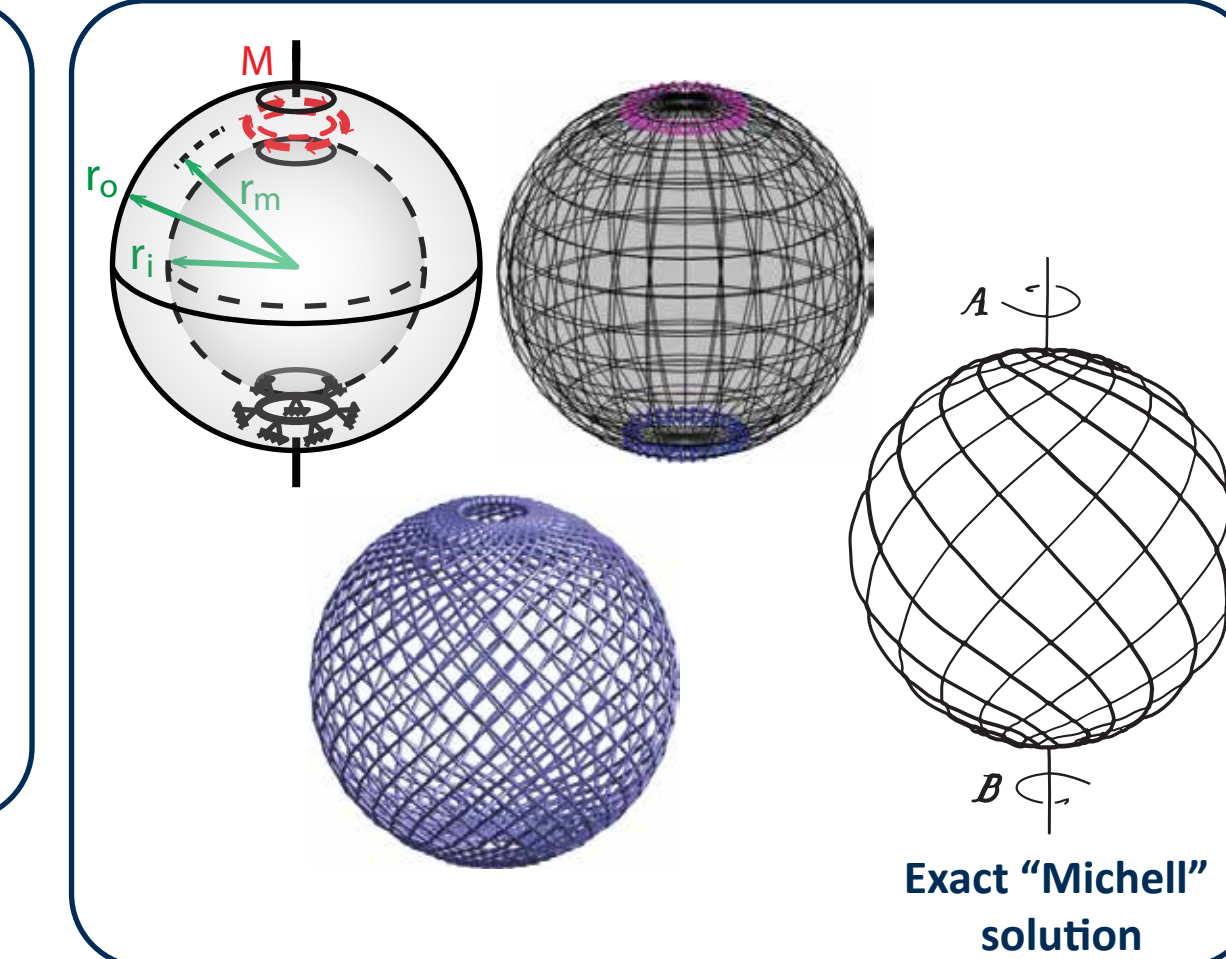
$$\begin{aligned} \min_{\mathbf{s}^+, \mathbf{s}^-} \quad & V = \mathbf{l}^T \left(\mathbf{s}^+ + \mathbf{s}^- \right) \\ \text{s.t.} \quad & \mathbf{B}^T (\mathbf{s}^+ - \mathbf{s}^-) = \mathbf{f} \\ & s_i^+, s_i^- \geq 0 \end{aligned}$$



Tall-building bracing design



Design tracing log spiral



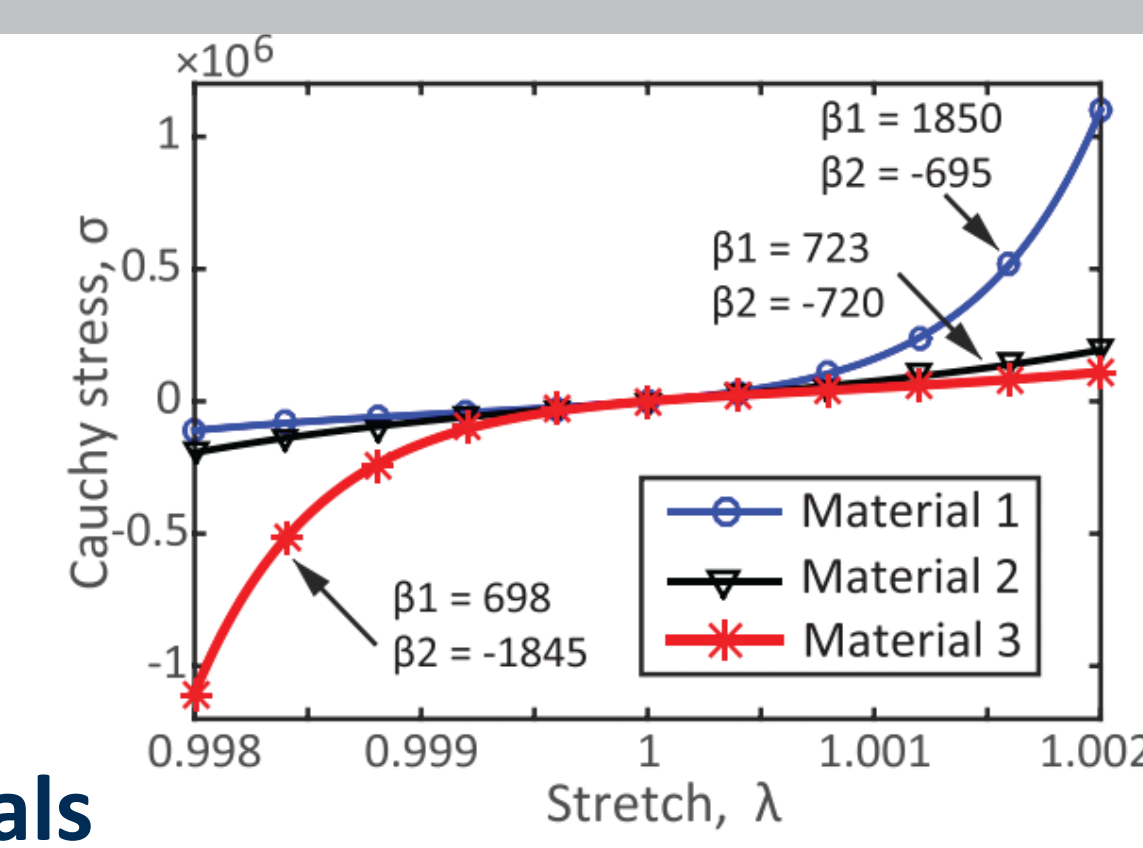
Zegard and Paulino. "GRAND - Ground structure based topology optimization for arbitrary 2D domains using MATLAB." *Structural and Multidisciplinary Optimization*. 50:861-882. 2014.

Zegard and Paulino. "GRAND3 - Ground structure based topology optimization for arbitrary 3D domains using MATLAB." *Structural and Multidisciplinary Optimization*. 52:1161-1184. 2015.

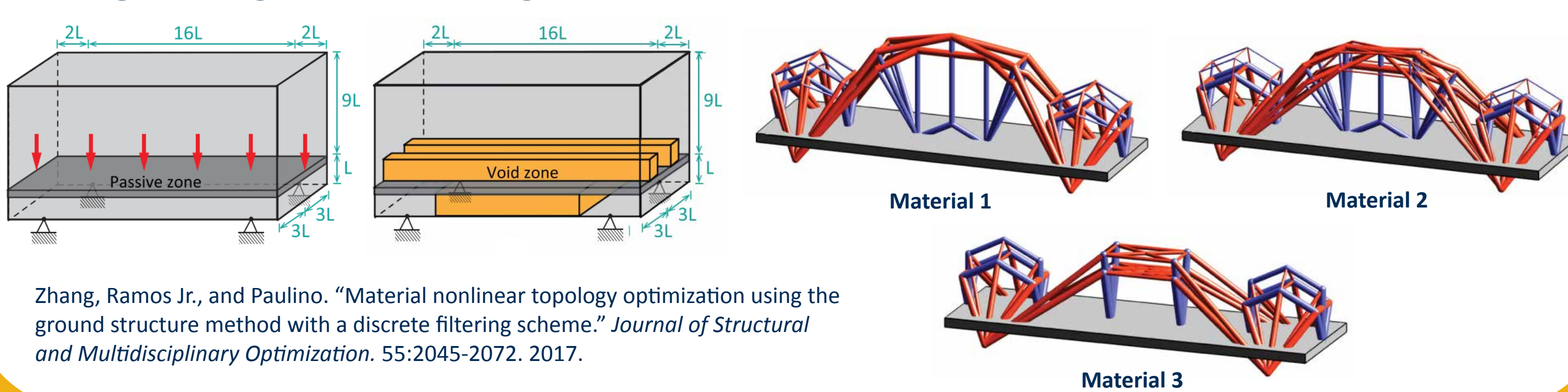
Michell. "The limits of economy of material in frame structures." *Philosophical Magazine Series 6*. 8:(47)589-597.1904.

Elastic formulation

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) = -\Pi_{\min}(\mathbf{x}, \mathbf{u}(\mathbf{x})) \\ \text{s.t.} \quad & g(\mathbf{x}) = \mathbf{L}^T \mathbf{x} - V^{\max} \leq 0 \\ & 0 \leq x_i \leq x_i^{\max} \quad i = 1, \dots, n \\ \text{with} \quad & \min_{\mathbf{u}} \left[\Pi(\mathbf{x}, \mathbf{u}(\mathbf{x})) + \frac{\eta}{2} \mathbf{u}(\mathbf{x})^T \mathbf{u}(\mathbf{x}) \right] \end{aligned}$$



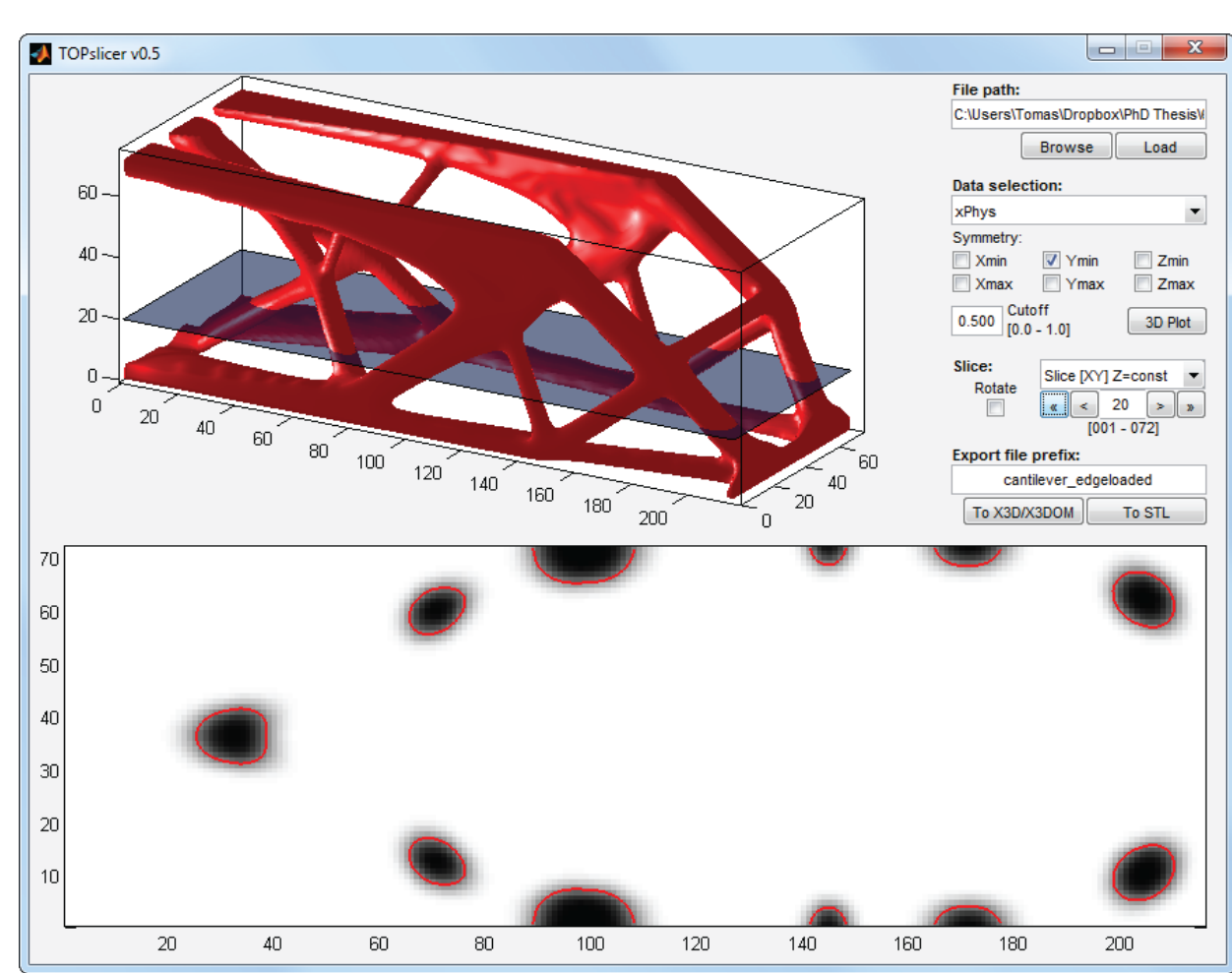
Bridge design considering nonlinear materials



Zhang, Ramos Jr., and Paulino. "Material nonlinear topology optimization using the ground structure method with a discrete filtering scheme." *Journal of Structural and Multidisciplinary Optimization*. 55:2045-2072. 2017.

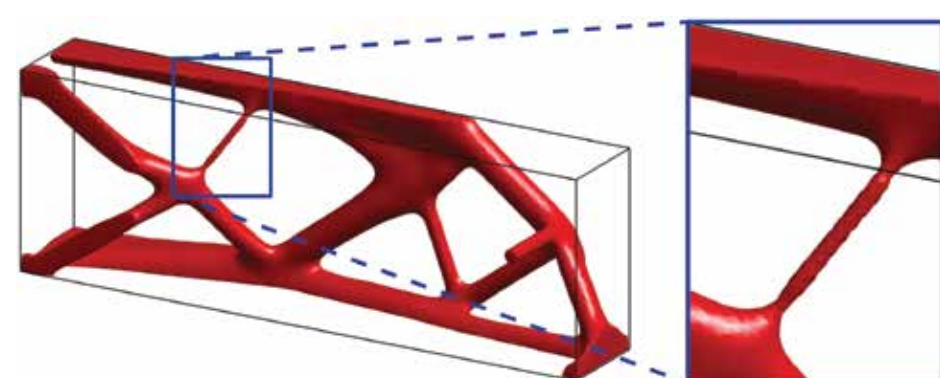
Topology optimization & additive manufacturing

Part inspection (TopSlicer)

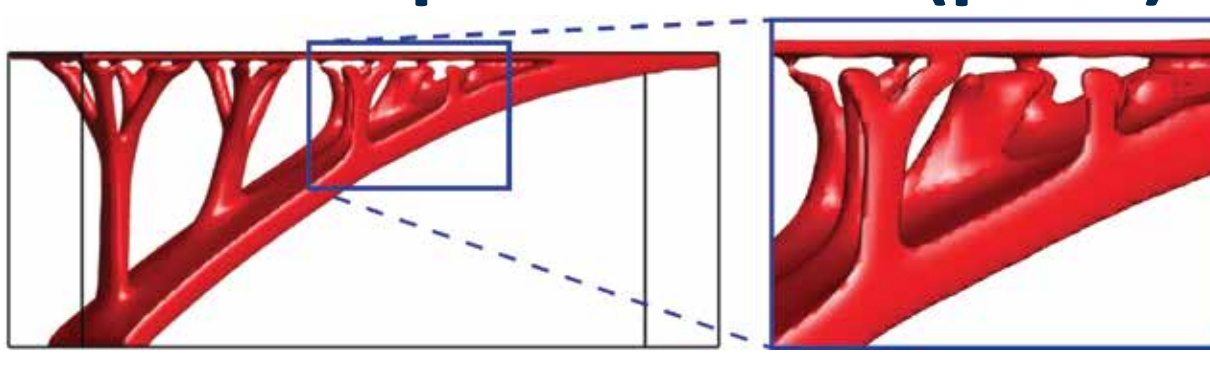


Zegard and Paulino. "Bridging topology optimization and additive manufacturing." *Journal of Structural and Multidisciplinary Optimization*. 53:175-192. 2016.

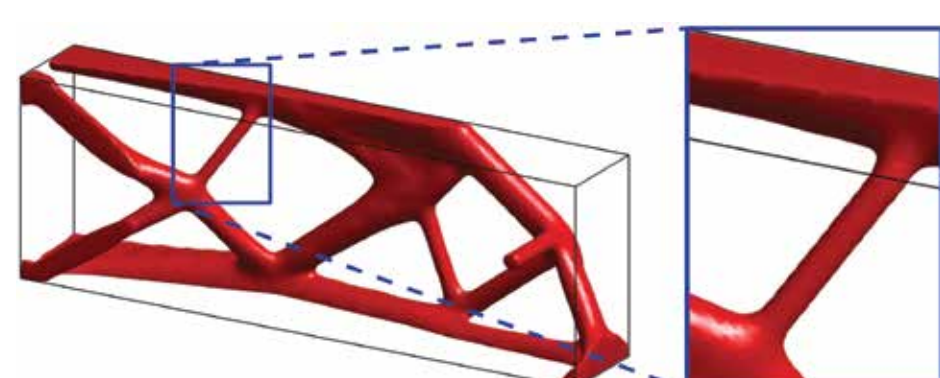
Linear filter



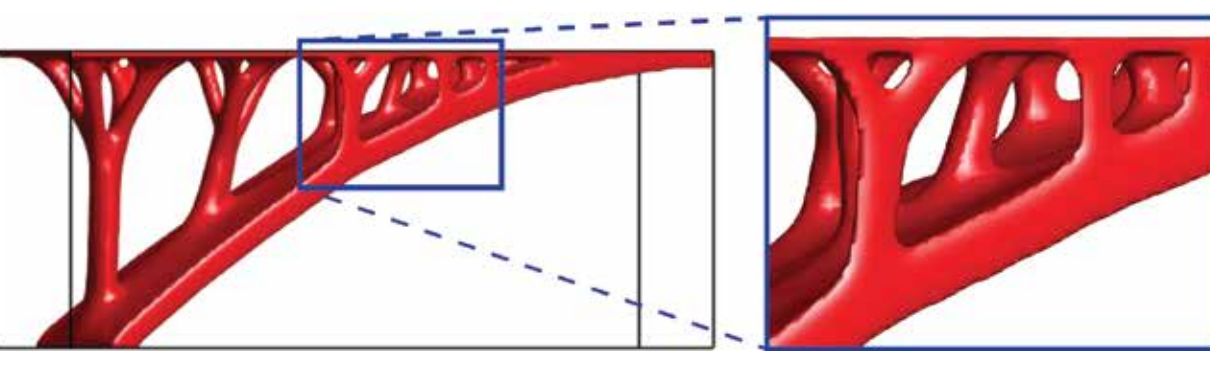
Constant penalization (p = 3)



Cubic filter

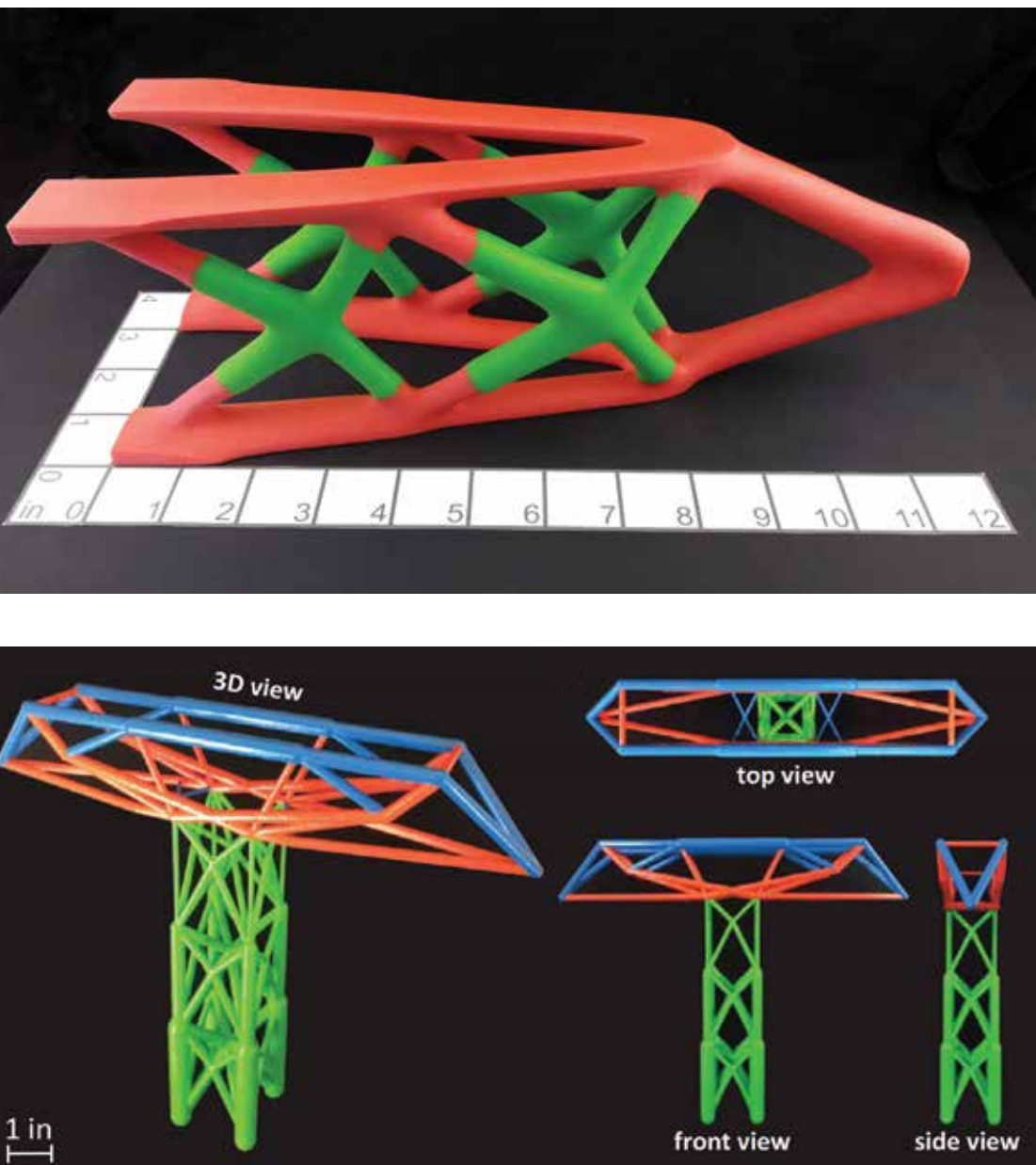


Continuation on penalization

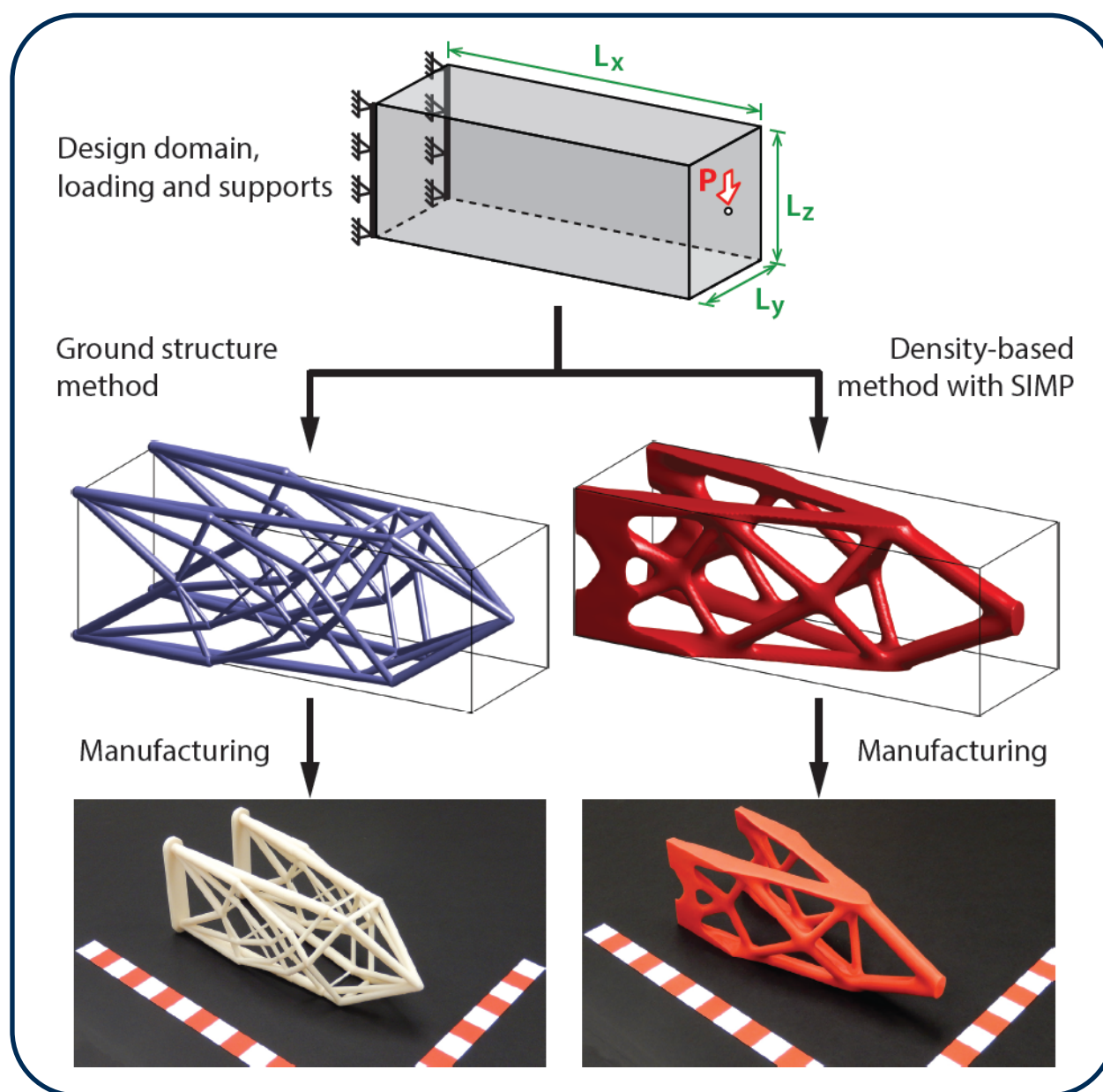


3D printed designs

Student projects



Education



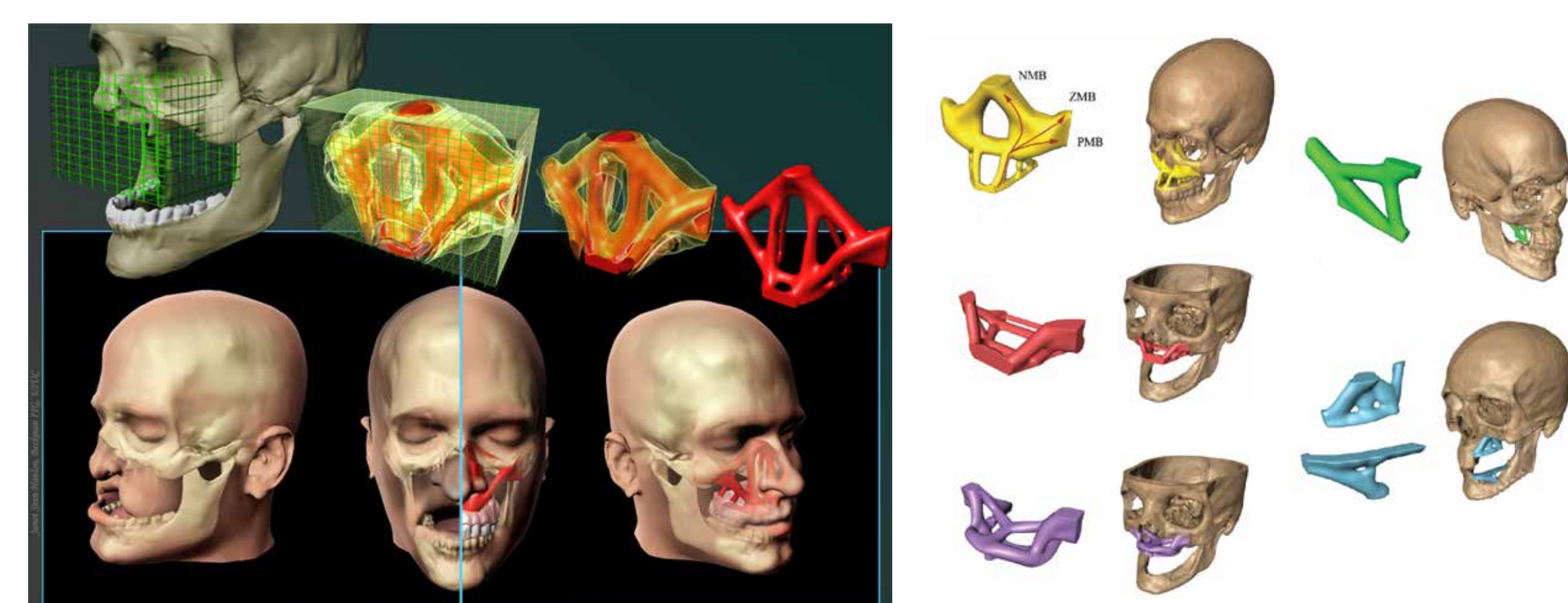
Zegard and Paulino. "Bridging topology optimization and additive manufacturing." *Journal of Structural and Multidisciplinary Optimization*. 53:175-192. 2016.

Zhang, Paulino, and Ramos Jr. "Multi-material topology optimization with multiple volume constraints: A general approach applied to ground structures with material nonlinearity." *Journal of Structural and Multidisciplinary Optimization*. Accepted.

Sanders, Aguiló, and Paulino. "Multi-material continuum topology optimization with arbitrary volume and mass constraints." Under review.

Applications

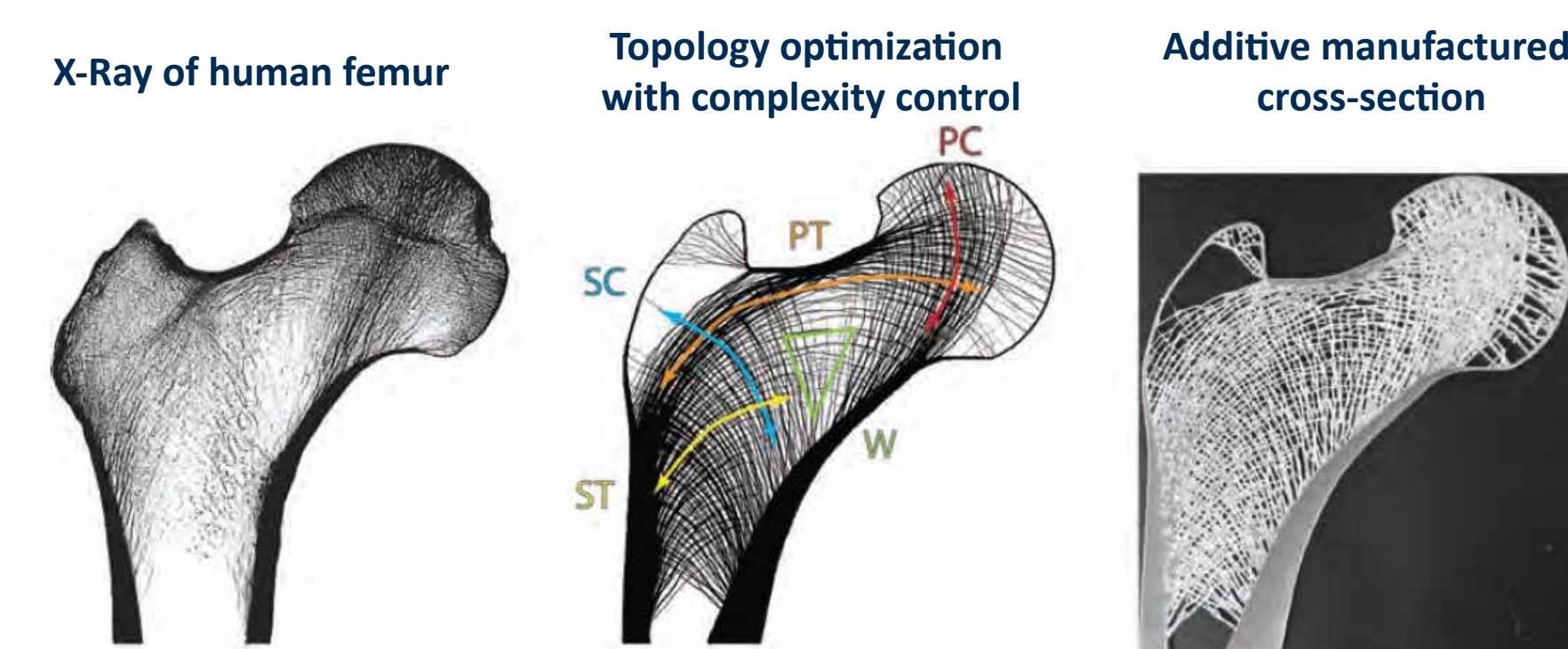
Craniofacial bone replacement



Sutradhar, Paulino, Miller, and Nguyen. "Topological optimization for designing patient-specific large craniofacial segmental bone replacements." *Proceedings of the National Academy of Sciences*. 107(30):13222-13227. 2010.

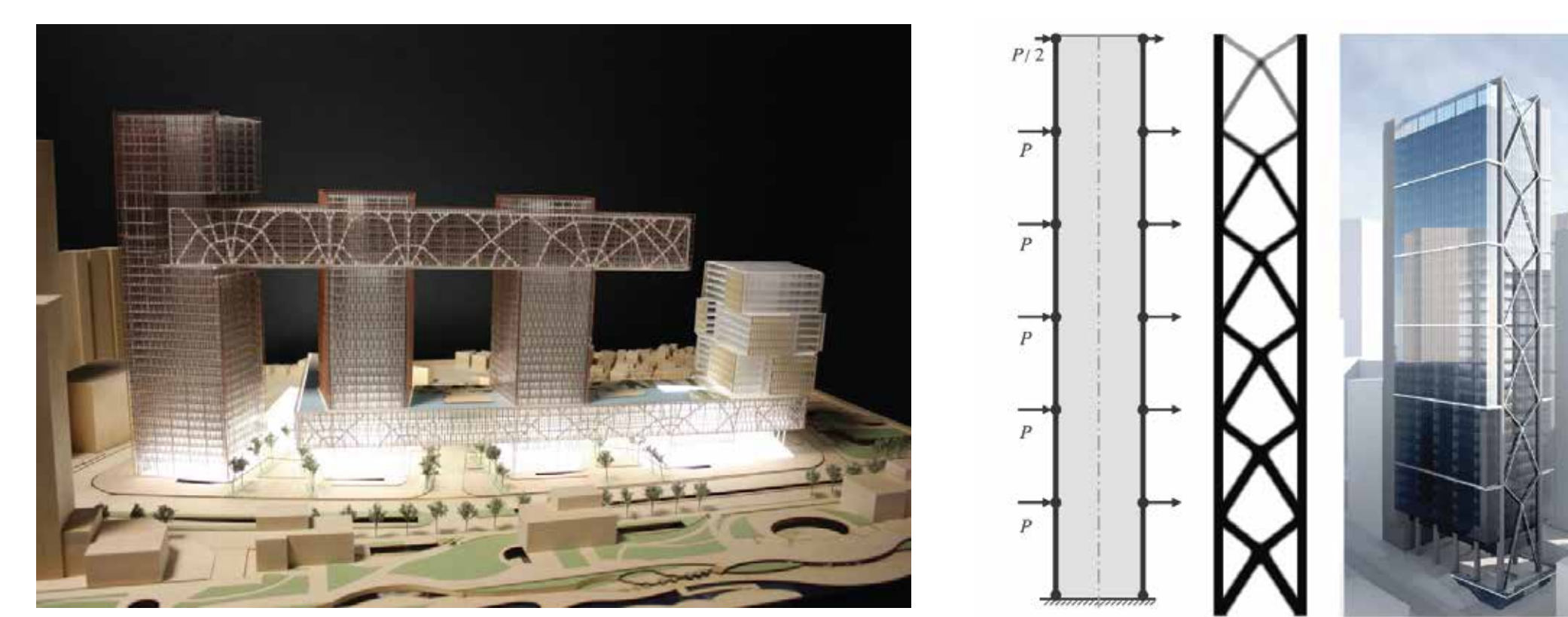
Sutradhar, Park, Carrau, Nguyen, Miller, and Paulino. "Designing patient-specific 3D printed craniofacial implants using a novel topology optimization method." *Medical and Biological Engineering and Computing*. 54:1123-1135. 2015.

Complex internal bone structure



Park, Sutradhar, Shah, and Paulino. "Design of complex internal structure of next generation bone implants using restricted topology optimization." *Computers in Biology and Medicine*. 00:1-24. 2017.

Connecting architecture and engineering



Beghini, Beghini, Katz, Baker, and Paulino. "Connecting architecture and engineering through structural topology optimization." *Engineering Structures*. 59:716-726. 2014.