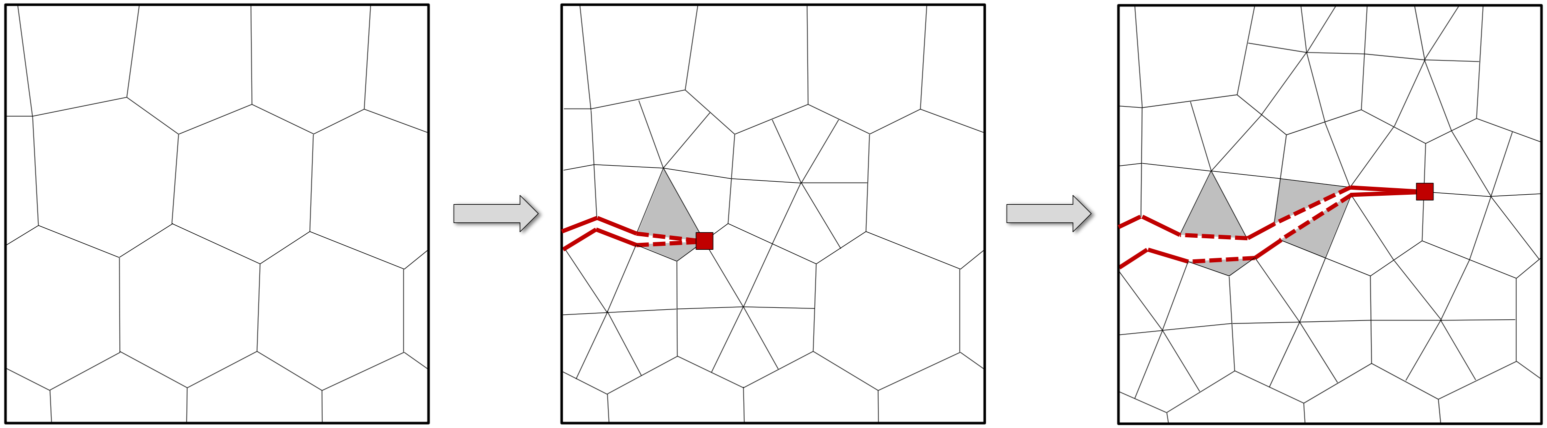
Unstructured Methods to Reduce Mesh Dependency in Dynamic Cohesive Fracture Simulations



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June 19th 2014

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Material Failure Pervades the Field of Engineering

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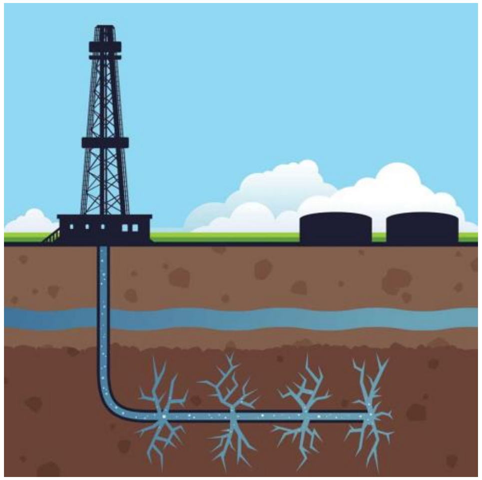

www.mlive.com

Biomechanical Applications

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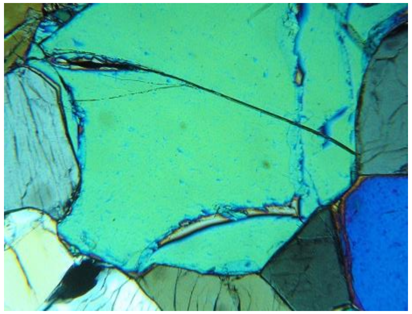
Failure Occurs at Various Scales and in Various Contexts

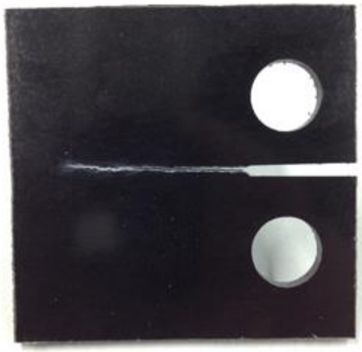
Range of Fracture Scale:



Intragranular Cracks

Earthquake Fault Lines

Range of Fracture Behavior:



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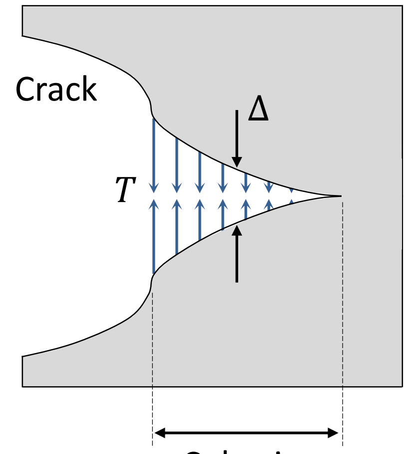
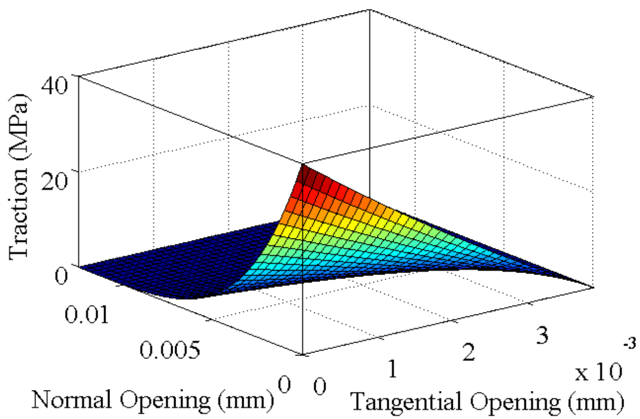
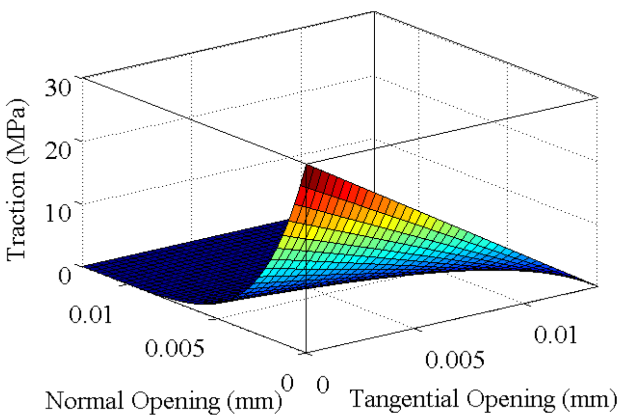
The Cohesive Element Method for Fracture

- The basic idea is to use cohesive elements to capture the inelastic failure zone in front of the crack tip.
- Cohesive elements initially have zero thickness and impart a traction on the surrounding bulk elements, as they separate.
- A macro-crack forms when the cohesive elements have fully separated (the traction-separation relation goes to zero).



The Park-Paulino-Roesler (PPR) model is one example of a cohesive model:



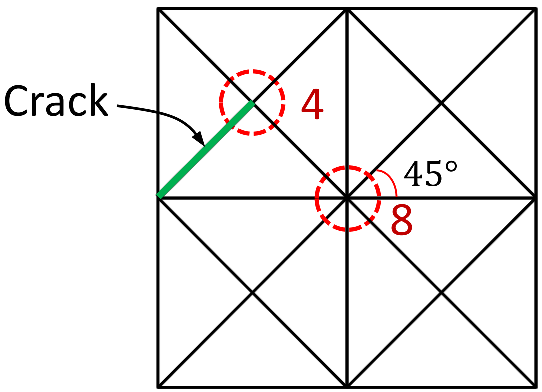


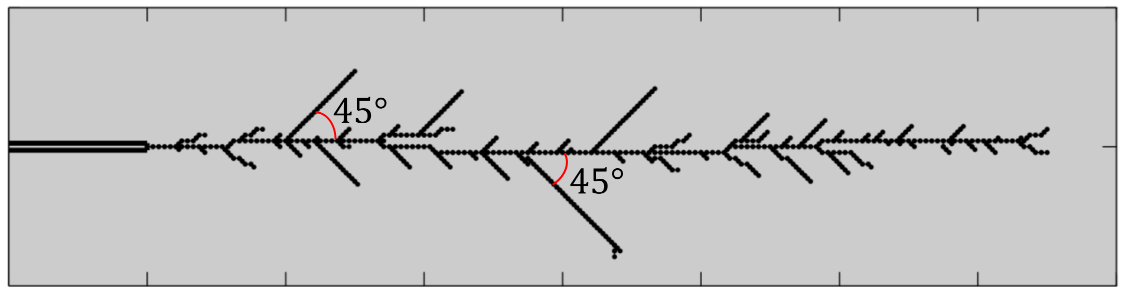
Park, K., Paulino, G. H., and Roesler, J. R., A unified potential-based cohesive model of mixed-mode fracture. *Journal of the Mechanics and Physics of Solids*, vol. 57, pp. 891–908, 2009.

The Argument for using Unstructured Meshes

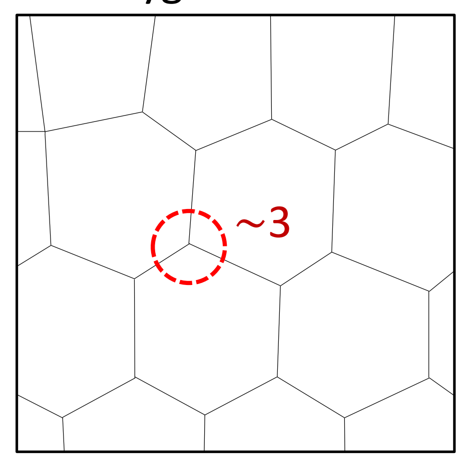
One of the primary critiques of the cohesive element method is its mesh dependency

4k Mesh





Polygonal Mesh



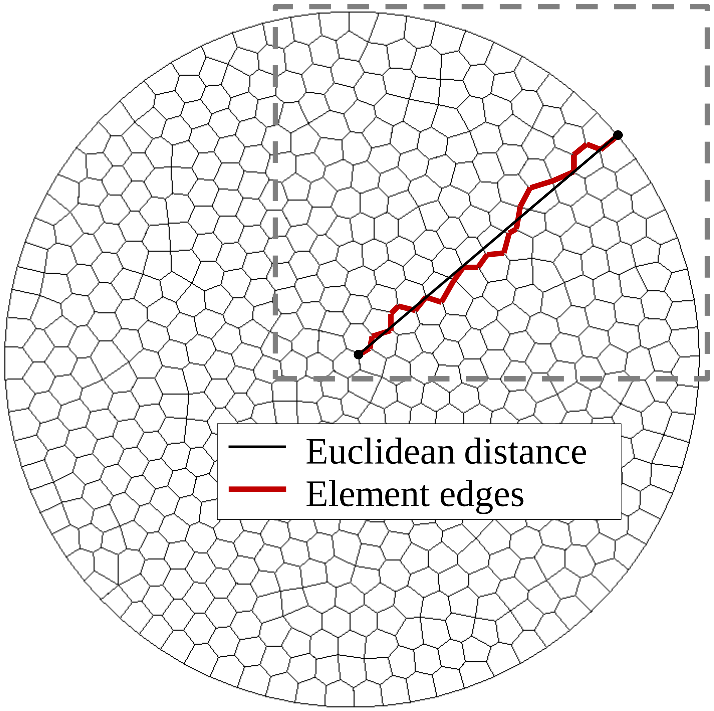
Structured 4k meshes are biased to cracks propagating at angles which are a multiple of 45°, but have a large number of crack paths at each node

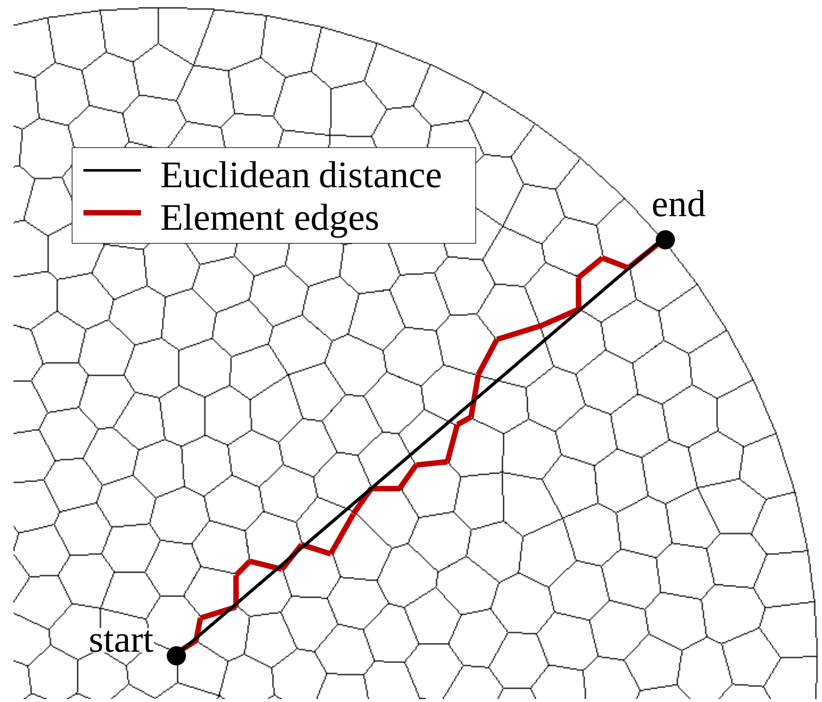
Polygonal meshes are unbiased, but limit number of crack paths at each node

Zhang, Z., Paulino, G.H., Celes, W., Extrinsic cohesive zone modeling of dynamic fracture and microbranching instability in brittle materials. *International Journal for Numerical Methods in Engineering*. vol. 72, pp. 893-923, 2007.

How to Quantify Mesh Isotropy/Anisotropy

Dijkstra’s algorithm is used to compute the shortest path between two points in the mesh



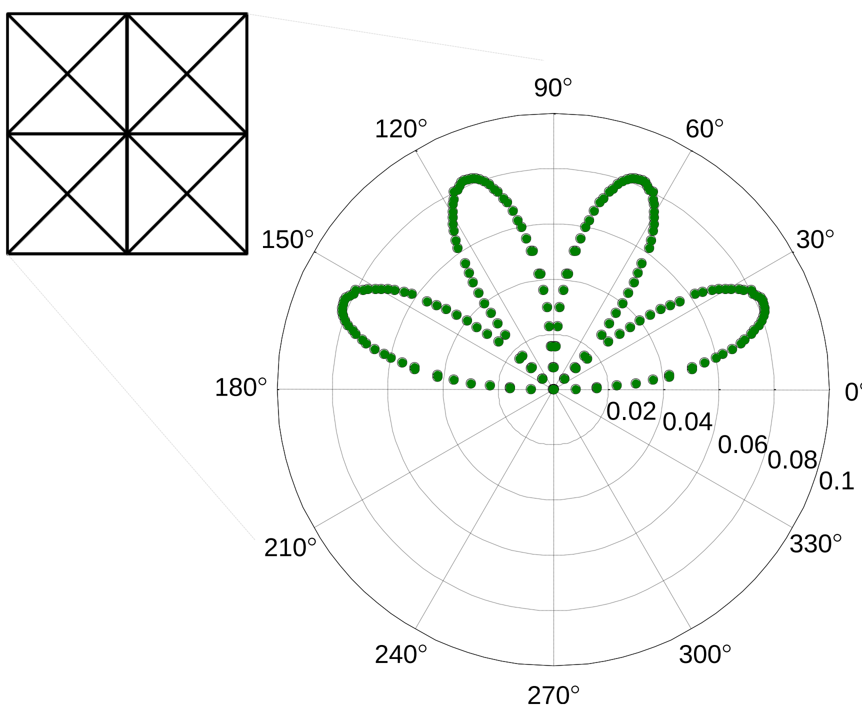


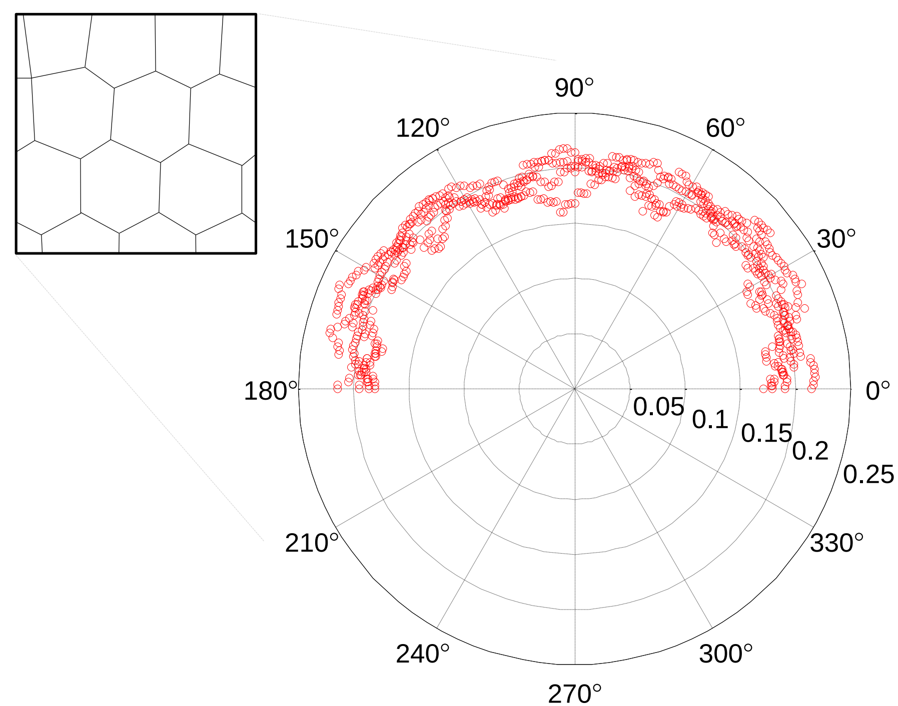
The path deviation is computed as: $\eta = \frac{L_g}{L_E} - 1$

Leon, S. E.*, Spring, D. W.*, Paulino, G. H., Reduction in mesh bias for dynamic fracture using adaptive splitting of polygonal elements. *Accepted to the International Journal for Numerical Methods in Engineering,* 2014.

Quantification of Mesh Isotropy/Anisotropy

A study was conducted on the path deviation over a range of 180°



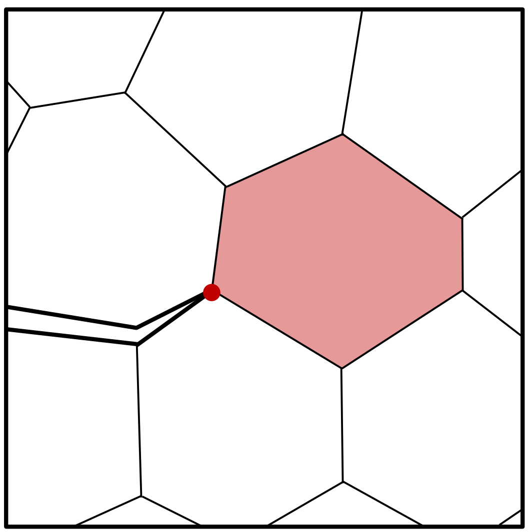


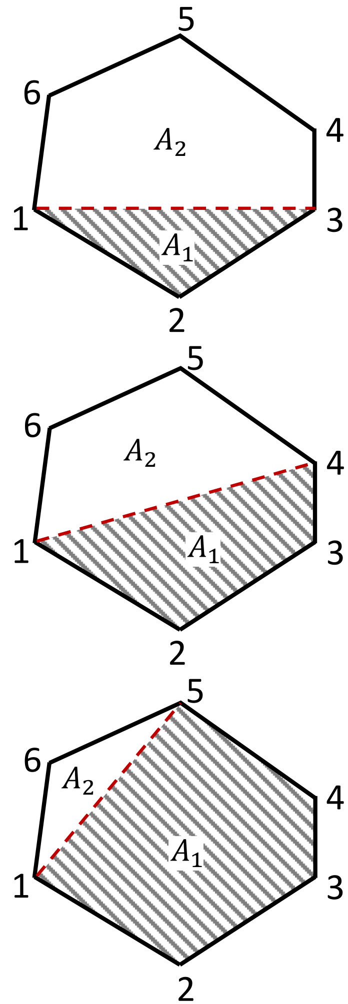
The structured 4K mesh is anisotropic, while the unstructured polygonal discretization is isotropic. However, the path deviation in the polygonal mesh is significantly higher that that in the 4k mesh.

Leon, S. E.*, Spring, D. W.*, Paulino, G. H., Reduction in mesh bias for dynamic fracture using adaptive splitting of polygonal elements. *Accepted to the International Journal for Numerical Methods in Engineering,* 2014.

The First Topological Operator: Element Splitting

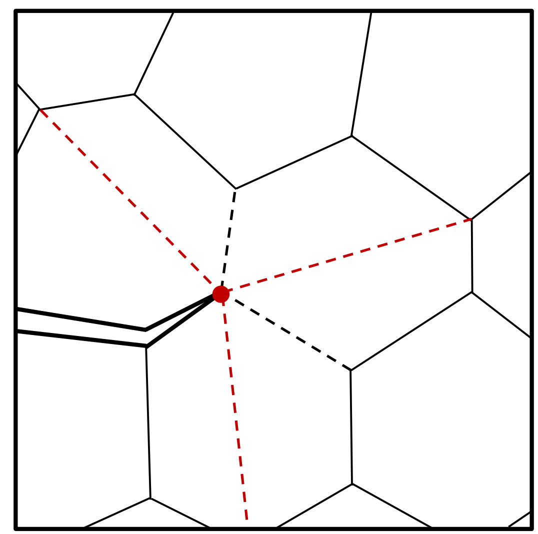
In order to reduce the path deviation in the polygonal mesh, we propose using an *element splitting* technique to increase the number of fracture paths at each node in the mesh.





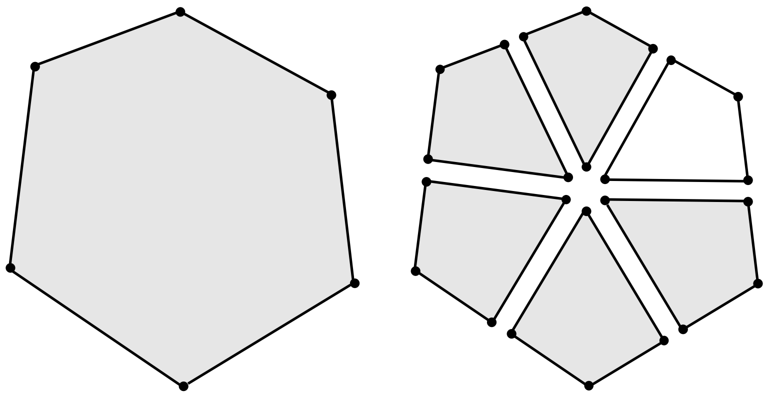
$$A = \frac{1}{2}\sum_{k=1}^{n}\left(v_x^{[k]}v_y^{[k+1]} - v_x^{[k+1]}v_y^{[k]}\right)$$

Allow elements to be split along the path which minimizes the difference between the two areas.



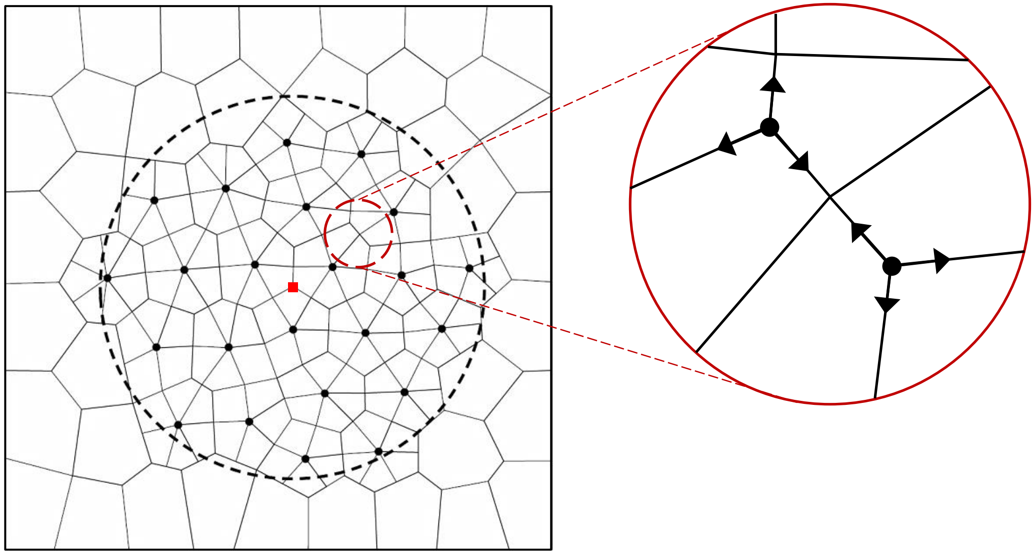
The Second Topological Operator: Adaptive Refinement

We propose the use of a quadrilateral refinement scheme, wherein each polygon around the crack tip is removed and replaced with a set of unstructured quads which meet at the centroid of the original polygon.



However, this refinement scheme does not increase the number of fracture paths at the original nodes.

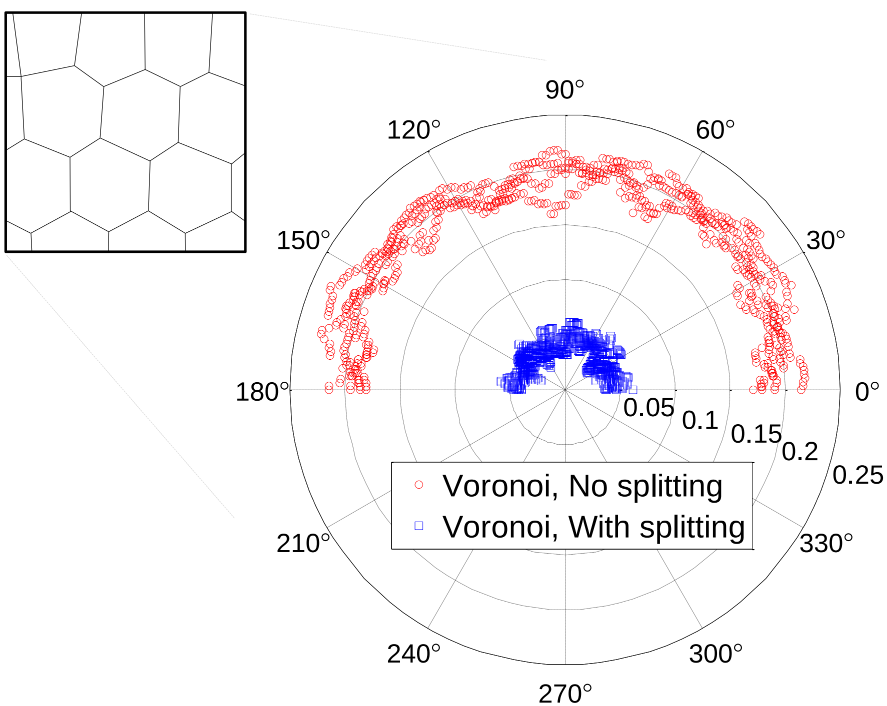
To account for this, we permit elements to be split.

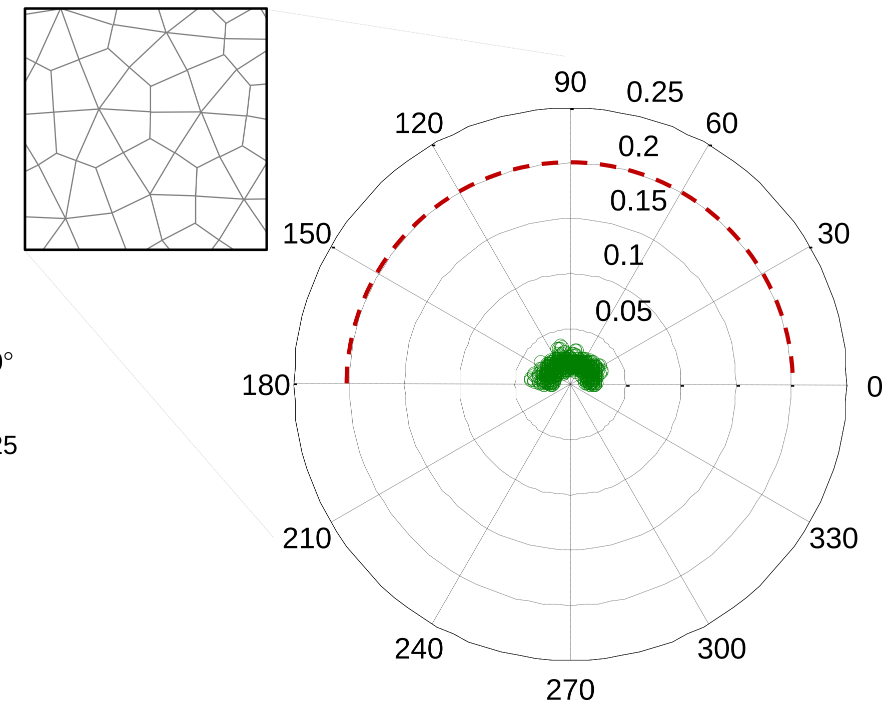


Spring, D. W., Leon, S. E., and Paulino, G. H., Unstructured adaptive refinement on polygonal meshes for the numerical simulation of dynamic cohesive fracture. Submitted to the *International Journal of Fracture*, 2014.

Quantification of Improvement in Path Deviation

Path deviation studies with both topological operators





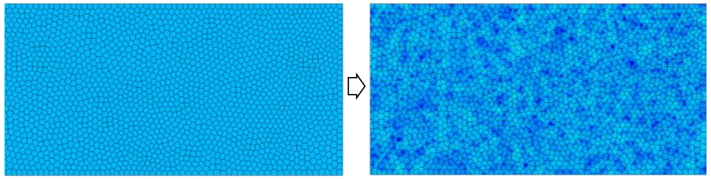
The results for 5 distinct meshes are illustrated.

Spring, D. W., Leon, S. E., and Paulino, G. H., Unstructured adaptive refinement on polygonal meshes for the numerical simulation of dynamic cohesive fracture. Submitted to the *International Journal of Fracture*, 2014.

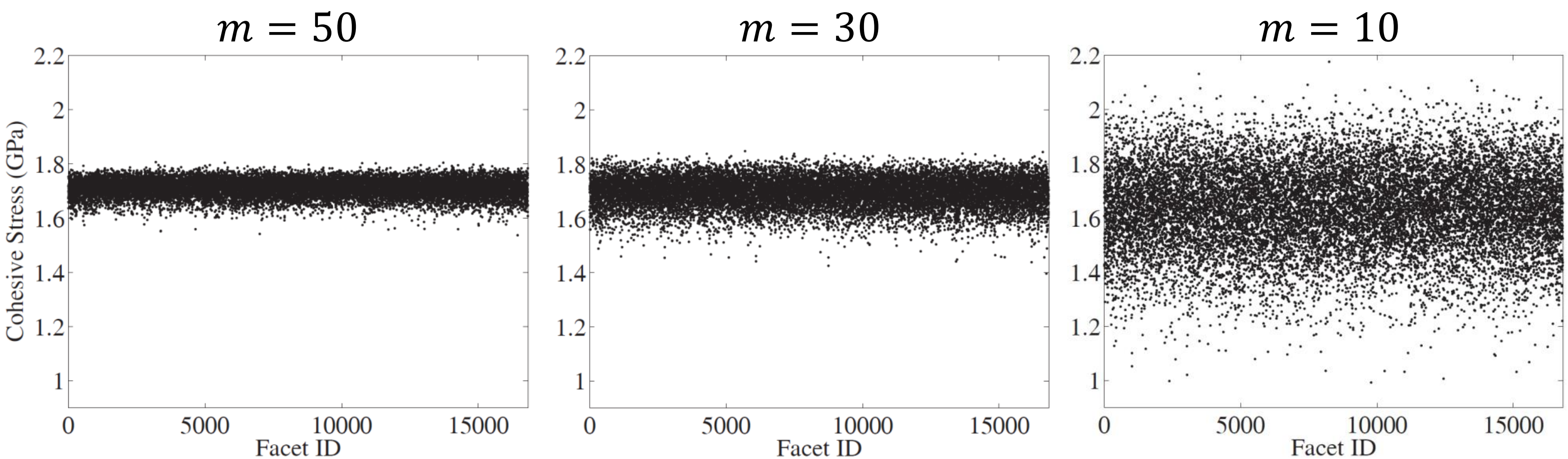
Unstructured Constitutive Models (Weibull Distribution)

To account for microscale inhomogeneities in our simulations, we prescribe a statistical distribution of material properties:

$$V = \frac{L_s^{1/m}}{L_f^{1/m}} V_s\left(-\ln(1-\rho)^{1/m}\right) \qquad V_s = [\sigma, \phi, E]$$



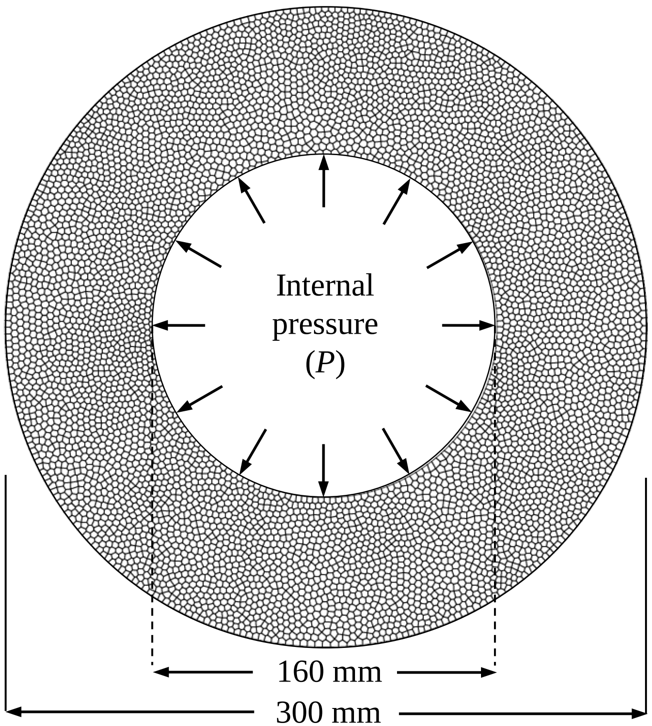
where: m is the Weibull modulus, V_s is the average material property, ρ is a randomly generated number between 0 and 1, L_f is the length of the cohesive element, and L_s is a scaling parameter

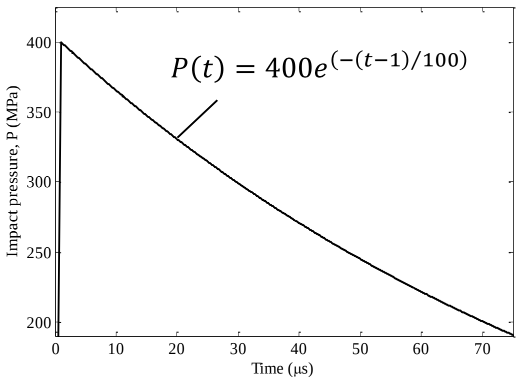


Zhou, F. and Molinari, J.F., Dynamic crack propagation with cohesive elements: a methodology to address mesh dependency. *International Journal for Numerical Methods in Engineering*, vol. 59, pp. 1-24, 2004.

Example: Illustrating Influence of Element Splitting

Pervasive fracture of an internally impacted thick cylinder



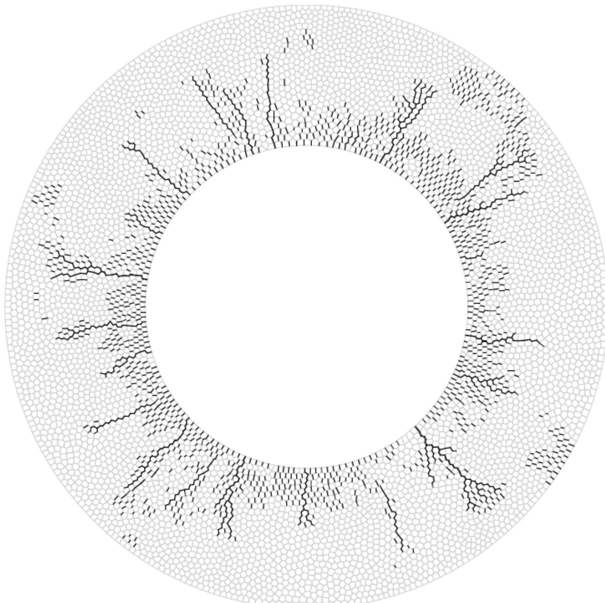


E	$210GPa$
ρ	$7850\,kg/m^3$
ϕ	$2000\,N/m$
σ	$850MPa$
α	2

Leon, S. E.*, Spring, D. W.*, Paulino, G. H., Reduction in mesh bias for dynamic fracture using adaptive splitting of polygonal elements. *Accepted to the International Journal for Numerical Methods in Engineering,* 2014.

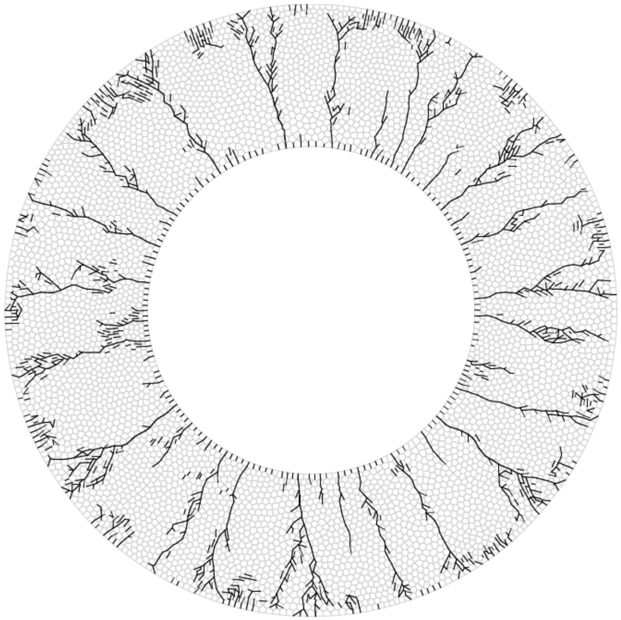
Example: Illustrating Influence of Element Splitting

When we only use a **geometrically** unstructured mesh, we get unbiased fracture behavior, but *unrealistic fracture patterns*.



Without element splitting

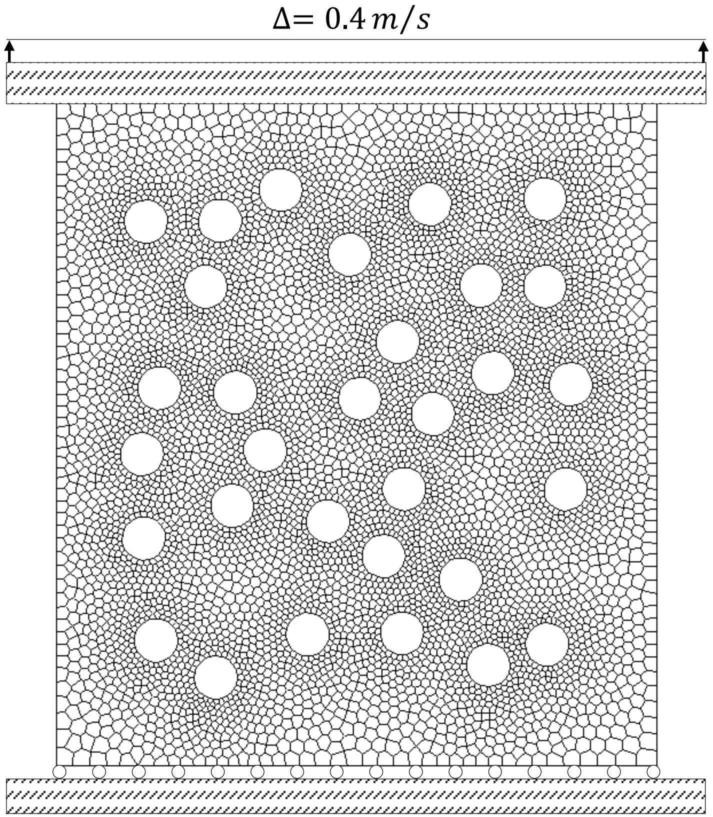
When we use both a **geometrically** and **topologically** unstructured mesh, we get unbiased fracture behavior and *realistic fracture patterns*.



With element splitting

Leon, S. E.*, Spring, D. W.*, Paulino, G. H., Reduction in mesh bias for dynamic fracture using adaptive splitting of polygonal elements. *Accepted to the International Journal for Numerical Methods in Engineering,* 2014.

Example: Illustrating Influence of Material Heterogeneity



Coordinates of Hole Locations			
(14.86,18.57)	(26.60,55.30)	(72.46,70.99)	(40.31,36.25)
(23.70,13.09)	(45.03,54.38)	(63.01,71.14)	(48.54,31.07)
(13.03,33.66)	(22.18,70.99)	(50.67,62.76)	(59.97,27.56)
(37.26,19.49)	(13.34,80.59)	(64.69,58.19)	(51.28,19.64)
(26.14,38.53)	(24.31,80.74)	(76.27,56.51)	(72.77,17.96)
(12.73,46.15)	(43.51,75.71)	(55.85,52.10)	(64.08,14.91)
(31.01,46.76)	(55.40,83.18)	(75.51,40.97)	(33.30,85.31)
(15.32,55.90)	(72.46,83.94)	(51.59,40.97)	

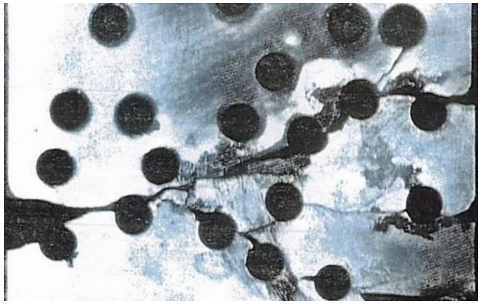
E (GPa)	ν	ρ (kg/m^3)	σ (MPa)	ϕ (N/m)	α
3.26	0.38	1100	62.8	100.0	2

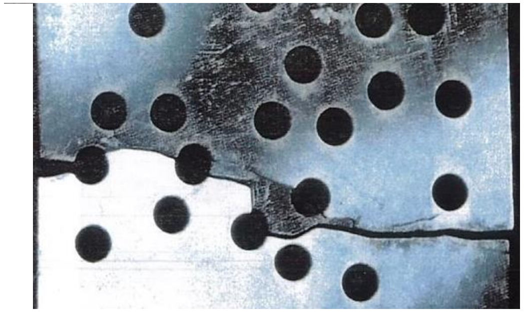
Al-Ostaz, A. and Jasiuk, I., Crack initiation and propagation in materials with randomly distributed holes. *Engineering Fracture Mechanics*, vol. 58, pp. 395-420, 1997.

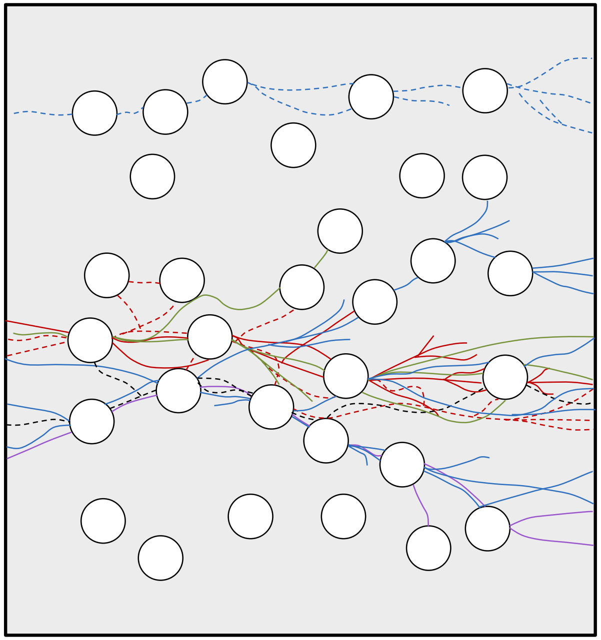
Motivation

This problem was investigated experimentally on seven plates with the same geometry and macroscopic material parameters. The observed fracture patterns were different in each plate.

The authors noted that the different fracture patterns for macroscopically similar plates may be due to microscale heterogeneities.





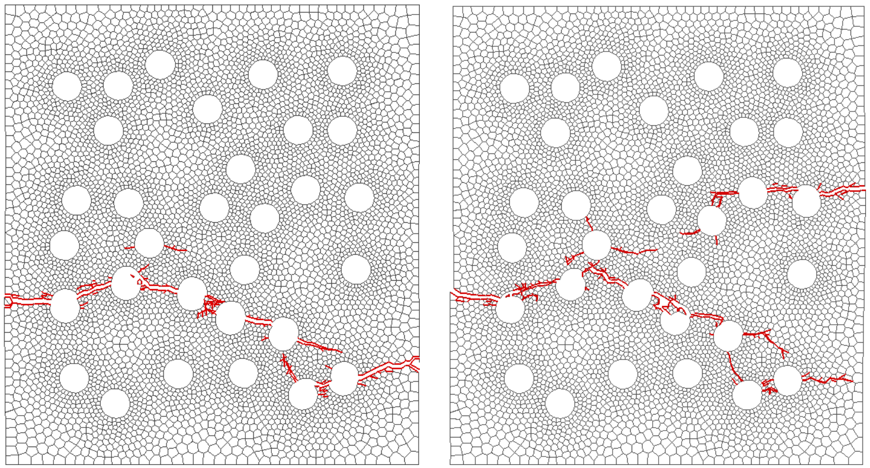


Combined experimental results

Al-Ostaz, A. and Jasiuk, I., Crack initiation and propagation in materials with randomly distributed holes. *Engineering Fracture Mechanics*, vol. 58, pp. 395-420, 1997.

Influence of Random Material Parameters

The results shown are for a Weibull modulus of m=10, and are all for the same mesh.

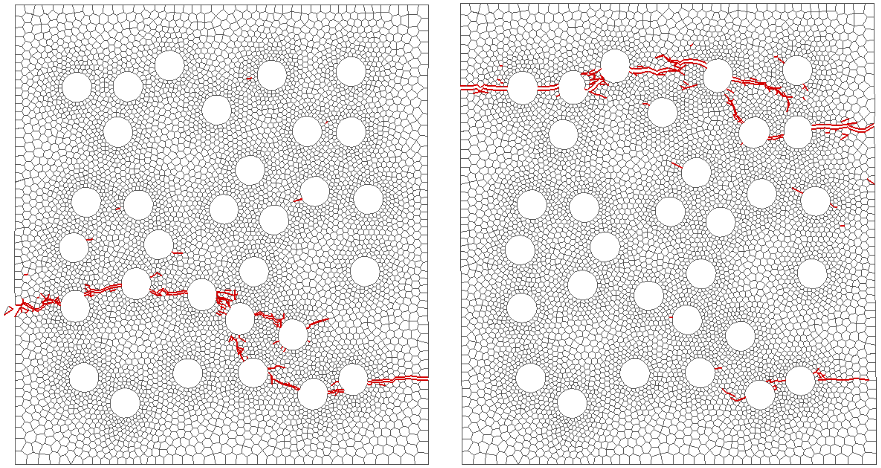


Variation in Elastic Modulus

- The bulk elastic modulus variation captures some of the fracture trends, but the cracks are limited to the lower portion of the plate
- When a smaller distribution of material properties is considered, little variation in the fracture patterns is observed.

Variation in Cohesive Strength

- The variation in cohesive strength captures more of the experimental trends, in particular the crack along the upper portion of the plate.
- When a smaller distribution of material properties is considered, less variation in the fracture patterns is observed.



Results: Illustrating Influence of Material Heterogeneity

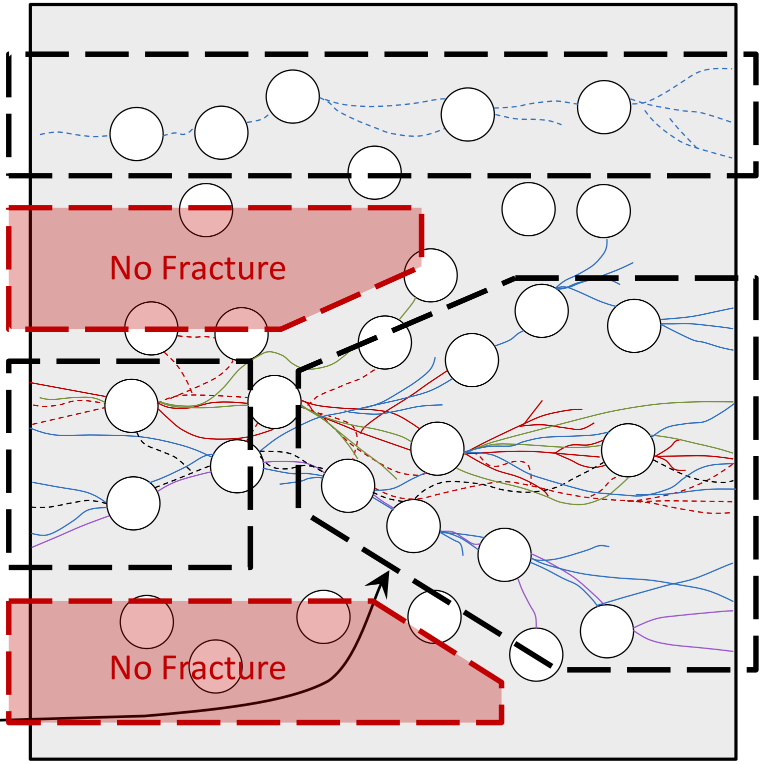
The authors who investigated this example experimentally noted that the different fracture patterns for macroscopically similar plates may be due to microscale heterogeneities.

The numerical investigations we provide here support the authors conclusions.

When the cohesive strength is randomly assigned, with a large range of heterogeneity, cracking in the upper portion of the plate can be captured

The cracks almost exclusively propagate through this section of the plate.

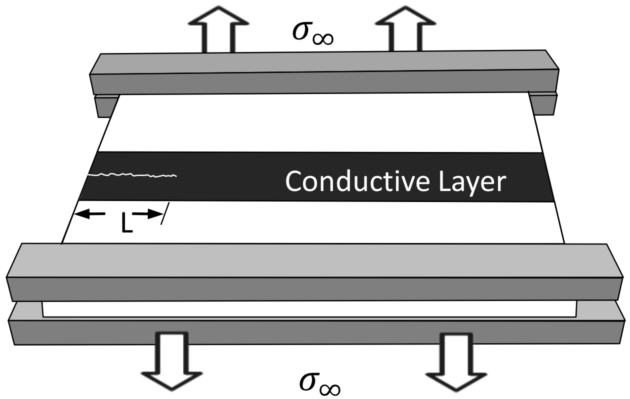
There are a wide range of fracture patterns spanning this portion of the plate. This behavior can be reproduced numerically

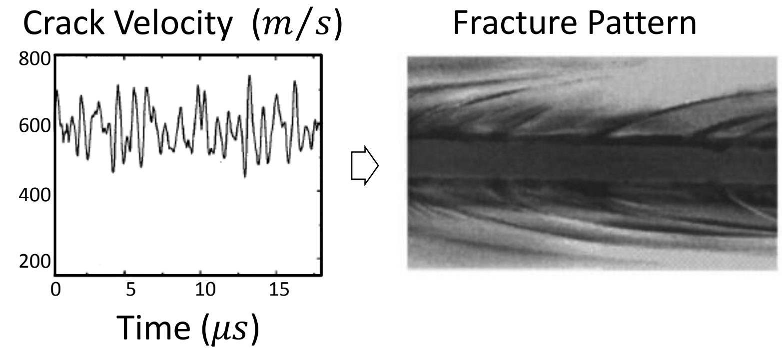


Summary of Experimental Results

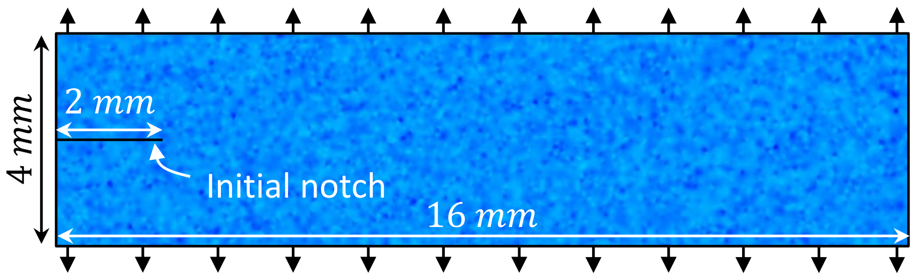
Example: Single Dominant Crack with Microbranching

Experimental problem:





Numerical model:



E	$3.24 GPa$
ρ	$1190\, kg/m^3$
ϕ	$352.4\, N/m$
σ	$129.6 MPa$
α	2

Sharon, E. and Fineberg, J., Microbranching instability and the dynamic fracture of brittle materials. *Physical Review B*, vol. 54, pp. 7128-7139, 1996.

Fracture Patterns: Different Material Parameters

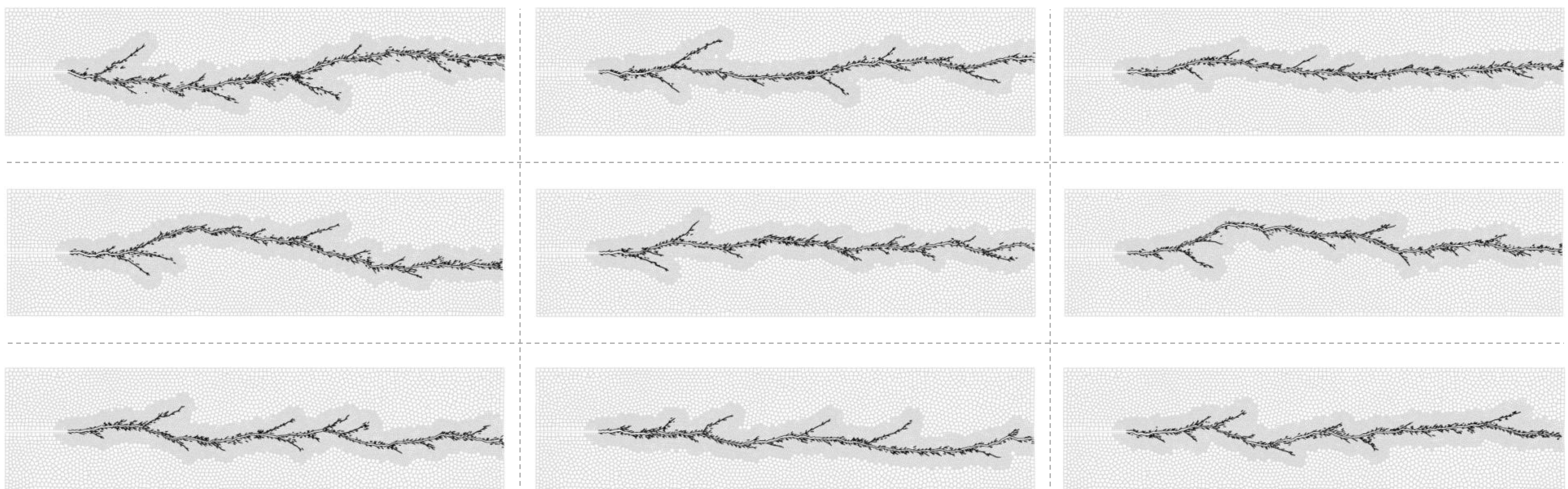
Various material properties and ranges of properties were considered. For each scenario, multiple cases were run, and the following illustrate typical results.

Cohesive Stress | Cohesive Energy | Elastic Modulus

$m = 10$

$m = 30$

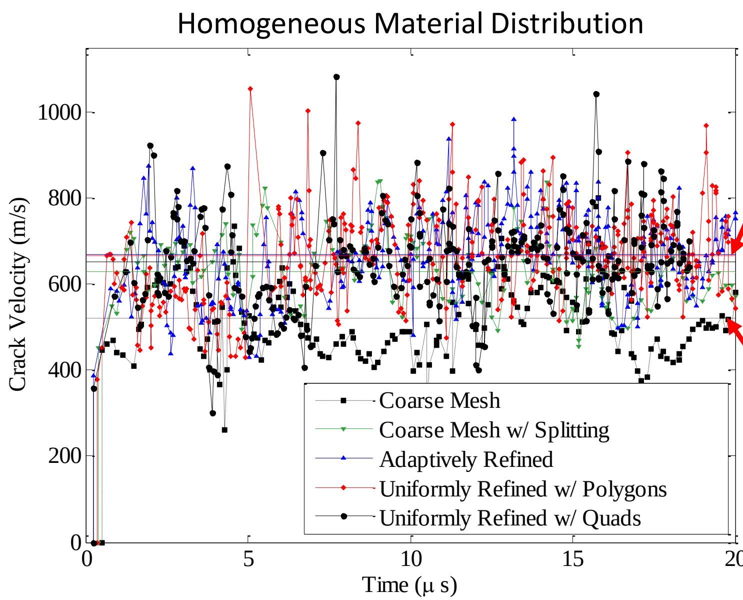
$m = 50$

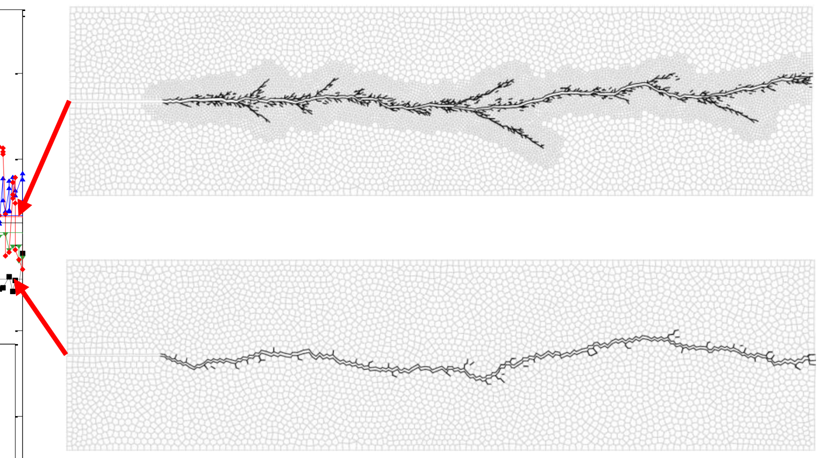


In general, the overall crack path changes for different material property distributions, but the fracture characteristics (microbranching and macrobranching) stay the same.

Spring, D. W., Leon, S. E., and Paulino, G. H., Unstructured adaptive refinement on polygonal meshes for the numerical simulation of dynamic cohesive fracture. Submitted to the *International Journal of Fracture*, 2014.

Comparison of Crack-Tip Velocities





		Distribution of Material			
		Homogeneous	m=50	m=30	m=10
Polygonal	Cohesive Stress (σ)	546	562	547	547
	Cohesive Energy (φ)		557	531	551
	Elastic Modulus (E)		533	521	504
Element Splitting	Cohesive Stress (σ)	602	630	624	638
	Cohesive Energy (φ)		582	594	605
	Elastic Modulus (E)		607	589	565
Adaptive	Cohesive Stress (σ)	672	681	662	689
	Cohesive Energy (φ)		677	682	673
	Elastic Modulus (E)		654	669	650

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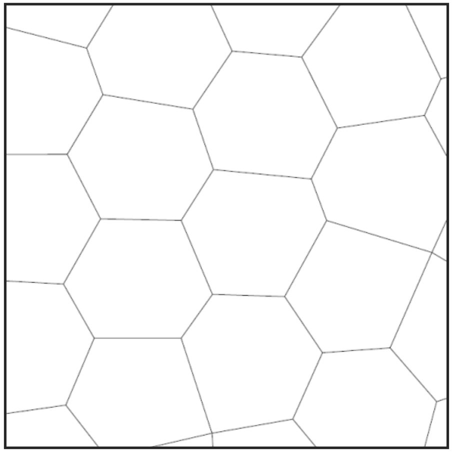
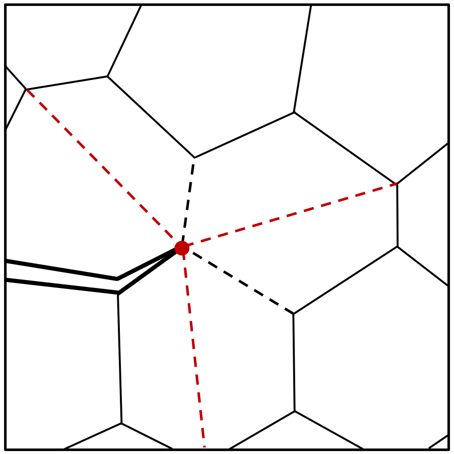
Summary: Techniques for overcoming mesh dependency

We review three methods for overcoming mesh dependency:

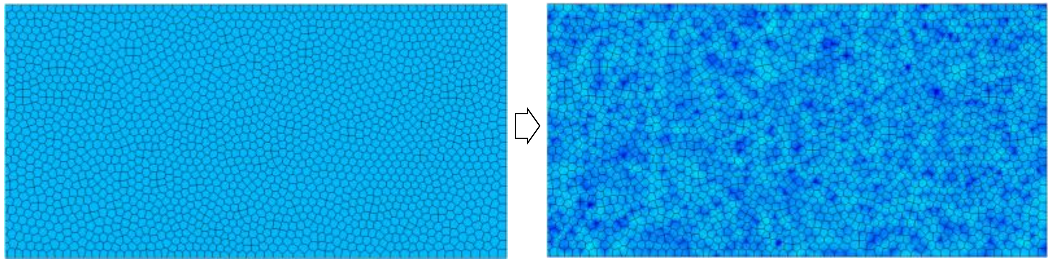
Geometrically Unstructured
Unstructured meshes



Topologically Unstructured
On-the-fly mesh modifications



Constitutivey Unstructured
Statistically distributing material properties



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Summary

- Unstructured polygonal meshes produce an isotropic discretization.
- The two new topological operators (element splitting and adaptive refinement) reduce the mesh induced path deviation.
- For pervasive fracture problems the element splitting operator improves overall fracture behavior.
- Material heterogeneity may be the cause of dissimilar fracture patterns in similar test specimens.
- For problems with a single dominant crack, material heterogeneity has a minimal influence on the fracture behavior.
- The crack velocity is influenced by the mesh induced restrictions, but isn't significantly altered when the material parameters are statistically distributed.
- By combining unstructured geometries, topologies and material distributions the model is truly random and minimizes numerically induced restrictions.

Thank You!

spring2@illinois.edu