# TRUSS LAYOUT OPTIMIZATION EMBEDDED IN A CONTINUUM

TOMAS ZEGARD – GLAUCIO H. PAULINO

ILLINOIS
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

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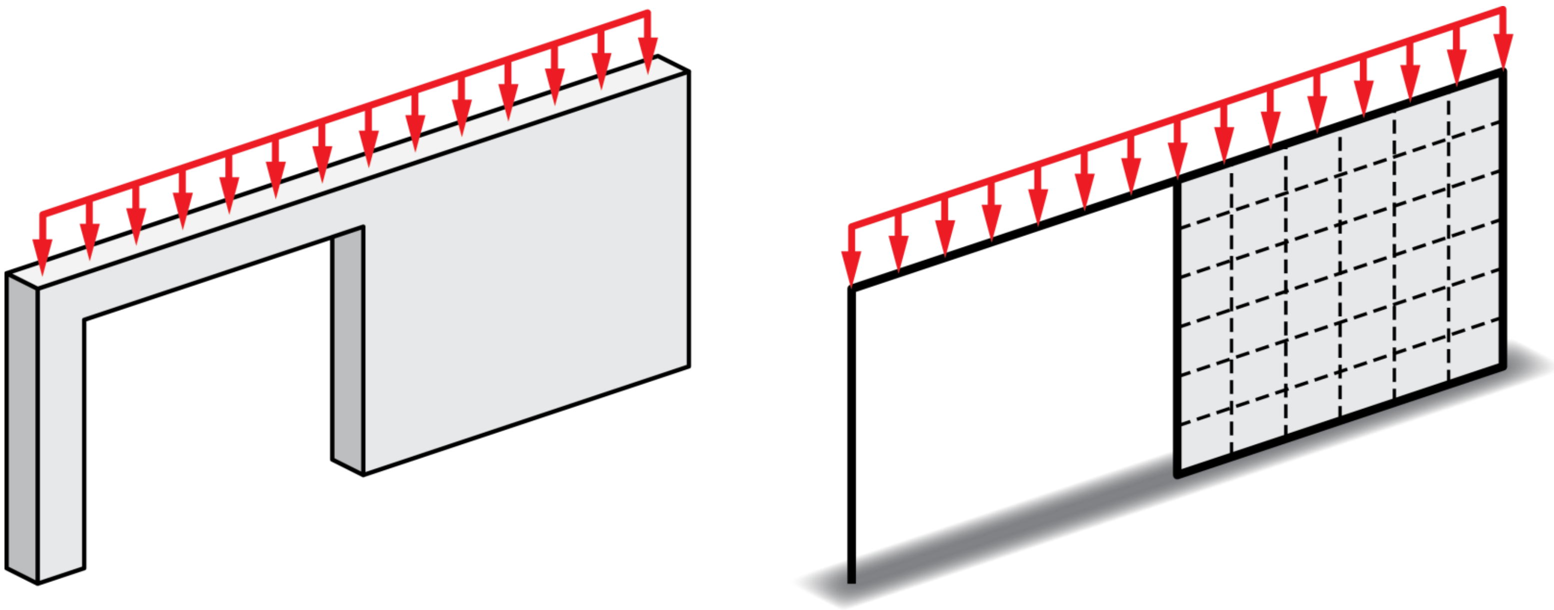
# 1 – INTRODUCTION & MOTIVATION

- WHY DISCRETE—CONTINUUM?
  - LIMITED MODELING CAPABILITY
  - REASONABLE SIMPLIFICATIONS OF REALITY

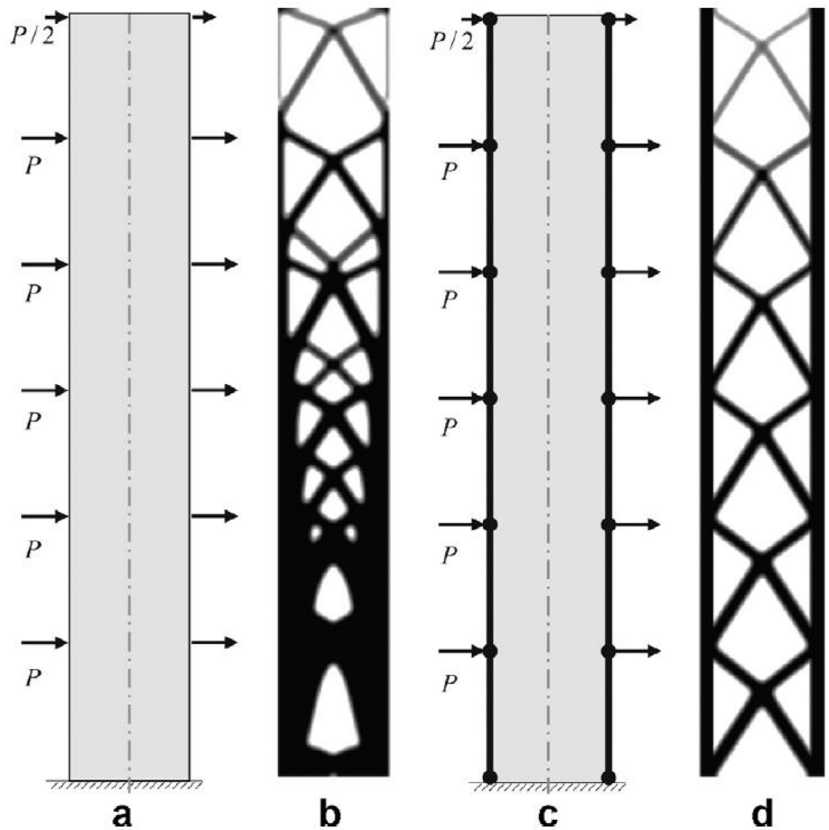


REAL FRAME

SIMPLIFIED FRAME MODEL

# 1 – Introduction & Motivation

- Continuum opt w. discrete elements

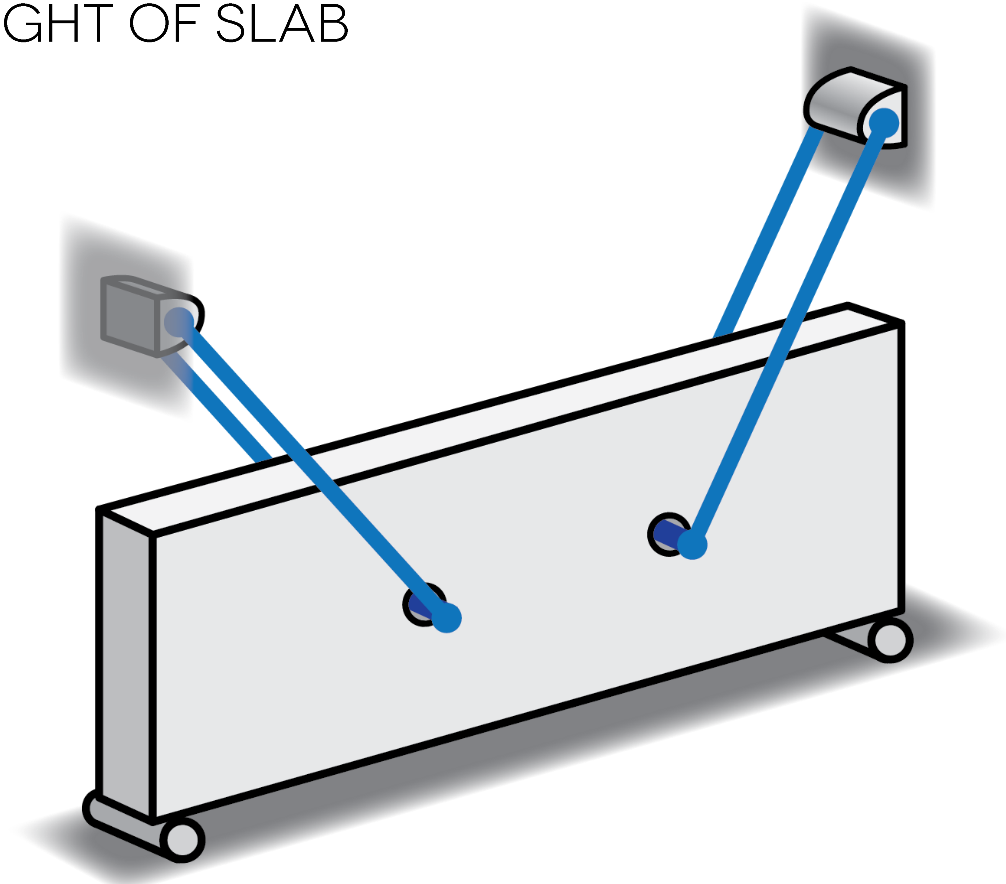



Stromberg L L, Beghini A, Baker W F, Paulino G H (2012) – "Topology optimization for braced frames: combining continuum and beam/column elements", Engineering Structures 27, pp 106–124

# 1 – INTRODUCTION & MOTIVATION

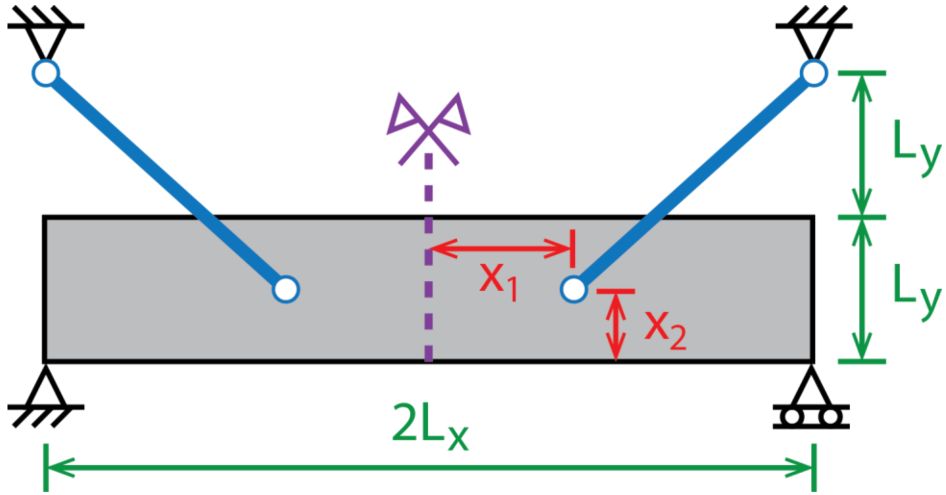
- SLAB WITH SUPPORTING CABLES

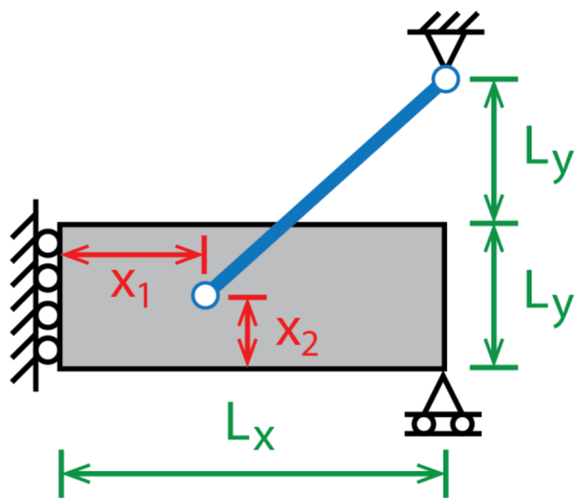
SELF WEIGHT OF SLAB



# 1 – INTRODUCTION & MOTIVATION

- SLAB WITH SUPPORTING CABLES



# 2 - LINKAGE FORMULATION

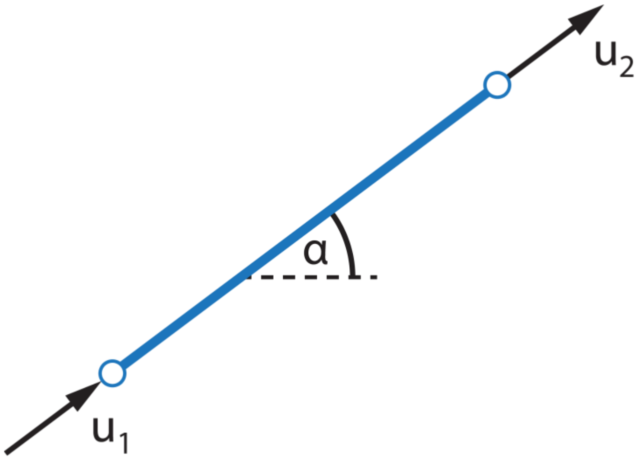
- TRUSS ELEMENT

$$\mathbf{K}^{\star} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

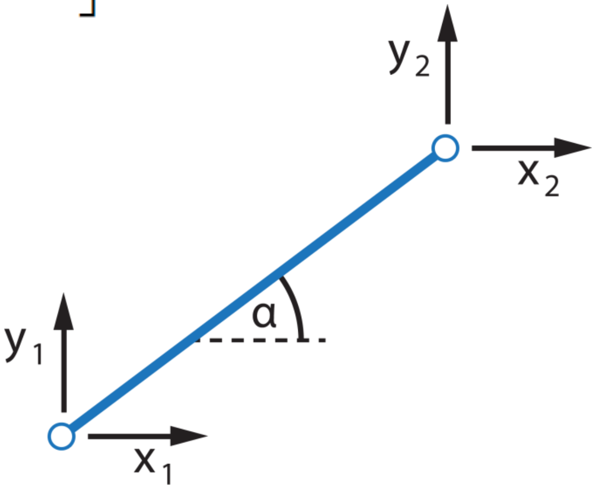
$$\mathbf{K}_e = \mathbf{T}_e^T \mathbf{K}_e^{\star} \mathbf{T}_e$$

$$\mathbf{d} = \frac{1}{L}\left[x_2 - x_1, y_2 - y_1, z_2 - z_1\right]$$

$$\mathbf{T} = \begin{bmatrix} \mathbf{d} & \mathbf{0} \\ \mathbf{0} & \mathbf{d} \end{bmatrix}$$



LOCAL COORDINATES



GLOBAL COORDINATES

# 2 – LINKAGE FORMULATION

- SENSITIVITY W.R.T COORD 'N' OF NODE 'J'

$$\frac{\partial \mathbf{K}_e}{\partial n_j} = \frac{\partial \mathbf{T}_e^T}{\partial n_j}\mathbf{K}_e^\star \mathbf{T}_e + \mathbf{T}_e^T \frac{\partial \mathbf{K}_e^\star}{\partial L}\frac{\partial L}{\partial n_j}\mathbf{T}_e + \mathbf{T}_e^T \mathbf{K}_e^\star \frac{\partial \mathbf{T}_e}{\partial n_j}$$

$$\frac{\partial L}{\partial n_1} = -\mathbf{d}_n \qquad \frac{\partial L}{\partial n_2} = \mathbf{d}_n$$

$$\frac{\partial \mathbf{K}^\star}{\partial L} = -\frac{AE}{L^2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\mathbf{J}_{(1)}(\mathbf{d}) = \frac{1}{L}\left(\mathbf{d}^T\mathbf{d} - \mathbf{I}\right) \qquad \mathbf{J}_{(2)}(\mathbf{d}) = -\mathbf{J}_{(1)}(\mathbf{d})$$

HANSEN S R, VANDERPLAATS G N (1988) – "AN APPROXIMATION METHOD FOR CONFIGURATION OPTIMIZATION OF TRUSSES", AAIA JOURNAL 28(1), PP 161–168

# 2 – LINKAGE FORMULATION

- DISPLACEMENTS "U" ANYWHERE IN Ω USING FEM SHAPE FUNCTIONS

$$\mathbf{u} = \mathbf{N}\mathbf{u}_c$$

- CONFORMING COUPLING
  - TRUSS MEMBER'S $\mathbf{K}_e$ IS COUPLED BY AN EQUIVALENT $\mathbf{K}_e^+$ MATRIX

$$\mathbf{u}^T\mathbf{K}_e\mathbf{u} = \mathbf{u}_c^T\mathbf{K}_e^+\mathbf{u}_c$$

$$(\mathbf{N}\mathbf{u}_c)^T\,\mathbf{K}_e\,(\mathbf{N}\mathbf{u}_c) = \mathbf{u}_c^T\mathbf{K}_e^+\mathbf{u}_c$$

$$\mathbf{u}_c^T\,\left(\mathbf{N}^T\mathbf{K}_e\mathbf{N}\right)\mathbf{u}_c = \mathbf{u}_c^T\mathbf{K}_e^+\mathbf{u}_c$$

$$\boxed{\mathbf{N}^T\mathbf{K}_e\mathbf{N} = \mathbf{K}_e^+}$$

# 2 – LINKAGE FORMULATION

- COMPLIANCE FORMULATION

$$\min_{\mathbf{A},\mathbf{x}} \quad C = \mathbf{u}^T\mathbf{K}\mathbf{u} = \mathbf{u}^T\mathbf{f}$$

$$\text{s.t.} \quad \textstyle\sum_i A_i L_i \leq \bar{V}$$

← REQUIRED IF MEMBERS ARE ALSO SIZED

$$\text{with} \quad \mathbf{K}\mathbf{u} = \mathbf{f}$$

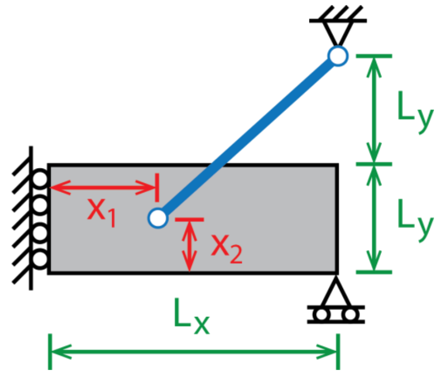
# 3 – OPTIMIZATION DIFFICULTIES

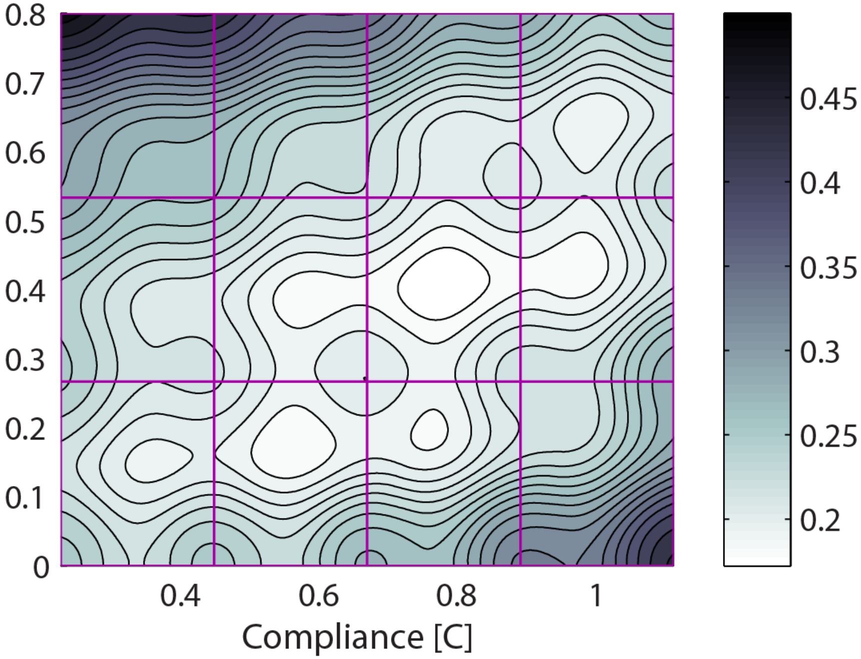
- 3X9 MESH (Q4 ELEMS)

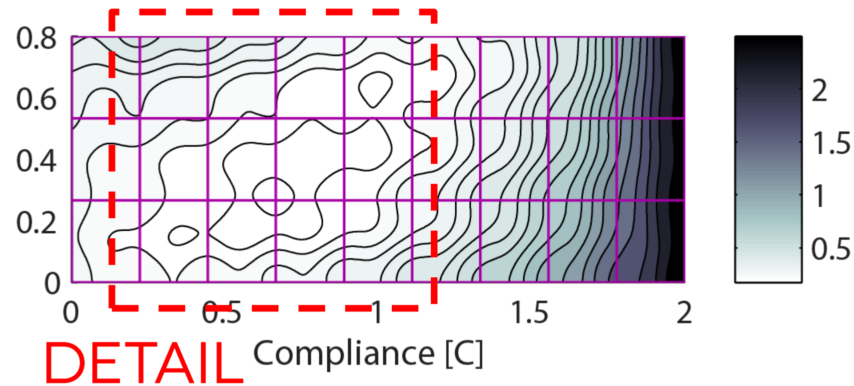
  SLAB: LX=2 LY=0.8 E=100 $\nu$ =0.3

  CABLE: AE=300

  LOAD: B=[0 −2]



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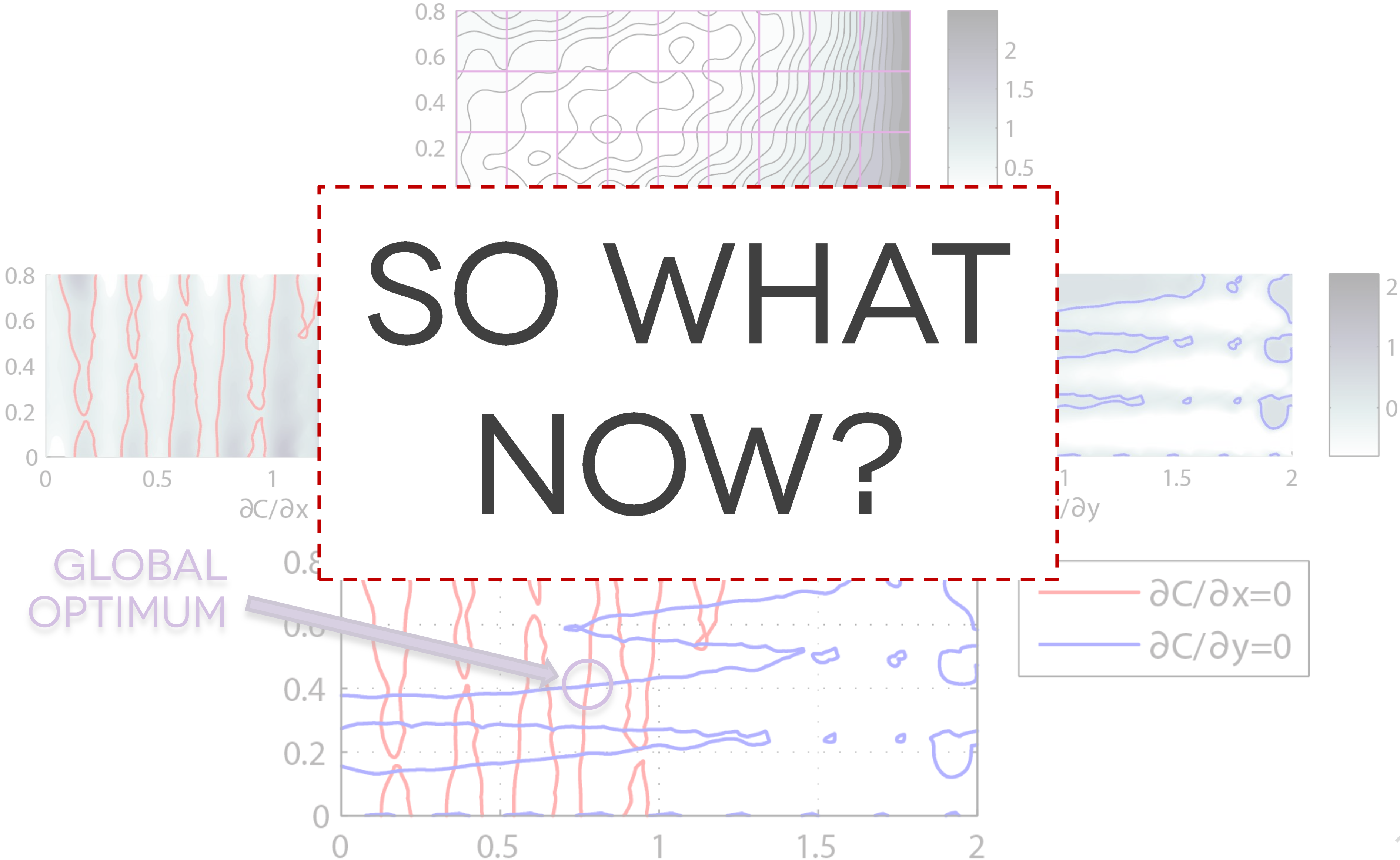



**SO WHAT NOW?**

# 4 – CONVOLUTION FORMULATION

- GAUSSIAN BLUR
  - CONVOLUTION WITH A GAUSSIAN FUNCTION

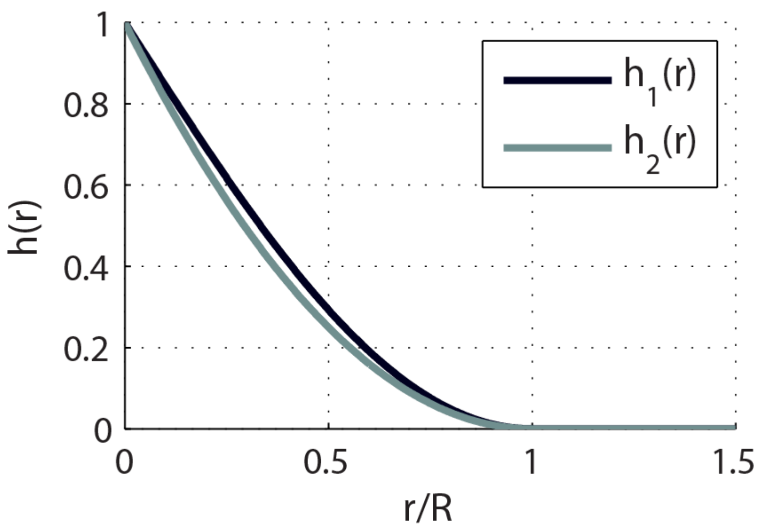

HTTP://EN.WIKIPEDIA.ORG/WIKI/GAUSSIAN_BLUR

# 4 – CONVOLUTION FORMULATION

- ARBITRARILY SMOOTH CONVOLUTION?

$$h(0) = 1$$
$$h(r \geq R) = 0$$
$$\left.\frac{dh}{dr}\right|_{r=R} = 0$$

REQUIREMENTS



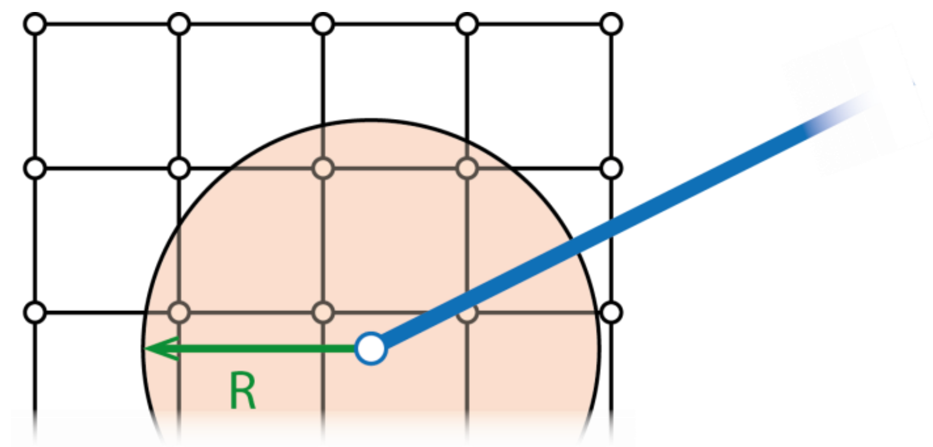

$$h_1(r) = \begin{cases} 1 - \sin\left(\frac{r\pi}{2R}\right) & r \leq R \\ 0 & r > R \end{cases}$$

$$h_2(r) = \begin{cases} \left(\frac{r}{R}\right)^2 - 2\left(\frac{r}{R}\right) + 1 & r \leq R \\ 0 & r > R \end{cases}$$

# 4 - CONVOLUTION FORMULATION

- CONVOLUTION-BASED SHAPE FUNCTIONS
  - PARTITION OF UNITY REQUIREMENT

$$\tilde{N}_a = \frac{h\left(r_a\right)}{\sum_k h\left(r_k\right)}$$

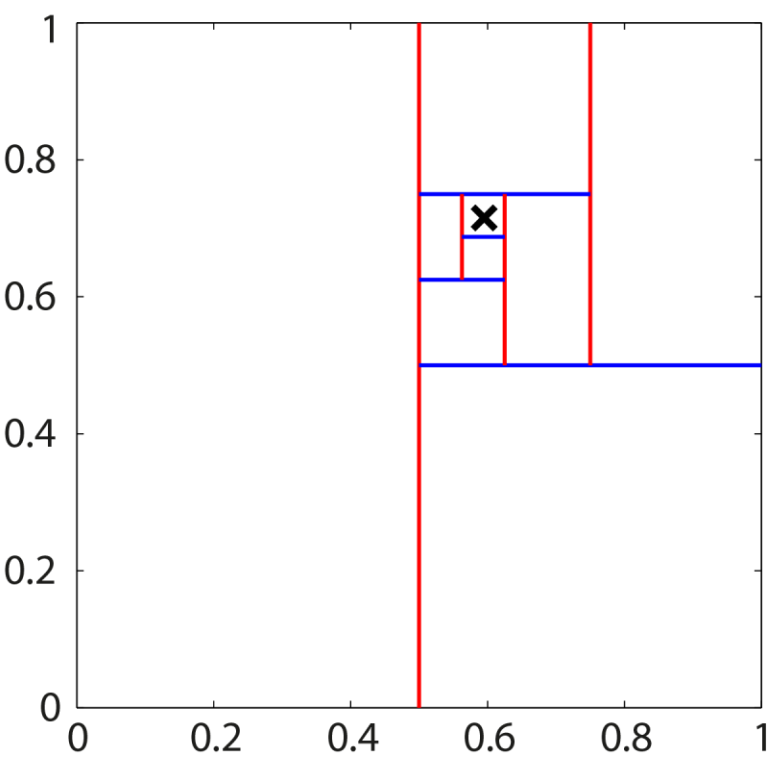


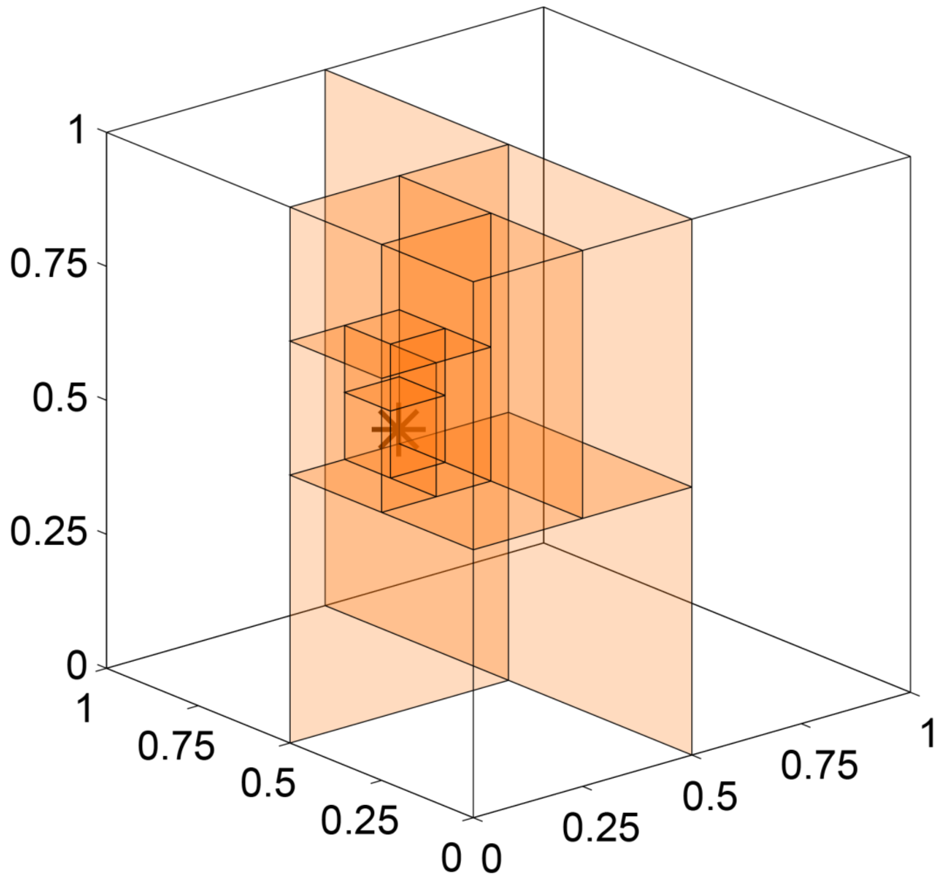

$$\mathbf{K}_e^+ = \tilde{\mathbf{N}}^T \mathbf{K}_e \tilde{\mathbf{N}}$$

$$\frac{\partial \mathbf{K}_e^+}{\partial n_j} = \frac{\partial \tilde{\mathbf{N}}^T}{\partial n_j} \mathbf{K}_e \tilde{\mathbf{N}} + \tilde{\mathbf{N}}^T \frac{\partial \mathbf{K}_e}{\partial n_j} \tilde{\mathbf{N}} + \tilde{\mathbf{N}}^T \mathbf{K}_e \frac{\partial \tilde{\mathbf{N}}}{\partial n_j}$$

# 4 - CONVOLUTION FORMULATION

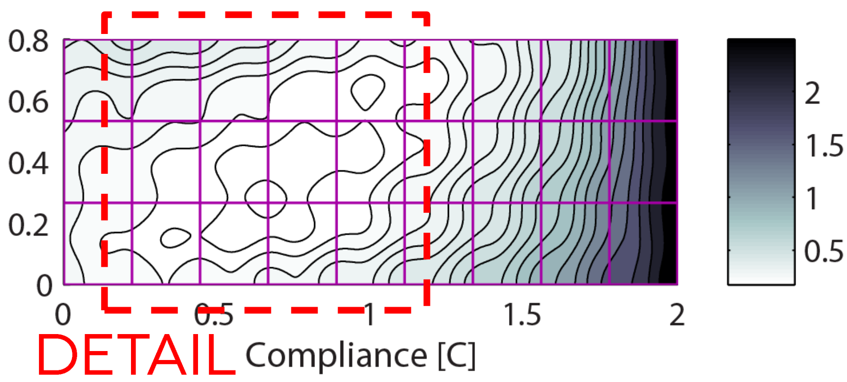
- EFFICIENT CONVOLUTION SEARCH
  - BINARY, QUAD & OCT TREES



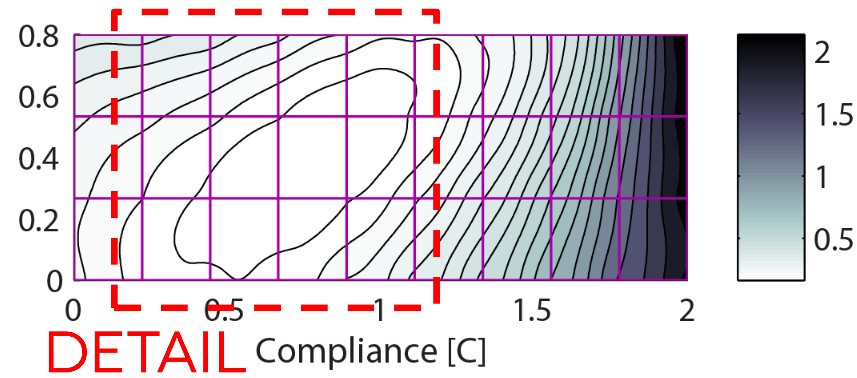



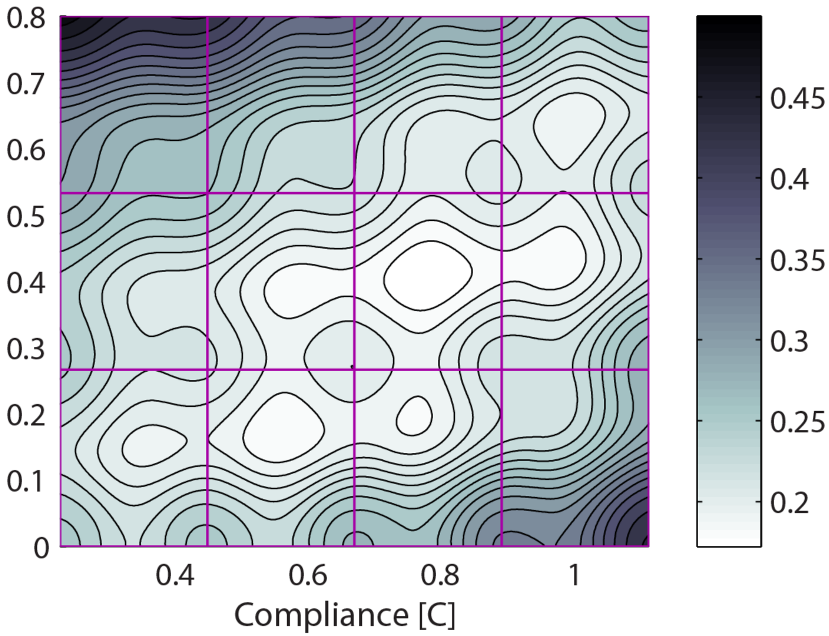
# 5 - COMPARISON & ANALYSIS

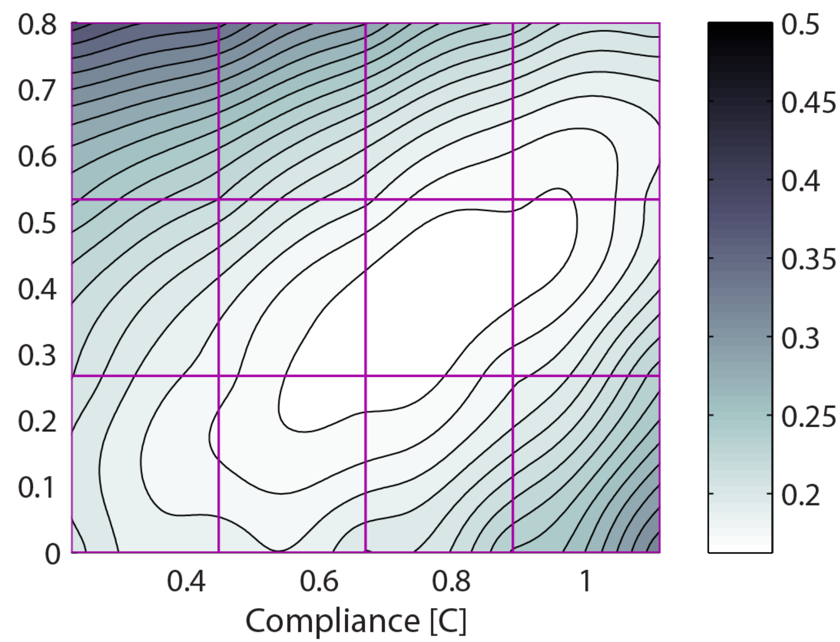
## FEM SHAPE FUNCTIONS




## CONVOLUTION (R=0.5)



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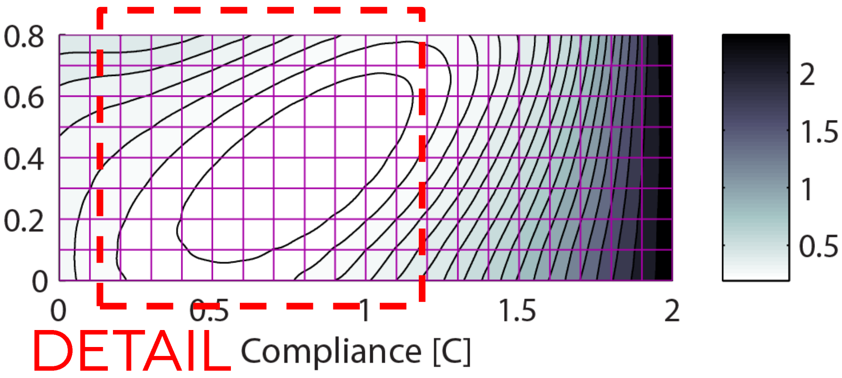
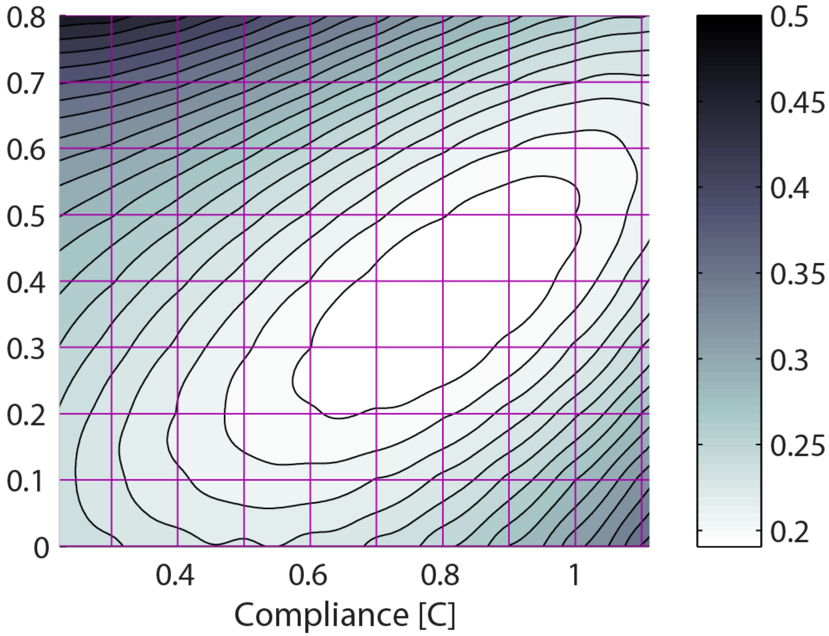
## CONVOLUTION (R=0.3)

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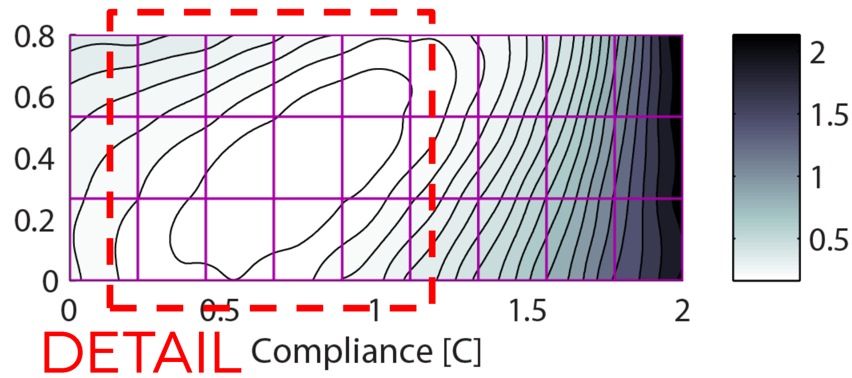


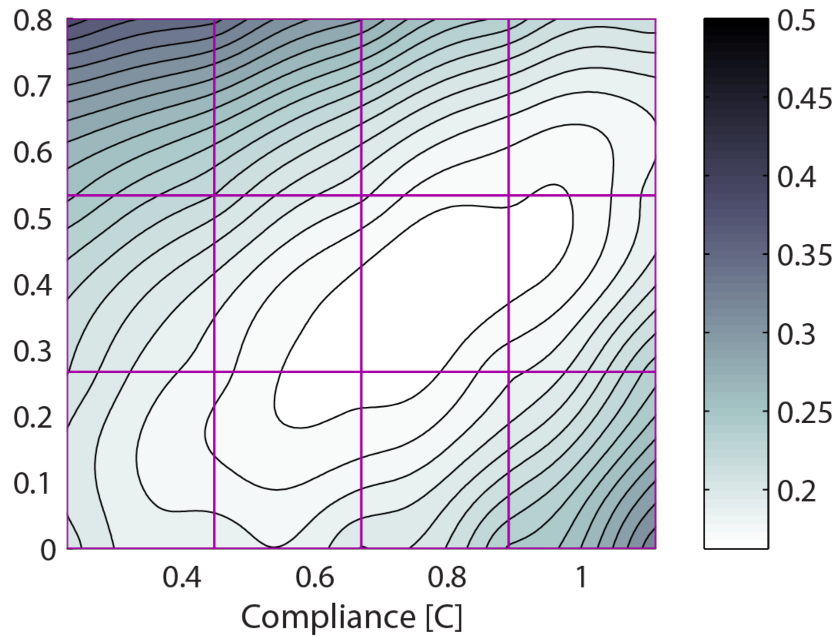

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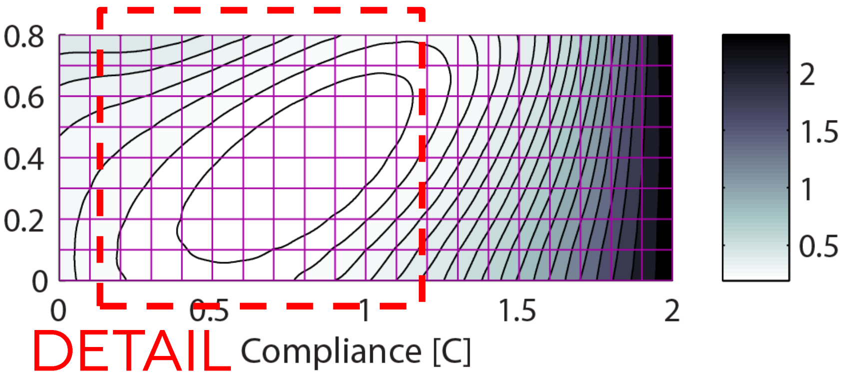
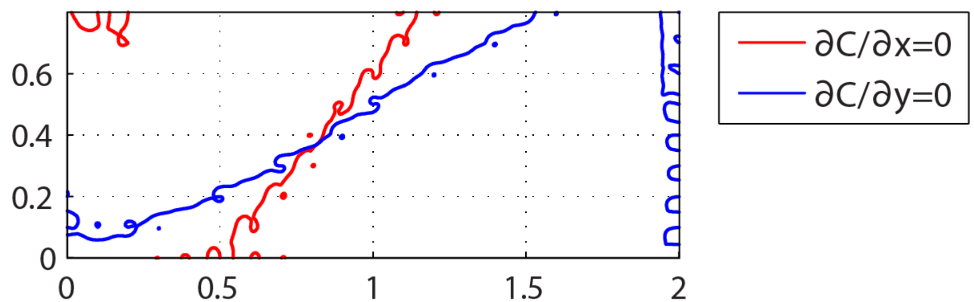
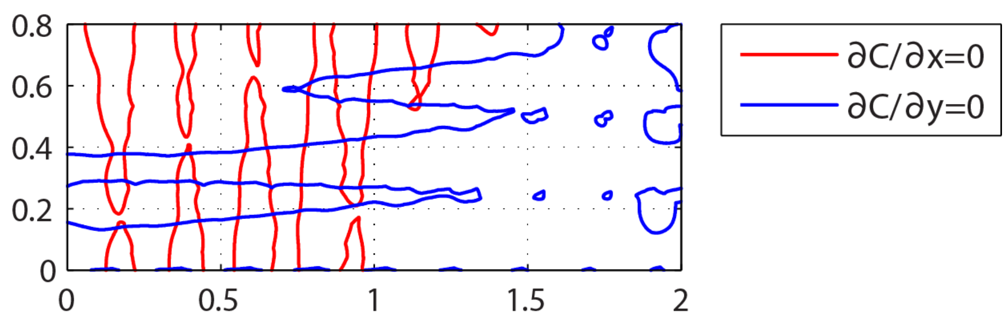
## CONVOLUTION (R=0.3)



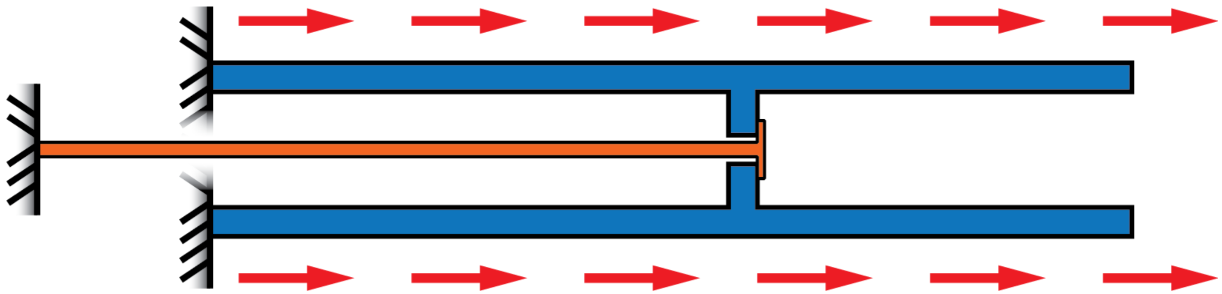




PREVIOUS CASE

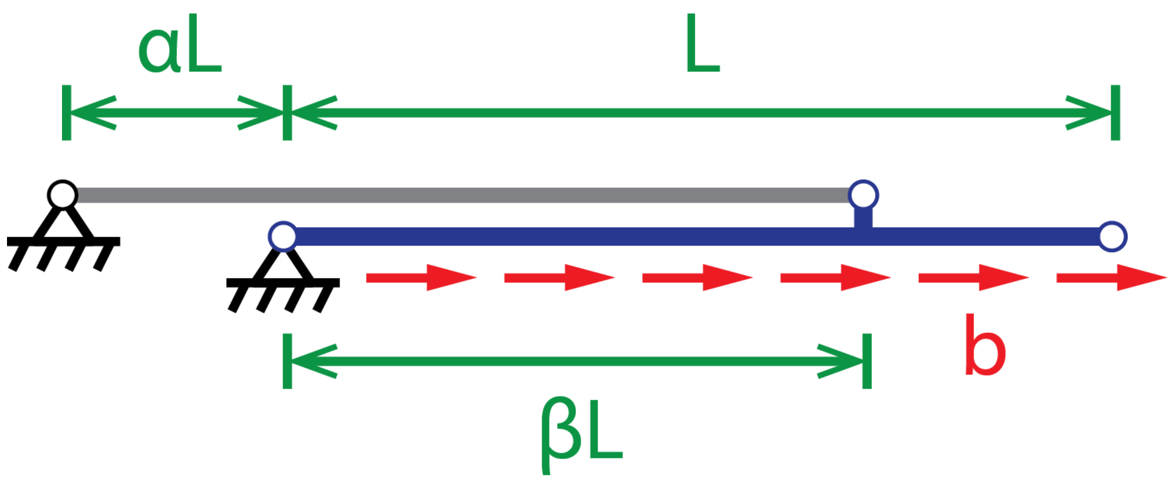


# 6 – VERIFICATION

- 1D PROBLEM



- MODEL IDEALIZATION



# 6 – VERIFICATION

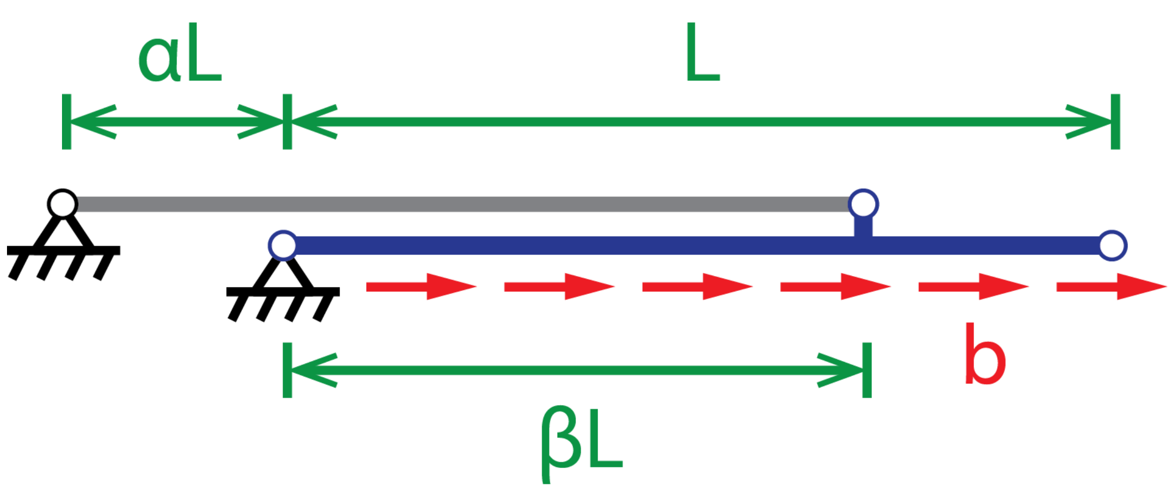
- ANALYTICAL SOLUTION

GIVEN

$$\gamma = EA/E_cA_c$$

COMPLIANCE IS

$$C = \frac{b^2L^3}{12EA}\frac{4\alpha + 4\beta + 4\gamma\beta - 12\gamma\beta^2 + 12\gamma\beta^3 - 3\gamma\beta^4}{\alpha + \beta + \gamma\beta}$$

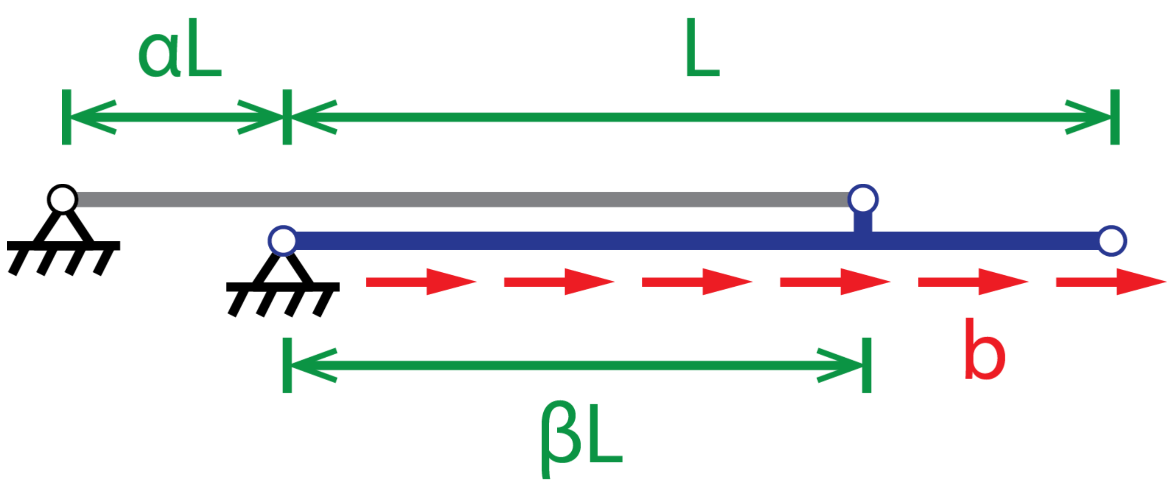


# 6 – VERIFICATION

- OPTIMAL ANCHOR POINT

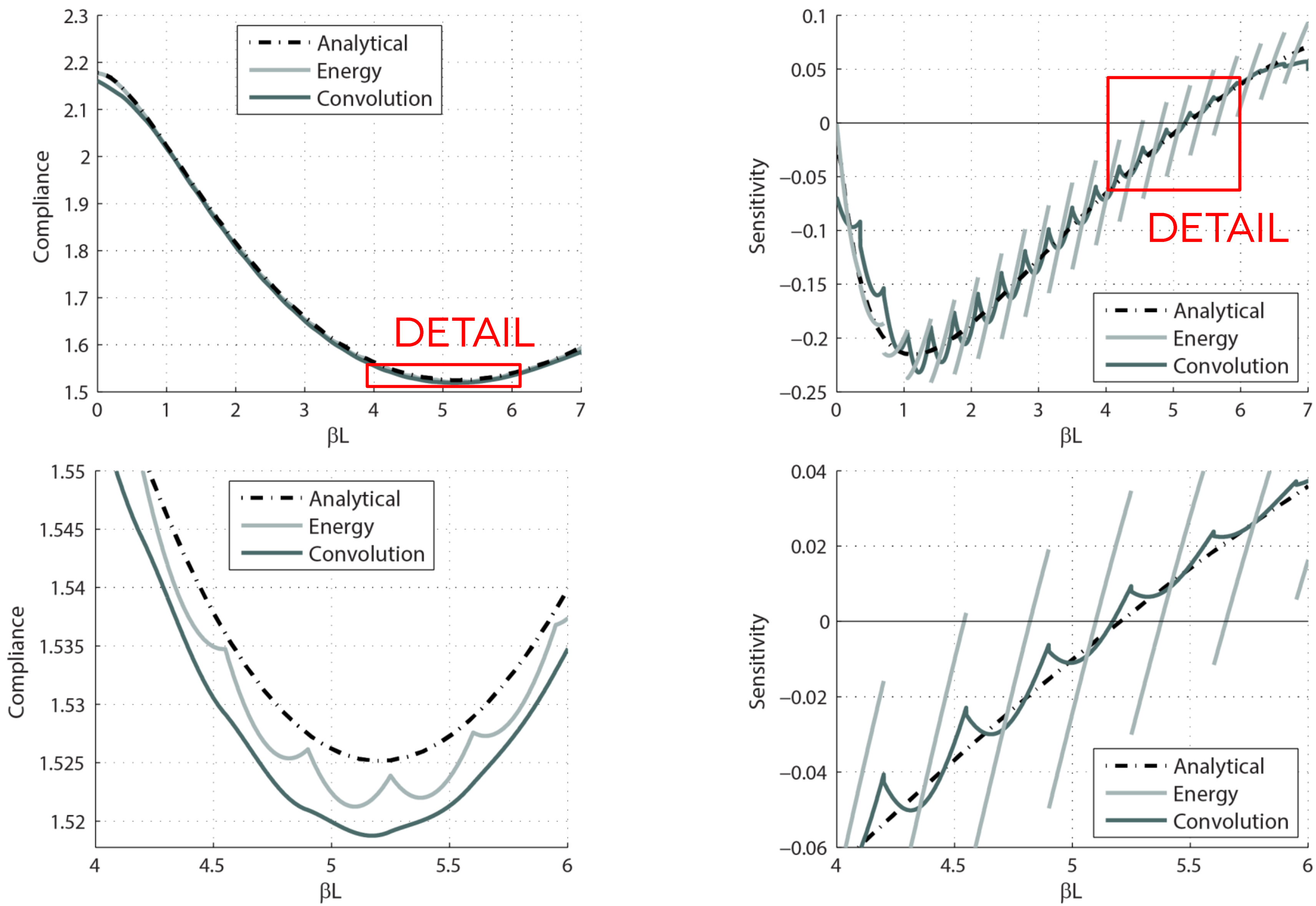
$$\beta_c = \frac{1 + \gamma - 2\alpha + \sqrt{1 + 8\alpha + 2\gamma + 4\alpha^2 + 8\alpha\gamma + \gamma^2}}{3 + 3\gamma}$$

, for $\beta_c \leq 1$, and at $\beta_c = 1$ otherwise.



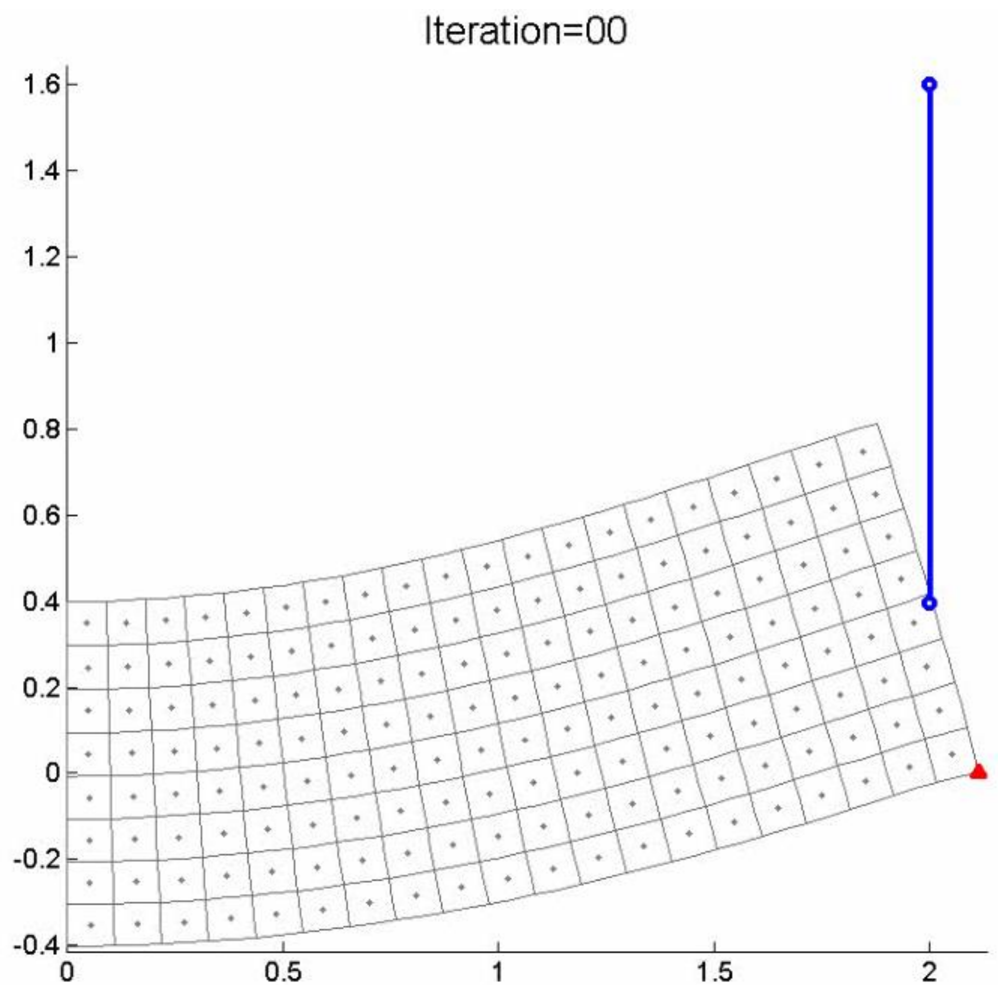
# 6 – VERIFICATION

20 L2 ELEMS LOAD: B=2
ROD: L=7 $\alpha$=2/7 EA=150
CABLE: AE=210



# 7 - EXAMPLES & RESULTS

- SLAB WITH SUPPORTING CABLES (8X21 Q9)

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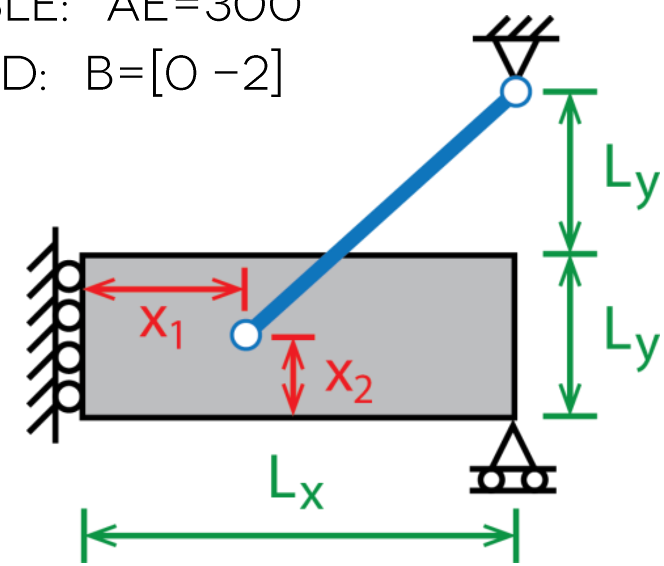
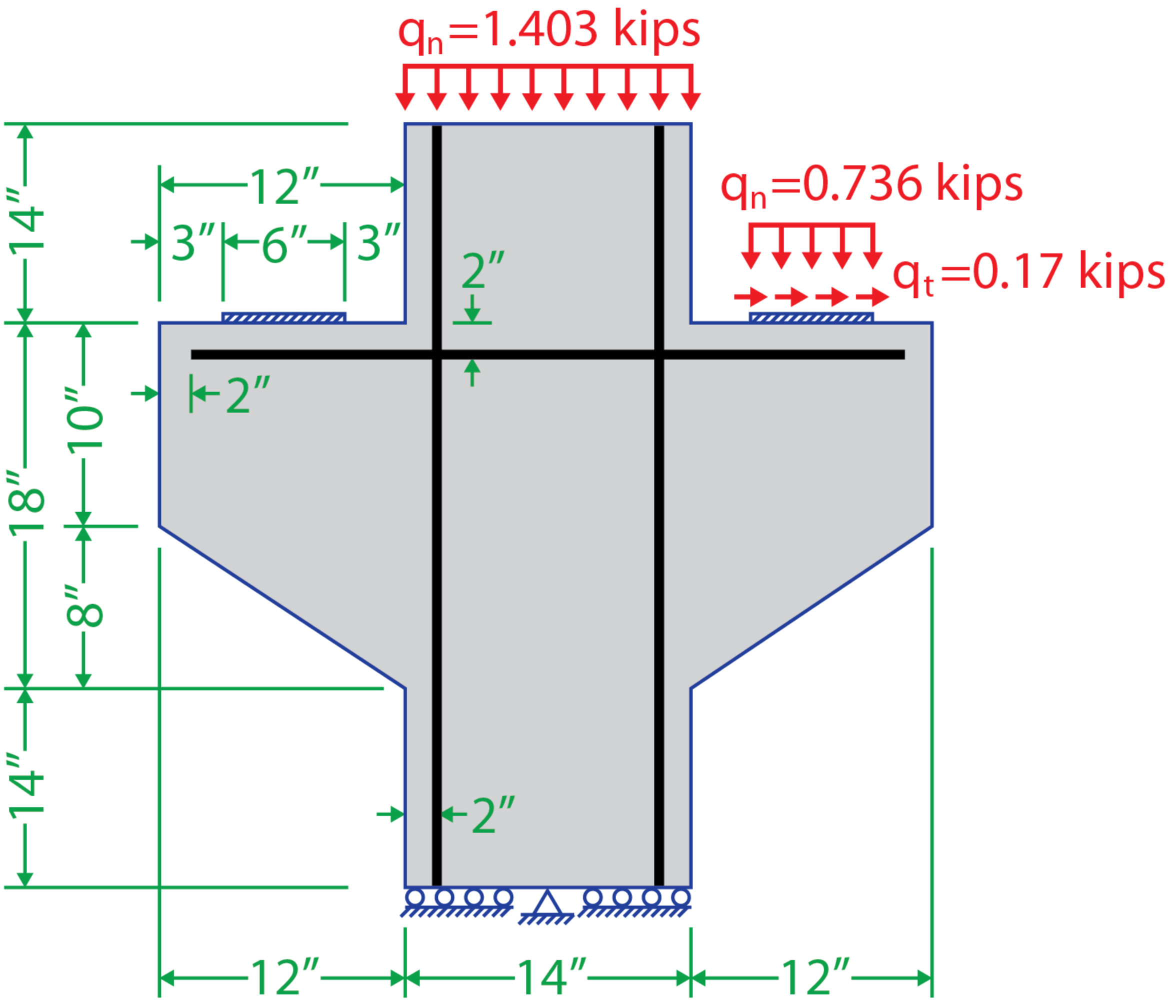


# 7 - EXAMPLES & RESULTS

- DOUBLE CORBEL (UNSTRUCTURED MESH)



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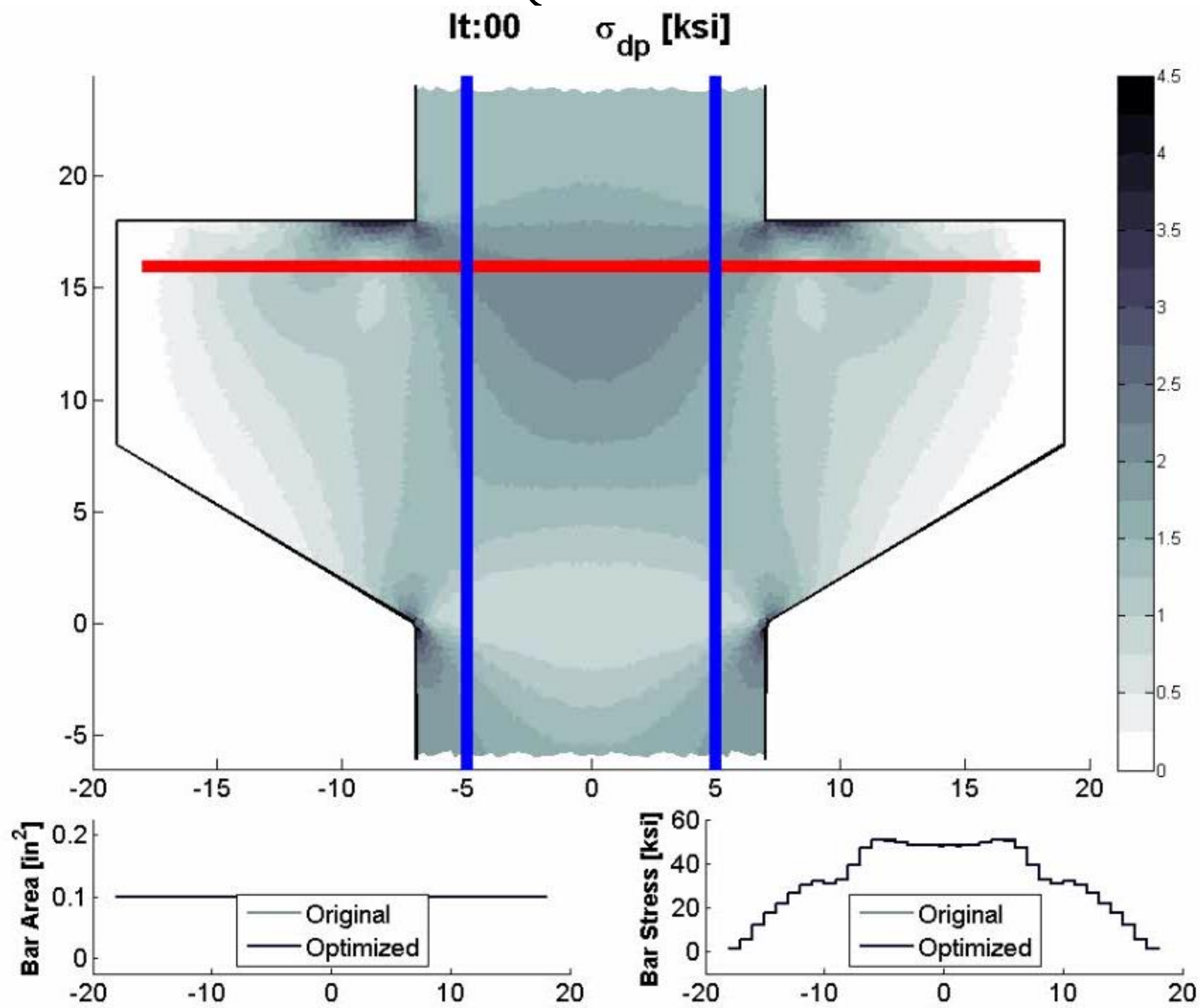
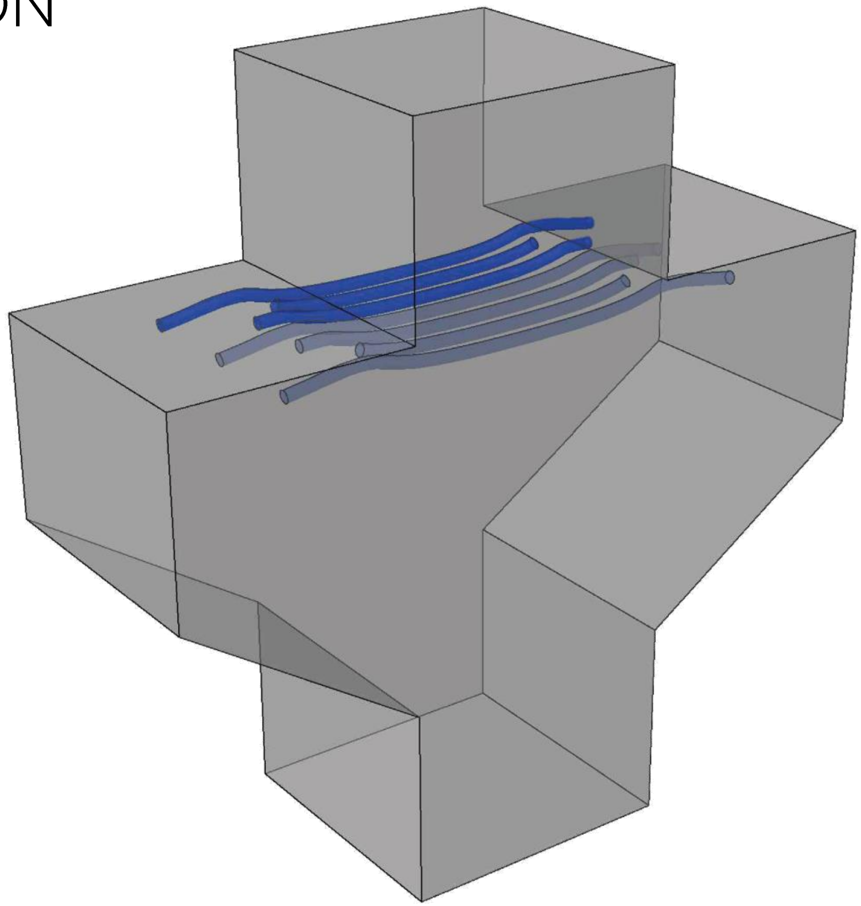


# 7 - EXAMPLES & RESULTS

- DOUBLE CORBEL (UNSTRUCTURED MESH)
  - INTERPRETATION



# 8 – CONCLUSIONS

- PROBLEM WITH TRADITIONAL COUPLING
- CONVOLUTION COUPLING
  - SIMPLE
  - FAST
  - ERROR DECREASES WITH REFINEMENT
  - IT WORKS!!!
- HELPS BRIDGE THE GAP BETWEEN CONTINUUM AND DISCRETE OPT

# THE END

ZEGARD T, PAULINO G H (2013) – "TRUSS LAYOUT OPTIMIZATION WITHIN A CONTINUUM"
STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 48(1), PP. 1–16