Design of Functionally Graded Piezocomposite Materials Using Topology Optimization with Polygonal Mesh

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Acknowledgements:

Outline

- **Introduction and Motivation**
- **Objective**
- **Theoretical Topics**
  - Polygonal Mesh
  - Topology Optimization
- **Numerical Results**
- **Conclusions and Future Works**

Piezoelectric Materials

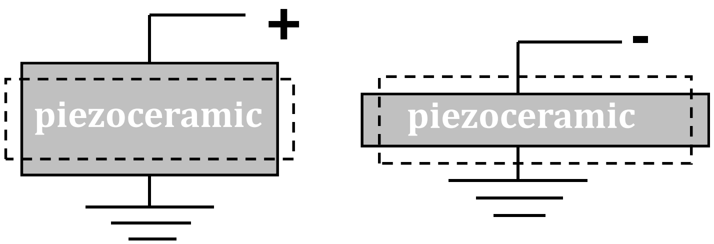
Force

Mechanical Energy ⟷ Piezoelectric Material ⟷ Electrical Energy

Electric potential

Displacement

Electric charge



Constitutive Equations:

$$\begin{cases} [T] = \left[c^E\right][S] - [e]^t\{E\} \\ \{D\} = \left[\varepsilon^S\right]\{E\} + [e][S] \end{cases}$$

elastic: $[T] = \left[c^E\right][S]$

electric: $\{D\} = \left[\varepsilon^S\right]\{E\}$

electromechanical: $-[e]^t\{E\}$ and $+[e][S]$

$[T]$ – stress tensor

$[S]$ – strain tensor

$\{E\}$ – electric field vector

$\{D\}$ – electric displacement vector

$[c^E]$ – elastic tensor obtained at constant electric field

$[e]$ – piezoelectric tensor

$[\varepsilon^S]$ – dielectric tensor obtained at constant strain

Examples:

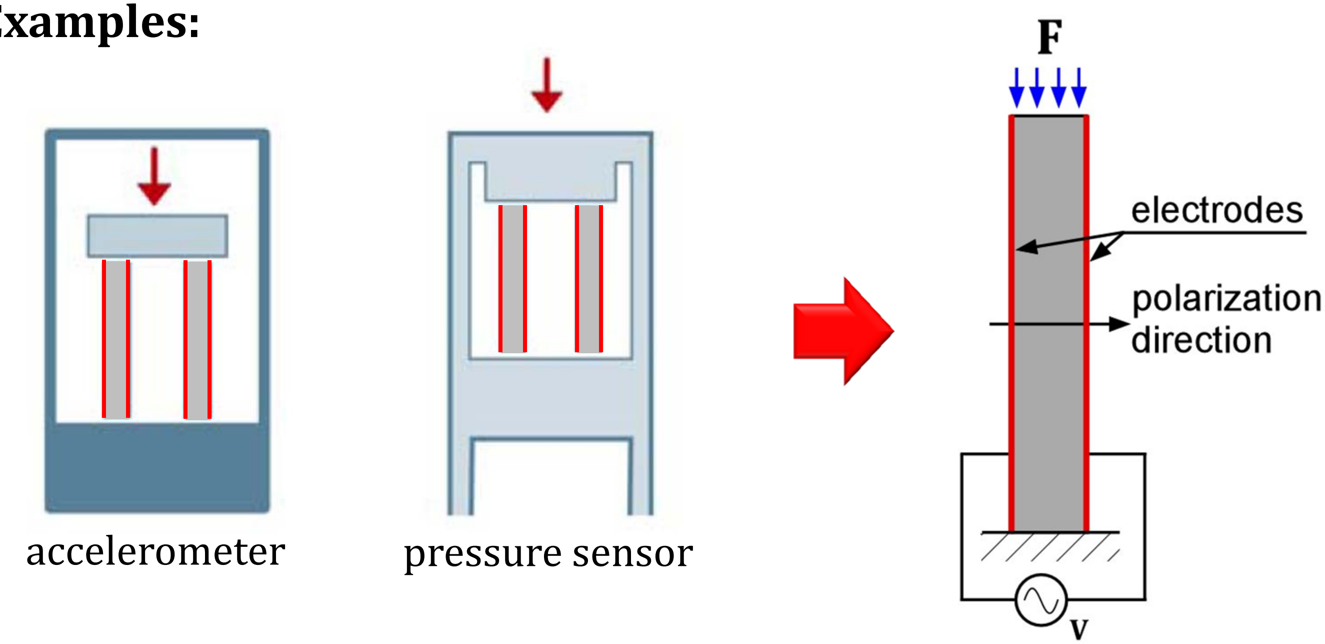
- Quartz (natural)
- Ceramic (PZT5A, PMN)
- Polymer (PVDF)

Applications: ultrasonic transducers, actuators, pressure sensors, accelerometers, sonar, hydrophones, MEMS, etc...

Motivation

Motivation: to design materials used in piezoelectric sensors

Examples:



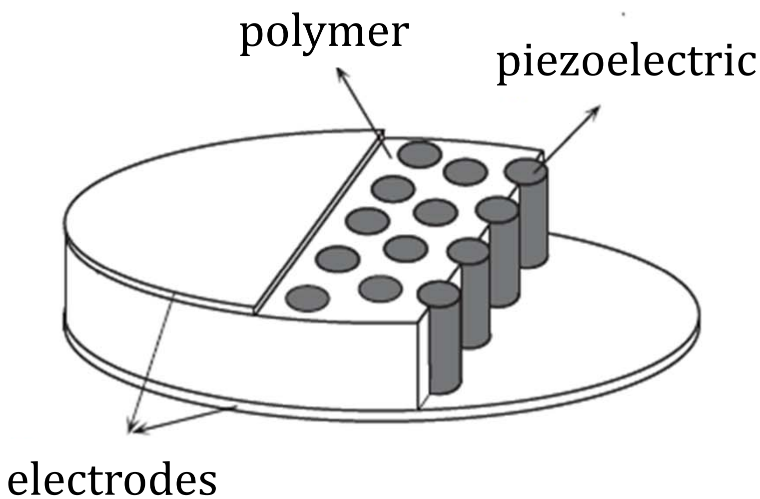
Main objective
maximize the output voltage

→ Piezoelectric materials are too stiff

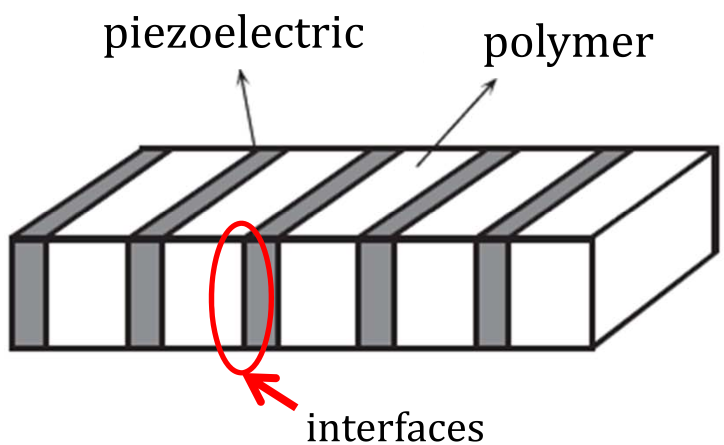
→ Need to reduce their stiffness

Piezocomposite Materials

1-3 piezocomposite



2-2 piezocomposite



- Combines piezoelectric material → non-piezoelectric materials
- Better performance than pure materials
- Depends on: volume fractions, material properties, shape of inclusions
- Interfaces: might present stress concentrations, which may cause material fracture and fatigue.

Newnham RE, Skinner DP, Cross LE, "Connectivity And Piezoelectric-Pyroelectric Composites", Materials Research Bulletin, Pergamon-Elsevier Science Ltd, 1978, 13, 525-536.

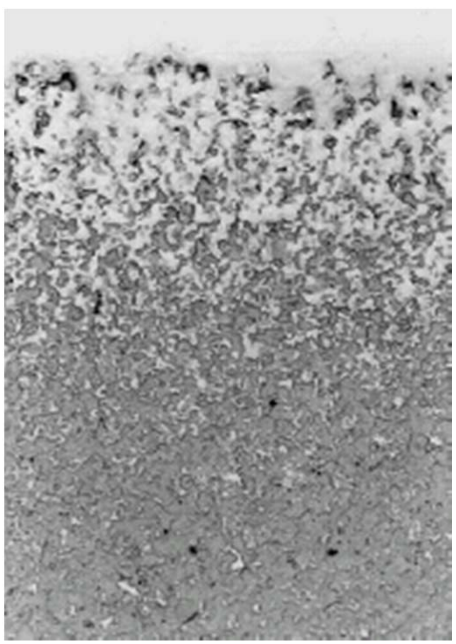
Functionally Graded Materials (FGM)

FGMs possess continuously graded properties with gradual change in microstructure which avoids interface problems, such as, stress concentrations.

Miyamoto, Y., Kaysser, W. A., Rabin, B. H., and and R. G. Ford, A. K., "Functionally Graded Materials: Design, Processing and Applications", Kluwer Academic Publishers, Dordrecht, 1999.

Microstructure

T_{Hot}

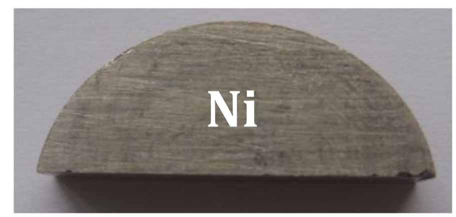


T_{Cold}

- Ceramic Phase
- Ceramic matrix with metallic inclusions
- Transition region
- Metallic matrix with ceramic inclusions
- Metallic Phase

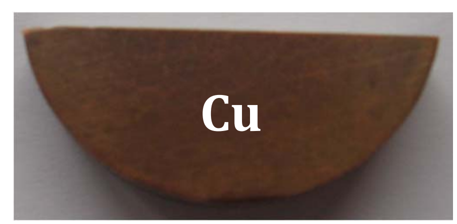
Example: Cu-Ni FGM disk

Top View



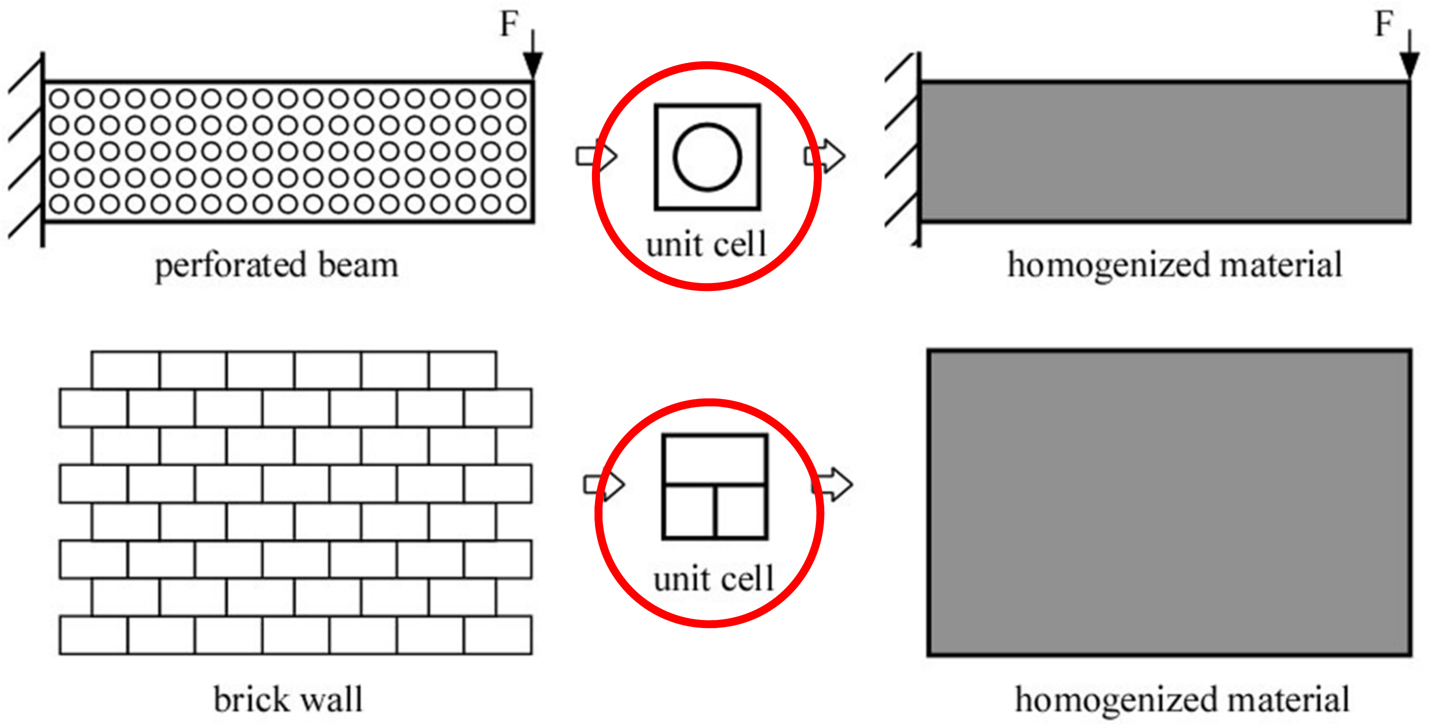
Front View

Bottom View



Homogenization Method

→ Calculation of the effective properties

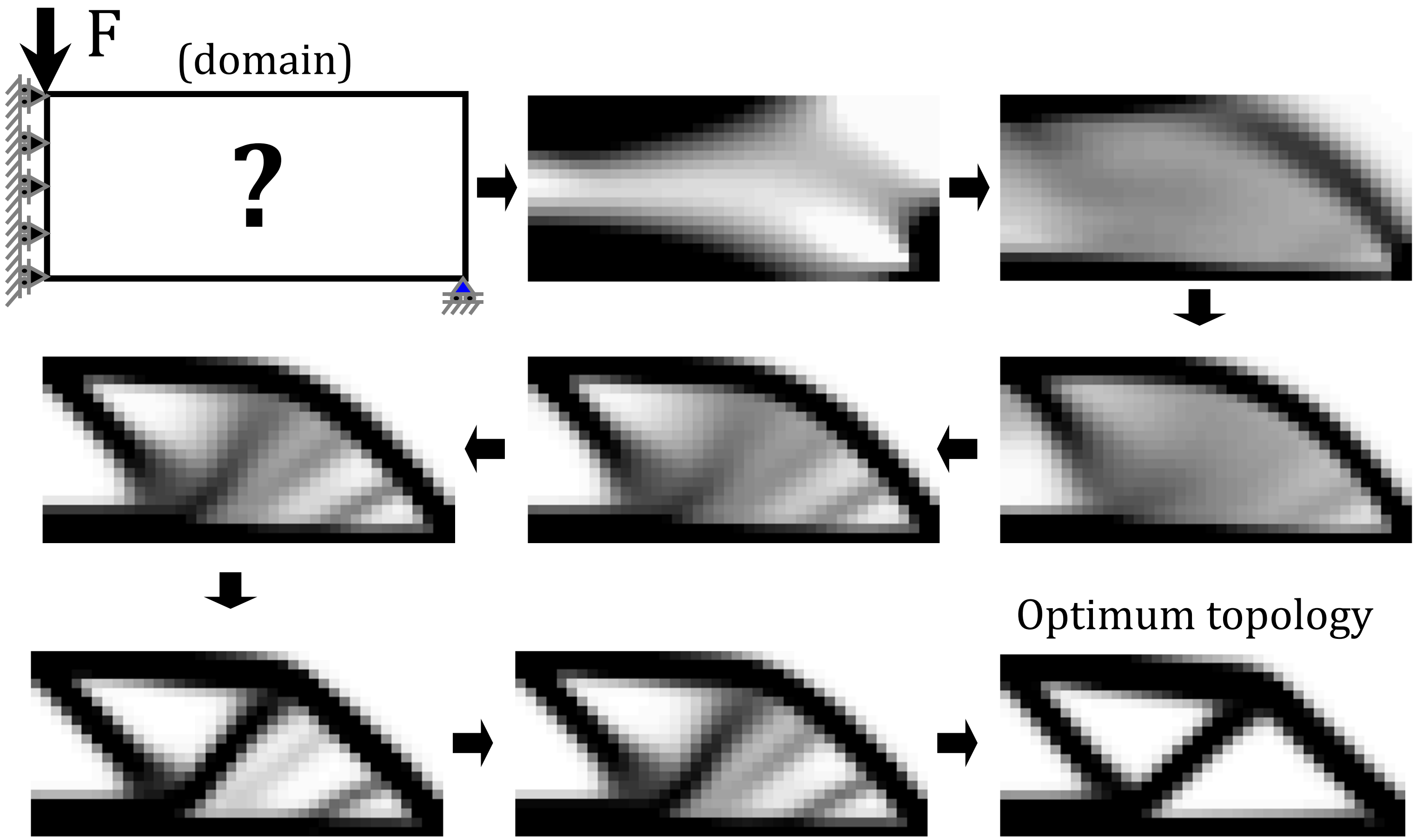


→ The homogenized properties depend on the volume fractions of constituent materials, its properties, and shape of inclusions in the unit cell.

Sanchez-Palencia E, "Non-Homogeneous Media and Vibration Theory", Lectures Notes in Physics 127, Springer, Berlin, 1980.

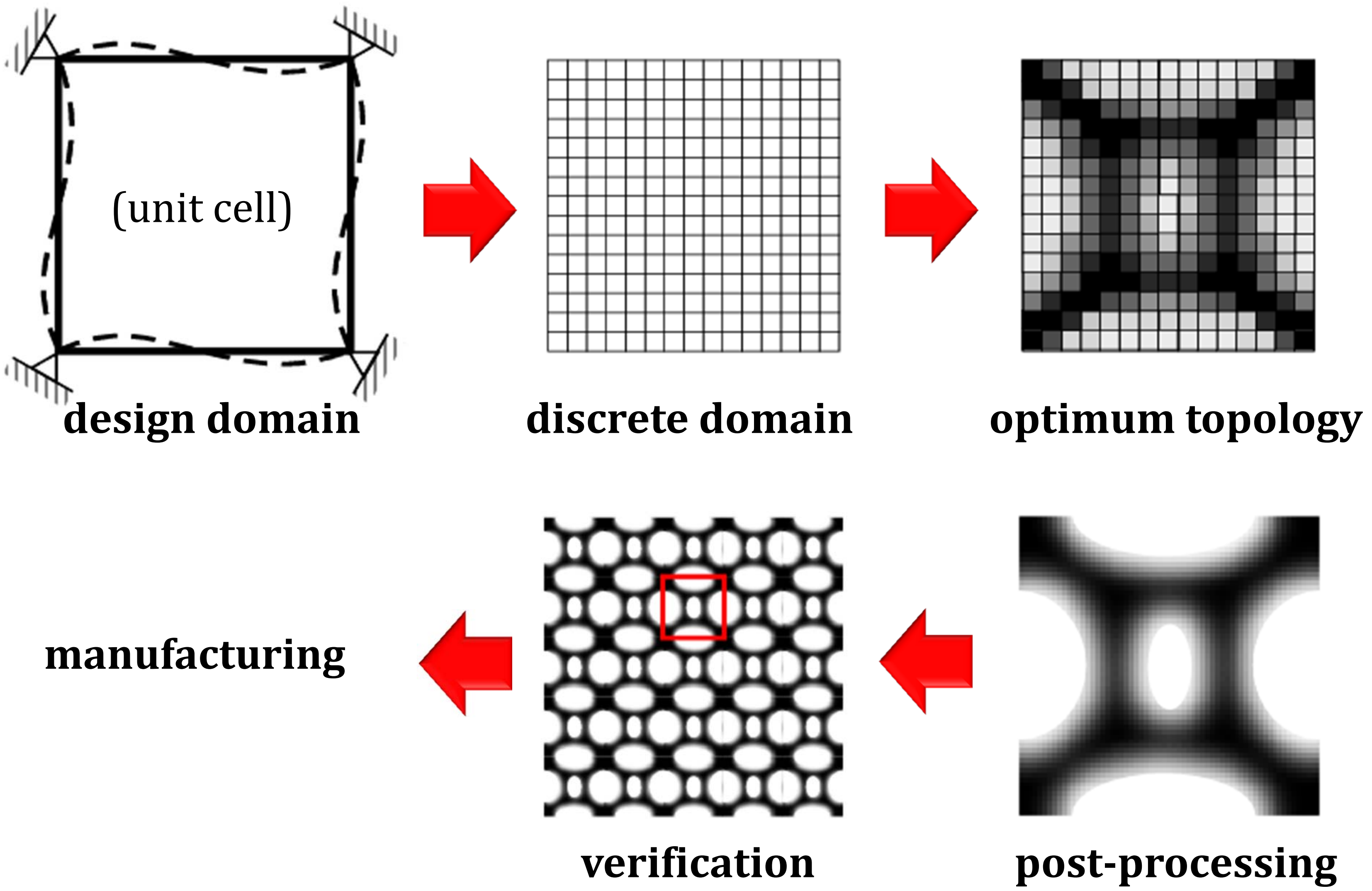
Topology Optimization Method (TOM)

→Design of piezocomposites FGM



Bendsoe MP, Sigmund O, "Topology Optimization - Theory, Methods and Applications". Springer, New York, EUA, 2003.

Piezocomposite Design Using TOM



Previous works

Material Design

- Distribution of material phases in a unit cell that optimizes the properties of a composite (Cherkaev, Kohn, 1997)
- Design of materials with prescribed parameters:
  - elastic materials (Sigmund, 1995)
  - piezoelectric materials (Silva et al, 1999)
  - thermoelastic materials (Torquato et al, 2003)

Stress Calculation In Unit Cells

- Preprocessing And Postprocessing For Materials Based On The Homogenization Method With Adaptive Finite-Element Methods (Guedes and Kikuchi, 1990)
- Determination of the micro stress field in composite by homogenization method (Ni et al, 2006)

FGM Material Design

- Optimal design of FGM composites with prescribed properties (Paulino et al, 2008)

Objective

To design piezocomposite materials
based on FGM concept using
topology optimization and homogenization methods,
in order to maximize the output voltage
of piezoelectric sensors.

Outline

- Introduction and Motivation
- Objective
- **Theoretical Topics**
    - Polygonal Mesh
    - Topology Optimization
- **Numerical Results**
- **Conclusions and Future Works**

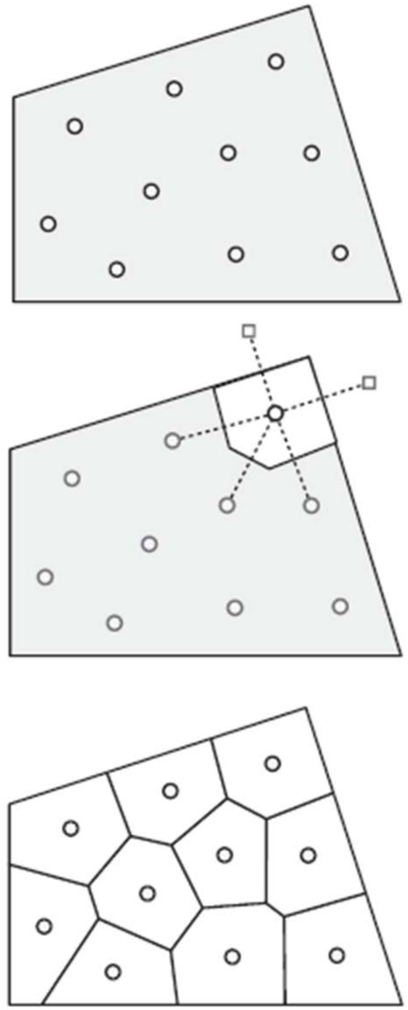
Polygonal Mesh

Generation:

a) Populate the domain with a desired number of 'seeds'

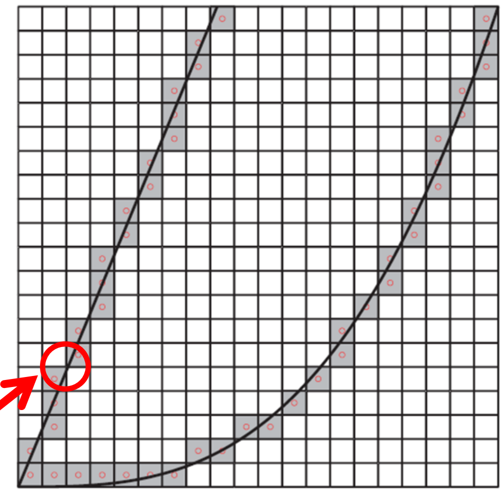
b) Calculate auxiliary points in the boundary domain

c) Construct the Voronoi diagram



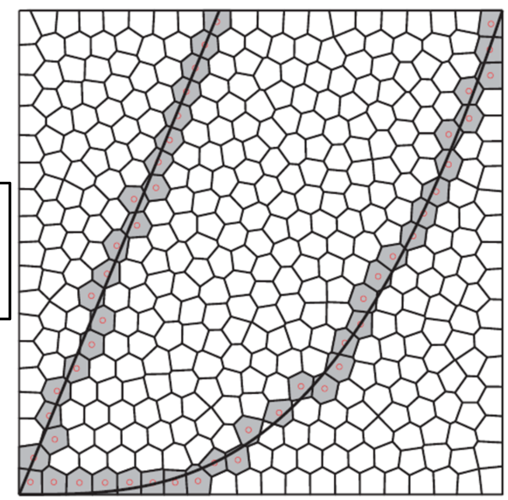
Example:

regular square mesh



one-node connections

polygonal mesh

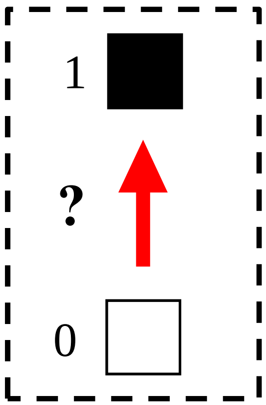


Talischi C, Paulino GH, Pereira A, Menezes, IFM, "Polygonal finite elements for topology optimization: A unifying paradigm", *International Journal for Numerical Methods in Engineering*, 2009, 28

Topology Optimization Method

→ How to change the material from zero to one?

Bendsoe MP, Sigmund O, "Topology Optimization - Theory, Methods and Applications". Springer, New York, EUA, 2003.

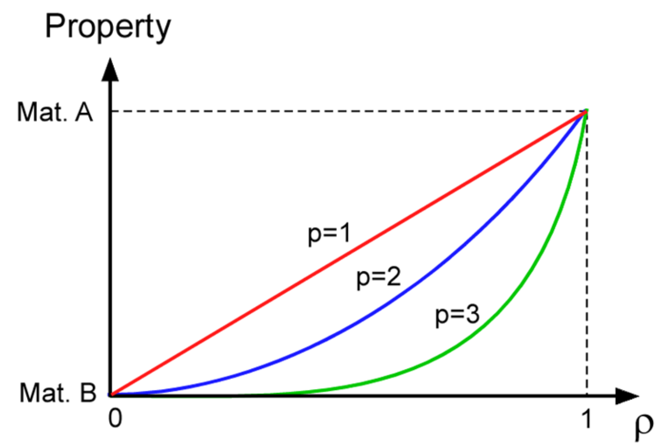


$$\boldsymbol{\Gamma}^H = \rho^p \boldsymbol{\Gamma}_A + \left(1 - \rho^p\right) \boldsymbol{\Gamma}_B$$

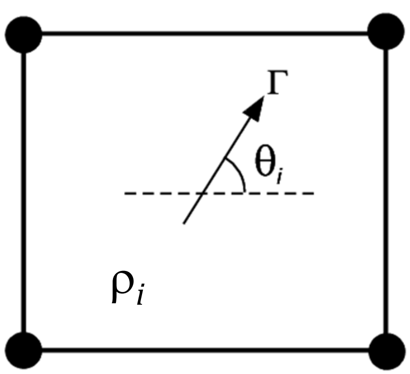
ρ: design variable for material distribution

Γ: material properties

- **c** – elastic properties
- **e** – piezoelectric properties
- **ε** – dielectric properties



→ Classical concept of orientation optimization in a finite element

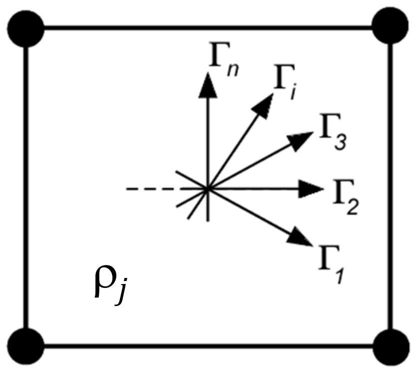


θ_i: design variable for angle

Common problem: large risk of obtaining a local optimum solution

Topology Optimization Method

→ Material Model: Discrete Material Optimization (DMO)



material properties

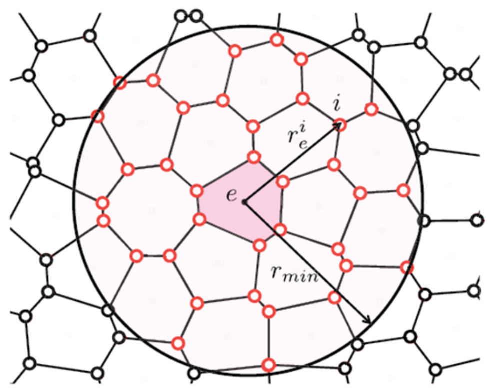
$$\boldsymbol{\Gamma}_0 = \sum_{i=1}^{n^c} w_i \boldsymbol{\Gamma}_i = w_1\boldsymbol{\Gamma}_1 + w_2\boldsymbol{\Gamma}_2 + \cdots + w_{n^c}\boldsymbol{\Gamma}_{n^c}$$

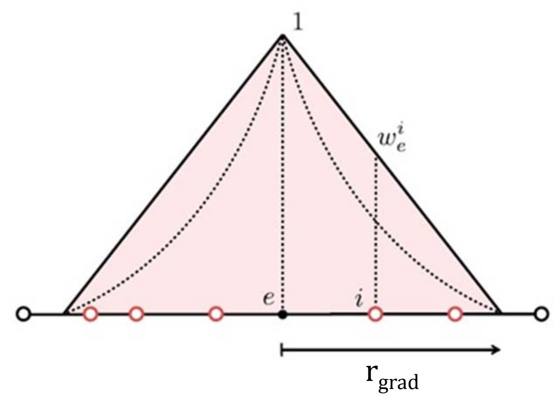
design variables

$$w_i = \frac{\hat{w}_i}{\sum_{k=1}^{n} \hat{w}_k}, \text{ where } \hat{w}_i = (\rho_i)^p \prod_{j=1,\, j\neq 1}^{n} \left[1-(\rho_i)^p\right]$$

Stegmann J, Lund E, "Discrete material optimization of general composite shell structures", International Journal for Numerical Methods in Engineering, 2005; 62, p. 2009–2027

How to control the gradation? Projection Functions



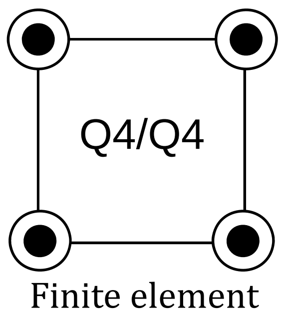


Talischi, C.; Paulino, G. H.; Pereira, A. & Menezes, I. F. M. "Polygonal finite elements for topology optimization: A unifying paradigm", International Journal for Numerical Methods in Engineering, 2009, 28.

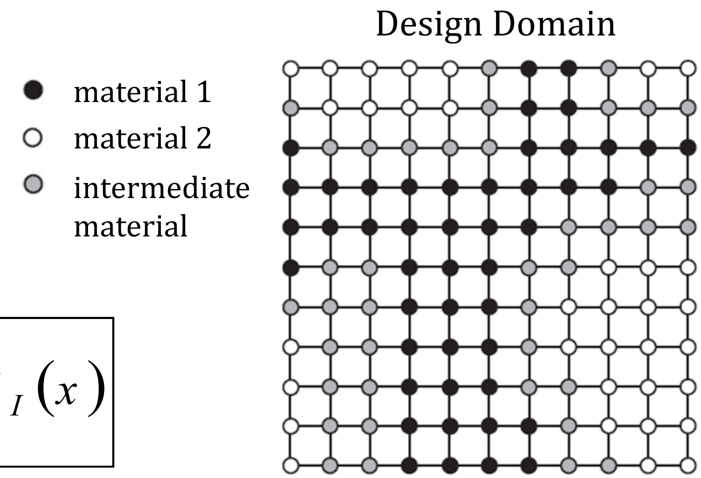
Guest, J. K.; Prévost, J. H. & Belytschko, T., "Achieving minimum length scale in topology optimization using nodal design variables and projection functions", International Journal for Numerical Methods in Engineering, 2004, 61, 238-254.

Topology Optimization Method

CAMD: Continuous Approximation of Material Distribution



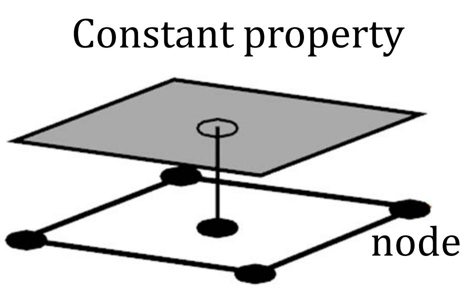
- Design variable
- Displacements (x and y) and electrical potential

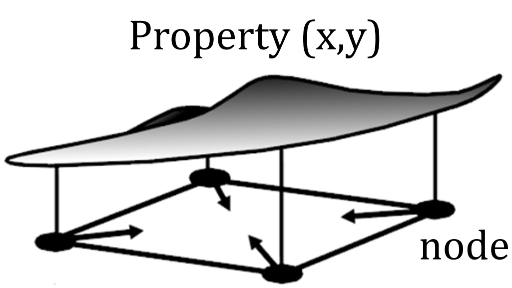


$$\rho(x) = \sum_{I=1}^{nnodes} \rho_I N_I(x)$$

Matsui, K., Terada, K., "Continuous Approximation of Material Distribution for Topology Optimization", International Journal for Numerical Methods in Engineering, 2004, 59, 1925-1944.

Grade Finite Element Concept





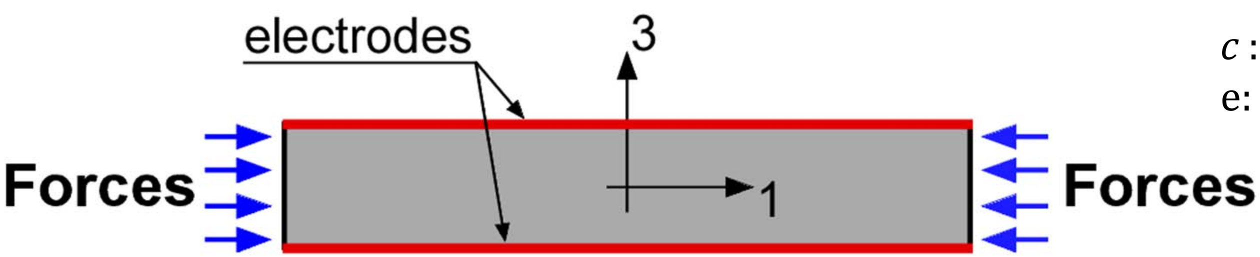
$$D^e(x,y) = \sum_{i=1}^{4} D_i^e N_i$$

Kim, J. H., Paulino, G. H., "Isoparametric Graded Finite Elements for Nonhomogeneous Isotropic and Orthotropic Materials", ASME Journal of Applied Mechanics, 2002, 69, 502-514.

Topology Optimization Method

How to measure it?

electrical energy	⟷	**mechanical energy**



Electromechanical Coupling Coefficient:

$$d = \frac{\left(c_{33} - c_{13}\right) e_{13}}{c_{11} c_{33} - c_{13}^{2}}$$

c : elastic coefficients
e: piezoelectric coefficients

Formulation of the Optimization Problem:

$\underset{\rho}{\text{Maximize:}} \quad F(\rho, d)$

subject to: $0 \le \rho \le 1$
symmetry conditions
gradation control

ρ: design variables

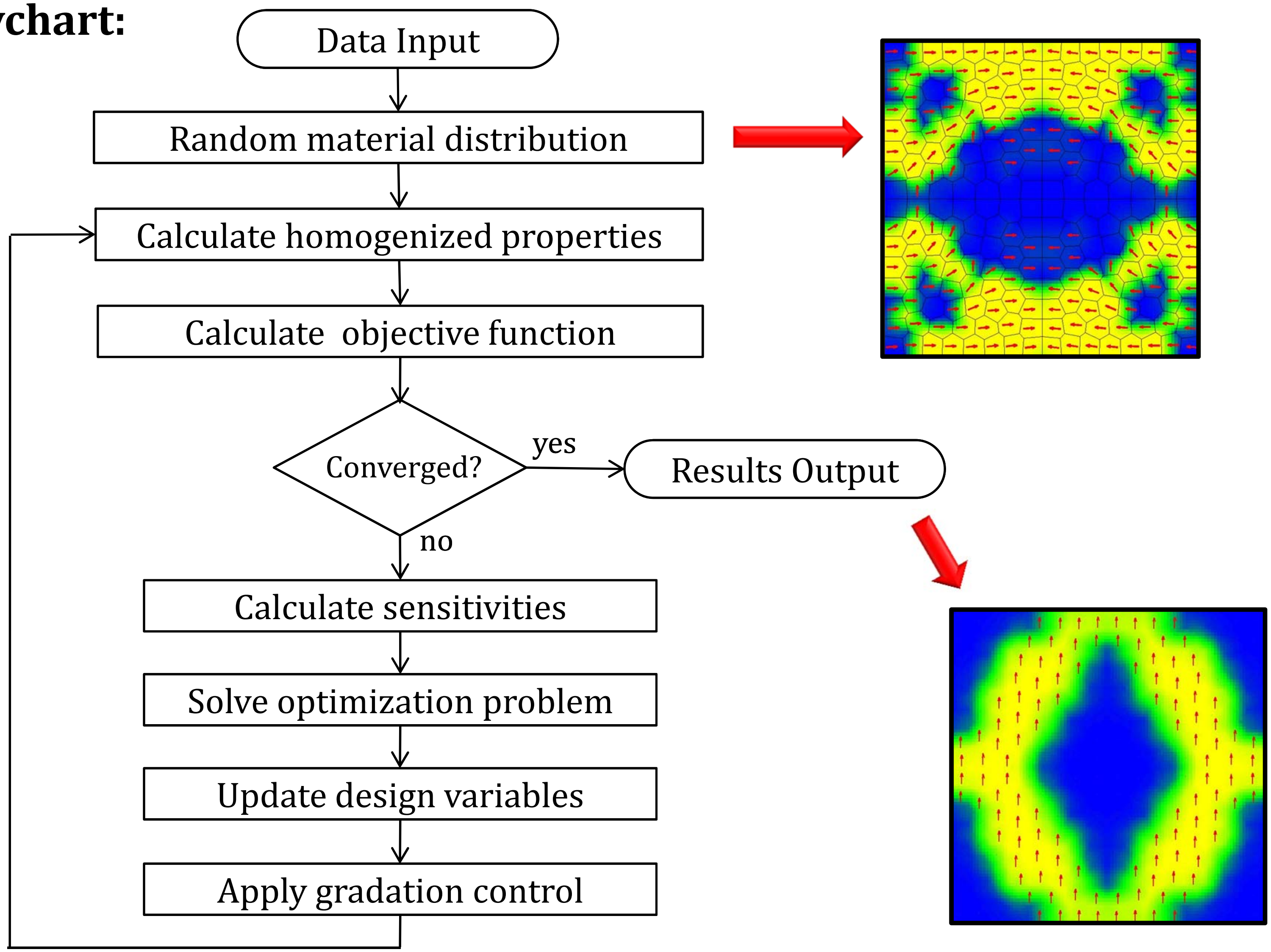
Solver:

Method of Moving Asymptotes (MMA)

Svanberg, K., "The method of moving asymptotes - A new method for structural optimization," International Journal for Numerical Methods in Engineering, Vol. 24, 1987, pp. 359–373.

Topology Optimization Method

Flowchart:



Numerical Results: Parameters

Materials:

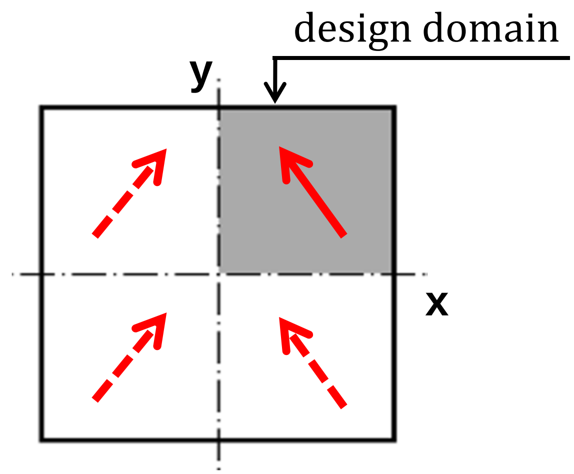
- PZT-5A ceramic
- Epoxy polymer

Mesh: 60 elements

Initial guess: random

Boundary Conditions:

- material distribution is symmetric in x and y
- polarization direction is symmetric in y

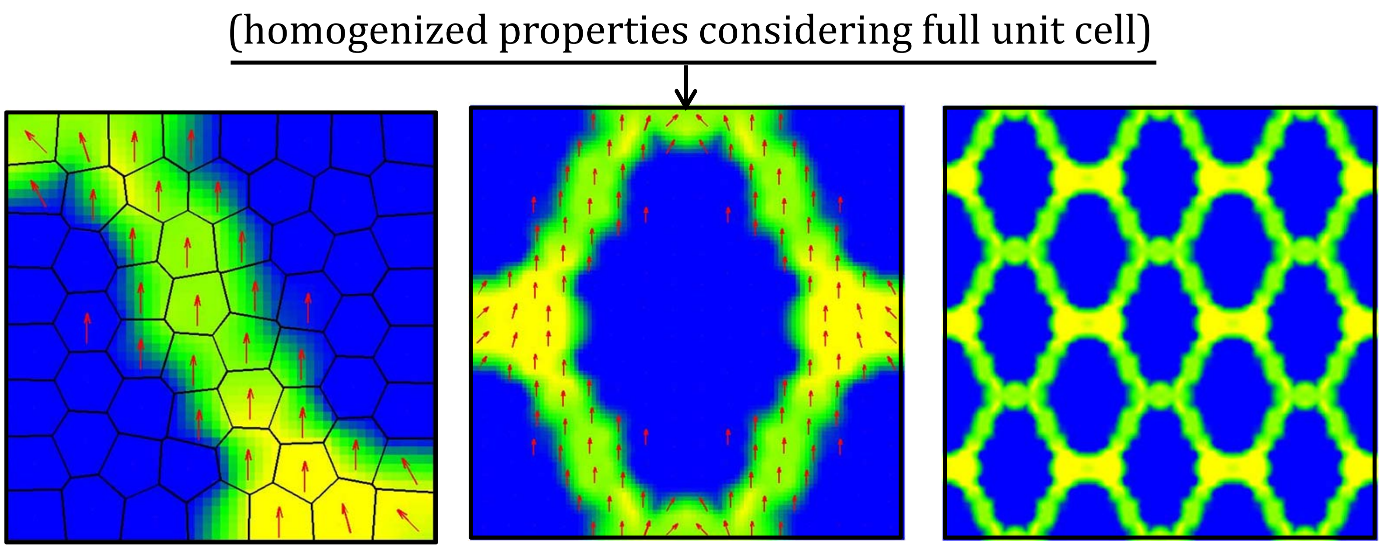


Example:

r_{grad} = 2.5%

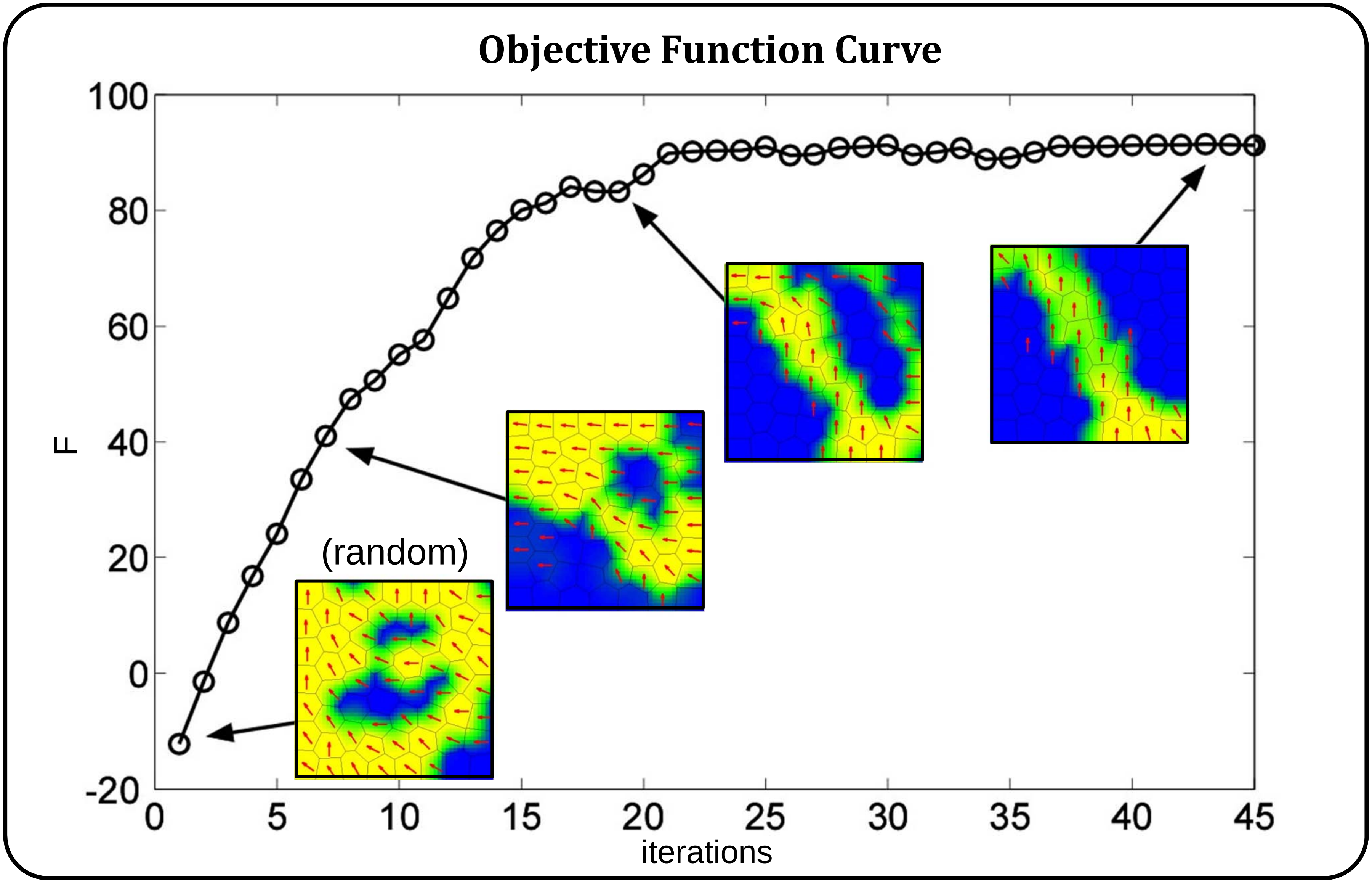
PZT-5A

Epoxy



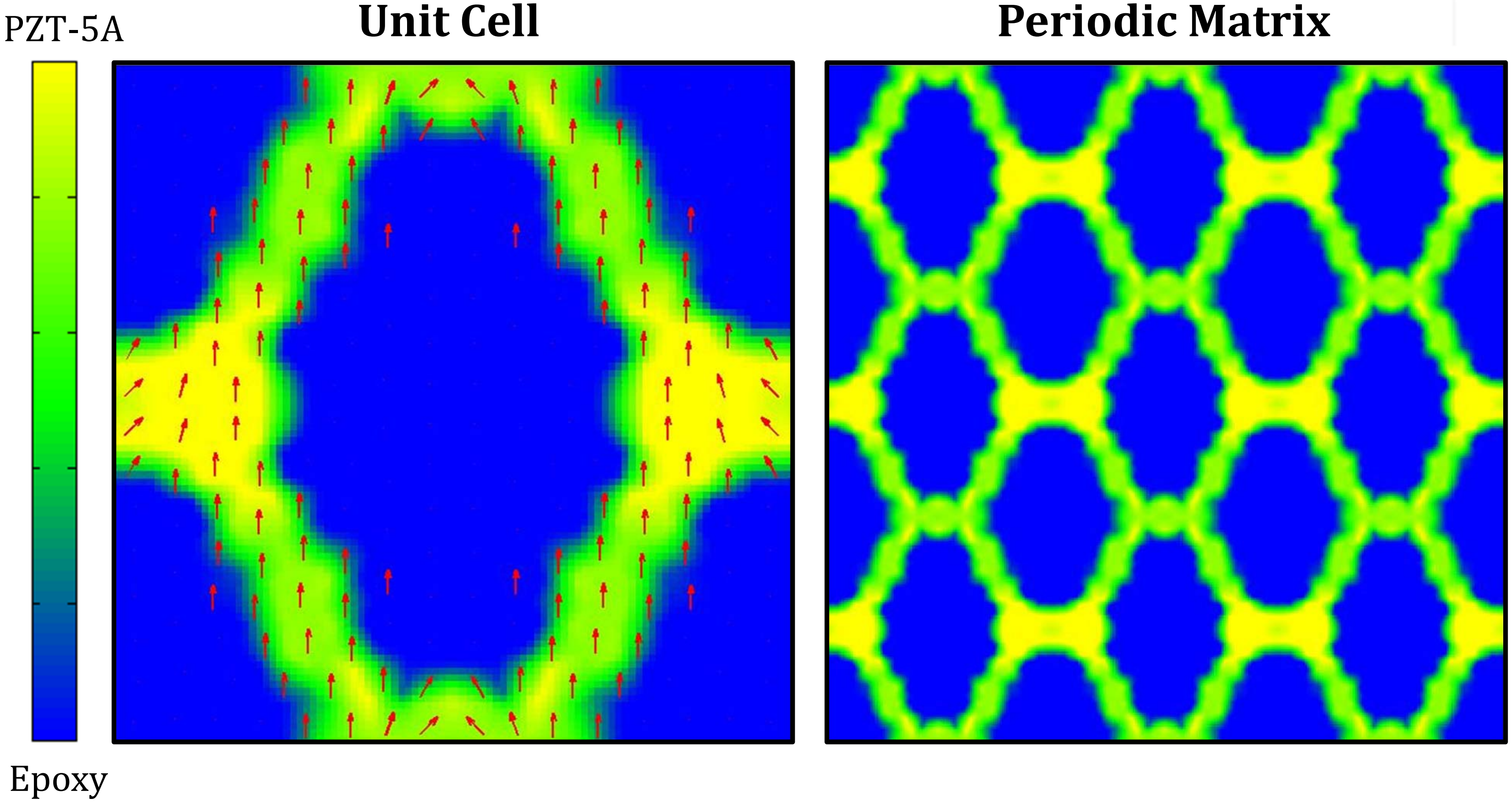
design domain unit cell periodic matrix

Numerical Results



Numerical Results: Optimized Unit Cells

r_{grad} = 2.5% | r_{grad} = 5% | r_{grad} = 7.5% | r_{grad} = 10% | r_{grad} = 12.5%



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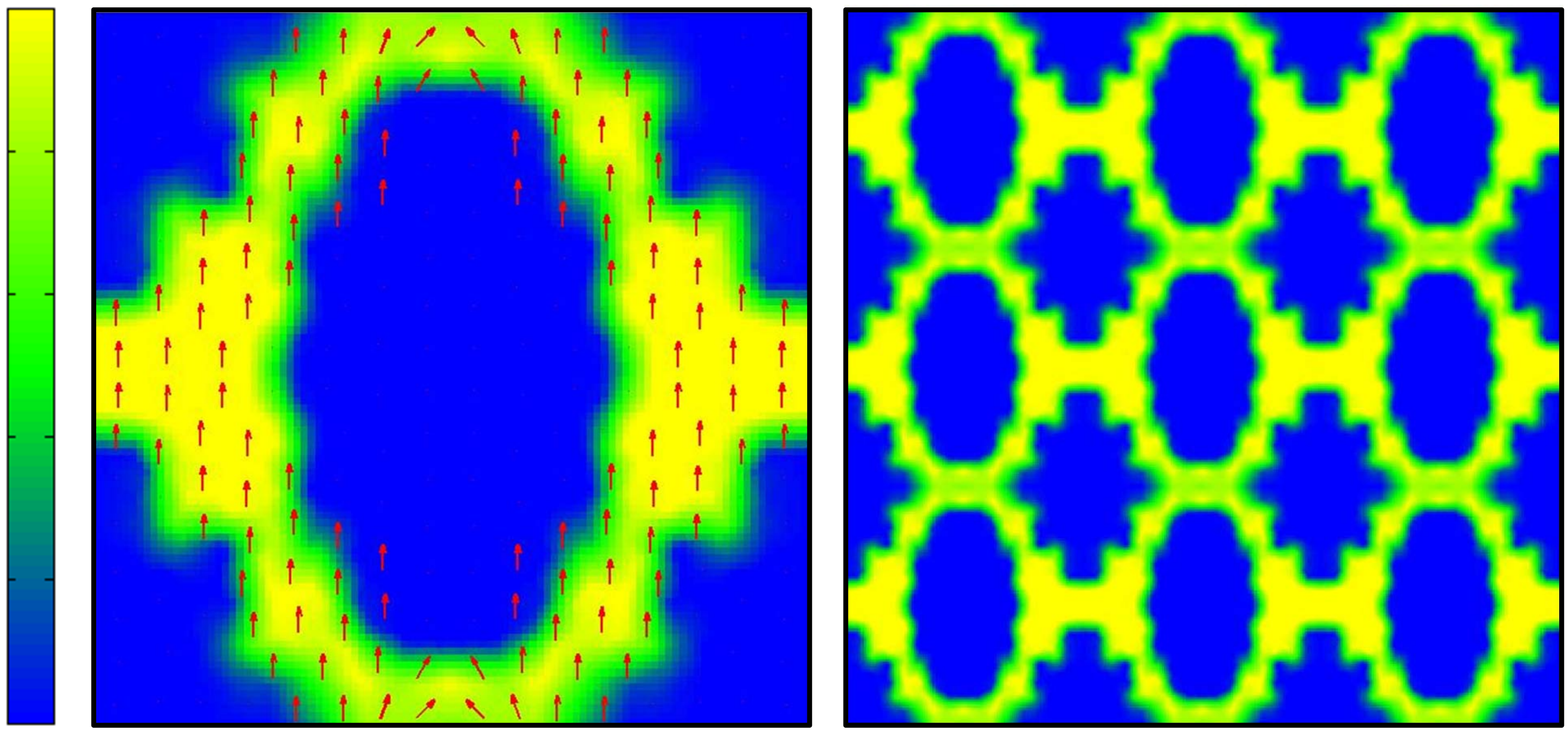
PZT-5A

Unit Cell **Periodic Matrix**



Epoxy

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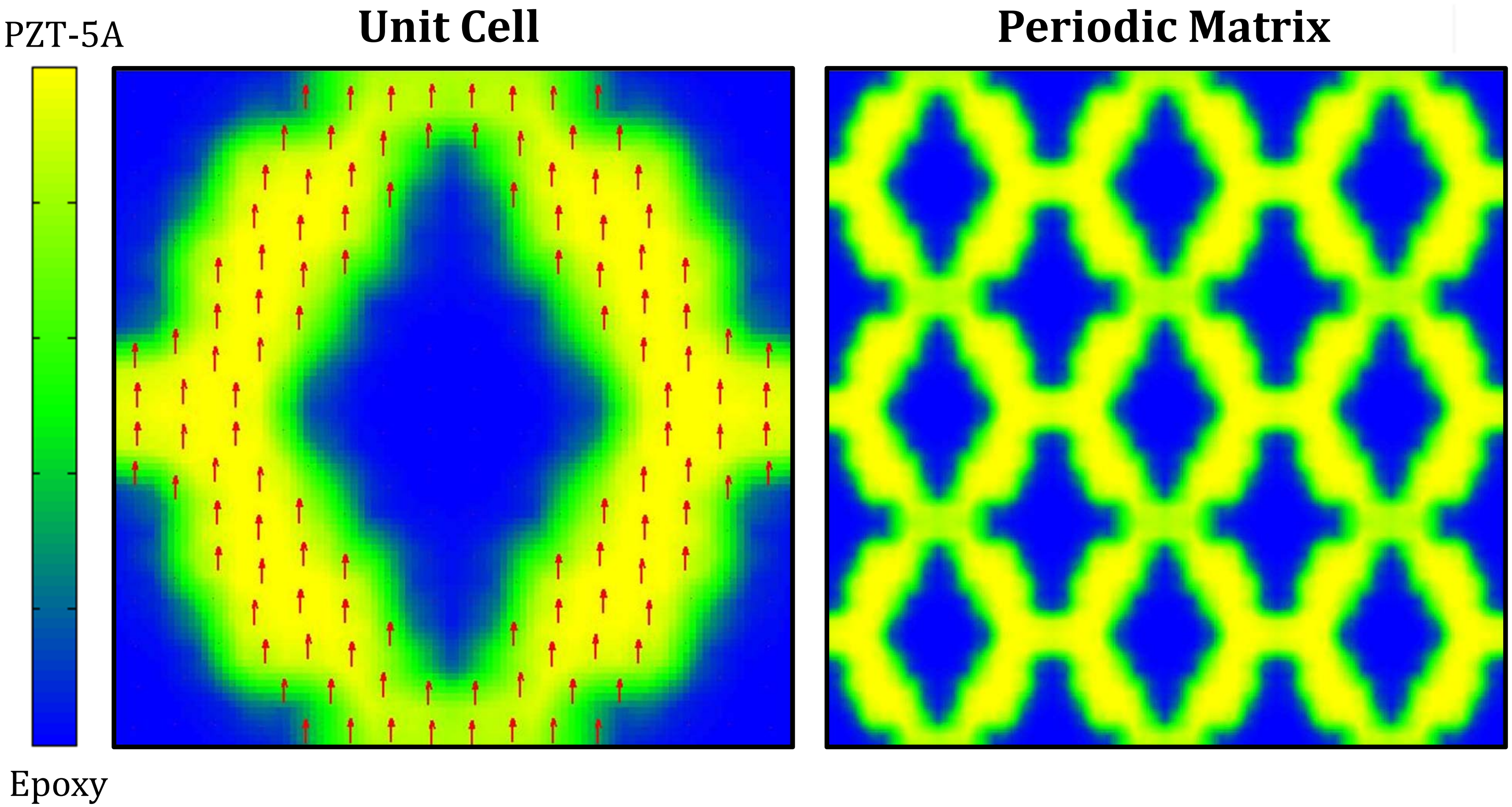

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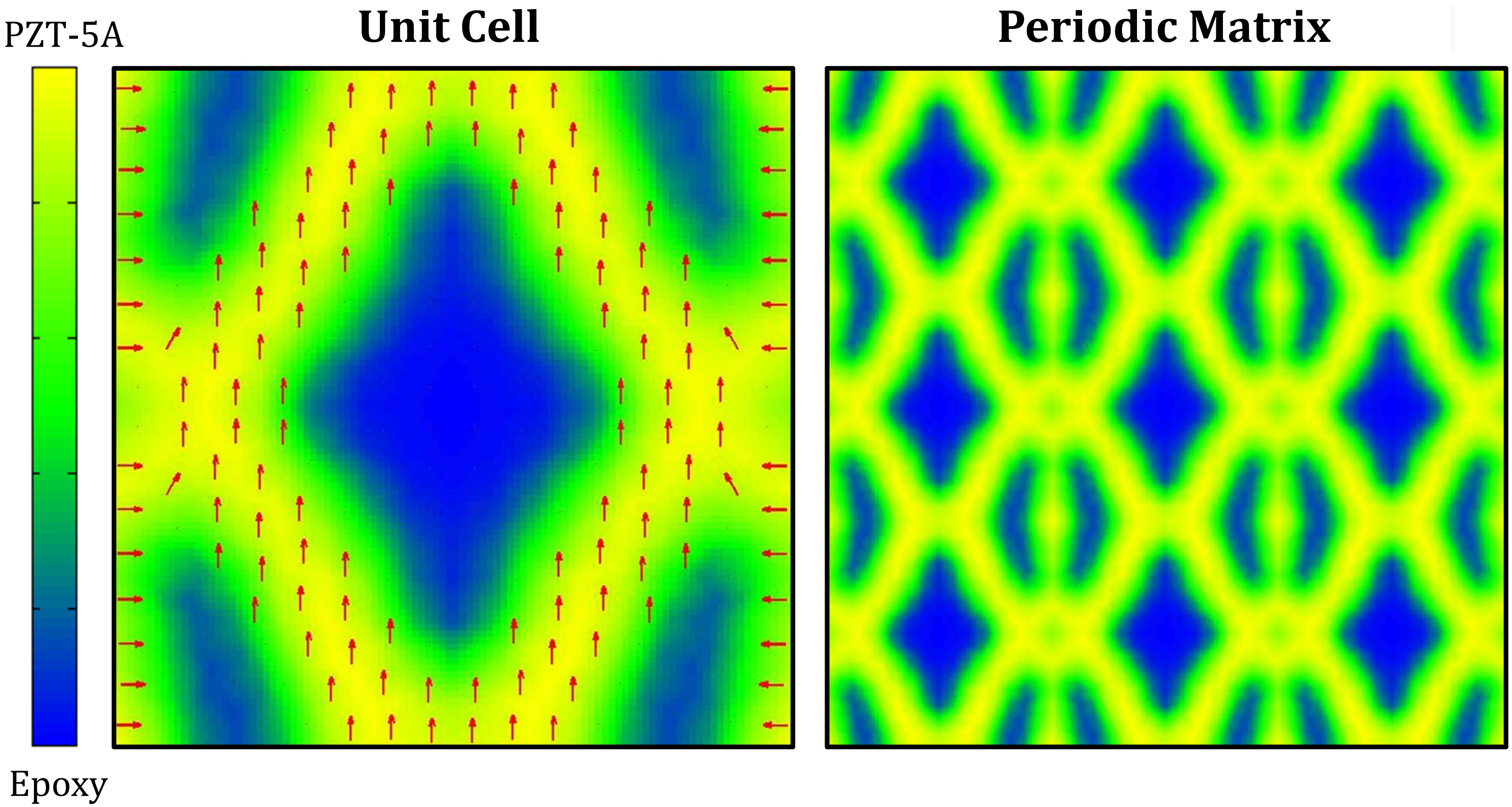

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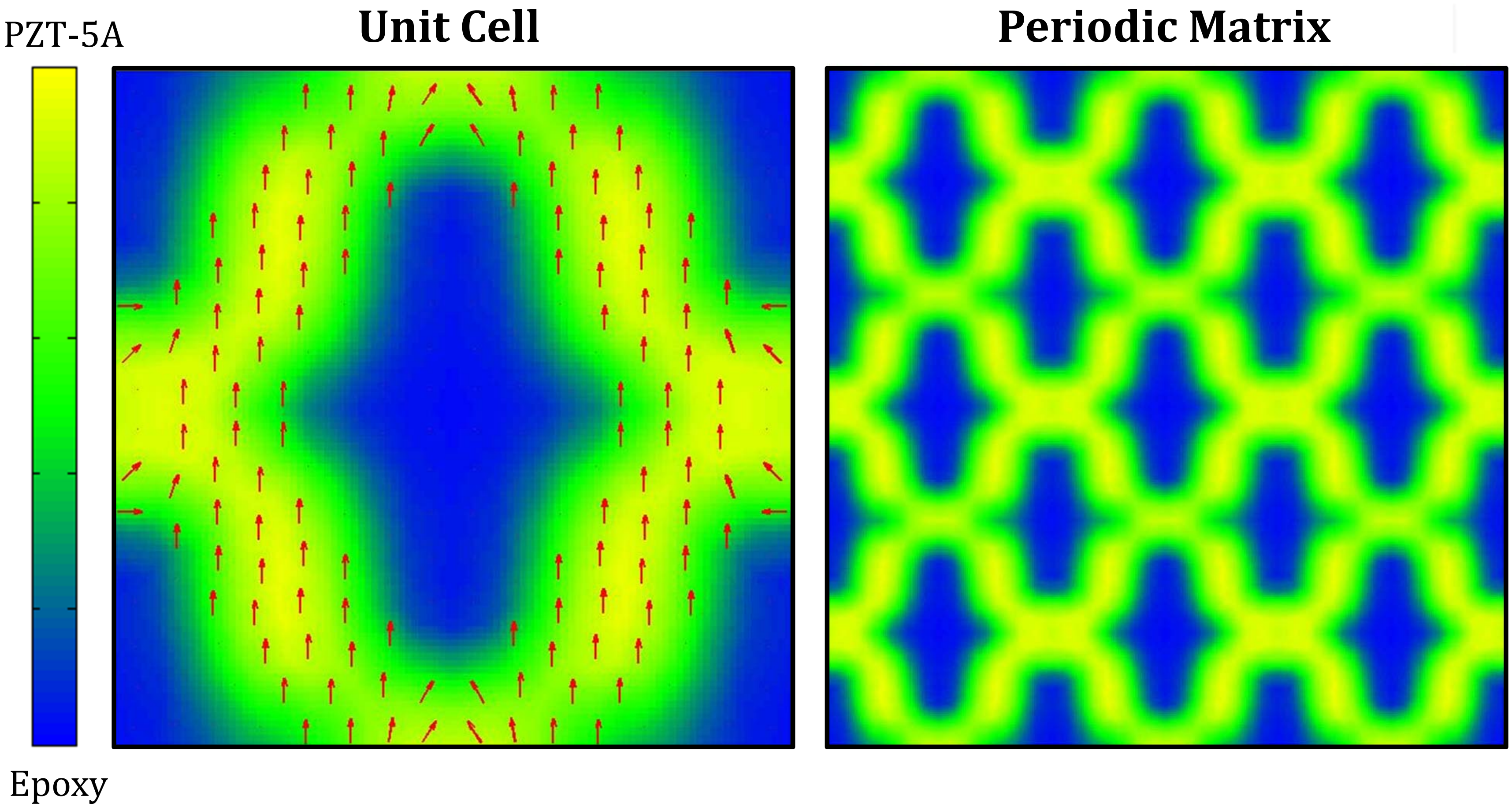
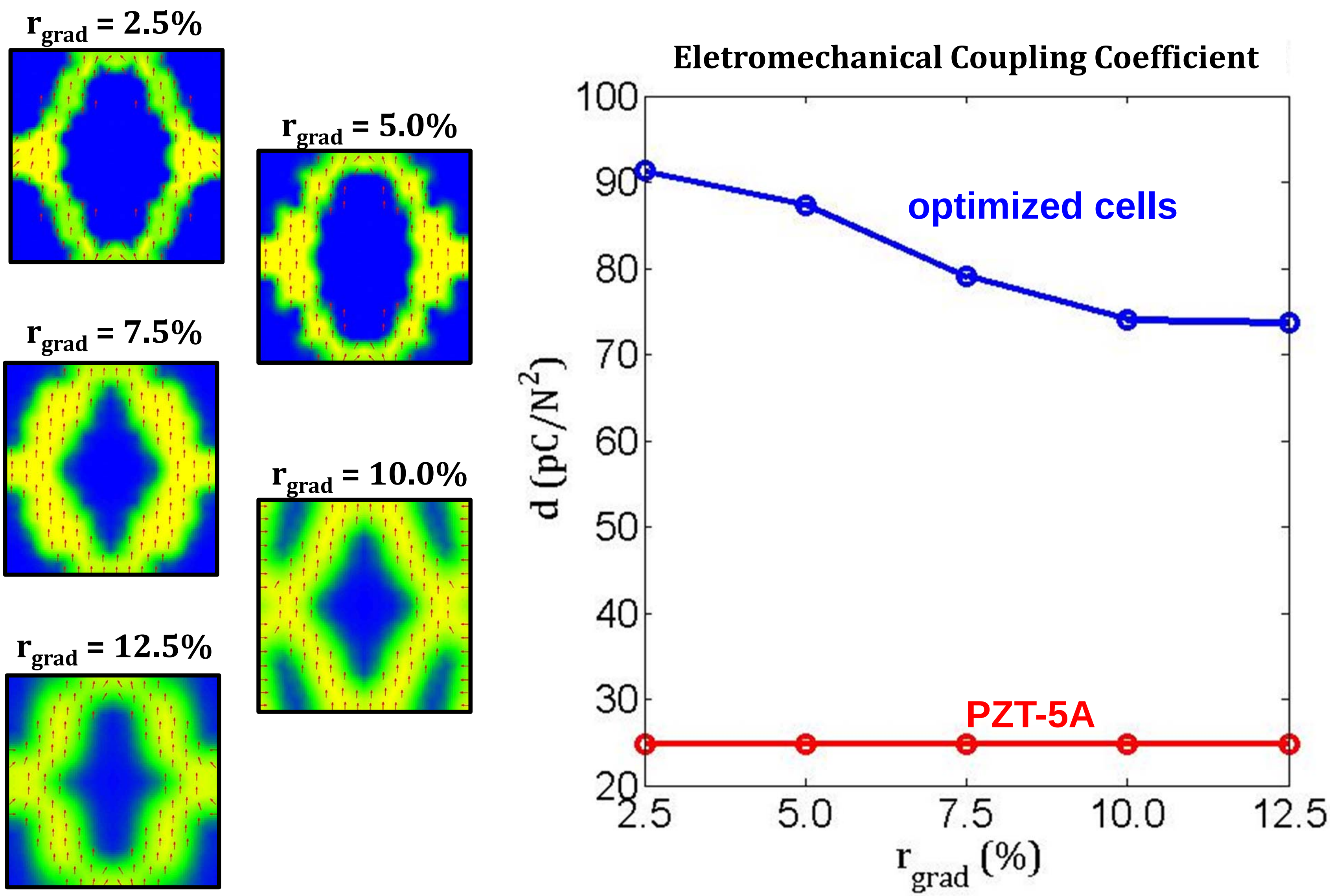
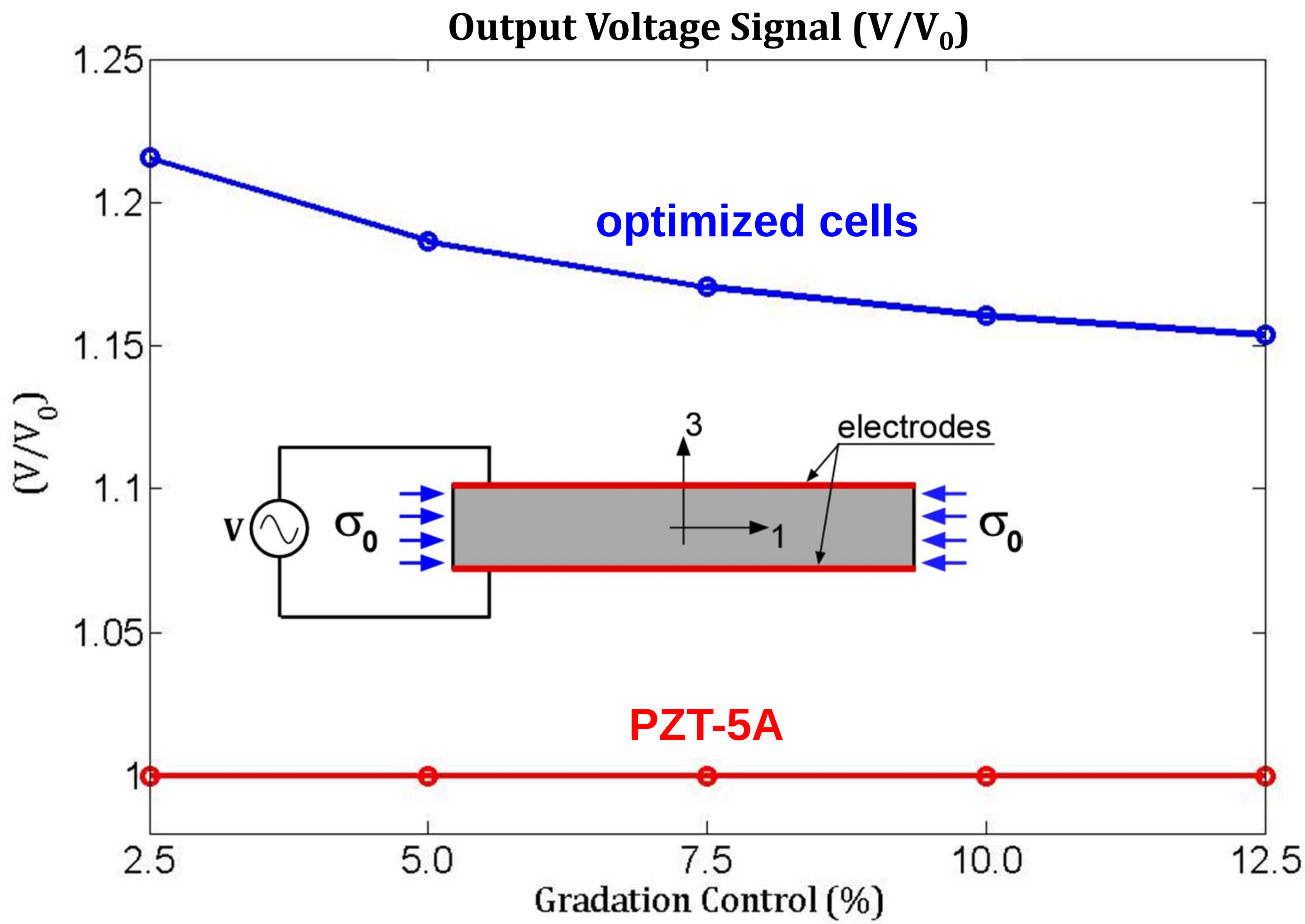


Numerical Results



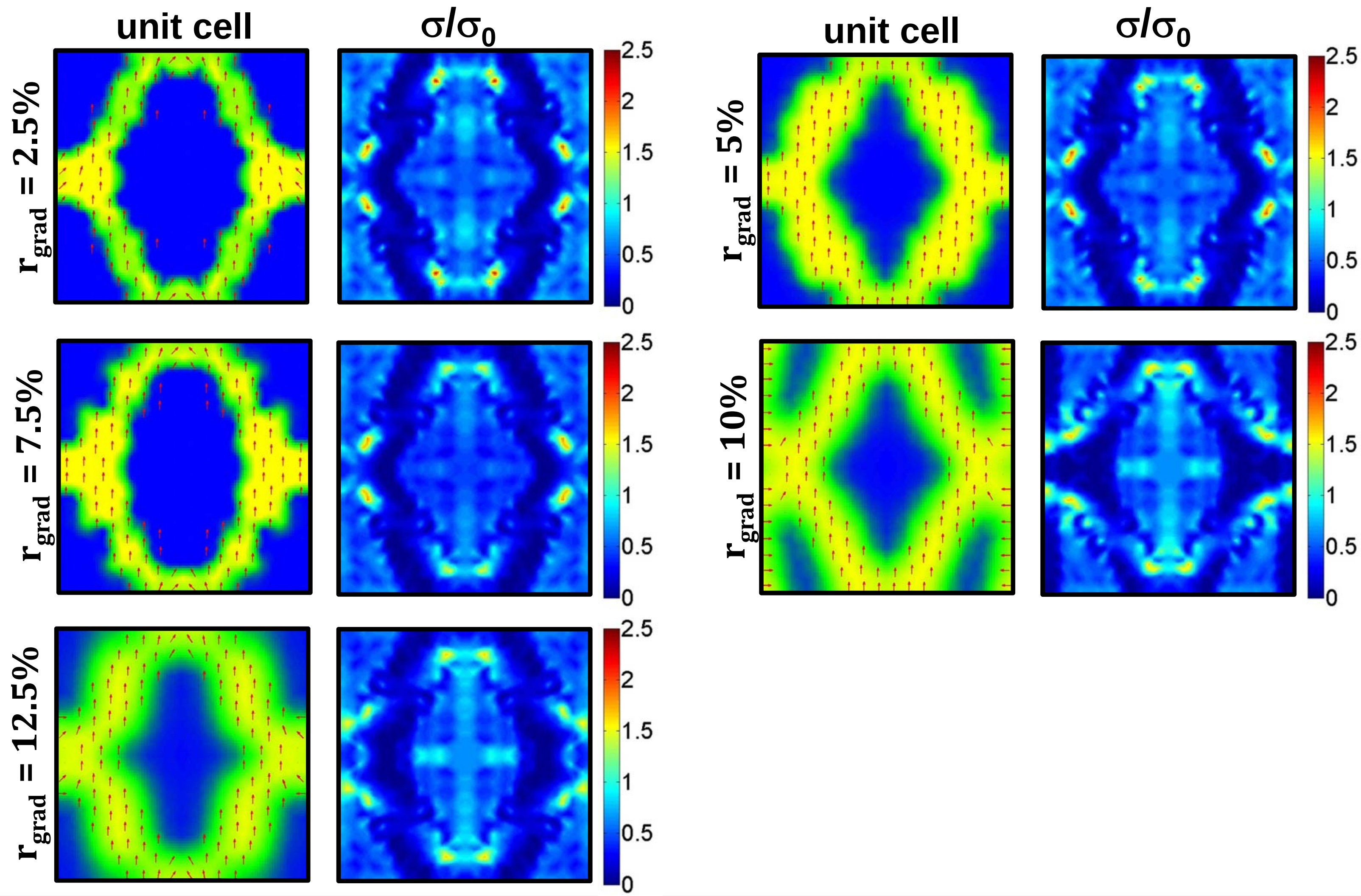
Numerical Results

Output voltage signal | Microscopic Stress | Micro Stress Graphics



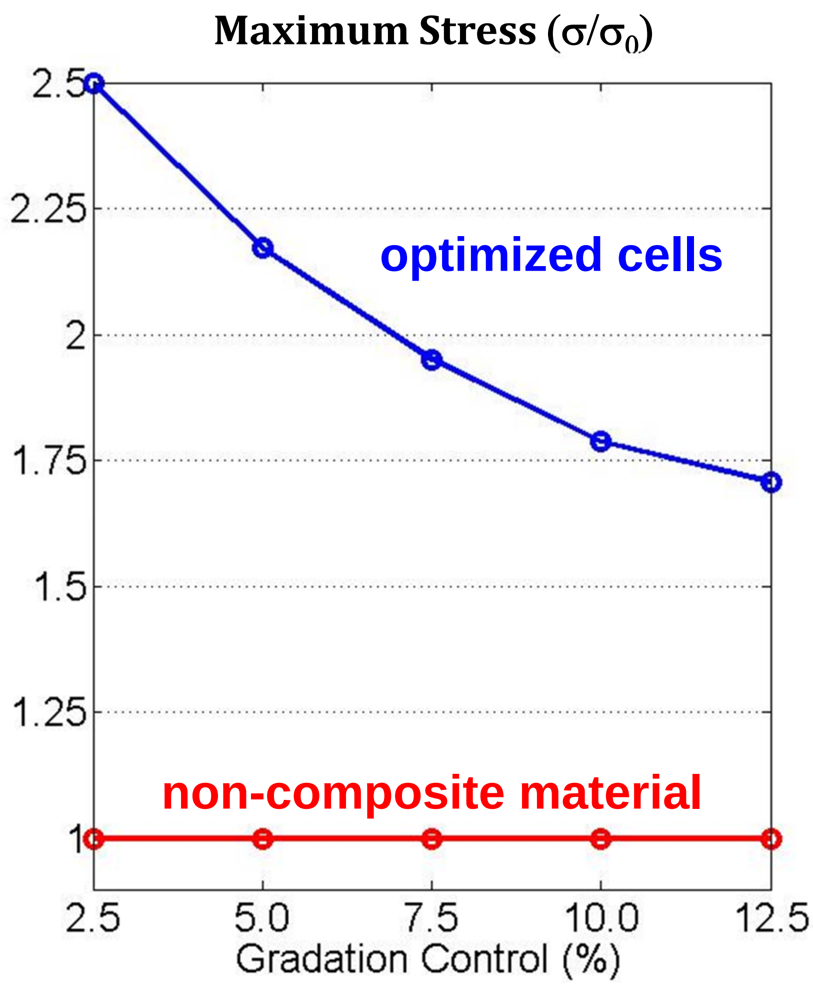
Numerical Results

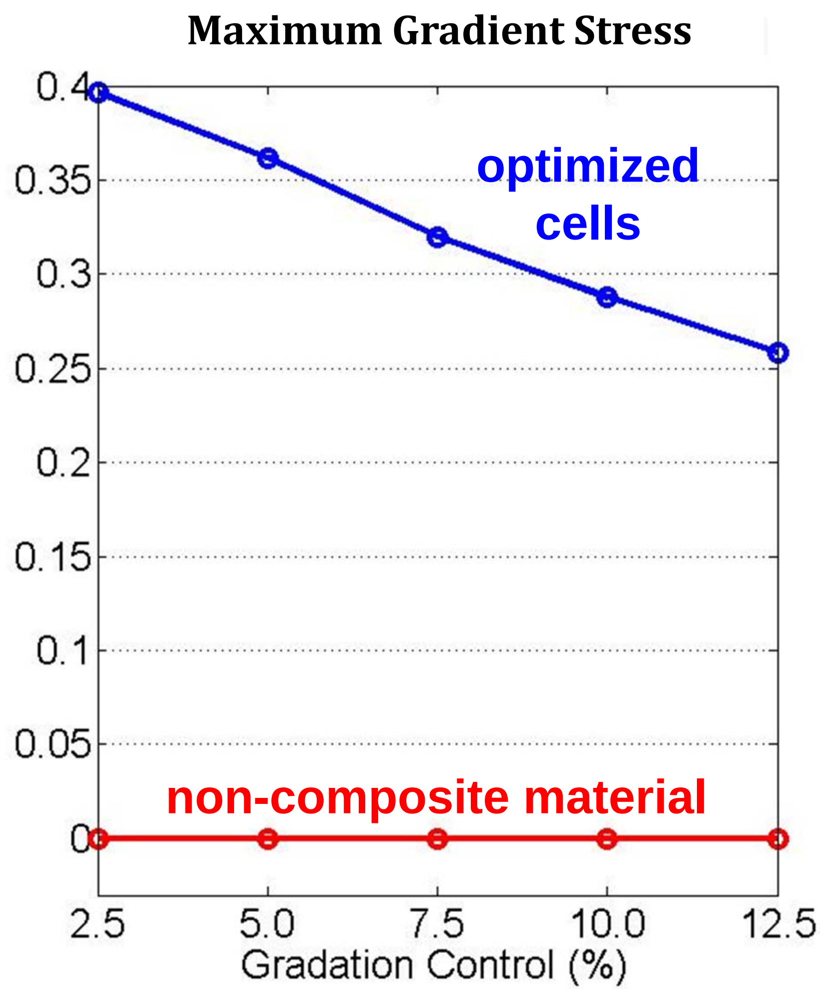
Output voltage signal | **Microscopic Stress** | Micro Stress Graphics



Numerical Results

Output voltage signal | Microscopic Stress | **Micro Stress Graphics**

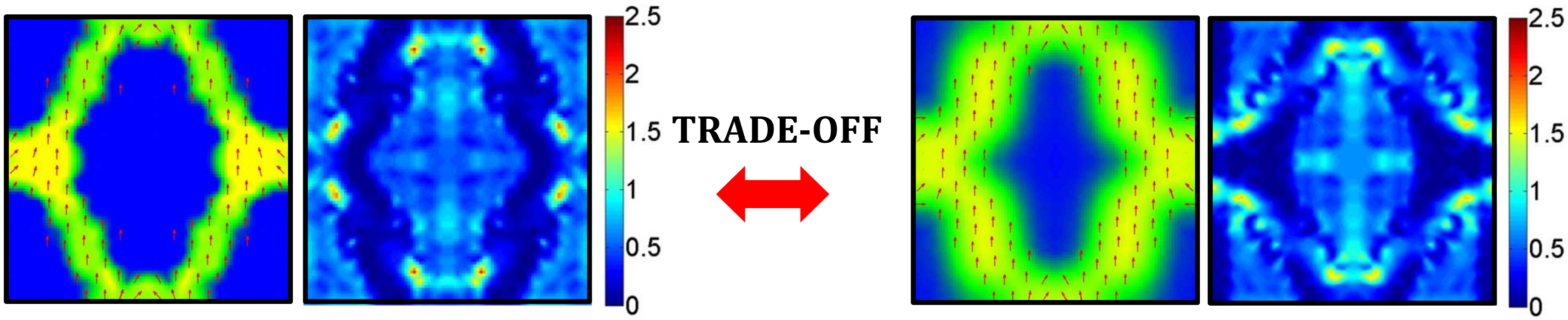




Conclusions

FGM concept

- decreases the objective function values
- reduces maximum microscopic stress
- reduces microscopic stress concentrations



DMO material model

- The variation of the polarization directions inside the unit cell helps to increase the objective function.

The End

Thank you!

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