10th World Congress on Structural and Multidisciplinary Optimization

TOPOLOGY OPTIMIZATION OF STRUCTURES UNDER STOCHASTIC EXCITATIONS

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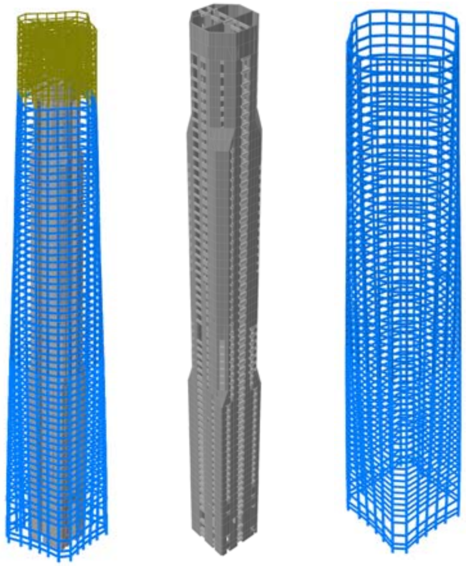
Structural engineering in Natural hazards and risks

San Francisco Earthquake, 1907
http://www.documentingreality.com

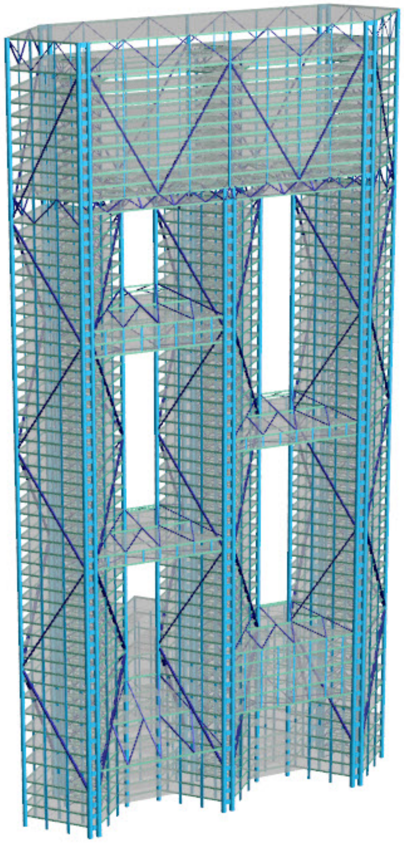
Tacoma bridge, 1940
http://failuremag.com

Structural engineering in Natural hazards and risks

Courtesy of Skidmore, Owing and Merrill, LLP



Motivation

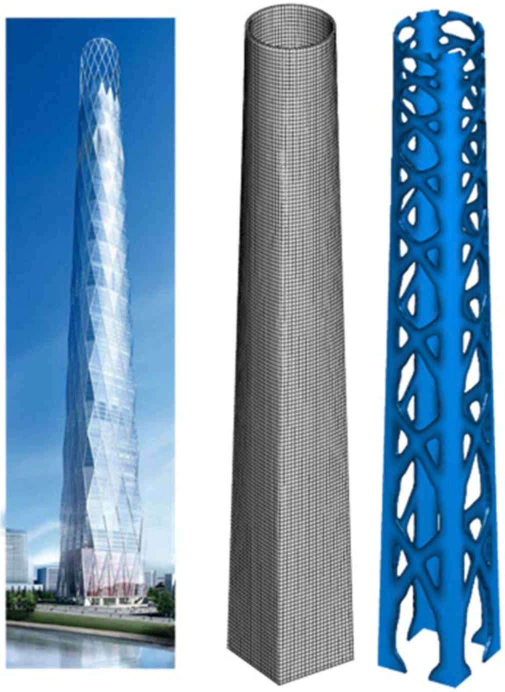


John Hancock Center
http://en.wikipedia.org/wiki/John_Hancock_Center

Ssiger International Plaza
Courtesy of Skidmore, Owing and Merrill, LLP

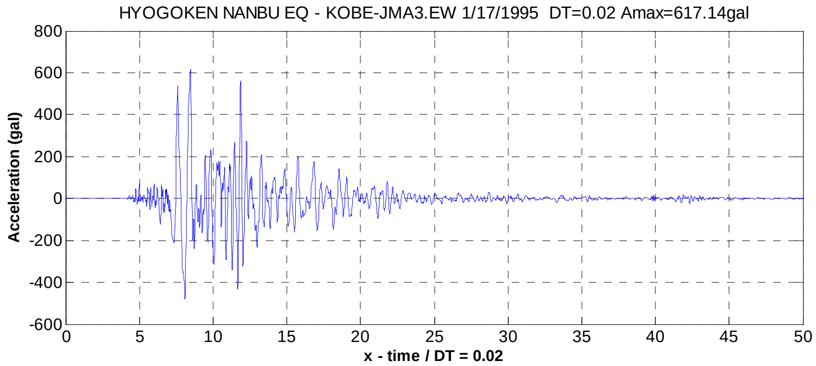
Motivation

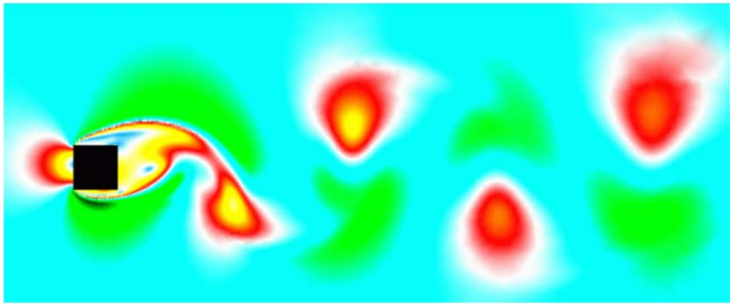
☐ Topology optimization



Stromberg, Beghini, Baker and Paulino (2011)

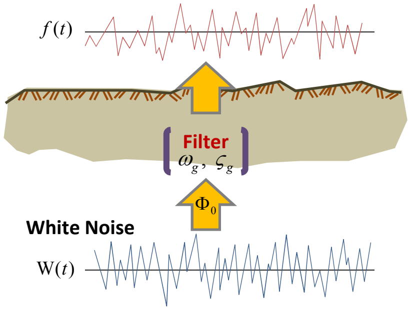
☐ Stochastic excitation



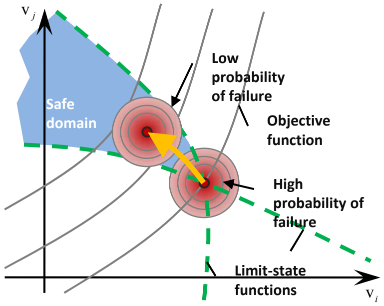


"Application of layout and topology optimization using pattern gradation for the conceptual design of buildings." L.L . Stromberg, A. Beghini, W. F. Baker, and G. H. Paulino. Structural and Multidisciplinary Optimization. Vol 43, No. 2, pp. 165-180, 2011.

1. Discrete representation method



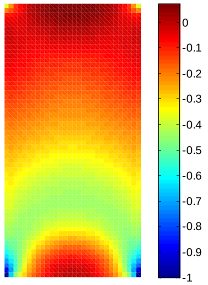
2 . Instantaneous failure probability



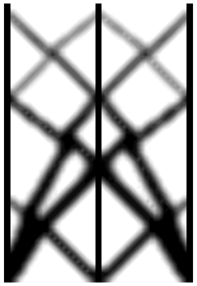
3. Stochastic topology optimization

$$\begin{aligned} &\min_{\mathbf{d}} && f_{obj}(\tilde{\boldsymbol{\rho}}(\mathbf{d})) \\ &s.t && P(E_f) \le P_f^{\text{target}} \\ &with && \mathbf{M}(\tilde{\boldsymbol{\rho}})\ddot{\mathbf{u}}(t,\tilde{\boldsymbol{\rho}}) + \mathbf{C}(\tilde{\boldsymbol{\rho}})\dot{\mathbf{u}}(t,\tilde{\boldsymbol{\rho}}) + \mathbf{K}(\tilde{\boldsymbol{\rho}})\mathbf{u}(t,\tilde{\boldsymbol{\rho}}) = \mathbf{f}(t,\tilde{\boldsymbol{\rho}}) \end{aligned}$$

4. Sensitivity



5. Numerical examples

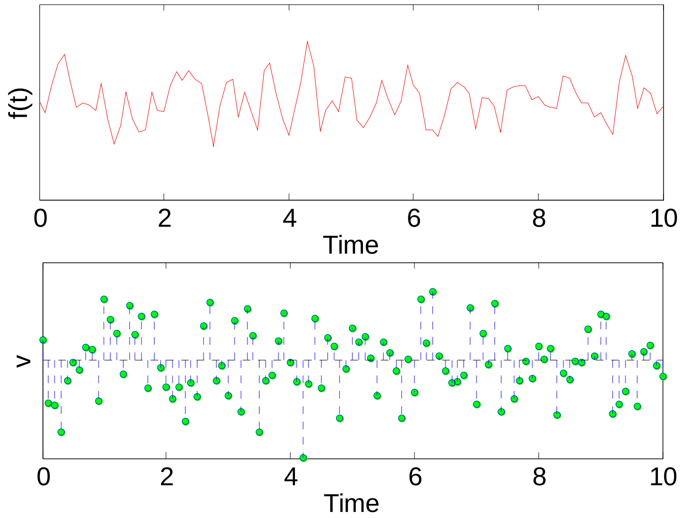


Discrete representation method

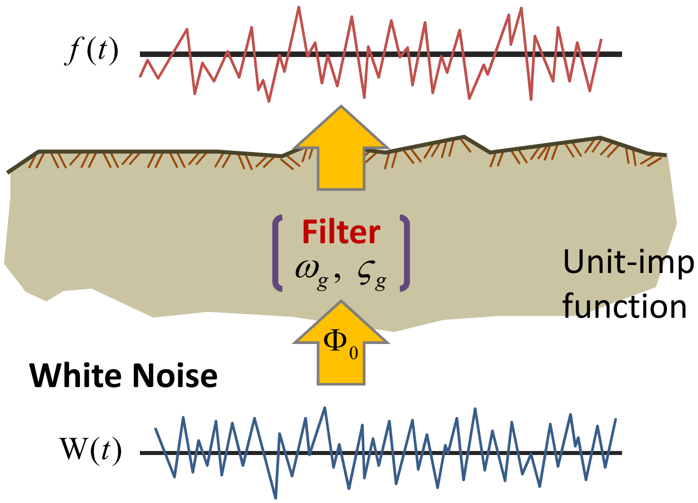
□ General (random process)

$$f(t)=\mu(t)+\sum_{i=1}^{n} v_i s_i(t)=\mu(t)+\mathbf{s}(t)^T\mathbf{v}$$

- v: uncorrelated standard normal random variables
- $\mathrm{s}(t)$: deterministic basis functions based on the spectral characteristics of the process



□ Example : Filtered white noise (earthquake)



$$f(t)=\int_0^t v(\tau)s(t-\tau)d\tau$$

$$\cong\sum_{i=1}^{n} v_i s_i(t)=\sum_{i=1}^{n} W_i\cdot h_f(t-t_i)\Delta t$$

$$=\sum_{i=1}^{n}\sqrt{2\pi\Phi_0/\Delta t}\cdot v_i\cdot h_f(t-t_i)\Delta t\ =\ \mathbf{s}(t)^{\mathrm{T}}\mathbf{v}$$

Unit-impulse response function of the filter

Magnitude of WN impulse

Der Kiureghian, A. (2000). The geometry of random vibrations and solutions by FORM and SORM. *Probabilistic Engineering Mechanics*, 15(1),: 81-90.

Response of Linear System to Stochastic Excitation

Linear system + Gaussian

☐ Duhamel's Integral

$$u(t)=\int_{0}^{t} f(\tau)h_s(t-\tau)d\tau$$

- $h_s(t)$: the unit-impulse response function of the system

☐ Response

$$u(t)=\int_{0}^{t}\sum_{i=1}^{n} v_i s_i(\tau)h_s(t-\tau)d\tau=\sum_{i=1}^{n} v_i a_i(t)=\mathbf{a}(t)^{\mathrm{T}}\mathbf{v}$$

$$a_i(t)=\int_{0}^{t} s_i(\tau)h_s(t-\tau)d\tau,\quad i=1,\ldots,n$$

Deterministic, time-dependent
- filter + structure

Random, time-independent

☐ MDOF in FEM settings

$$\begin{pmatrix} u(t_1)\\ u(t_2)\\ \vdots\\ u(t_{n-1})\\ u(t_n)\end{pmatrix}=\begin{pmatrix} u(\Delta t)\\ u(2\Delta t)\\ \vdots\\ u(t_0-\Delta t)\\ u(t_0)\end{pmatrix}=\begin{bmatrix} 0 & 0 & \cdots & 0 & v_1\\ 0 & 0 & \cdots & v_1 & v_2\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & v_1 & \cdots & v_{n-2} & v_{n-1}\\ v_1 & v_2 & \cdots & v_{n-1} & v_n\end{bmatrix}\begin{pmatrix} a_1(t_0)\\ a_2(t_0)\\ \vdots\\ a_{n-1}(t_0)\\ a_n(t_0)\end{pmatrix}$$

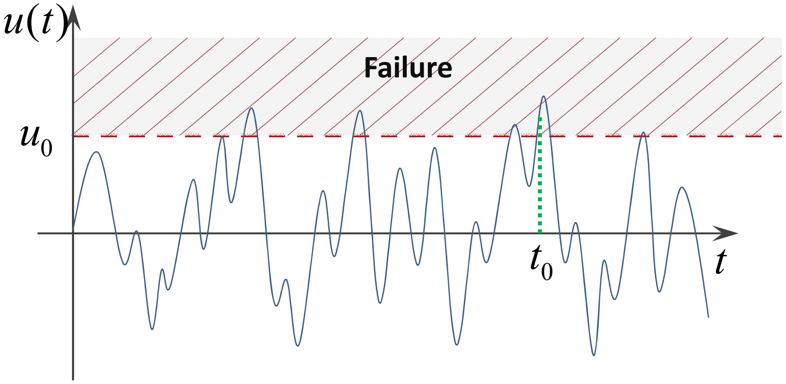
Numerical time integration

Inversely computed

Instantaneous failure probability

☐ ‘Instantaneous’ failure events of a linear system

$$E_f = \{u(t_0) \geq u_0\} = \{\mathbf{a}(t_0)^{\mathrm{T}}\mathbf{v} \geq u_0\}$$

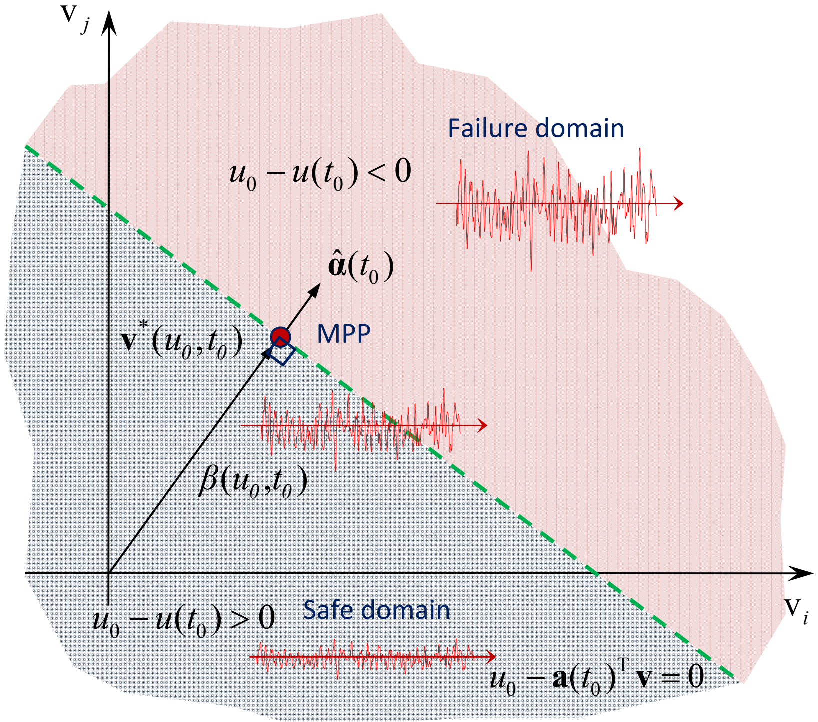


☐ Failure Probability, $P(E_f)$

$$P\left(u(t_0) \geq u_0\right) = P\left(u_0 - \mathbf{a}^T(t_0)\mathbf{v} \leq 0\right) = P\left(g(u) \leq 0\right)$$

$$g(u) = \frac{u_0}{\left\|\mathbf{a}(t_0)\right\|} - \frac{\mathbf{a}^T(t_0)}{\left\|\mathbf{a}(t_0)\right\|}\mathbf{v} = \beta(u_0,t_0) - \hat{\boldsymbol{\alpha}}(t_0)\cdot\mathbf{v}$$

$$\boxed{\begin{array}{l} P\left(u(t_0) \geq u_0\right) = \Phi[-\beta(u_0,t_0)] \\ \beta(u_0,t_0) = u_0 / \left\|\mathbf{a}(t_0)\right\| = \hat{\boldsymbol{\alpha}}(t_0)\cdot\mathbf{v}^* \end{array}}$$



Stochastic topology optimization formulation

$$\begin{aligned} &\min_{\mathbf{d}} && f_{obj}(\tilde{\boldsymbol{\rho}}(\mathbf{d})) \\ &s.t && \boxed{P(E_f) \leq P_f^{\text{target}}} \\ &with && \mathbf{M}(\tilde{\boldsymbol{\rho}})\ddot{\mathbf{u}}(t,\tilde{\boldsymbol{\rho}}) + \mathbf{C}(\tilde{\boldsymbol{\rho}})\dot{\mathbf{u}}(t,\tilde{\boldsymbol{\rho}}) + \mathbf{K}(\tilde{\boldsymbol{\rho}})\mathbf{u}(t,\tilde{\boldsymbol{\rho}}) = \mathbf{f}(t,\tilde{\boldsymbol{\rho}}) \end{aligned}$$

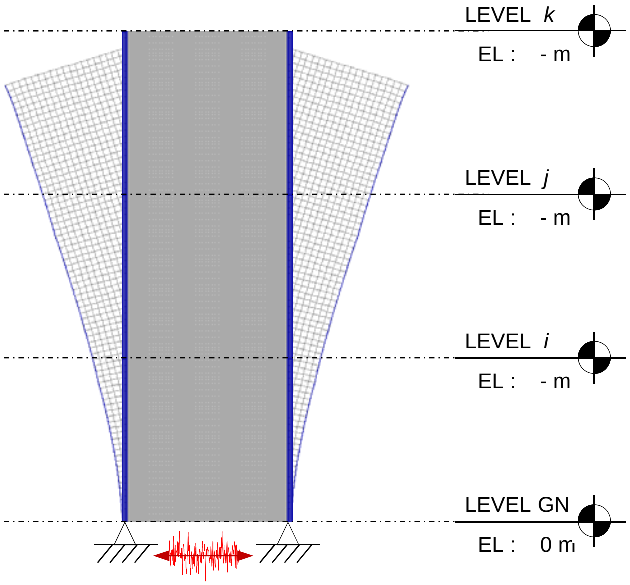
How to compute sensitivity in an efficient way

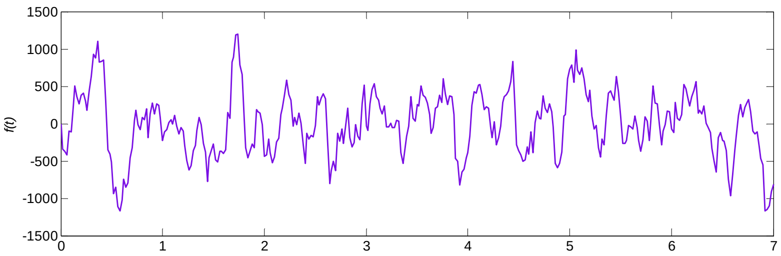
$\tilde{\boldsymbol{\rho}}$: Filtered density

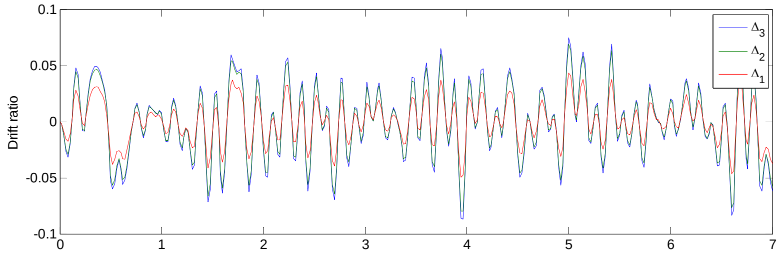
M: Mass matrix, **C**: Damping matrix, **K**: Stiffness matrix

$f(t)$: Stochastic process (e.g., for earthquake loading $\mathbf{f}(t) = -\mathbf{M}\mathbf{l}\ddot{u}_g(t)) = -\mathbf{M}\mathbf{l}f(t))$

E_f: Failure event, P_f^{target}: Target failure probability







Sensitivity calculation

Computing the sensitivity of structural responses with respect to various design parameters is essential for efficient gradient-based optimization

- Probabilistic constraint on of an instantaneous failure event

$$P(E_f) = P(u_0 - \mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}\,\mathbf{v} \le 0) \le P_f^{\text{target}}$$

- Alternatively,

$$\Phi[-\beta(u_0,t_0,\tilde{\boldsymbol{\rho}})] \le \Phi(-\beta^{\text{target}})$$

- Sensitivity of the reliability index with respect to the design variable is derived as

$$\boxed{\beta(u_0,t_0) = \frac{u_0}{\left\|\mathbf{a}(t_0)\right\|}}$$

$$\frac{\partial\beta(u_0,t_0,\tilde{\boldsymbol{\rho}})}{\partial d_e} = \sum_{j=1}^{n_e}\frac{\partial\beta(u_0,t_0,\tilde{\boldsymbol{\rho}})}{\partial\tilde{\rho}_j}\cdot\frac{\partial\tilde{\rho}_j}{\partial d_e}$$

$$= \left[-\frac{u_0}{\left(a_1(t_0,\tilde{\boldsymbol{\rho}})^2+\cdots+a_n(t_0,\tilde{\boldsymbol{\rho}})^2\right)^{\frac{3}{2}}}\right]\cdot\sum_{j=1}^{n_e}\sum_{i=1}^{n}\left(a_i(t_0,\tilde{\boldsymbol{\rho}})\cdot\frac{\partial a_i(t_0,\tilde{\boldsymbol{\rho}})}{\partial\tilde{\rho}_j}\right)\cdot\frac{\partial\tilde{\rho}_j}{\partial d_e}$$

$$= \sum_{j=1}^{n_e}\sum_{i=1}^{n}\left(c_i(u_0,t_0,\tilde{\boldsymbol{\rho}})\cdot\boxed{\frac{\partial a_i(t_0,\tilde{\boldsymbol{\rho}})}{\partial\tilde{\rho}_j}}\right)\cdot\frac{\partial\tilde{\rho}_j}{\partial d_e}$$

Implicitly defined term

Sensitivity calculation-cont’d

☐ Adjoint method by introducing adjoint system equation (Newmark method)

$$\begin{aligned}\left(\frac{1}{\eta(\Delta t)^2}\mathbf{M}(\tilde{\boldsymbol{\rho}})+\frac{\gamma}{\eta\Delta t}\mathbf{C}(\tilde{\boldsymbol{\rho}})+\mathbf{K}(\tilde{\boldsymbol{\rho}})\right)\mathbf{u}(t_{j+1},\tilde{\boldsymbol{\rho}})&=\mathbf{f}(t_{j+1},\tilde{\boldsymbol{\rho}})\\&+\mathbf{C}(\tilde{\boldsymbol{\rho}})\left[\frac{\gamma}{\eta\Delta t}\mathbf{u}(t_j,\tilde{\boldsymbol{\rho}})+\left(\frac{\gamma}{\eta}-1\right)\dot{\mathbf{u}}(t_j,\tilde{\boldsymbol{\rho}})+\Delta t\left(\frac{\gamma}{2\eta}-1\right)\ddot{\mathbf{u}}(t_j,\tilde{\boldsymbol{\rho}})\right]\\&+\mathbf{M}(\tilde{\boldsymbol{\rho}})\left[\frac{1}{\eta(\Delta t)^2}\mathbf{u}(t_j,\tilde{\boldsymbol{\rho}})+\frac{1}{\eta\Delta t}\dot{\mathbf{u}}(t_j,\tilde{\boldsymbol{\rho}})+\left(\frac{1}{2\eta}-1\right)\ddot{\mathbf{u}}(t_j,\tilde{\boldsymbol{\rho}})\right]\end{aligned}$$

☐ From a general recurrence relation associated with three sequential displacements (Chan et al. 1962, Zienkiewicz 1977)

$$\begin{aligned}\left(\mathbf{M}(\tilde{\boldsymbol{\rho}})+\gamma\Delta t\mathbf{C}(\tilde{\boldsymbol{\rho}})+\eta(\Delta t)^2\mathbf{K}(\tilde{\boldsymbol{\rho}})\right)\mathbf{u}(t_{j+1},\tilde{\boldsymbol{\rho}})&=\eta(\Delta t)^2\mathbf{f}(t_{j+1},\tilde{\boldsymbol{\rho}})+(0.5+\gamma-2\eta)(\Delta t)^2\mathbf{f}(t_j,\tilde{\boldsymbol{\rho}})\\&+(0.5-\gamma+\eta)(\Delta t)^2\mathbf{f}(t_{j-1},\tilde{\boldsymbol{\rho}})\\&-\left[-2\mathbf{M}(\tilde{\boldsymbol{\rho}})+(1-2\gamma)\Delta t\mathbf{C}(\tilde{\boldsymbol{\rho}})+(0.5+\gamma-2\eta)(\Delta t)^2\mathbf{K}(\tilde{\boldsymbol{\rho}})\right]\mathbf{u}(t_j,\tilde{\boldsymbol{\rho}})\\&-\left[\mathbf{M}(\tilde{\boldsymbol{\rho}})+(\gamma-1)\Delta t\mathbf{C}(\tilde{\boldsymbol{\rho}})+(0.5-\gamma+\eta)(\Delta t)^2\mathbf{K}(\tilde{\boldsymbol{\rho}})\right]\mathbf{u}(t_{j-1},\tilde{\boldsymbol{\rho}})\end{aligned}$$

Sensitivity calculation-cont'd

- By pre-multiplying the discretized adjoint system with the dimensional adjoint variable vector and adding to right-hand side terms of the original sensitivity equation

$$
\begin{aligned}
\frac{\partial \beta(u_0, t_0, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} = & \sum_{i=1}^{n}\left(T_i \cdot \mathbf{z}^{\mathrm{T}} \frac{\partial \mathbf{u}(t_i, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \right) \\
& + \sum_{j=1}^{n} \boldsymbol{\lambda}_{n-j+1}^{\mathrm{T}} \left[\frac{\partial \underline{\underline{\mathbf{A}}}(\tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \cdot \mathbf{u}(t_j, \tilde{\boldsymbol{\rho}}) - \eta \left(\Delta t\right)^2 \frac{\partial \mathbf{f}(t_j, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} - (0.5 + \gamma - 2\eta)\left(\Delta t\right)^2 \frac{\partial \mathbf{f}(t_{j-1}, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \right. \\
& \left. -(0.5 - \gamma + \eta)\left(\Delta t\right)^2 \frac{\partial \mathbf{f}(t_{j-2}, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} + \frac{\partial \underline{\underline{\mathbf{B}}}(\tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \cdot \mathbf{u}(t_{j-1}, \tilde{\boldsymbol{\rho}}) + \frac{\partial \underline{\underline{\mathbf{E}}}(\tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \cdot \mathbf{u}(t_{j-2}, \tilde{\boldsymbol{\rho}}) \right] \\
& + \sum_{j=1}^{n} \boldsymbol{\lambda}_{n-j+1}^{\mathrm{T}} \left[\underline{\underline{\mathbf{A}}}(\tilde{\boldsymbol{\rho}}) \cdot \frac{\partial \mathbf{u}(t_j, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} + \underline{\underline{\mathbf{B}}}(\tilde{\boldsymbol{\rho}}) \cdot \frac{\partial \mathbf{u}(t_{j-1}, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} + \underline{\underline{\mathbf{E}}}(\tilde{\boldsymbol{\rho}}) \cdot \frac{\partial \mathbf{u}(t_{j-2}, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \right]
\end{aligned}
$$

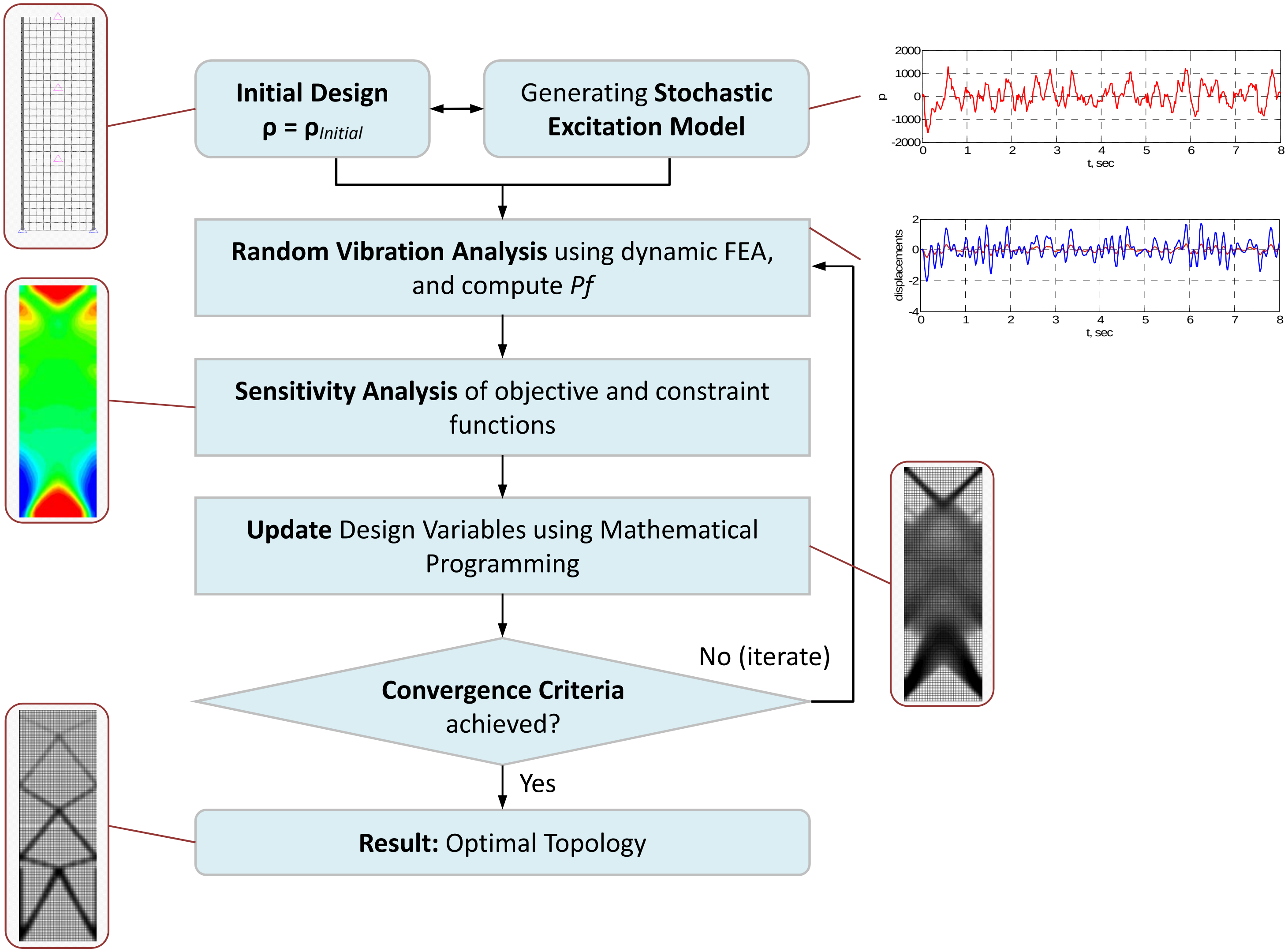
$$
\boxed{\sum_{i=1}^{n}\left(c_i(t_0, \tilde{\boldsymbol{\rho}}) \cdot \frac{\partial a_i(t_0, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \right) = \sum_{i=1}^{n}\left(T_i \cdot \mathbf{z}^{\mathrm{T}} \cdot \frac{\partial \mathbf{u}(t_i, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \right)}
$$

$$
\boxed{
\begin{aligned}
& \underline{\underline{\mathbf{A}}}(\tilde{\boldsymbol{\rho}}) = \mathbf{M}(\tilde{\boldsymbol{\rho}}) + \gamma \Delta t \cdot \mathbf{C}(\tilde{\boldsymbol{\rho}}) + \eta \left(\Delta t\right)^2 \mathbf{K}(\tilde{\boldsymbol{\rho}}) \\
& \underline{\underline{\mathbf{B}}}(\tilde{\boldsymbol{\rho}}) = -2\mathbf{M}(\tilde{\boldsymbol{\rho}}) + (1 - 2\gamma)\Delta t \mathbf{C}(\tilde{\boldsymbol{\rho}}) + (0.5 + \gamma - 2\eta)\left(\Delta t\right)^2 \mathbf{K}(\tilde{\boldsymbol{\rho}}) \\
& \underline{\underline{\mathbf{E}}}(\tilde{\boldsymbol{\rho}}) = \mathbf{M}(\tilde{\boldsymbol{\rho}}) + (\gamma - 1)\Delta t \mathbf{C}(\tilde{\boldsymbol{\rho}}) + (0.5 - \gamma + \eta)\left(\Delta t\right)^2 \mathbf{K}(\tilde{\boldsymbol{\rho}})
\end{aligned}
}
$$

- After solving adjoint system problem

$$
\begin{aligned}
\frac{\partial \beta(u_0, t_0, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} = & \sum_{j=1}^{n} \boldsymbol{\lambda}_{n-j+1}{}^{T} \left[\frac{\partial \underline{\underline{\mathbf{A}}}(\tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \cdot \mathbf{u}(t_j, \tilde{\boldsymbol{\rho}}) - \eta \left(\Delta t\right)^2 \frac{\partial \mathbf{f}(t_j, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} - (0.5 + \gamma - 2\eta)\left(\Delta t\right)^2 \frac{\partial \mathbf{f}(t_{j-1}, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \right. \\
& \left. -(0.5 - \gamma + \eta)\left(\Delta t\right)^2 \frac{\partial \mathbf{f}(t_{j-2}, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} + \frac{\partial \underline{\underline{\mathbf{B}}}(\tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \cdot \mathbf{u}_{j-1}(\tilde{\boldsymbol{\rho}}) + \frac{\partial \underline{\underline{\mathbf{E}}}(\tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \cdot \mathbf{u}(t_{j-2}, \tilde{\boldsymbol{\rho}}) \right] \\
& + \boldsymbol{\lambda}_{n}{}^{T} \left[\underline{\underline{\mathbf{B}}}(\tilde{\boldsymbol{\rho}}) \cdot \frac{\partial \mathbf{u}(0, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} + \underline{\underline{\mathbf{E}}}(\tilde{\boldsymbol{\rho}}) \cdot \frac{\partial \mathbf{u}(t_{-1}, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \right] + \boldsymbol{\lambda}_{n-1}{}^{T} \left[\underline{\underline{\mathbf{E}}}(\tilde{\boldsymbol{\rho}}) \cdot \frac{\partial \mathbf{u}(0, \tilde{\boldsymbol{\rho}})}{\partial \tilde{\rho}_j} \right]
\end{aligned}
$$

Flow chart for topology optimization under stochastic excitations



Initial Design
$\rho = \rho_{Initial}$

Generating **Stochastic Excitation Model**

Random Vibration Analysis using dynamic FEA, and compute *Pf*

Sensitivity Analysis of objective and constraint functions

Update Design Variables using Mathematical Programming

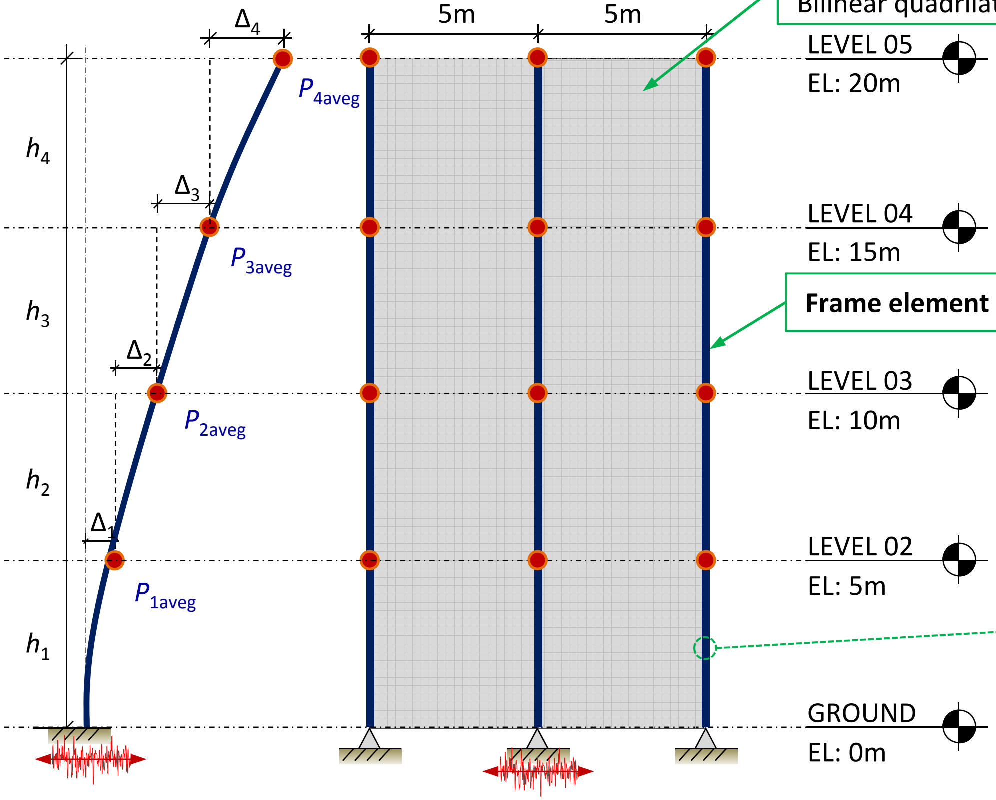
Convergence Criteria achieved?

No (iterate)

Yes

Result: Optimal Topology

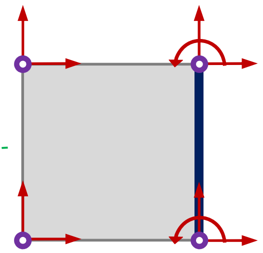
Numerical example : Building structure



Design domain:
Bilinear quadrilateral element, **Q4**

Frame element

Stromberg, L., Beghini, A., Baker, W. F. and Paulino, G. H. (2012)



Material	Concrete
Thickness	0.10 m
Time, t_0	10 sec
Threshold values u_0	0.02
Volf.	0.65
Column	0.4m x 0.4m
ζ_f	0.45
ω_f	5π

$$\min_{\mathbf{d}} \quad V(\tilde{\boldsymbol{\rho}}(\mathbf{d}))$$

$$s.t \quad P\boxed{(E_f)} \le P_f^{\text{target}}$$

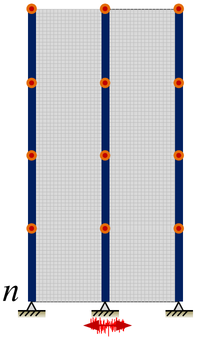
$$with \ \mathbf{M}(\tilde{\boldsymbol{\rho}})\ddot{\mathbf{u}}(t,\tilde{\boldsymbol{\rho}}) + \mathbf{C}(\tilde{\boldsymbol{\rho}})\dot{\mathbf{u}}(t,\tilde{\boldsymbol{\rho}}) + \mathbf{K}(\tilde{\boldsymbol{\rho}})\mathbf{u}(t,\tilde{\boldsymbol{\rho}}) = -\mathbf{MI}\boxed{f(t))}$$

Inter-story drift ratio

Filtered Gaussian process using **Kanai-Tajimi** filter

"Topology Optimization for Braced Frames: Combining Continuum and Discrete Elements." L.L . Stromberg, A. Beghini, W. F. Baker, and G. H. Paulino. Engineering Structures. Vol 37. pp. 106-124. 2012.

Failure event - Inter-story drift ratio



$$E_{f_i} = \begin{cases} \left\{ u_{i0} - \left(\dfrac{(\mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{i,L} + \mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{i,M} + \mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{i,R})\mathbf{v}}{nL_i} \right) \le 0 \right\} & \text{for } i = 2 \\ \left\{ u_{i0} - \left(\dfrac{(\mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{i,L} + \mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{i,M} + \mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{i,R})\mathbf{v}}{nL_i} - \dfrac{(\mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{(i-1),L} + \mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{(i-1),M} + \mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{(i-1),R})\mathbf{v}}{nL_i} \right) \le 0 \right\} & \text{for } i = 3,4,,..n \end{cases}$$

Filtered Gaussian process using KT filter

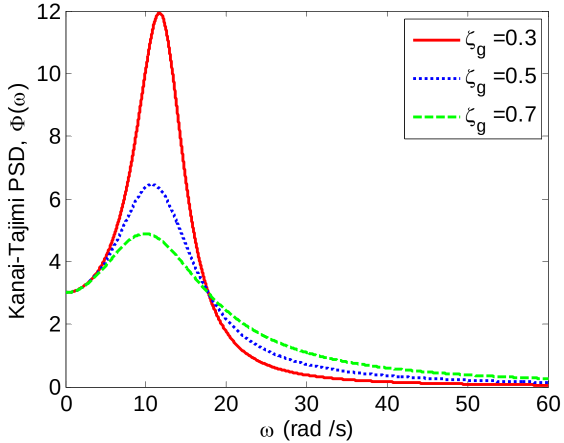
- ☐ Kanai-Tajimi filter model

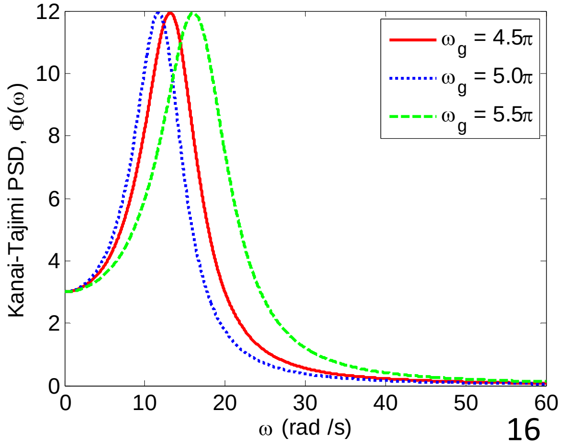
$$h_f(t) = \exp(-\zeta_f \omega_f t)\left[\frac{(2\zeta_f^2 - 1)\omega_f}{\sqrt{1-\zeta_f^2}} \sin(\omega_f \sqrt{1-\zeta_f^2} \cdot t) - 2\zeta_f \omega_f \cos(\omega_f \sqrt{1-\zeta_f^2} \cdot t) \right]$$

ζ_f : damping ratio of the filter
ω_f : predominant frequency of the filter

$$\Phi(\omega) = \frac{1 + 4\zeta_f^2 (\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2 + (2\zeta_f \omega/\omega_g)^2} \Phi_o$$

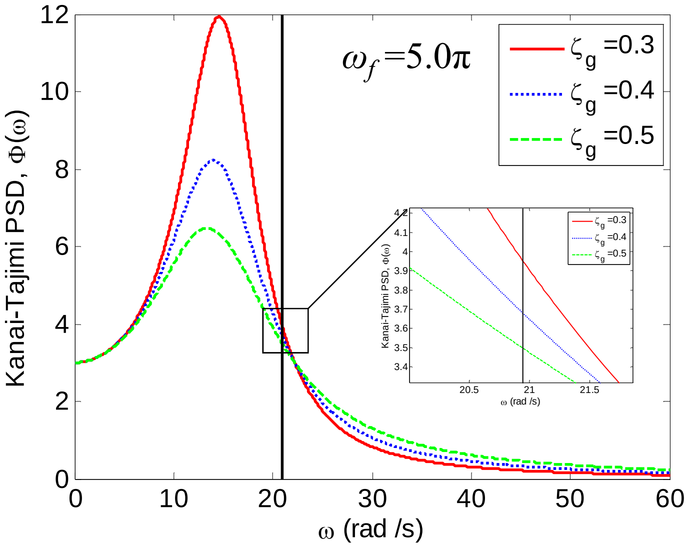
$$f(t) = \sum_{i=1}^{n} \sqrt{2\pi\Phi_0/\Delta t} \cdot v_i \cdot h_f(t - t_i)\Delta t$$

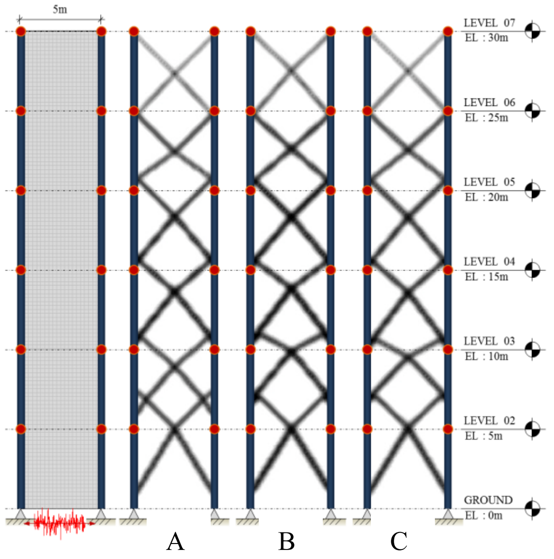




Clough, R. and Penzien, J. (1993). Dynamics of structures. New York: McGraw Hill; 1993.

Parametric study on impact of ground motion characteristics



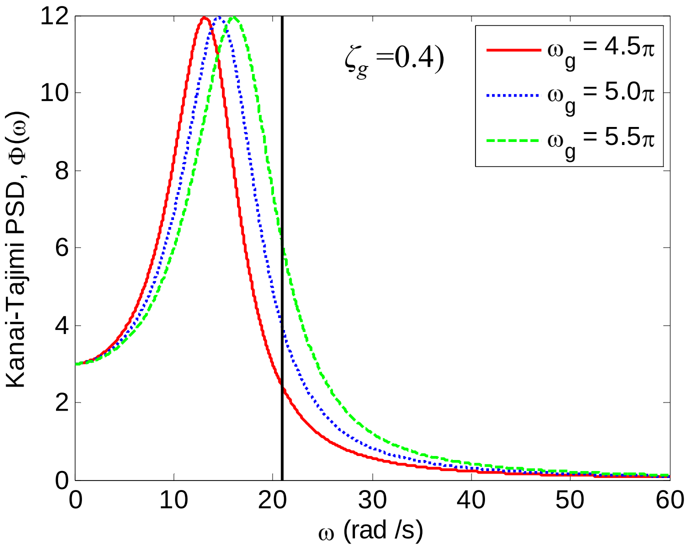


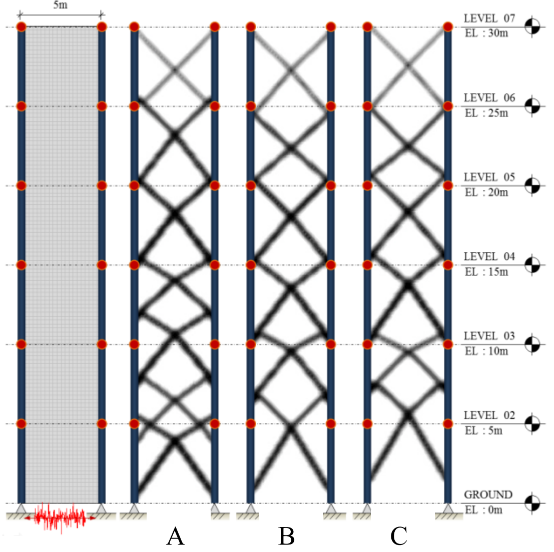
A (Φ_0=3, β^{target}=1.5)	
ζ_f	0.3
Volume	2.81 m^3 (**18.7%**)

B (Φ_0=3, β^{target}=1.5)	
ζ_f	0.4
Volume	2.42 m^3 (**16.1%**)

C (Φ_0=3, β^{target}=1.5)	
ζ_f	0.5
Volume	2.27 m^3 (**15.1%**)

β^{target}=1.5 = (P_f=6.68%)





A (Φ_0=3, β^{target}=1.5)	
ω_f	4.5π
Volume	2.08 m^3 (**13.9%**)

B (Φ_0=3, β^{target}=1.5)	
ω_f	5.0π
Volume	2.42 m^3 (**16.1%**)

C (Φ_0=3, β^{target}=1.5)	
ω_f	5.5π
Volume	2.82 m^3 (**18.8%**)

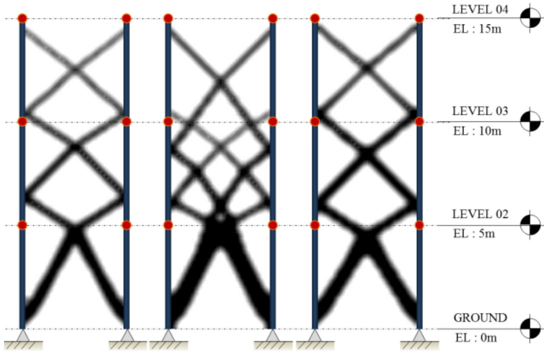
Conclusions

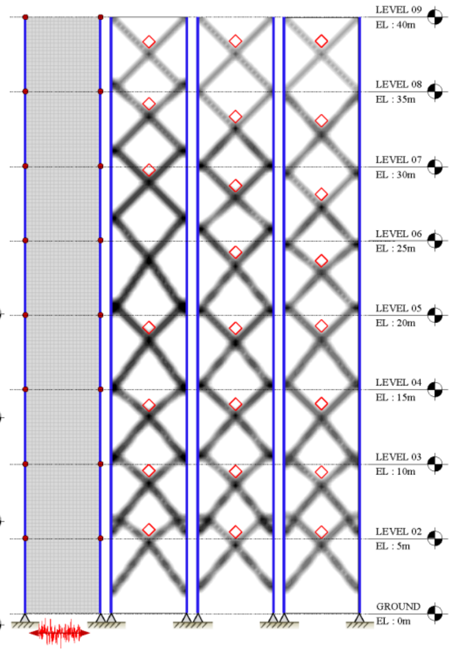
- Introduction of a new approach incorporating random vibration theories into topology optimization using a discrete representation method for stochastic processes

- Computation of the failure probability regarding stochastic responses by a closed-form solution

- Implementation of the Adjoint method to evaluate the sensitivity of dynamic responses modeled by the discrete representation method

- Application of the proposed stochastic topology optimization method to achieve optimal solutions satisfying probabilistic constraints on stochastic responses of structural systems





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Question and Comments?

Performance of sensitivity methods

- Input stochastic process

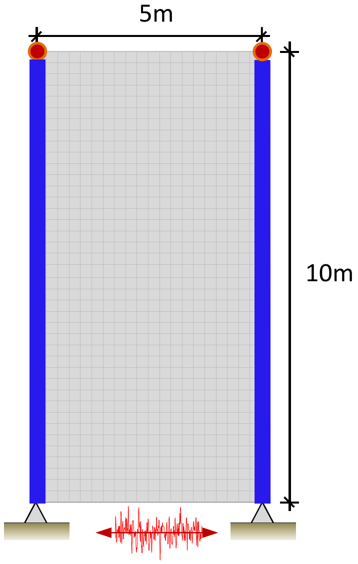
$$f(t) = \mathbf{s}(t)^{\mathrm{T}}\,\mathbf{v} \qquad s_i(t) = \exp[-2.4\pi(t-t_i)]\sin[3.2\pi(t-t_i)]H(t-t_i)/\left\|\mathbf{s}(t)\right\|$$

- Failure event

$$E_f = u_0 - \left(\frac{(\mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{Left} + \mathbf{a}(t_0,\tilde{\boldsymbol{\rho}})^{\mathrm{T}}_{Right})\mathbf{v}}{2}\right) \le 0$$

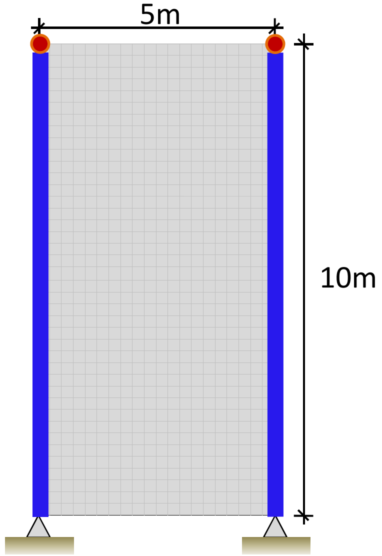
- Sensitivity

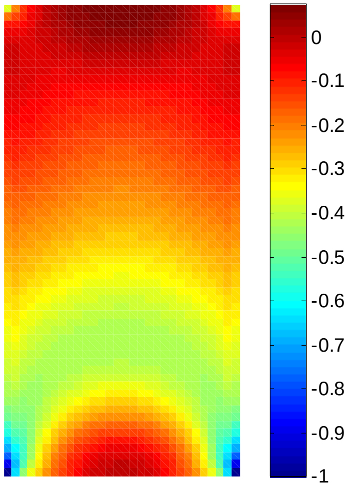
$$\frac{\partial\beta(u_0,t_0,\tilde{\boldsymbol{\rho}})}{\partial d_e}$$



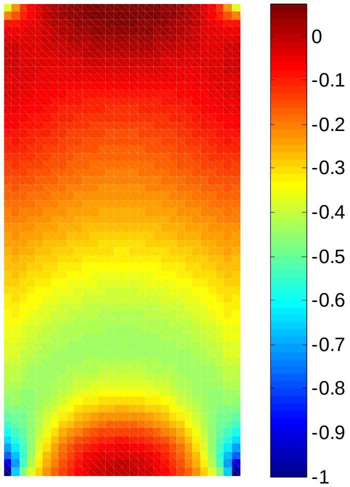
E	21,000Mpa
ρ	2,400kg/m^3
Thickness	0.10 m
Column	0.35m x 0.35m
Time, t_0	7 sec
Threshold values u_0	0.02
Initial.Volf	0.5

Performance of sensitivity methods-cont'd

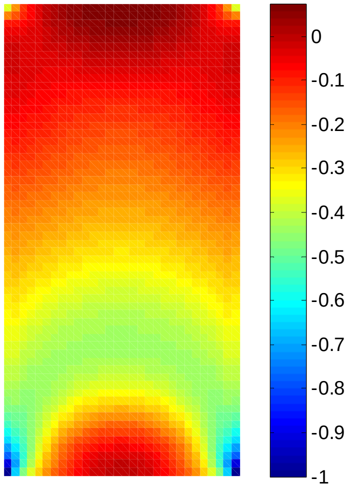




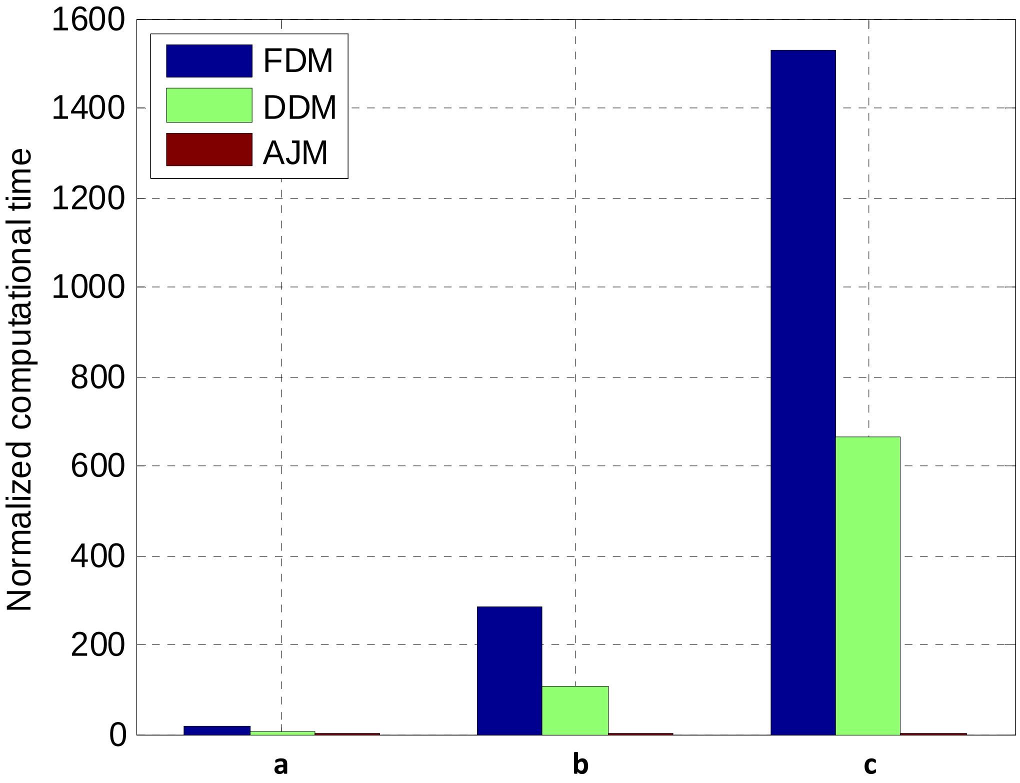
Finite difference method



Direct differentiation method



Adjoint method



a 200 finite elements, **b** 800 finite elements, and **c** 1800 finite elements

Reliability-Based Design Optimization

☐ Deterministic design optimization (DO)

$$\min_{\mathbf{d}} \quad f_{obj}(\mathbf{d})$$

$$s.t \quad g_i(\mathbf{d}) > 0,\ i = 1,\ldots,n_c$$

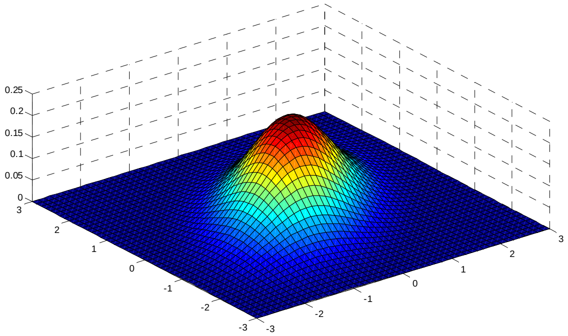
$$\mathbf{d}^{lower} \le \mathbf{d} \le \mathbf{d}^{upper}$$

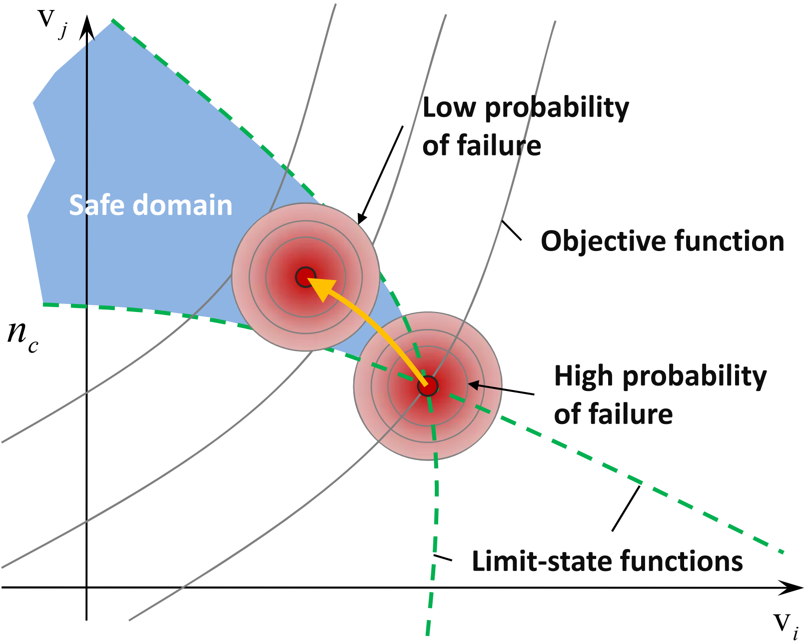
☐ Reliability-based design optimization (RBDO)

$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X}} \quad f_{obj}(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$

$$s.t \quad P(E_{sys}) = P\left[\bigcup\bigcap g_i(\mathbf{d},\boldsymbol{\mu}_\mathbf{X}) \le 0\right] \le P_{sys}^{\text{target}}, \quad i = 1,\ldots,n_c$$

$$\mathbf{d}^{lower} \le \mathbf{d} \le \mathbf{d}^{upper},\ \boldsymbol{\mu}^{lower} \le \boldsymbol{\mu}_\mathbf{X} \le \boldsymbol{\mu}^{upper}$$





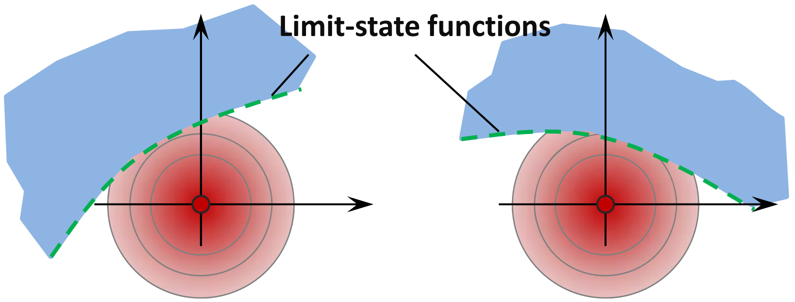
System Reliability-Based Design Optimization

☐ Component Reliability Based Design Optimization (CRBDO)

$$\min_{\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}}} \ f_{obj}(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}})$$

$$s.t \quad P\left[g_i(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}}) \le 0\right] \le P_f^{\text{target}}, \quad i = 1,\ldots,n_c$$

$$\mathbf{d}^{lower} \le \mathbf{d} \le \mathbf{d}^{upper}, \ \boldsymbol{\mu}^{lower} \le \boldsymbol{\mu}_{\mathbf{X}} \le \boldsymbol{\mu}^{upper}$$



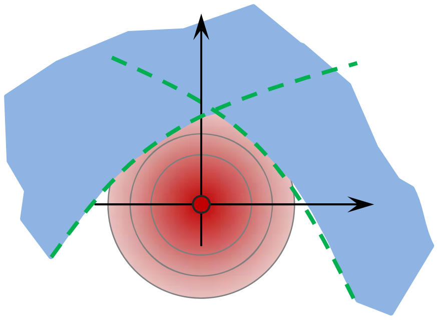
☐ System Reliability Based Design Optimization (SRBDO)

$$\min_{\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}}} \ f_{obj}(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}})$$

$$s.t \quad \boxed{P(E_{sys}) = P\left[\bigcup\bigcap g_i(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}}) \le 0\right] \le P_{sys}^{\text{target}}, \quad i = 1,\ldots,n_c}$$

$$\mathbf{d}^{lower} \le \mathbf{d} \le \mathbf{d}^{upper}, \ \boldsymbol{\mu}^{lower} \le \boldsymbol{\mu}_{\mathbf{X}} \le \boldsymbol{\mu}^{upper}$$

MSR method
(Song and Kang 2009, Kang *et al*. 2012)



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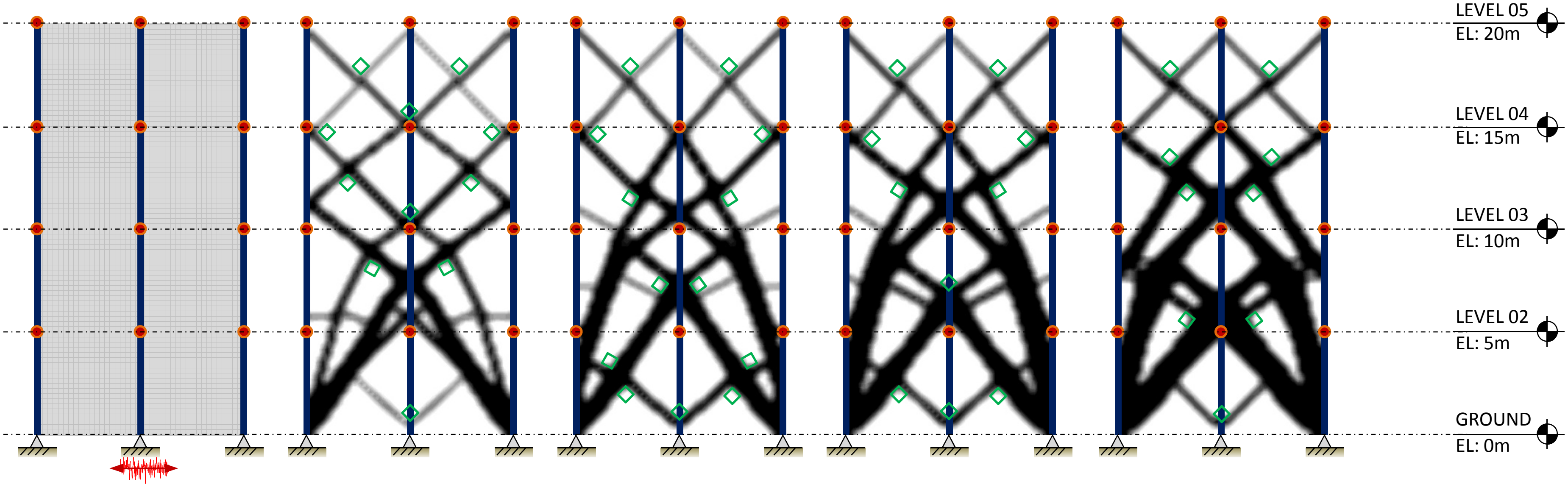
Kang, W.-H., Y.-J. Lee, J. Song, and B. Gencturk (2012). Further development of matrix-based system reliability method and applications to structural systems. Structure and Infrastructure Engineering: Maintenance, Management, Life-cycle Design and Performance. Vol. 8(5), 441-457.

Optimal topologies

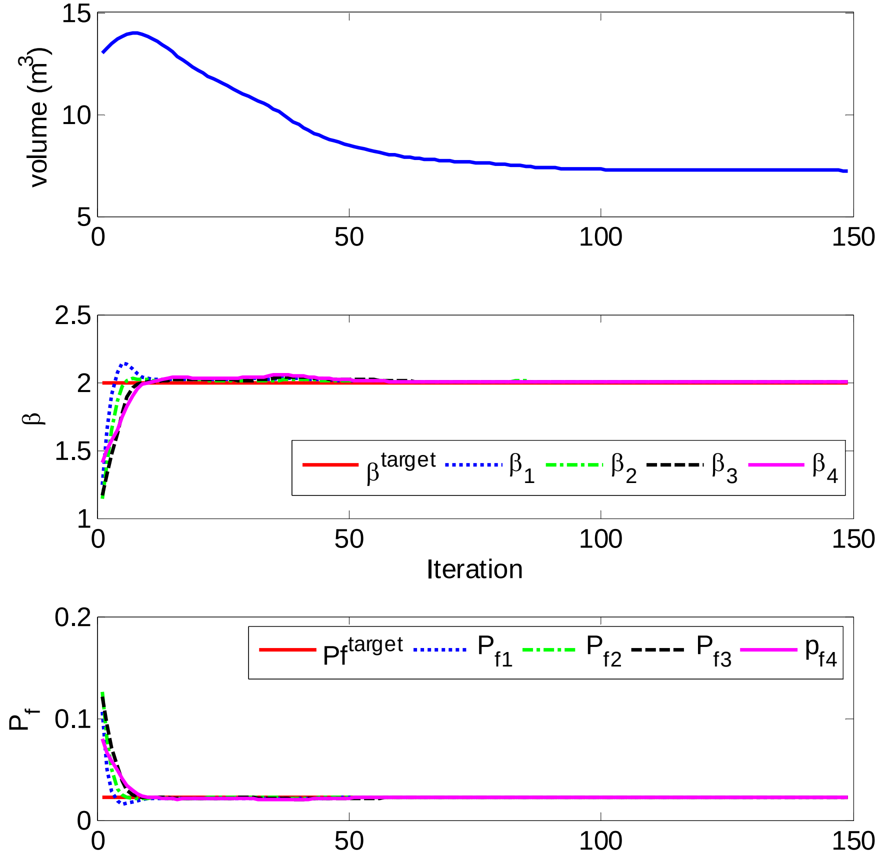
☐ Varying target failure probabilities



A (Φ_0=700)		B (Φ_0=700)		C (Φ_0=700)		D (Φ_0=700)	
β^{target}	1.4 = (P_f=8.1%)	**β^{target}**	1.7 = (P_f=4.5%)	**β^{target}**	2.0 = (P_f=2.3%)	**β^{target}**	2.3 = (P_f=1.1%)
Volume	4.95 m^3 (**24.7%**)	**Volume**	6.15 m^3(**30.8%**)	**Volume**	7.19 m^3(**35.9%**)	**Volume**	8.34 m^3(**41.7%**)

Convergence and Dynamic response

☐ Convergence history



☐ Dynamic response comparison