GEOMETRICAL ASPECTS OF LATERAL BRACING SYSTEMS:

WHERE SHOULD THE OPTIMAL BRACE POINT BE?

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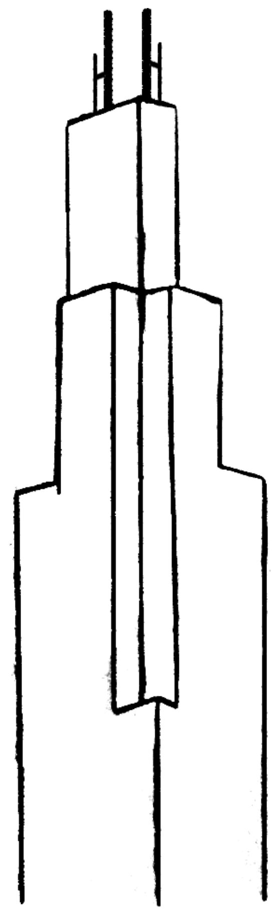
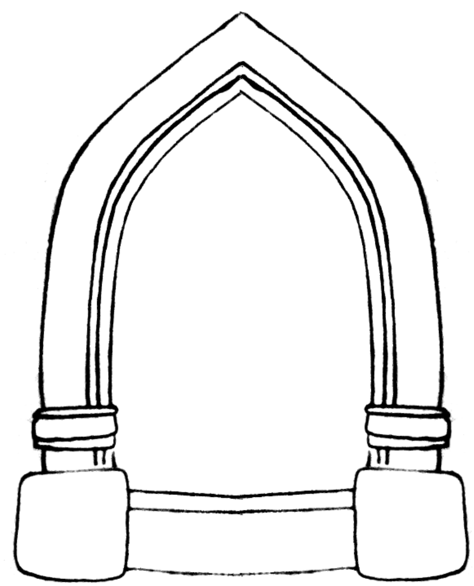
1 – INTRO & MOTIVATION

- WHY USE STRUCTURAL OPTIMIZATION?

LIMITED RESOURCES



EXTREME STRUCTURES



FUNCTIONAL

1 – INTRO & MOTIVATION

- BRACED BUILDINGS

JOHN HANCOCK CENTER
(CHICAGO, IL)


SULLIVAN
ANN
MARY
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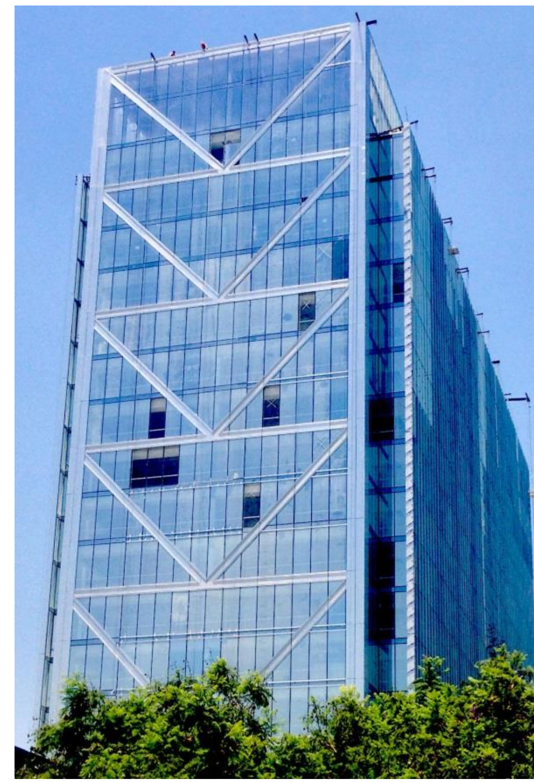
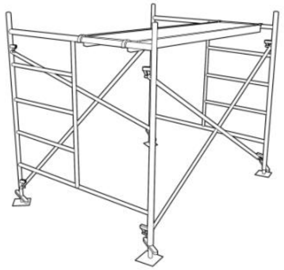
ALCOA BUILDING
(SAN FRANCISCO, CA)



BUILDING IN PDTE. RIESCO AVENUE
(SANTIAGO, CHILE)

1 – INTRO & MOTIVATION

- OTHER APPLICATIONS

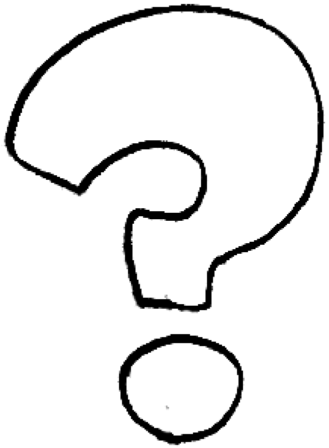


STAGE HIRE

CONSTRUCTION SCAFFOLDING

1 – INTRO & MOTIVATION

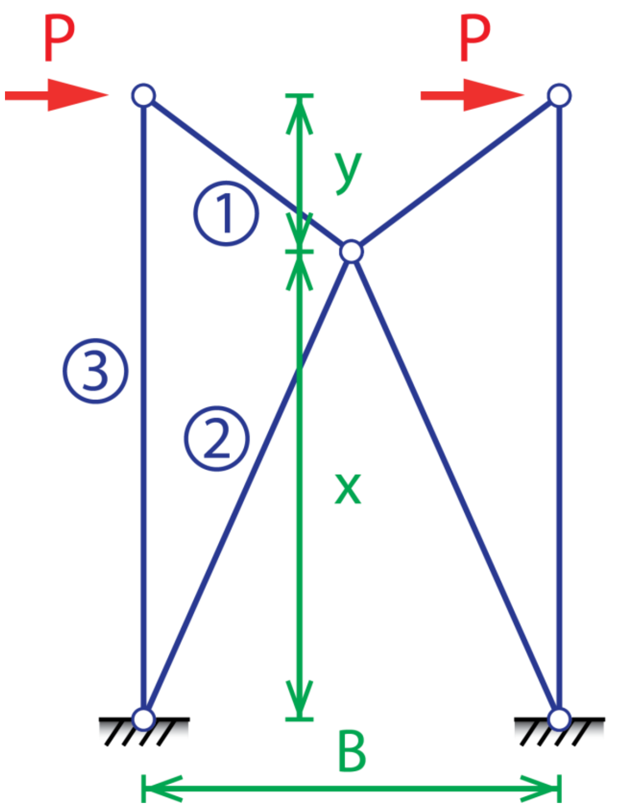
- WHAT IS OPTIMAL?
  - LEAST **WEIGHT**
  - MINIMUM **COMPLIANCE**
  - REDUCED **DISPLACEMENT**
  - OTHER…



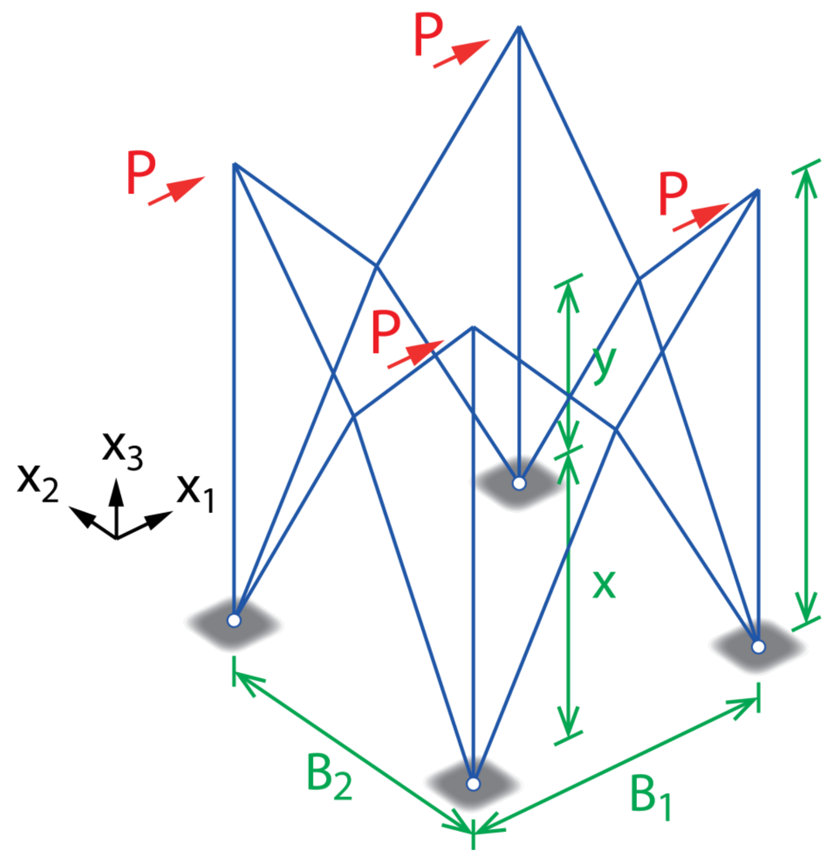
- ASSUMPTIONS
  - ZERO CONNECTION COST
  - STATIC, LINEAR & ELASTIC
  - TRUSS MEMBERS

2 – PROBLEM DESCRIPTION

- UNIT BRACES IN 2 AND 3-DIMENSIONS



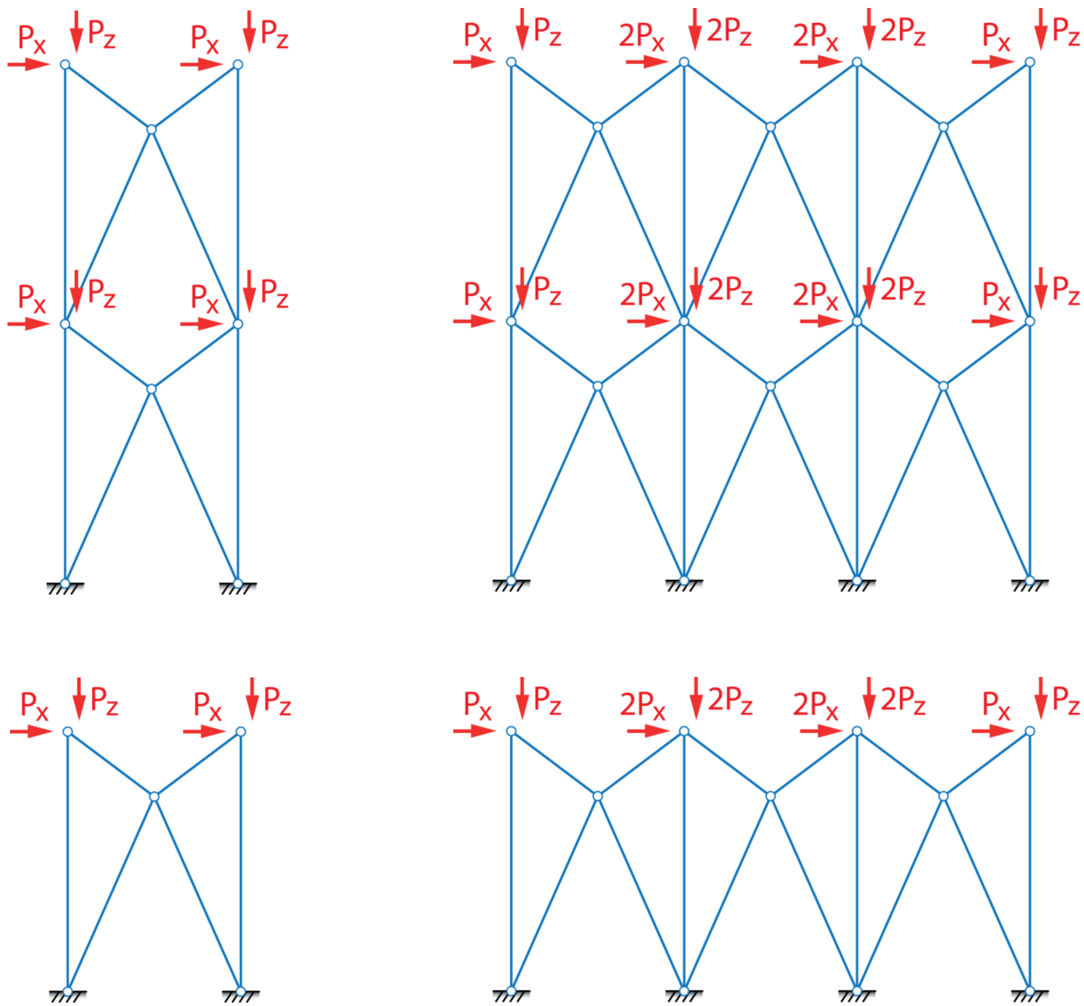
TWO-DIMENSIONAL BRACE



THREE-DIMENSIONAL BRACE

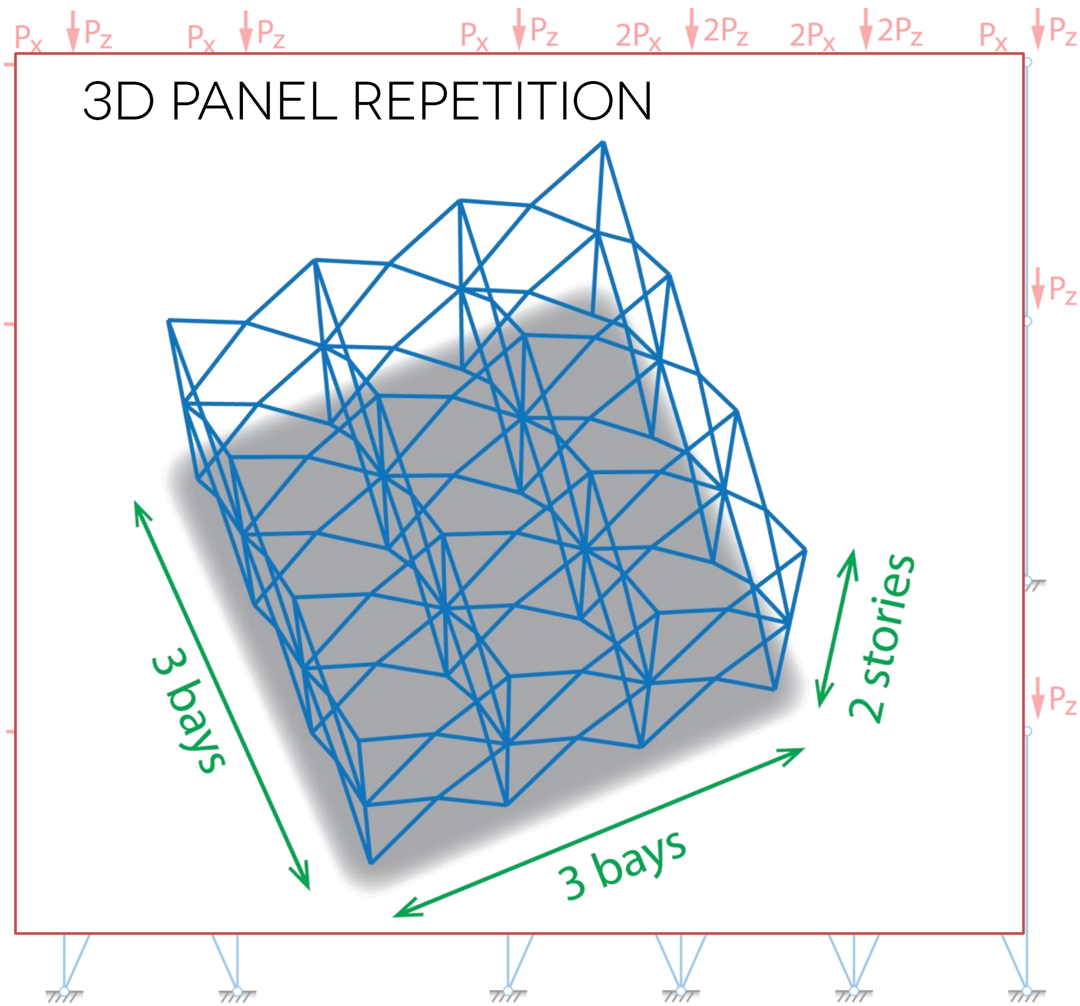
2 – Problem description

- Multiple stories – multiple bays



2 – PROBLEM DESCRIPTION

- MULTIPLE STORIES – MULTIPLE BAYS



2 – PROBLEM DESCRIPTION

- FORMULATIONS (1/2)
  - MINIMUM VOLUME

$$\begin{aligned} \min_{\mathbf{A},\mathbf{x}} \quad & V = \mathbf{A}^T\mathbf{L} \\ \text{s.t.} \quad & \sigma_c \leq \sigma_i \leq \sigma_t \qquad \forall i = 1 \ldots n_e \\ \text{with} \quad & \mathbf{K}\mathbf{u} = \mathbf{f} \end{aligned}$$

  - MINIMUM LOAD-PATH

$$\begin{aligned} \min_{\mathbf{A},\mathbf{x}} \quad & Z = \sum_i |N_i| \, L_i \\ \text{s.t.} \quad & \sum_i A_i L_i \leq \bar{V} \\ \text{with} \quad & \mathbf{K}\mathbf{u} = \mathbf{f} \end{aligned}$$

2 – PROBLEM DESCRIPTION

- FORMULATIONS (2/2)
  - MINIMUM COMPLIANCE

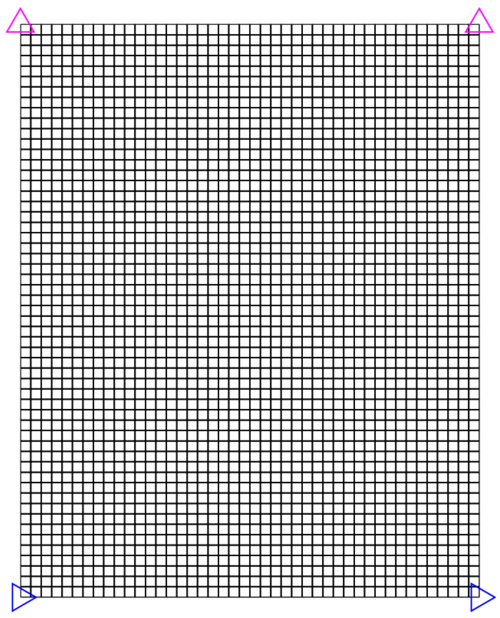
$$\begin{aligned} \min_{\mathbf{A},\mathbf{x}} \quad & C = \mathbf{u}^T\mathbf{K}\mathbf{u} = \mathbf{u}^T\mathbf{f} \\ \text{s.t.} \quad & \textstyle\sum_i A_i L_i \leq \bar{V} \\ \text{with} \quad & \mathbf{K}\mathbf{u} = \mathbf{f} \end{aligned}$$

  - MINIMUM DISPLACEMENT

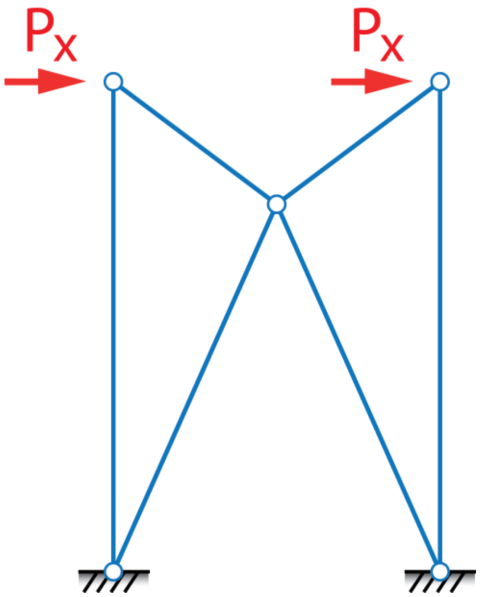
$$\begin{aligned} \min_{\mathbf{A},\mathbf{x}} \quad & \Delta = u_j \\ \text{s.t.} \quad & \textstyle\sum_i A_i L_i \leq \bar{V} \\ \text{with} \quad & \mathbf{K}\mathbf{u} = \mathbf{f} \end{aligned}$$

3 – EARLY RESULTS

- GROUND STRUCTURE METHOD
  – WEIGHT MINIMIZATION WITH SYMMETRY



BASE MESH



DORN W S, GOMORY R E, GREENBERG H J (1964) – "AUTOMATIC DESIGN OF OPTIMAL STRUCTURES", JOURNAL DE MECANIQUE 3(1), PP 25–52

3 – EARLY RESULTS

- GROUND STRUCTURE METHOD
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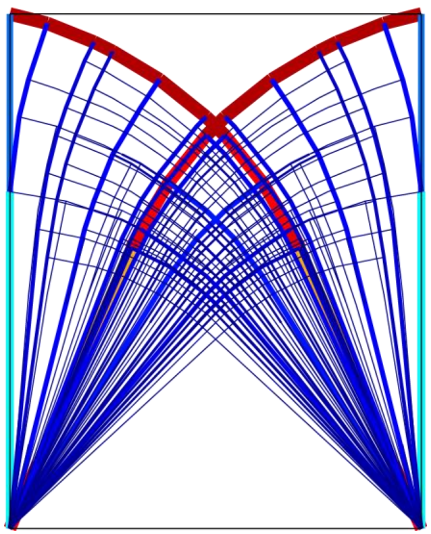
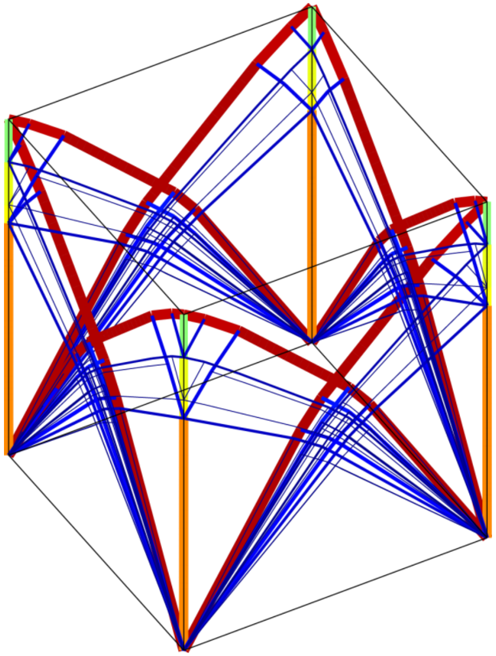


2D RESULT



3D RESULT

3 – EARLY RESULTS

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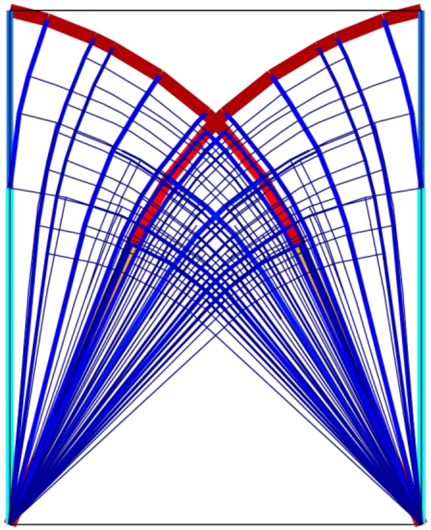
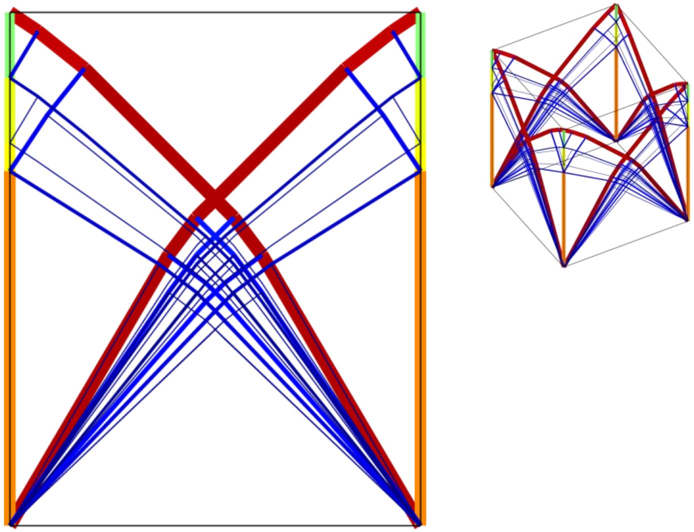


2D RESULT



3D RESULT

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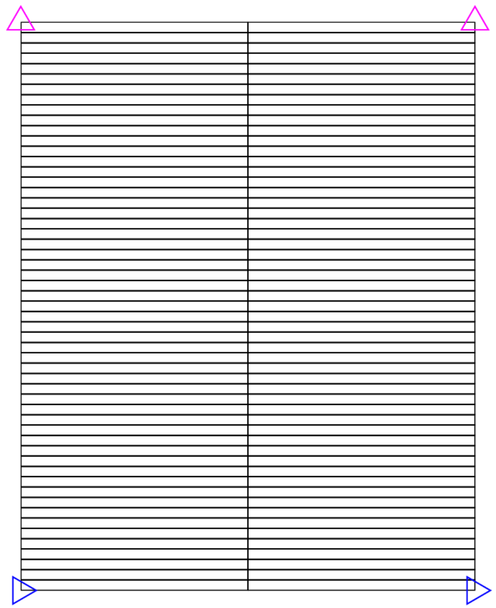
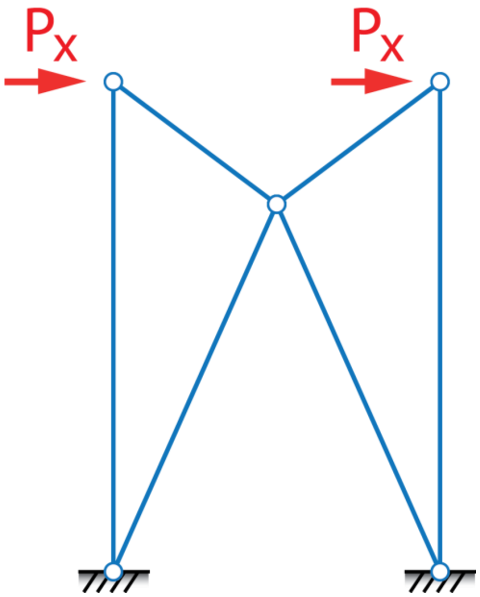


BASE MESH



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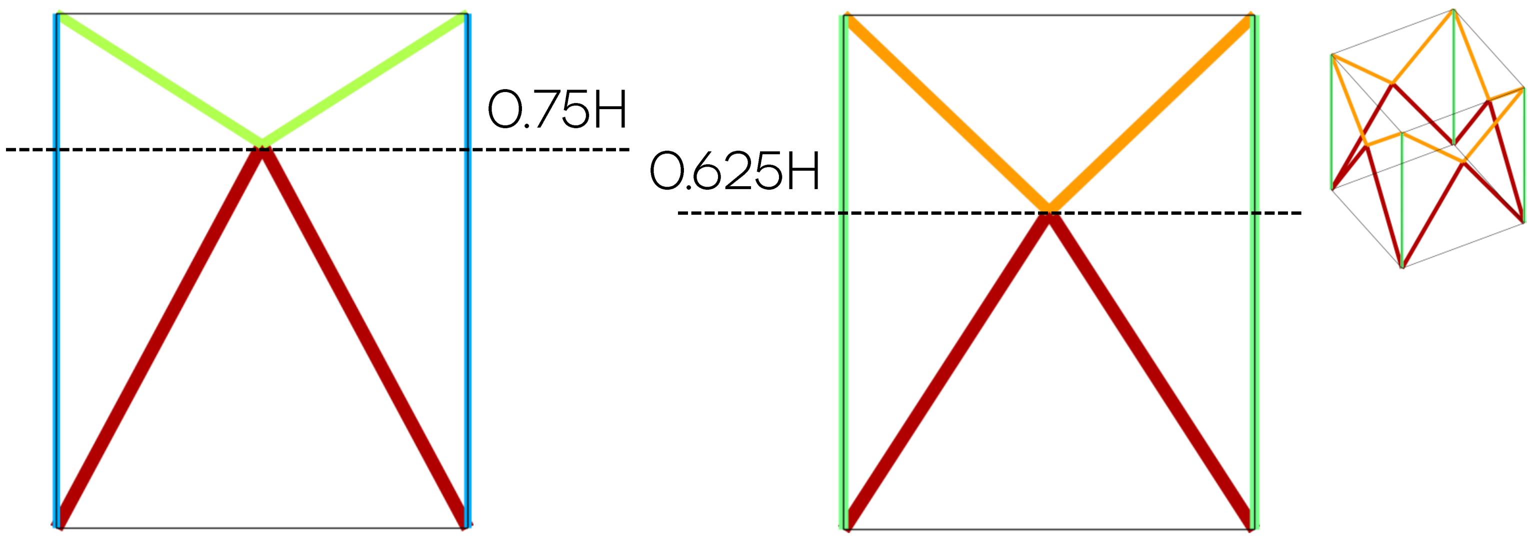


2D RESULT

3D RESULT

STROMBERG L L, BEGHINI A, BAKER W F, PAULINO G H (2012) – "TOPOLOGY OPTIMIZATION FOR BRACED FRAMES: COMBINING CONTINUUM AND BEAM/COLUMN ELEMENTS", ENGINEERING STRUCTURES 37, PP 106–124

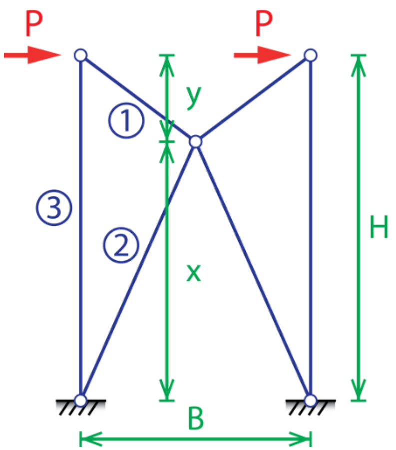
4 – DETAILED ANALYSIS

- MIN VOLUME **ANALYTICAL** SOLUTION
  - 2D BRACE: $\alpha=1$
  - 3D BRACE: $\alpha=2$

$$\mathcal{L} = \alpha A_1 L_1 + \alpha A_2 L_2 + A_3 H + \lambda_{11}\left(-A_1\bar{\sigma} - N_1\right) + \lambda_{12}\left(-A_1\bar{\sigma} + N_1\right) + \ldots$$

$$\lambda_{21}\left(-A_2\bar{\sigma} - N_2\right) + \lambda_{22}\left(-A_2\bar{\sigma} + N_2\right) + \ldots$$

$$\lambda_{31}\left(-A_3\bar{\sigma} - N_3\right) + \lambda_{32}\left(-A_3\bar{\sigma} + N_3\right)$$



$$\lambda_{11} = \alpha L_1/\bar{\sigma} \qquad \lambda_{12} = 0$$

$$\lambda_{21} = 0 \qquad \lambda_{22} = \alpha L_2/\bar{\sigma}$$

$$\lambda_{31} = 0 \qquad \lambda_{32} = H/\bar{\sigma}$$

$$x = \frac{2\alpha + 1}{4\alpha} H$$

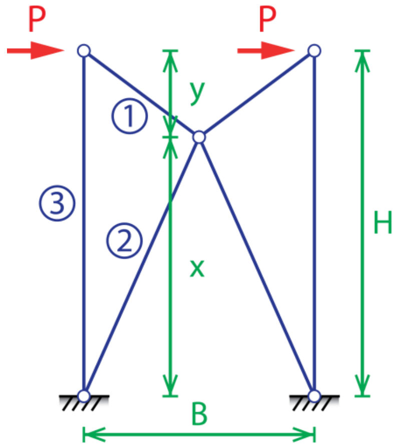
$$y = \frac{2\alpha - 1}{4\alpha} H$$

4 – DETAILED ANALYSIS

- MIN COMPLIANCE **ANALYTICAL** SOLUTION
  - 2D BRACE: α=1
  - 3D BRACE: α=2

$$C = \frac{4P^2}{EB^2}\left[\frac{L_1^3}{A_1} + \frac{L_2^3}{A_2} + \frac{L_3}{A_3}y^2\right]$$

$$\mathcal{L} = \frac{4P^2}{EB^2}\left[\frac{L_1^3}{A_1} + \frac{L_2^3}{A_2} + \frac{L_3}{A_3}y^2\right] + \lambda\left(\alpha A_1 L_1 + \alpha A_2 L_2 + A_3 H - \bar{V}\right)$$



$$\lambda = \frac{4P^2 y^2}{EB^2 A_3^2}$$

$$x = \frac{2\sqrt{\alpha}+1}{4\sqrt{\alpha}}H$$

$$y = \frac{2\sqrt{\alpha}-1}{4\sqrt{\alpha}}H$$

4 – DETAILED ANALYSIS

- ANALYTICAL SOLUTION FOR A SINGLE BAY
  - 2D BRACE: $\alpha=1$
  - 3D BRACE: $\alpha=2$

$$x = \frac{2\alpha + 1}{4\alpha} H$$

WEIGHT &
LOAD–PATH

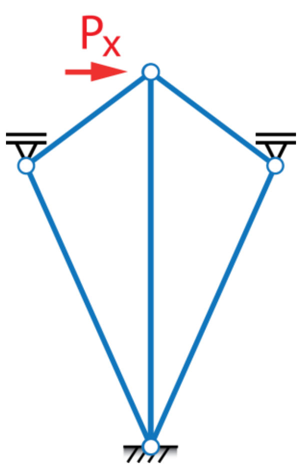
$$x = \frac{2\sqrt{\alpha} + 1}{4\sqrt{\alpha}} H$$

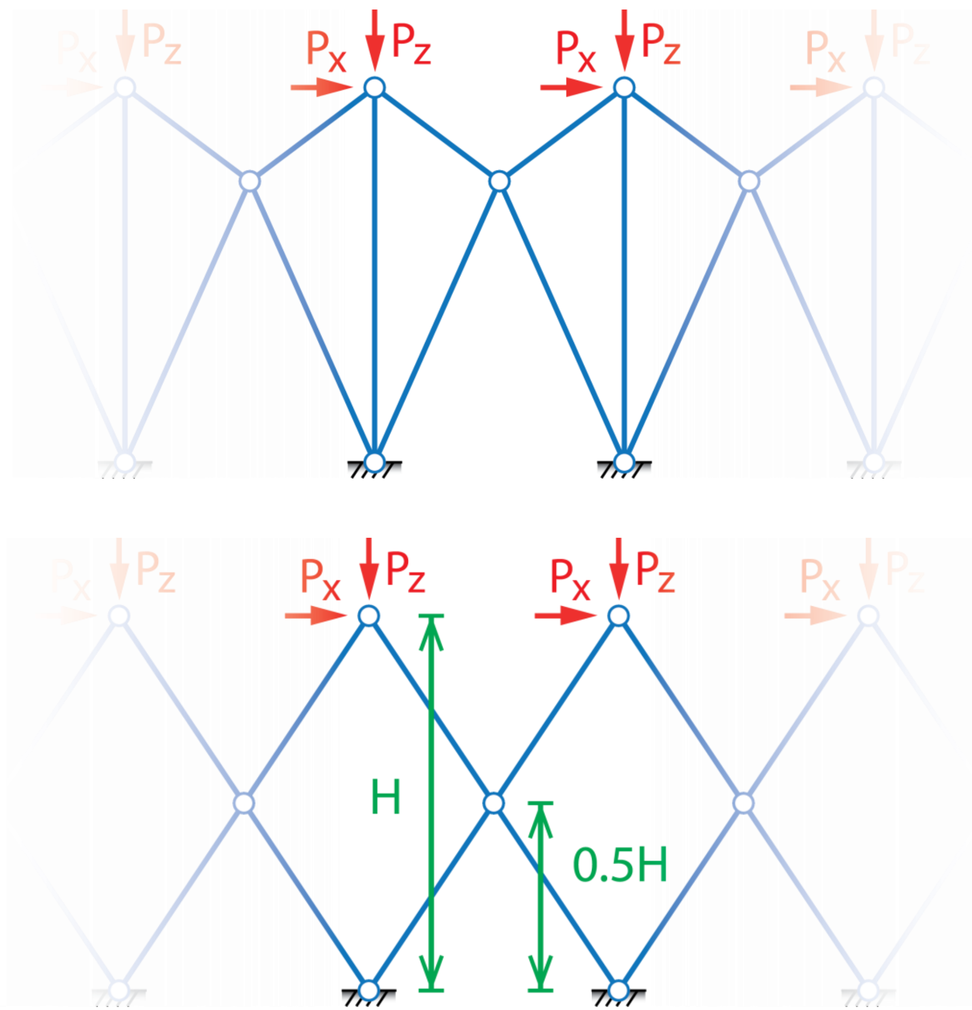
COMPLIANCE &
DISPLACEMENT

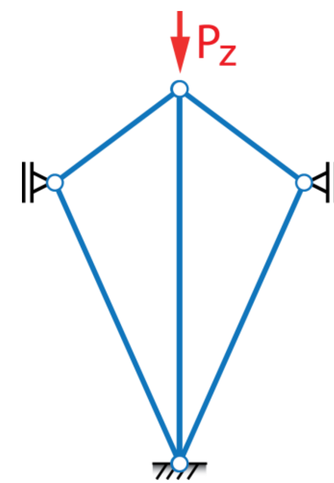
Height x	Weight - Cost		Performance	
	Volume	Load-Path	Compliance	Displacement
2D	$0.75H$	$0.75H$	$0.75H$	$0.75H$
3D	$0.625H$	$0.625H$	$0.6768H$	$0.6768H$

4 – DETAILED ANALYSIS

- LIMIT CASE OF ∞ BAYS







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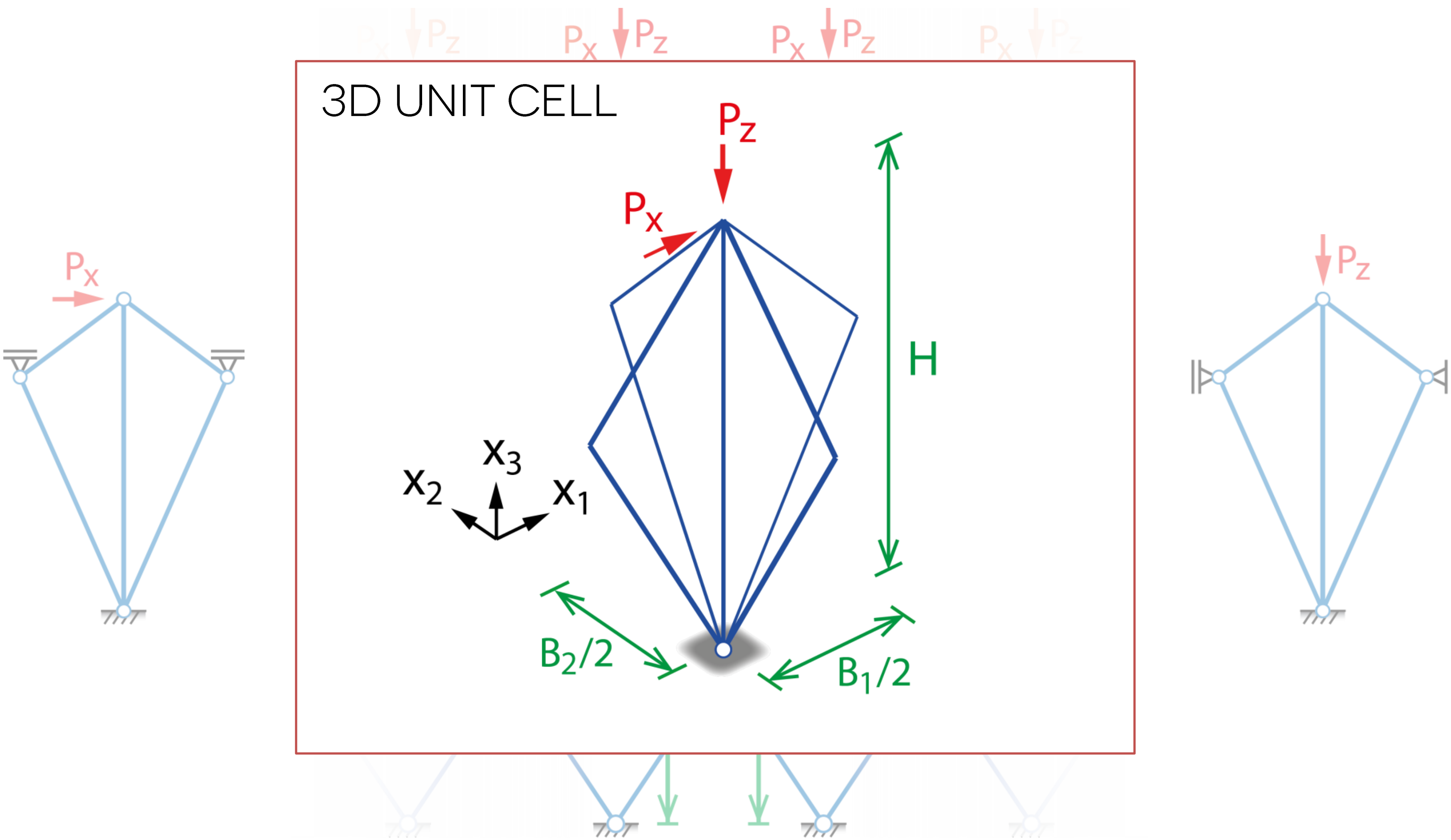
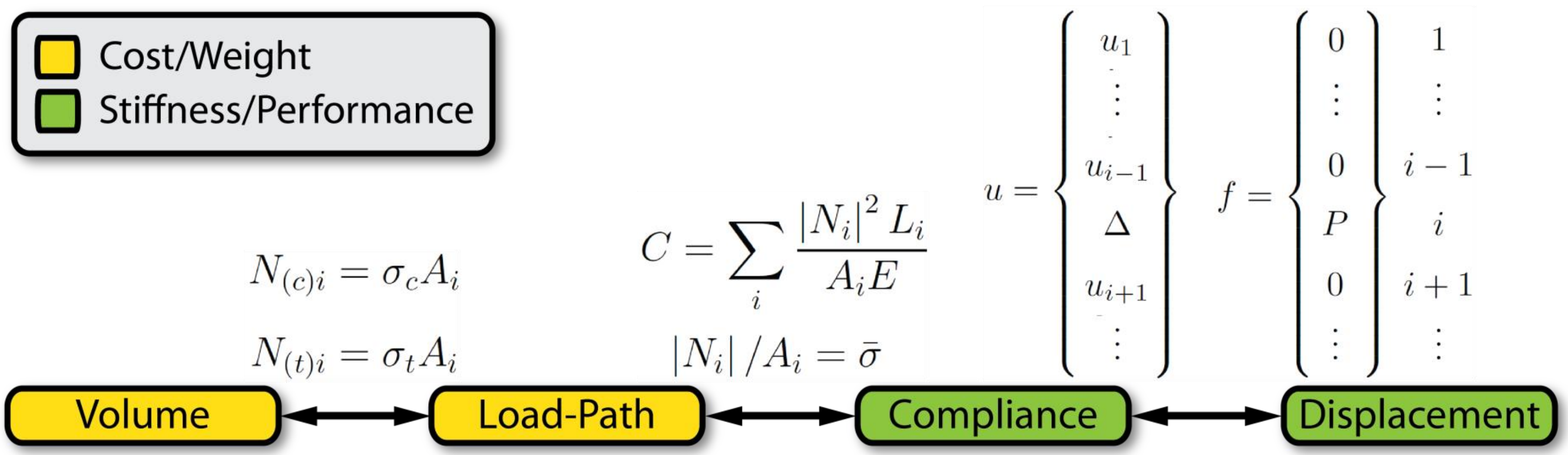


4 – DETAILED ANALYSIS

- EQUIVALENCY BETWEEN FORMULATIONS



Height x	Weight - Cost		Performance	
	Volume	Load-Path	Compliance	Displacement
2D	$0.75H$	$0.75H$	$0.75H$	$0.75H$
3D	$0.625H$	$0.625H$	$0.6768H$	$0.6768H$

4 – DETAILED ANALYSIS

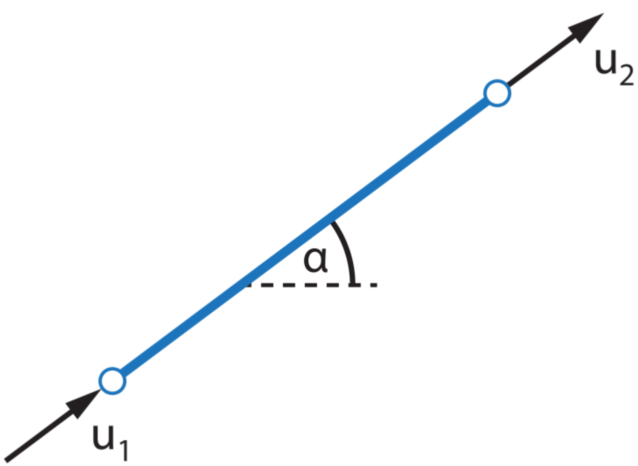
- TRUSS ELEMENT

$$\mathbf{K}^{\star} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

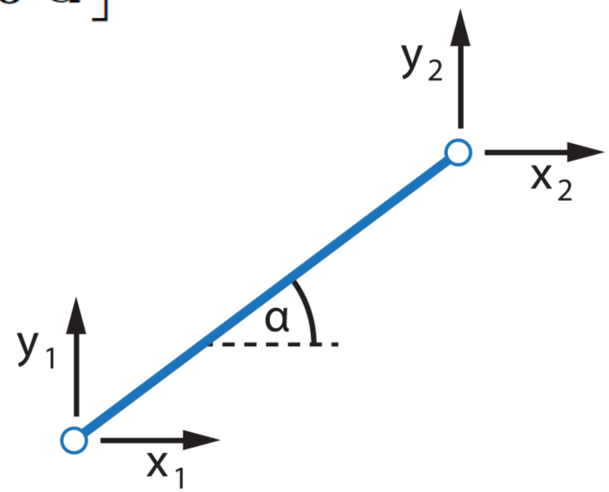
$$\mathbf{K}_e = \mathbf{T}_e^T \mathbf{K}_e^{\star} \mathbf{T}_e$$

$$\mathbf{d} = \frac{1}{L}\left[x_2 - x_1, y_2 - y_1, z_2 - z_1\right]$$

$$\mathbf{T} = \begin{bmatrix} \mathbf{d} & \mathbf{0} \\ \mathbf{0} & \mathbf{d} \end{bmatrix}$$



LOCAL COORDINATES



GLOBAL COORDINATES

4 – DETAILED ANALYSIS

- SENSITIVITY W.R.T COORD 'N' OF NODE 'J'

$$\frac{\partial \mathbf{K}_e}{\partial n_j} = \frac{\partial \mathbf{T}_e^T}{\partial n_j}\mathbf{K}_e^\star \mathbf{T}_e + \mathbf{T}_e^T \frac{\partial \mathbf{K}_e^\star}{\partial L}\frac{\partial L}{\partial n_j}\mathbf{T}_e + \mathbf{T}_e^T \mathbf{K}_e^\star \frac{\partial \mathbf{T}_e}{\partial n_j}$$

$$\frac{\partial L}{\partial n_1} = -\mathbf{d}_n \qquad \frac{\partial L}{\partial n_2} = \mathbf{d}_n$$

$$\frac{\partial \mathbf{K}^\star}{\partial L} = -\frac{AE}{L^2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\mathbf{J}_{(1)}(\mathbf{d}) = \frac{1}{L}\left(\mathbf{d}^T\mathbf{d} - \mathbf{I}\right) \qquad \mathbf{J}_{(2)}(\mathbf{d}) = -\mathbf{J}_{(1)}(\mathbf{d})$$

4 – DETAILED ANALYSIS

- SENSITIVITY W.R.T CROSS-SECTIONAL AREA

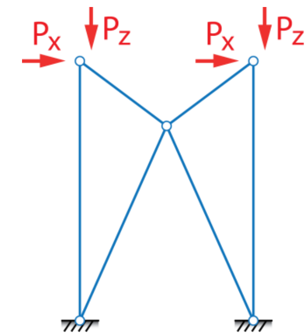
$$\frac{\partial \mathbf{K}_e}{\partial A_e} = \frac{1}{A_e}\mathbf{K}_e$$

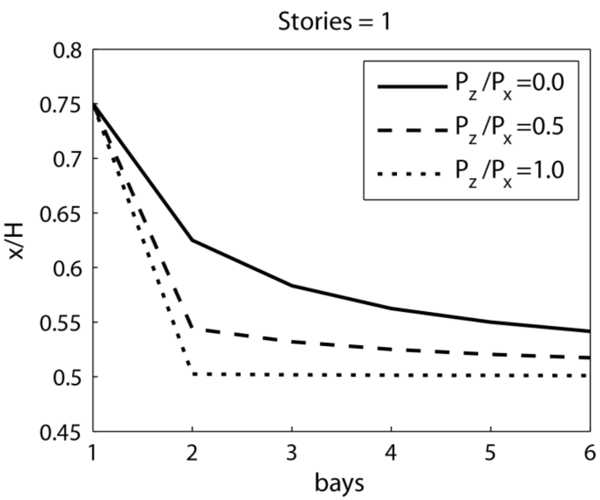
- THE DIFFICULTY LIES IN THE GEOMETRICAL OPTIMIZATION…

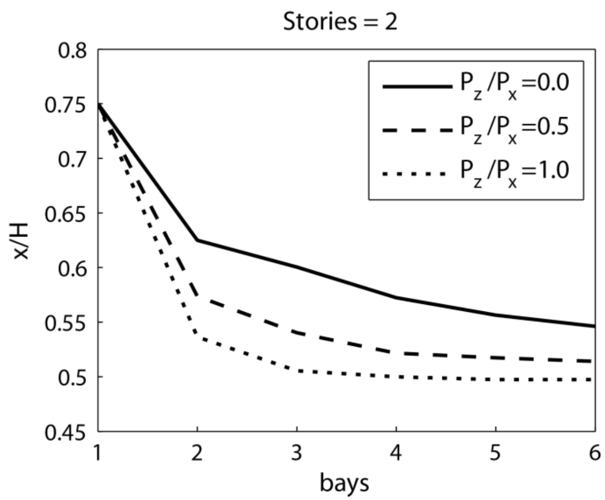
HANSEN S R, VANDERPLAATS G N (1988) – "AN APPROXIMATION METHOD FOR CONFIGURATION OPTIMIZATION OF TRUSSES", AAIA JOURNAL 28(1), PP 161–168

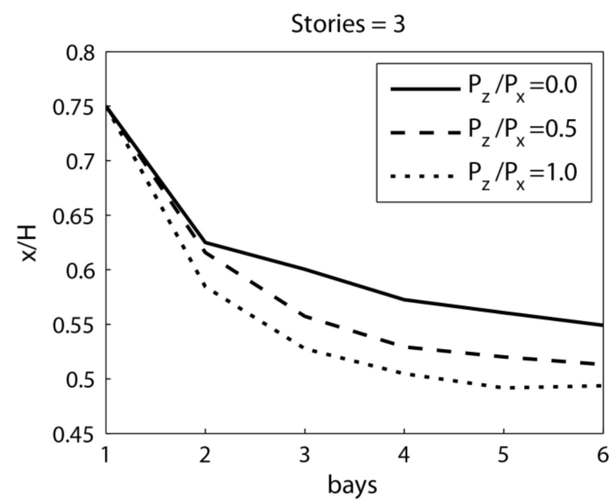
5 – RESULTS

- OPTIMAL BRACING POINT FOR TWO-DIMENSIONAL BRACES



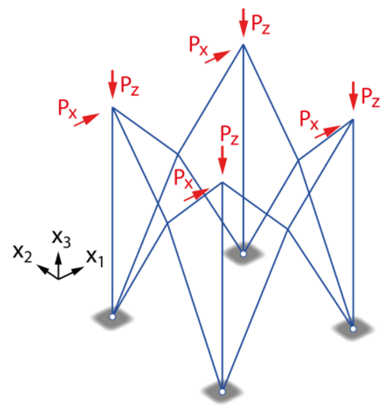


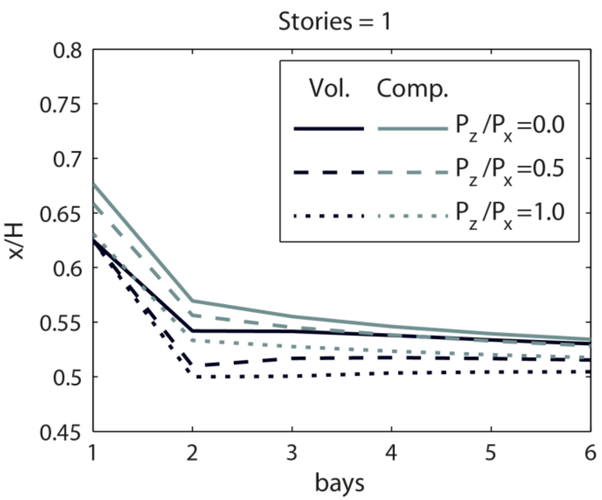


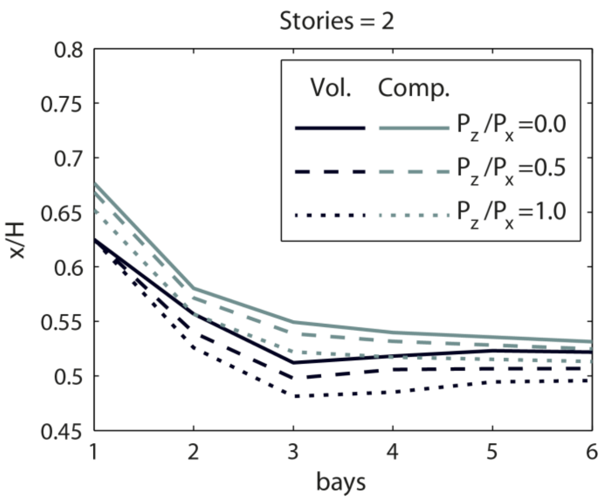


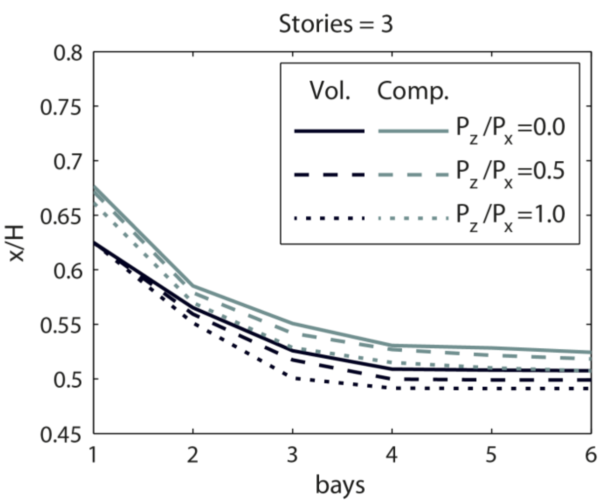
5 – RESULTS

- OPTIMAL BRACING POINT FOR THREE-DIMENSIONAL BRACES



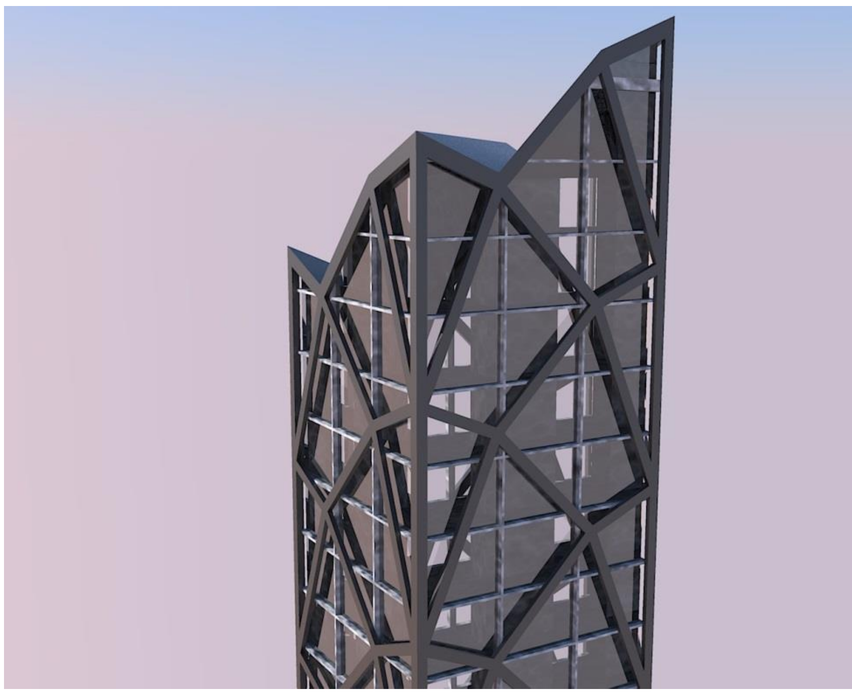






6 – FINAL REMARKS

- THE CROSS-BRACING IS NOT ALWAYS OPTIMAL
- OPTIMAL BRACING POINT LOCATION
  - BETWEEN **0.5H** AND **0.75H** IN 2D
  - BETWEEN **0.5H** AND **0.6768H** IN 3D



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THE END

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