Design of Structural Braced Frames Using Topology Optimization

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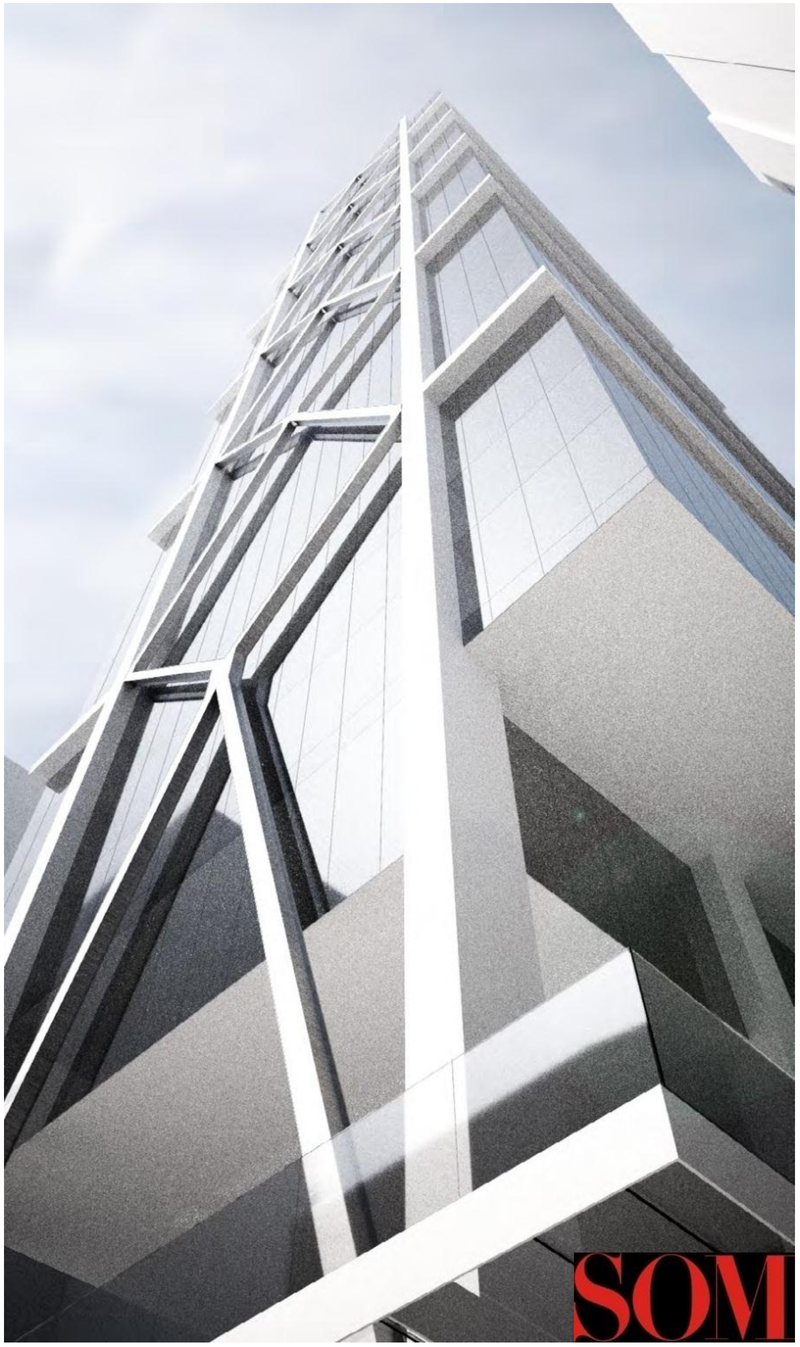
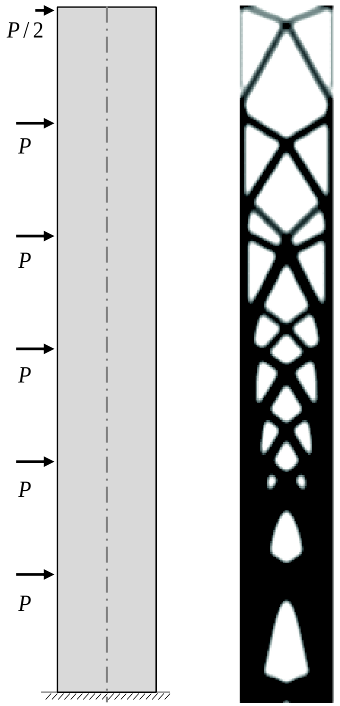
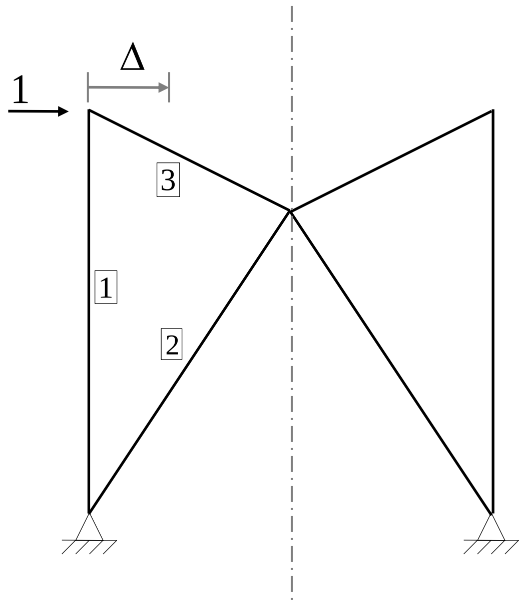


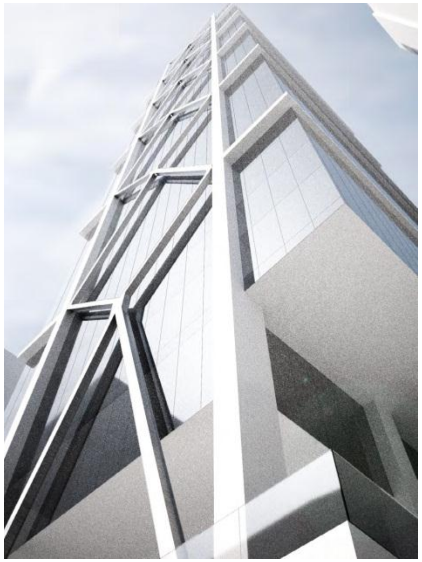
This presentation focuses on the use of topology optimization with combined elements in the building design industry



Motivation for new approach



Analytical aspects



Combining elements for structural braced frames

Architecture without evident engineering rationale is wasteful of resources and difficult to realize

Railway

http://en.wikipedia.org/wiki
/Hadid-Afragola

Olympic Stadium

http://en.wikipedia.org/wiki/Beijing_National_Stadium

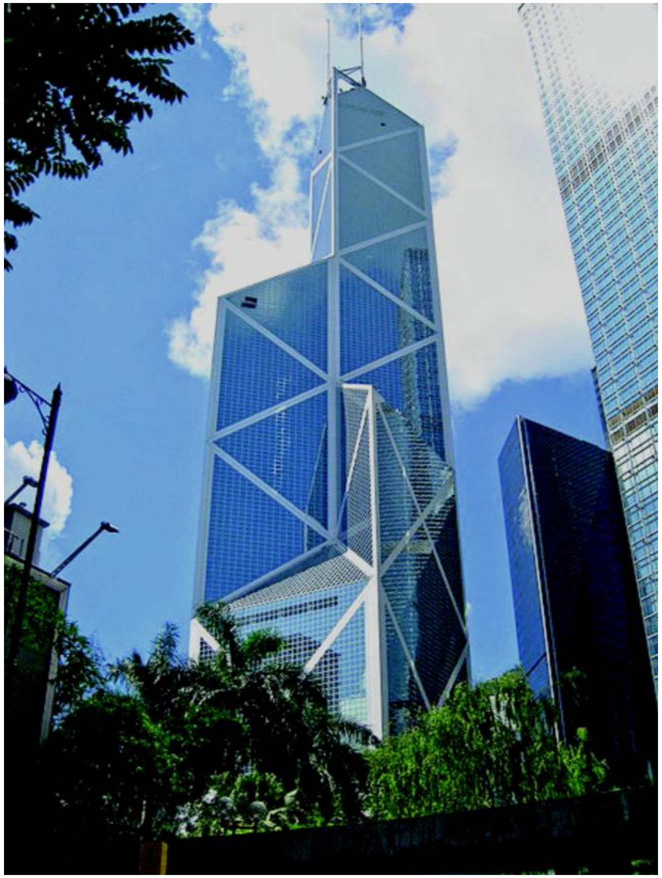
Convention Center

http://images.businessweek.com

Buildings can be designed using the lateral bracing for both engineering and architectural criteria

http://en.wikipedia.org/wiki/John_Hancock_Center

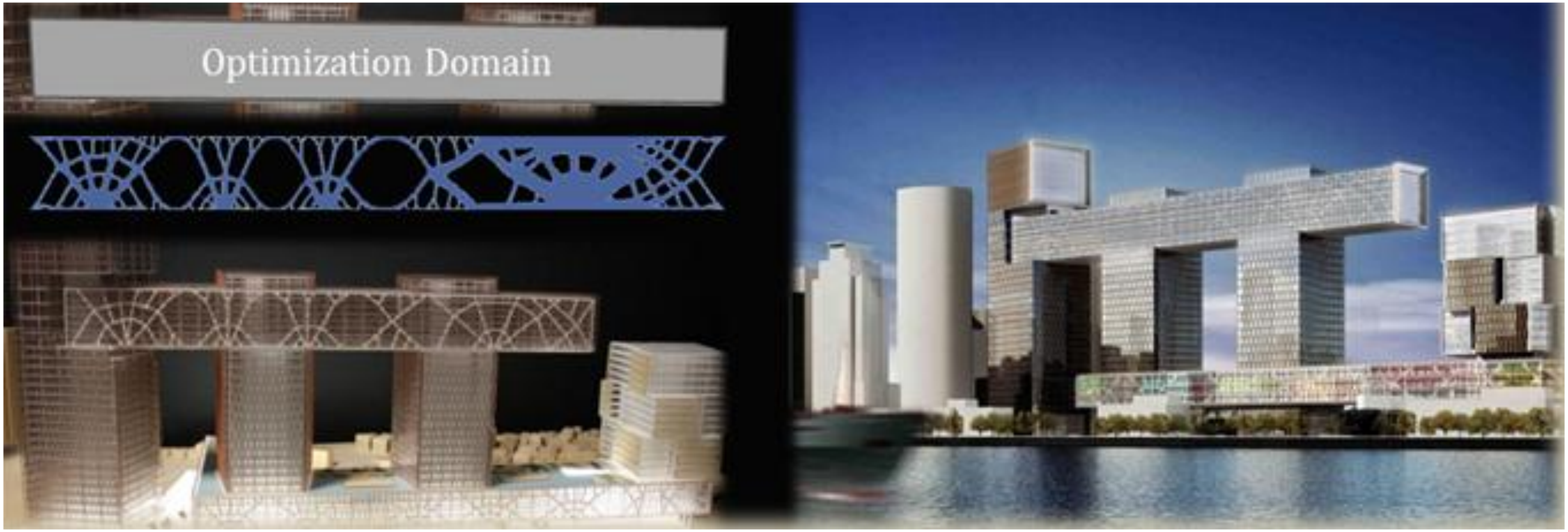
http://en.wikipedia.org/wiki/Broadgate_Tower



http://en.wikipedia.org/wiki/Bank_of_China

Stromberg, L.L., Beghini, A., Baker, W.F., Paulino, G.H. "Topology optimization for braced frames: Combining continuum and beam/column elements." *Engineering Structures* 2012, 37:106-124.

Topology optimization gives practical designs that satisfy principles from both structural engineering and architecture



*Images courtesy of SOM

Design for the "bridges" spanning between towers for the Zendai competition was a collaborative effort between UIUC and SOM

Pallasmaa, J., ed. *SOM Journal 7.* Ostfildern: Hatje Cantz Verlag, 2012.

Resulting designs resemble patterns from nature

Photography.nationalgeographic.com

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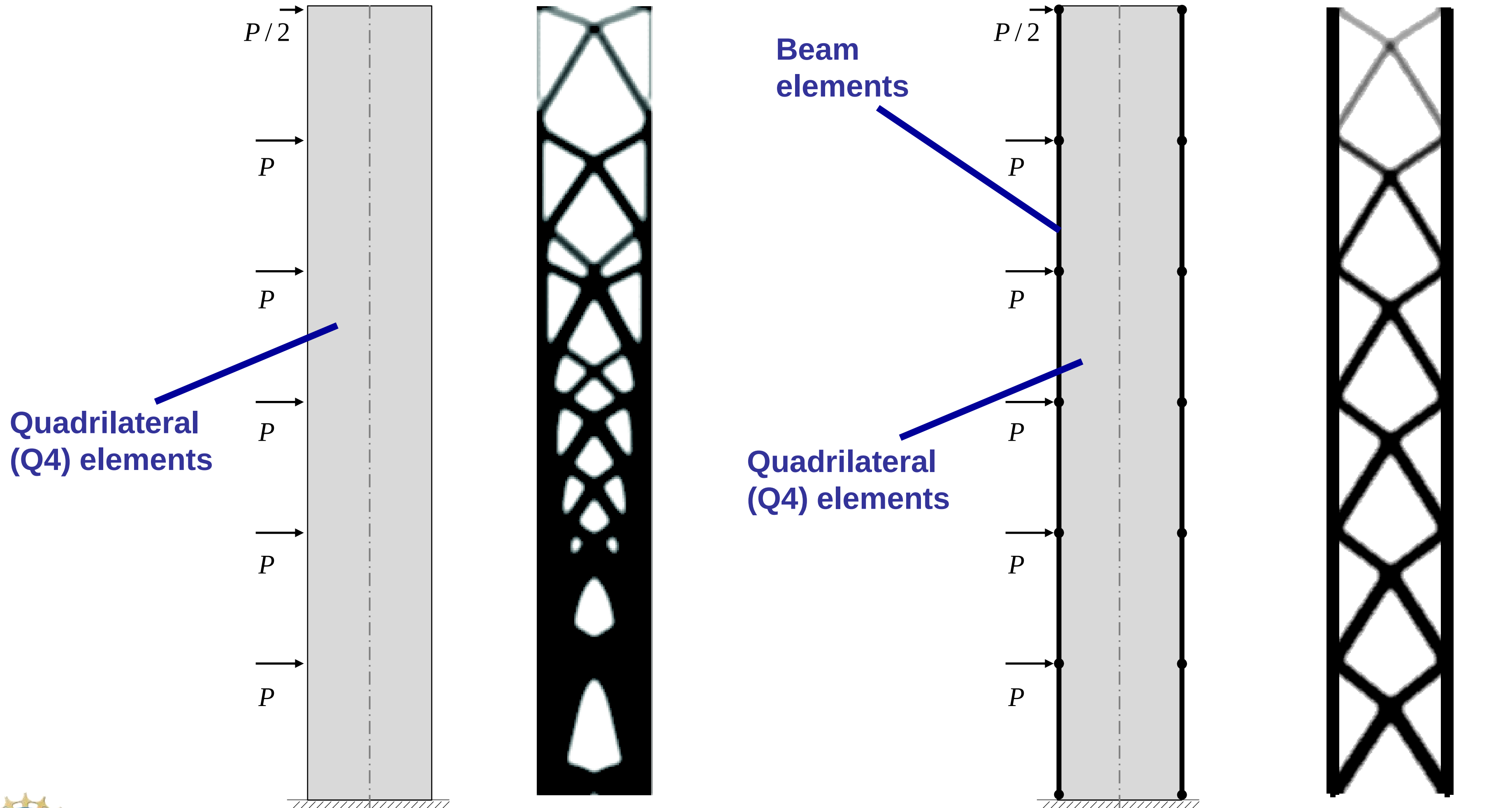
Current topology optimization techniques demonstrate limitations in design



Continuum approach **Combined approach**

Proposed Design Process

1. **Size vertical line elements according to gravity load combinations using technique of Baker (1992)[1]**

2. **Run topology optimization on the continuum elements for lateral load combinations (accounting for wind and seismic loads)**

3. **Identify optimal bracing layout to create frame model**

4. **Optimize final member sizes using virtual work methodology[1]**

5. **Iterate (if necessary)**

[1]Baker, W.F. "Energy-based design of lateral systems" *Struct Eng Int* 1992, 2:99-102.

Stromberg, L.L., Beghini, A., Baker, W.F., Paulino, G.H. "Topology optimization for braced frames: Combining continuum and beam/column elements." *Engineering Structures* 2012, 37:106-124.

Optimal design results in a state of constant stress

In terms of displacements u_i at each point of load application P_i the compliance can be expressed as

$$W_{ext} = \sum_i P_i u_i = \sum_j \frac{N_j^2 L_j}{E A_j} = W_{int}$$

By introducing the Lagrangian multiplier constraint on the volume,

$$W_{ext} = \sum_j \frac{N_j^2 L_j}{E A_j} + \lambda \left(\sum_j A_j L_j - V \right)$$

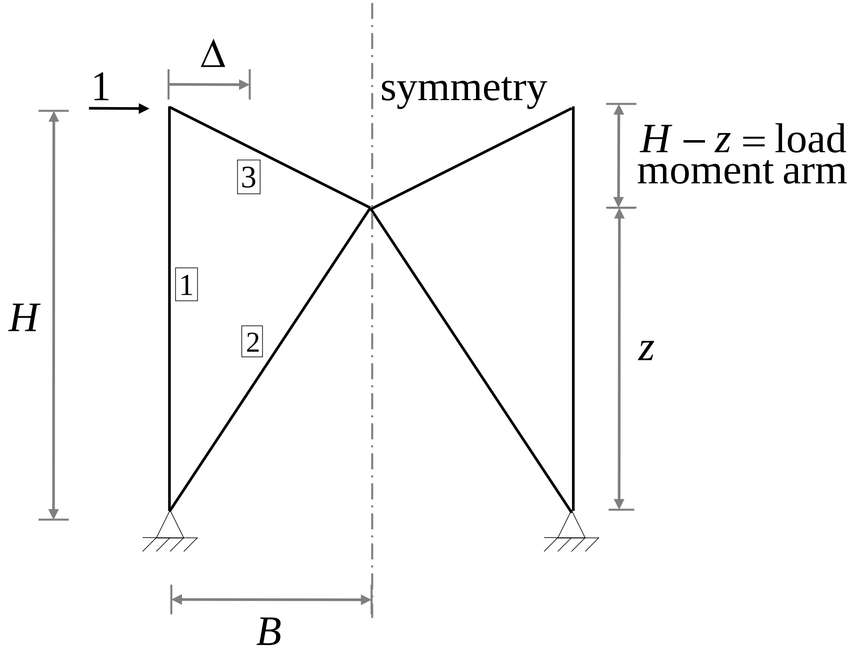
Differentiating with respect to the areas A_i and solving for the Lagrangian multiplier,

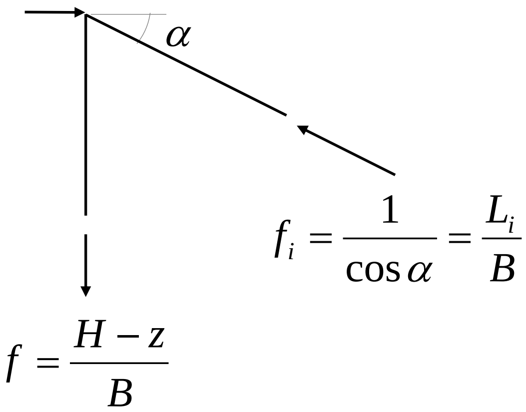
$$\lambda = \left(\frac{N_i}{A_i} \right)^2 \frac{1}{E} = \frac{\sigma^2}{E} = const$$

Optimal Single Module Bracing

The optimal bracing geometry for a single module is considered:

$$f_1 = \frac{H - z}{B}, f_2 = \frac{\sqrt{B^2 + z^2}}{B}, f_3 = \frac{-\sqrt{B^2 + (H - z)^2}}{B}$$





Optimal Single Module Bracing

Assuming each member to have a constant stress,

$$\Delta = \sum_i \frac{f_i F_i L_i}{E A_i} = \frac{\sigma B}{E} \sum_i \frac{f_i L_i}{B}$$

The tip deflection of the frame is minimal when

$$\frac{\partial \Delta}{\partial z} = \frac{\sigma B}{E} \frac{\partial}{\partial z} \left(\sum_i \frac{f_i L_i}{B} \right) = 0$$

$$= \frac{\sigma B}{E} \frac{\partial}{\partial z} \left(H \left(\frac{H - z}{B^2} \right) + \frac{B^2 + z^2}{B^2} + \frac{B^2 + (H - z)^2}{B^2} \right) = 0$$

Thus, the brace work point height is optimal at

$$\boxed{z = \frac{3}{4} H}$$

These rules can be generalized for multiple modules

Generalizing the previous equations for N modules,

$$
\begin{aligned}
\Delta &= \frac{\sigma B}{E} \sum_i \frac{f_i L_i}{B} \\
&= \frac{\sigma B}{E} \left[\sum_i \left(\frac{L_i^2}{B^2} \right)_{braces} + \sum_j \left(\frac{(H - z_j) L_j}{B^2} \right)_{columns} \right] \\
&= \frac{\sigma B}{E} \left[\sum_{n=1}^{N} (z_{2n} - z_{2n-2}) \frac{H - z_{n-1}}{B^2} + \frac{B^2 + (z_{2n-1} - z_{2n-2})^2}{B^2} + \frac{B^2 + (z_{2n-1} - z_{2n-2})^2}{B^2} \right]
\end{aligned}
$$

Differentiating,

$$
\frac{\partial}{\partial z_{2n-1}} \left(\frac{E\Delta}{\sigma B} \right) = 0 \rightarrow -3z_{2n} + 4z_{2n-1} - z_{2n-2} = 0
$$

$$
\frac{\partial}{\partial z_{2n}} \left(\frac{E\Delta}{\sigma B} \right) = 0 \rightarrow -z_{2n+1} + 4z_{2n} - 3z_{2n-1} = 0
$$

Stromberg, L.L., Beghini, A., Baker, W.F., Paulino, G.H. "Topology optimization for braced frames: Combining continuum and beam/column elements." *Engineering Structures* 2012, 37:106-124.

Optimal Multiple Modules Bracing

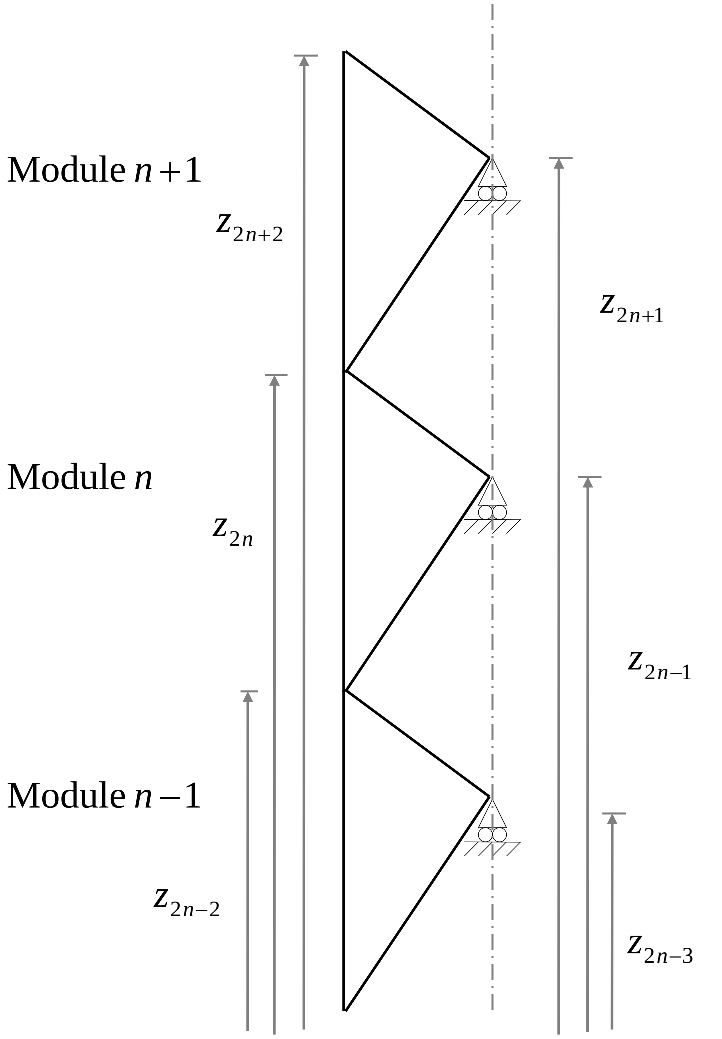
Rewriting these equations

$$z_{2n} = \frac{z_{2n-1} + z_{2n+1}}{2} - \frac{z_{2n+1} - z_{2n-1}}{4}$$

$$z_{2n-1} = \frac{z_{2n-2} + z_{2n}}{2} + \frac{z_{2n} - z_{2n-2}}{4}$$

This leads to two conclusions:

1. **The braced frame central work point is always at 75% of the module height.**
2. **The module heights are all equal, indicating that patterns are optimal.**



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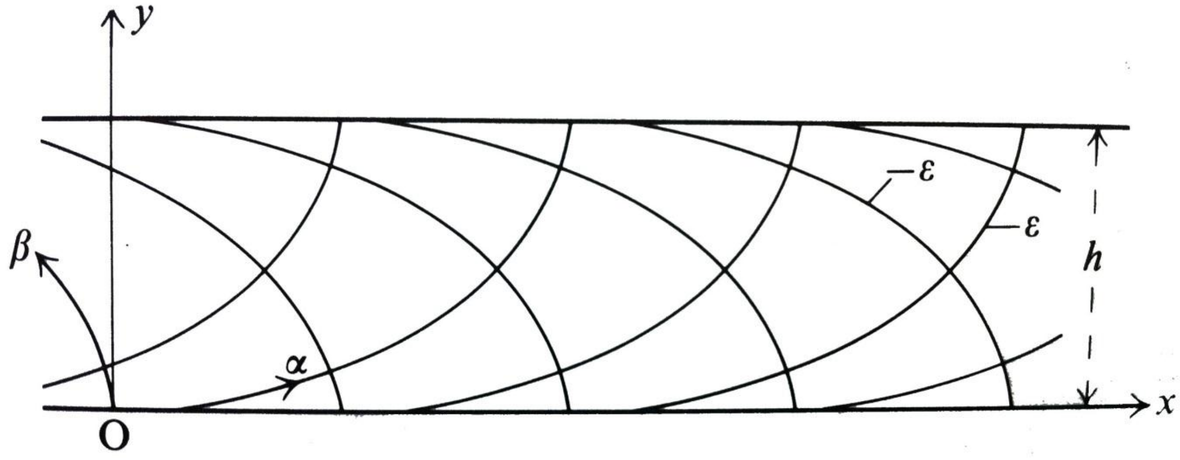
These results can be verified using Michell solutions



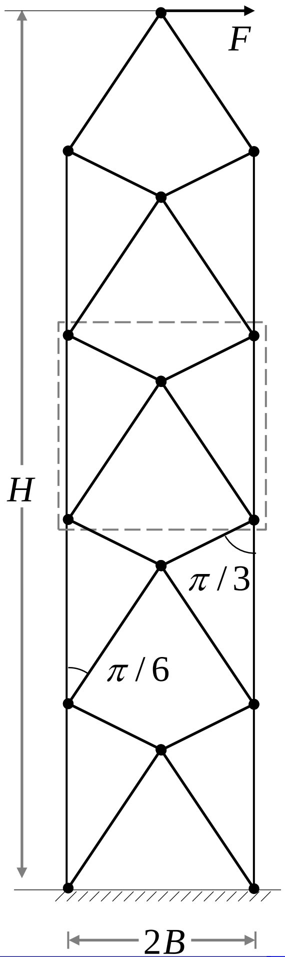
In 1904, Michell derived the mathematics behind structures of least volume, or *optimal structures*

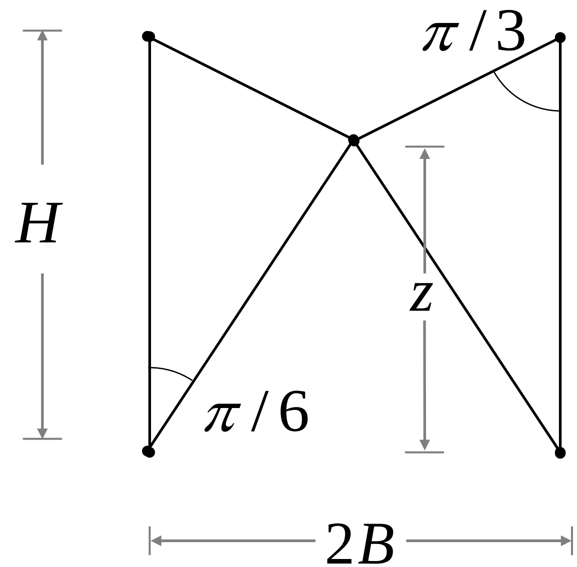
Hemp discusses the example of a cantilever beam subject to a point load with solutions formed by cycloids using the conditions set forth in Michell (1904).

Michell, A.G.M. "The Limits of Economy of Material in Frame-Structures." *Phil Mag* 8(47):589-597
Hemp, W.M. *Optimum Structures*. Oxford: Claredon Press, 1973.

These results can be verified using Michell solutions

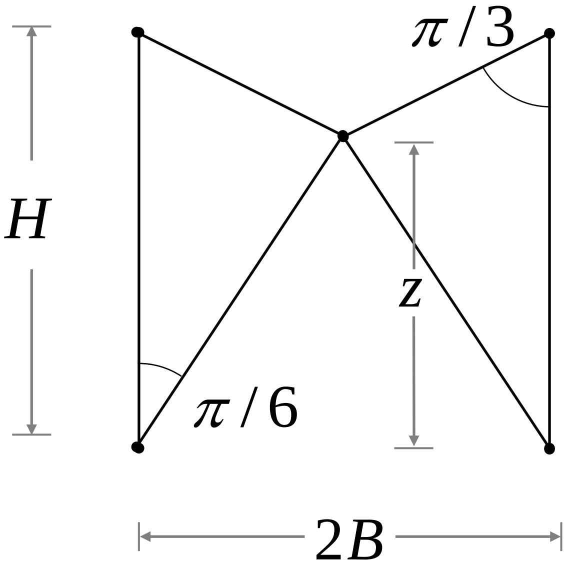
In Hemp (1973), an application of the Michell truss solution is given by the optimal shear bracing with the discretization of cycloids:

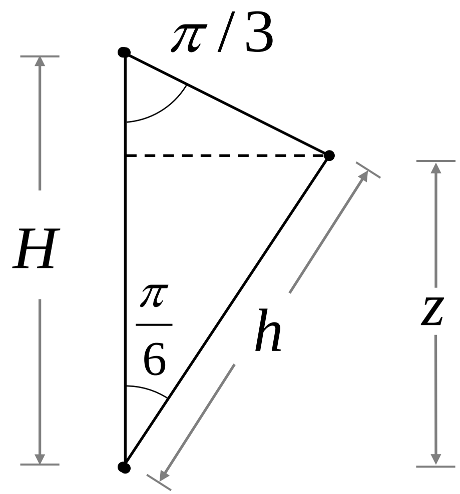




Optimal geometry given by angles – how does this correspond to the 75% rule?

These results can be verified using Michell solutions





$$h = H \cos(\pi / 6)$$

$$z = h \cos(\pi / 6) = H \left(\frac{\sqrt{3}}{2} \right)^2 = \boxed{\frac{3}{4} H}$$

Same results!

Topology Optimization Framework

Problem statement:

$$\min_{\rho,\boldsymbol{u}}: \quad c(\rho,\boldsymbol{u})$$
$$s.t.: \quad \boldsymbol{K}(\rho)\mathbf{u} = \mathbf{f}$$
$$\int_{\Omega} \rho \, dV \leq V_s$$

Objective function

Equilibrium constraint

Volume constraint

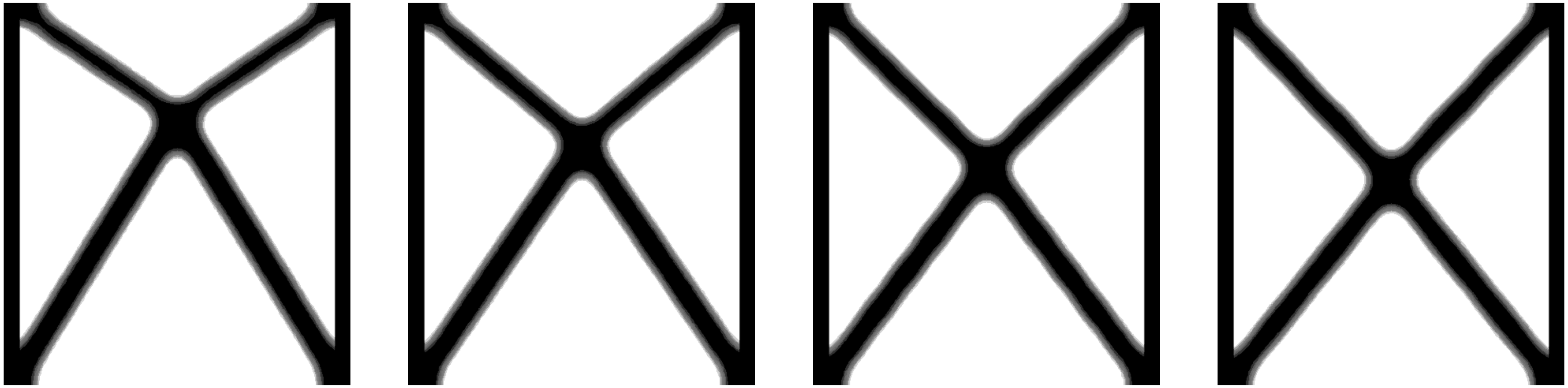
Using Solid Isotropic Material with Penalization (SIMP) model,

$$E(\boldsymbol{x}) = \rho(\boldsymbol{x})^p E^0, \quad p > 1$$

Bendsoe, M.P., Sigmund, O. *Topology Optimization: Theory, Methods and Applications*. Springer, 2002.

The optimal bracing point is highly influenced by the stress levels

Higher areas than constant stress state give lower bracing points

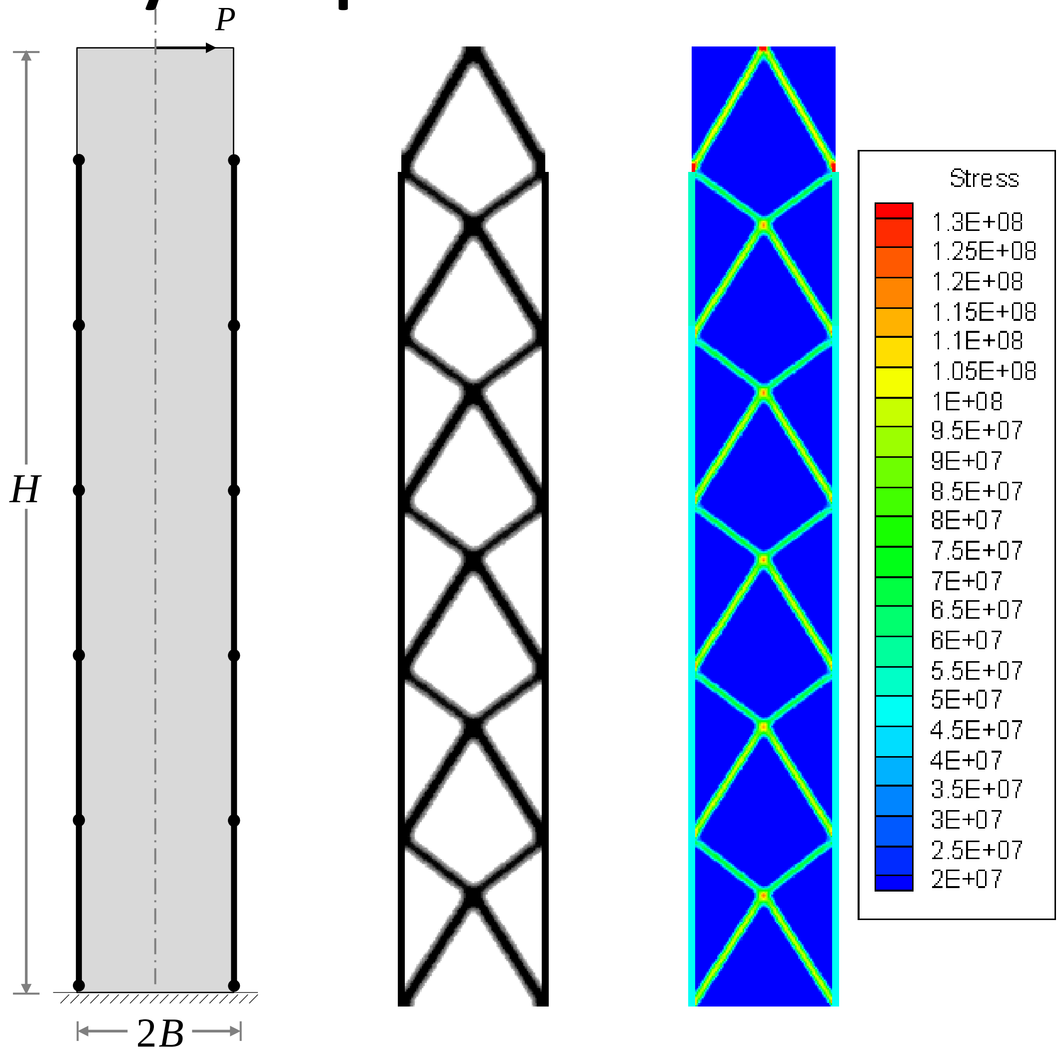


$A = A_0$ $\quad$ $A = 2A_0$ $\quad$ $A = 5A_0$ $\quad$ $A = 10A_0$

Area sized for constant stress

Stromberg, L.L., Beghini, A., Baker, W.F., Paulino, G.H. "Topology optimization for braced frames: Combining continuum and beam/column elements." ***Engineering Structures*** **2012, 37:106-124.**

Topology optimization results give practical structures and verify Hemp's solution



No pattern constraints

Each module has a 75% bracing point!

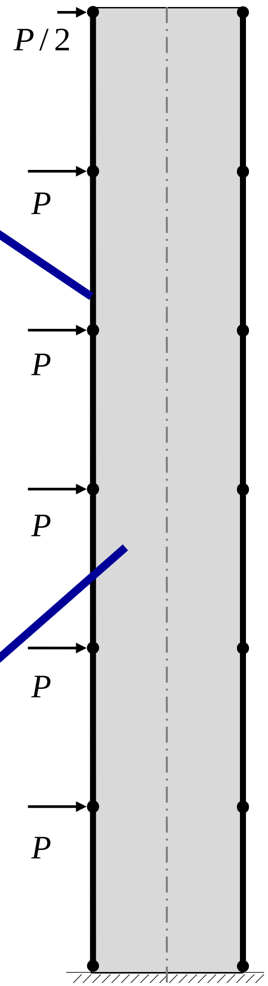
Stresses are constant.

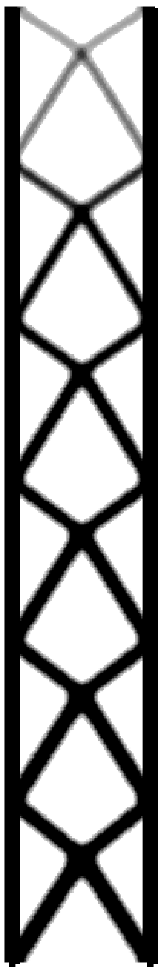
Stromberg, L.L., Beghini, A., Baker, W.F., Paulino, G.H. "Topology optimization for braced frames: Combining continuum and beam/column elements." *Engineering Structures* 2012, 37:106-124.

The combination of different element types is suitable for the design of lateral braced frames

Beam elements

Quadrilateral (Q4) Elements





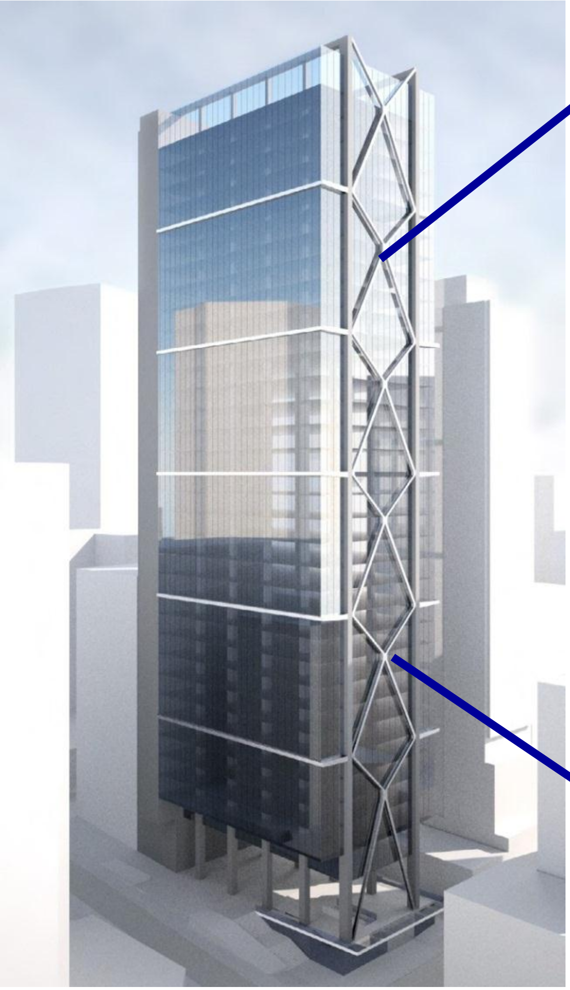
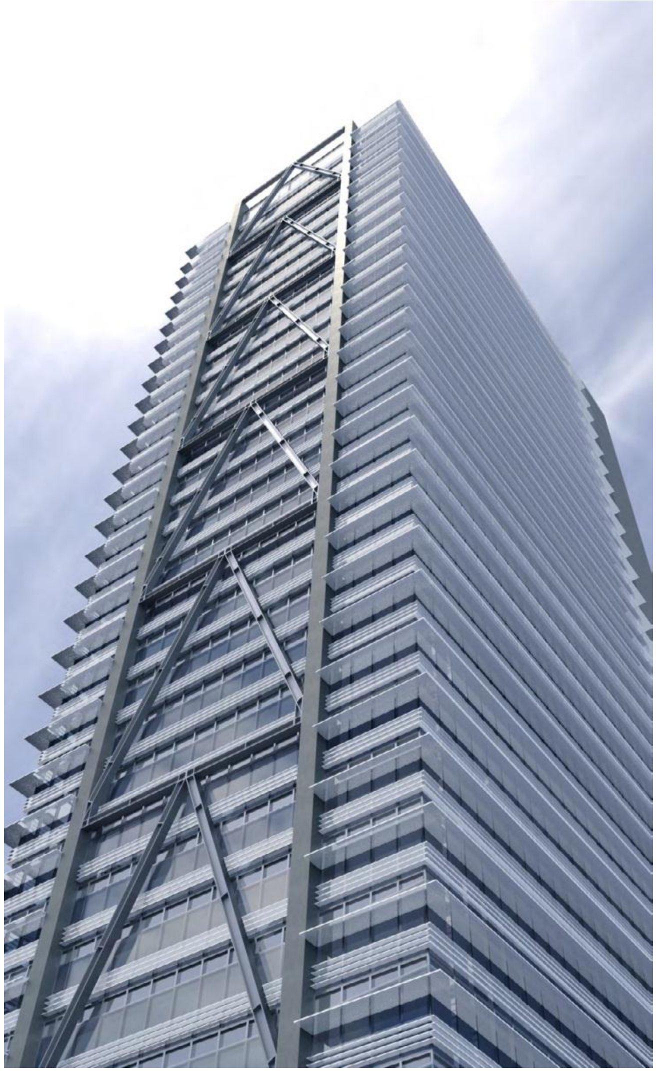


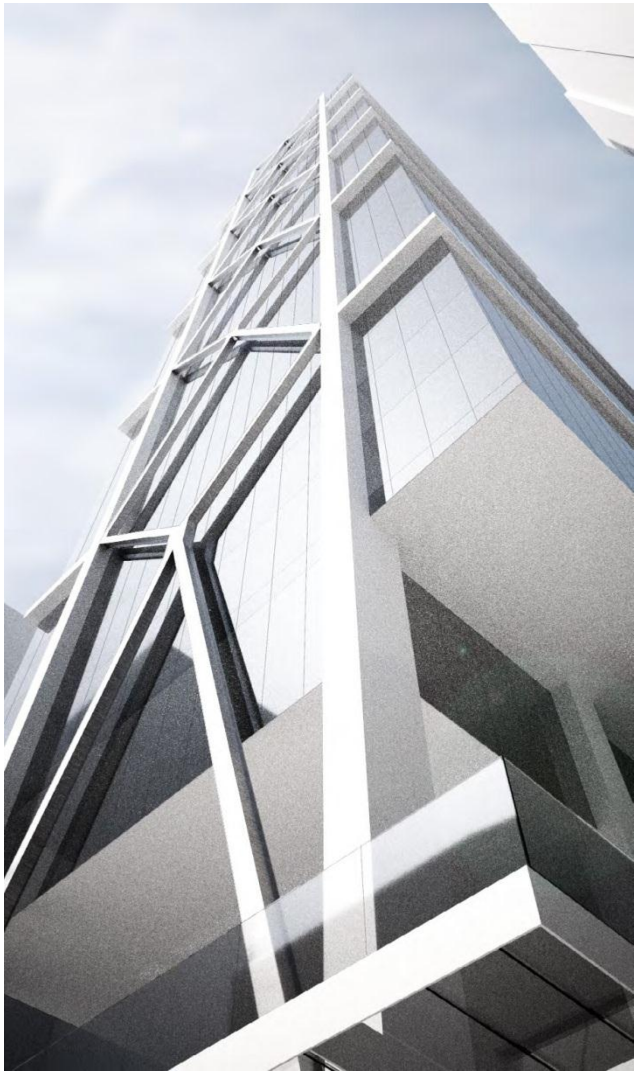
Image courtesy of SOM

75% bracing rule

Patterns

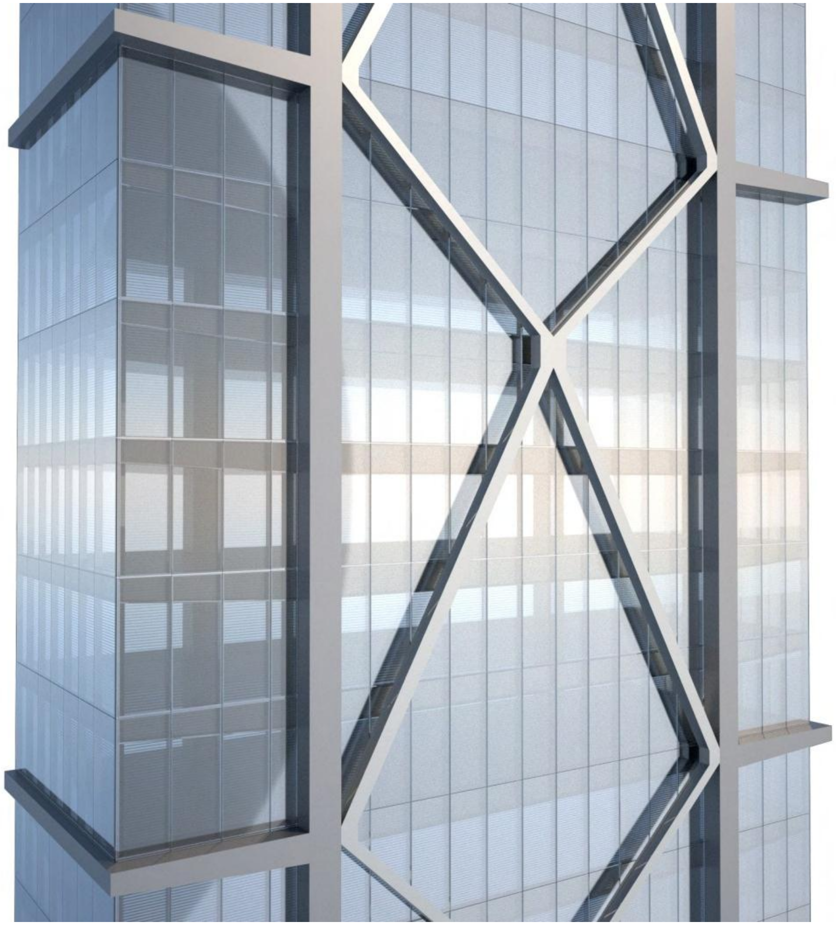
The combination of different element types is suitable for the design of lateral braced frames





Images courtesy of SOM

In summary, topology optimization with combined element types can and should be used for the practical design of buildings



Results show balance between engineering and architecture

Designs are innovative, aesthetically pleasing and structurally sound

Questions?