Topology Optimization with Design-dependent Loading: An Adaptive Sensitivity-separation Design Variable Update Scheme

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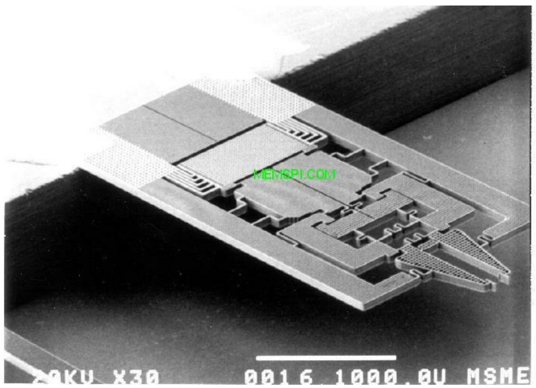
Outline

- Motivation
- Formulations of TopOpt with design-dependent loading
- Sensitivity-separation update scheme
- Numerical examples
- Concluding remarks

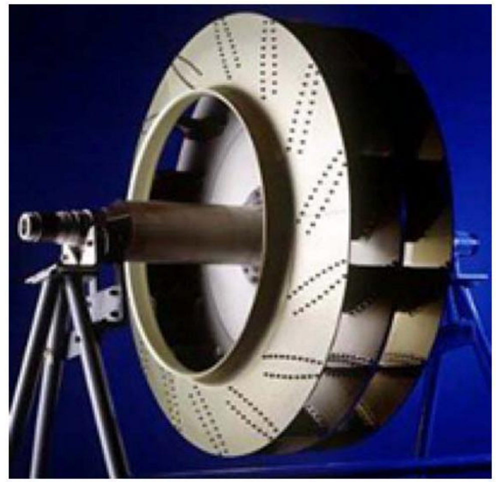
Y. Jiang, A. S. Ramos Jr. and G. H. Paulino (2019) Topology optimization with design-dependent loading: An adaptive sensitivity-separation design variable update scheme (submitted).

Motivation: Design-dependent loading

- **Design-dependent loads, including electromagnetic forces, heat load, centrifugal force and gravitational forces, can significantly influence the optimized design.**

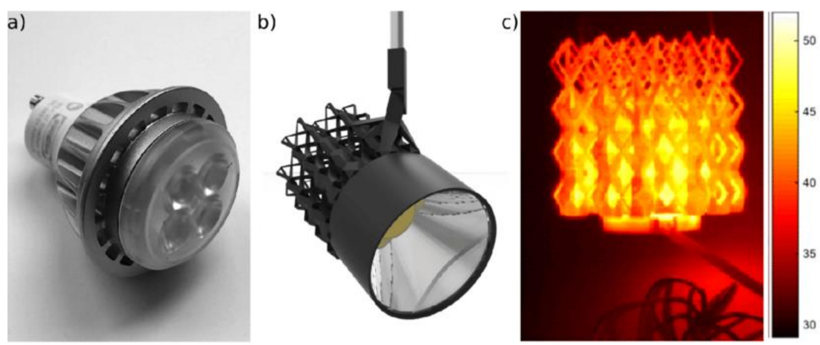


Micro-electro-mechanical system
(www.memspi.com)



Centrifugal fan
(Lee, et al)

Gateway Arch
(www.interestingamerica.com)



Heat sink
(Lazarov, et al)

Topology optimization with design-dependent loading: Solid formulation

- **Compliance minimization with a material volume constraint.**

$$
\begin{aligned}
\min_{\mathbf{z}} \quad & C = (\mathbf{f} + \mathbf{g}(\mathbf{z}))^T \mathbf{u} \\
\text{s.t.} \quad & G = \mathbf{v}^T \mathbf{x} - \bar{V} \leqslant 0 \\
& z_{min,i} \leqslant z_i \leqslant z_{max,i}, \quad i = 1, \ldots, n_e \\
\text{with} \quad & \mathbf{K}(\mathbf{z})\,\mathbf{u} = \mathbf{f} + \mathbf{g}(\mathbf{z}) \\
& \mathbf{P}\mathbf{z} = \mathbf{x}
\end{aligned}
$$

- **The objective is non-monotonous due to the design-dependent loading.**

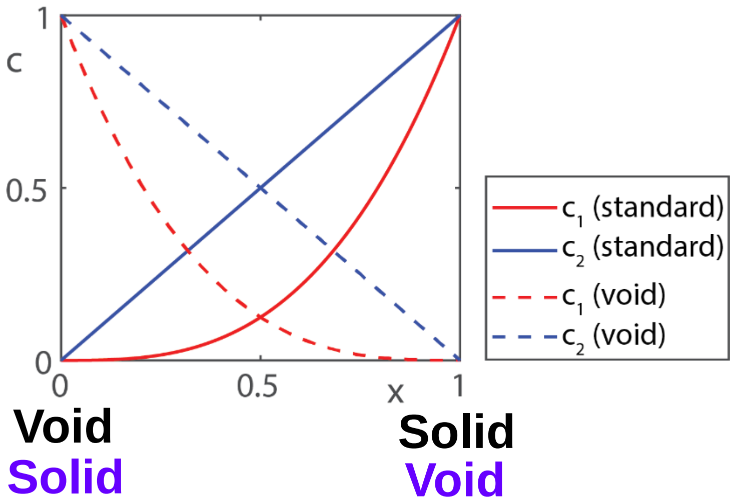
$$
\frac{\partial C}{\partial \mathbf{z}} = -\mathbf{u}^T \frac{\partial \mathbf{K}}{\partial \mathbf{z}} \mathbf{u} + \boxed{2\mathbf{u}^T \frac{\partial \mathbf{g}}{\partial \mathbf{z}}}
$$

Topology optimization with design-dependent loading: Void formulation

- **Void formulation applies an upper bound on the void region, i.e., a lower bound on material volume.**

$$\min_{\mathbf{z}} \quad C = (\mathbf{f} + \mathbf{g}(\mathbf{z}))^T \mathbf{u}$$
$$\text{s.t.} \quad G_{void} = \mathbf{v}^T \mathbf{x} - \bar{V}_{void} \leqslant 0$$
$$z_{min,i} \leqslant z_i \leqslant z_{max,i}, \quad i = 1, ..., n_e$$
$$\text{with} \quad \mathbf{K}(\mathbf{z})\mathbf{u} = \mathbf{f} + \mathbf{g}(\mathbf{z})$$
$$\mathbf{P}\mathbf{z} = \mathbf{x}$$

- **Void formulation uses a reflected material property interpolation function**



Solid formulation

$$E_k = x_k^p E_0, p \geqslant 1$$
$$g_{k,V} = -\frac{1}{n_{node,k}} \gamma v_k x_k.$$

Void formulation

$$E_k = (1 - x_k)^p E_0, p \geqslant 1$$
$$g_{k,V} = -\frac{1}{n_{node,k}} \gamma v_k (1 - x_k).$$

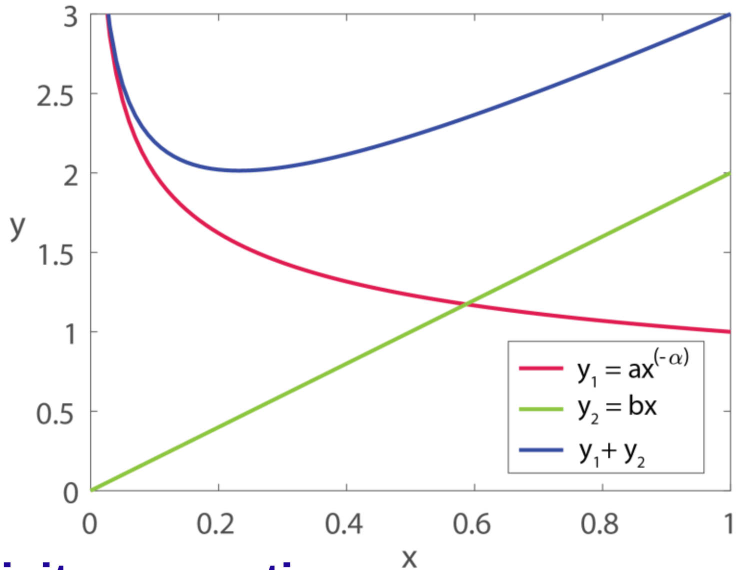
Sensitivity-separation update scheme: Constructing sub-problems

- **The objective function is approximated by the sum of non-monotonous separable functions.**

$$C(\mathbf{z}_c) \approx C_{app}(\mathbf{z}_c)$$

$$= \sum_{i=1}^{n_e} \left(a_i z_{c,i}^{-\alpha} + b_i z_{c,i} \right) + c$$

where $a_i \geqslant 0,\ b_i \geqslant 0,\ \alpha > 0.$



- **The construction is enabled by sensitivity separation.**

$$\frac{\partial C_{app}}{\partial z_i} = -\alpha a_i (z_{c,i})^{-\alpha-1} + b_i$$

$$\frac{\partial C}{\partial z_i} = \left(\frac{\partial C}{\partial z_i}\right)_n + \left(\frac{\partial C}{\partial z_i}\right)_p$$

$$a_i = -\left(\frac{\partial C}{\partial z_i}\right)_n (z_{c,i})^{\alpha+1} / \alpha$$

$$b_i = \left(\frac{\partial C}{\partial z_i}\right)_p$$

Sensitivity-separation update scheme: Solving the approximate sub-problem

- **The update is obtained by solving a Lagrangian dual problem.**

$$\min_{\mathbf{z}} \quad \sum_{i=1}^{n_e} \left(a_i z_i^{-\alpha} + b_i z_i\right)$$

$$\text{s.t.} \quad G = \mathbf{v}^T\mathbf{x} - \bar{V} \leqslant 0$$

$$z_{min,i} \leqslant z_i \leqslant z_{max,i}, \quad i = 1, \ldots, n_e$$

$$\text{with} \quad \mathbf{Pz} = \mathbf{x}$$

$$a_i = -\left(\frac{\partial C}{\partial z_i}\right)_n \left(z_{c,i}\right)^{\alpha+1} / \alpha$$

$$b_i = \left(\frac{\partial C}{\partial z_i}\right)_p$$

$$L(\mathbf{z}, \lambda) = \sum_{i=1}^{n_e} \left(a_i z_i^{-\alpha} + b_i z_i\right) + \lambda G$$

$$\Downarrow$$

$$\frac{\partial L}{\partial z_i} = -\alpha a_i \left(z_i\right)^{-\alpha-1} + b_i + \lambda \frac{\partial G}{\partial z_i} = 0$$

$$\Downarrow \quad \eta = 1/(\alpha+1)$$

$$z_{opt}(\lambda) = \left(\frac{a_i}{b_i + \lambda \frac{\partial G}{\partial z_i}}\right)^{\eta} = \left(\frac{-\left(\frac{\partial C}{\partial z_i}\right)_n}{\left(\frac{\partial C}{\partial z_i}\right)_p + \lambda \frac{\partial G}{\partial z_i}}\right)^{\eta} z_{i,c}$$

$$\Downarrow$$

$$z_i(\lambda) = \begin{cases} \underline{z_i}, & \text{if} \quad z_{opt} < \underline{z_i} \\ z_{opt}, & \text{if} \quad \underline{z_i} \leqslant z_{opt} \leqslant \bar{z}_i \\ \bar{z}_i, & \text{if} \quad z_{opt} > \bar{z}_i \end{cases}$$

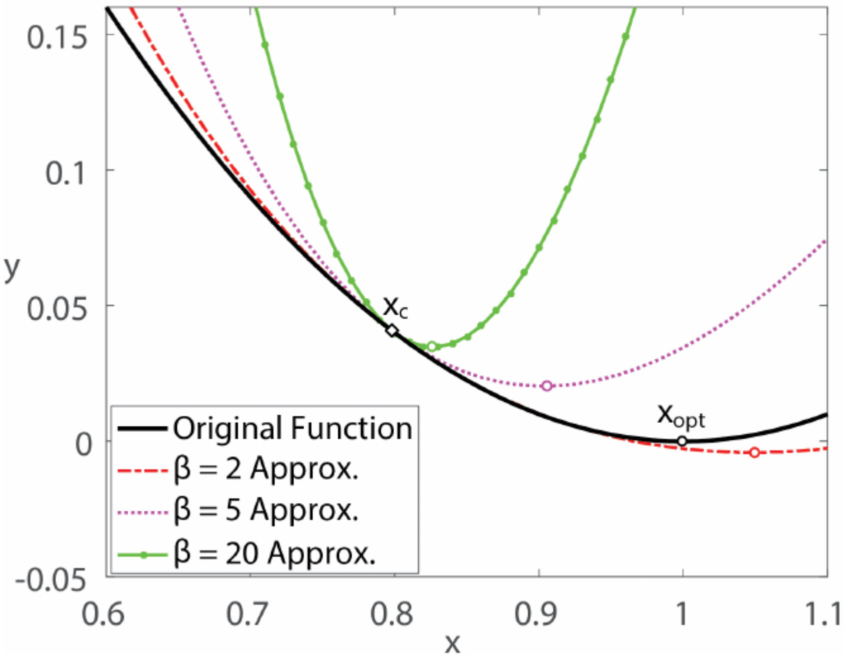
- **The single variable λ can be efficiently obtained by bisection method.**

Sensitivity-separation update scheme: Adaptive sensitivity separation by β

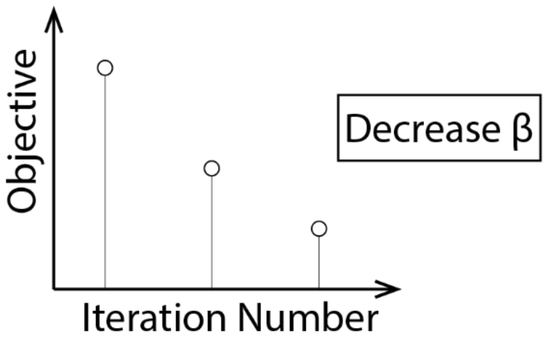
- **The larger β is, the more conservative the update is.**

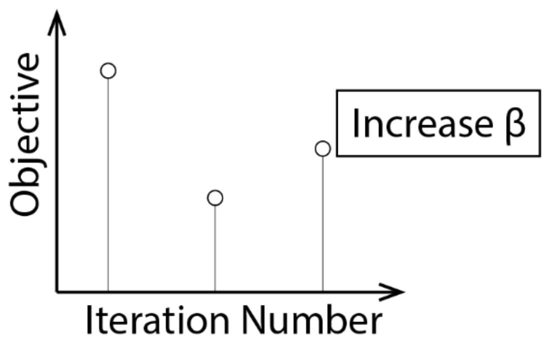
$$\left(\frac{\partial C}{\partial z_i}\right)_n = \begin{cases} (1+\beta)\frac{\partial C}{\partial z_i}, \text{ if } & \left(\frac{\partial C}{\partial z_i}\right) \leqslant 0 \\ (-\beta)\frac{\partial C}{\partial z_i}, \quad \text{if} & \left(\frac{\partial C}{\partial z_i}\right) > 0 \end{cases}$$

$$\left(\frac{\partial C}{\partial z_i}\right)_p = \begin{cases} (-\beta)\frac{\partial C}{\partial z_i}, \quad \text{if} & \left(\frac{\partial C}{\partial z_i}\right) \leqslant 0 \\ (1+\beta)\frac{\partial C}{\partial z_i}, \text{ if } & \left(\frac{\partial C}{\partial z_i}\right) > 0 \end{cases}$$

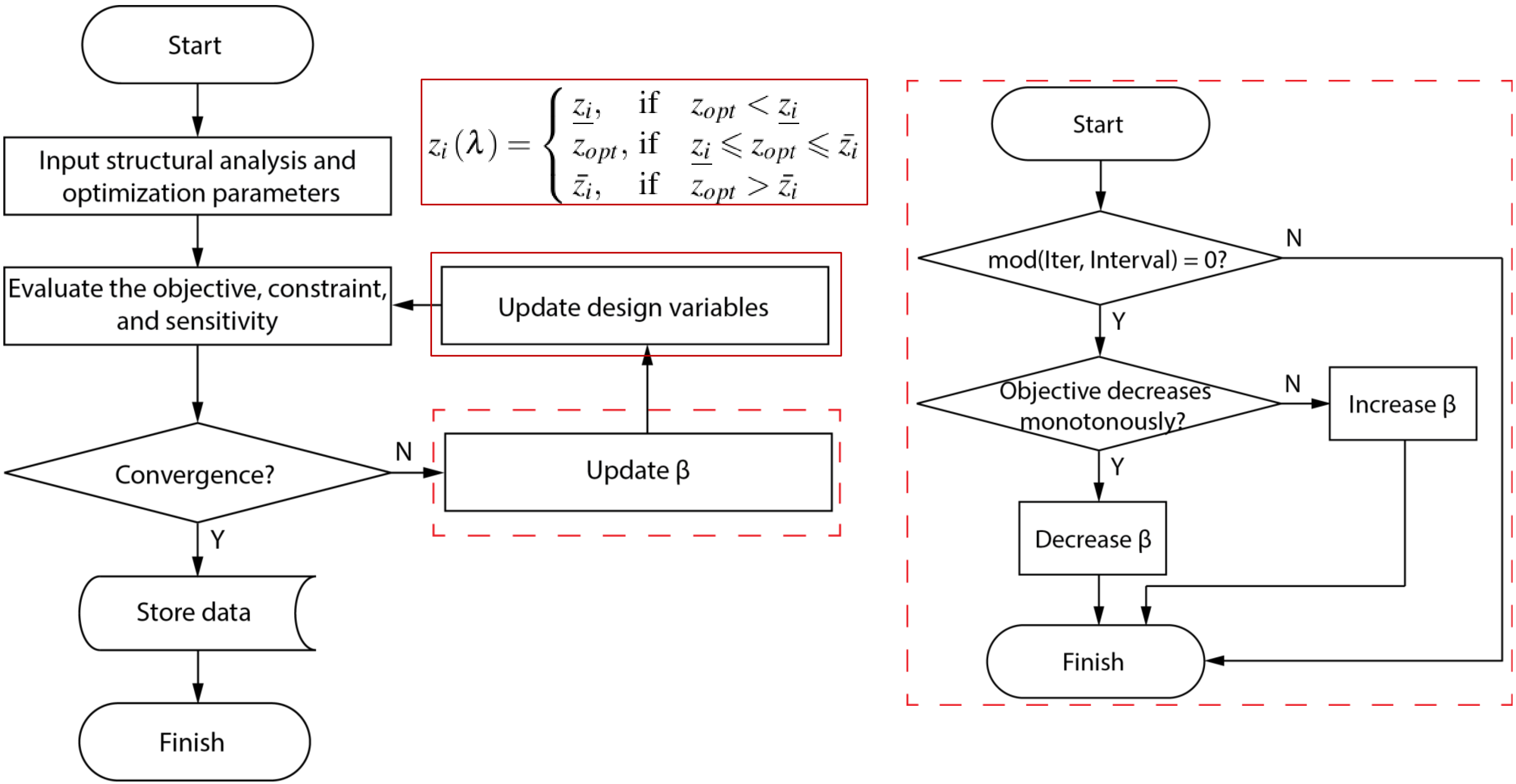


- **The sensitivity separation is adaptive as we update β adaptively.**



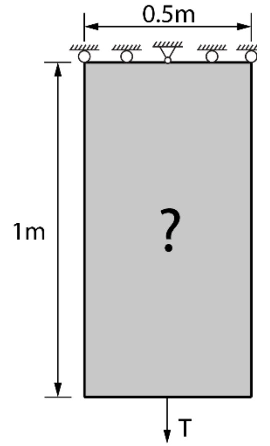


Algorithm: TopOpt with adaptive sensitivity separation



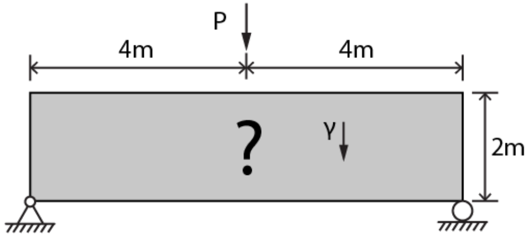
Numerical examples

Hanging bar



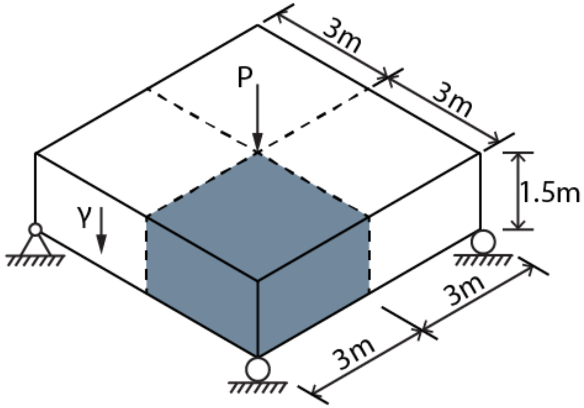
Verification problem

2D MBB beam



2D design under various loading scenarios

3D Dome

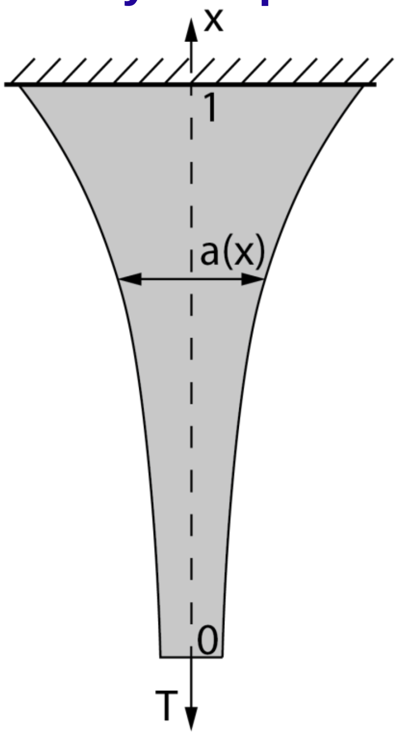


3D design extension

Hanging bar

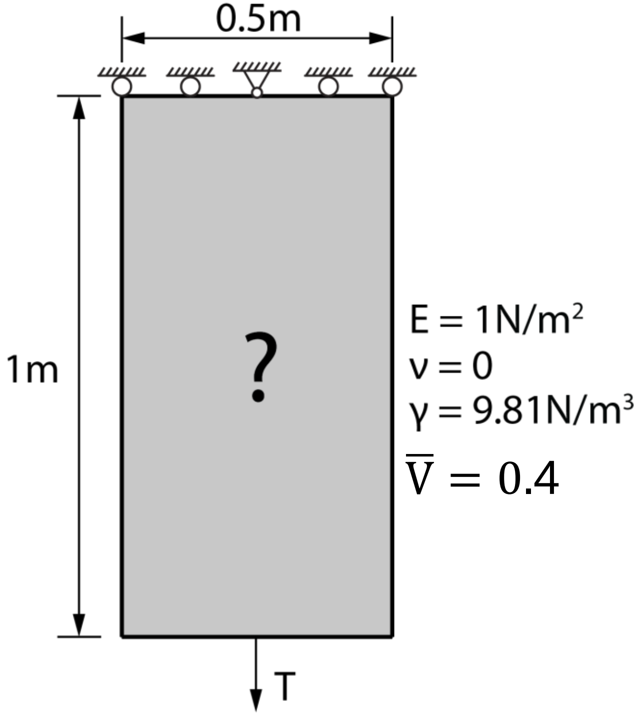
- **Cross-sectional area distribution for minimum tip displacement**

1D analytical problem



$$a(x) = \left(\frac{T}{\gamma}\right) ln\left(1 + \bar{V}\frac{\gamma}{T}\right) e^{xln\left(1+\bar{V}\frac{\gamma}{T}\right)}$$

2D numerical problem



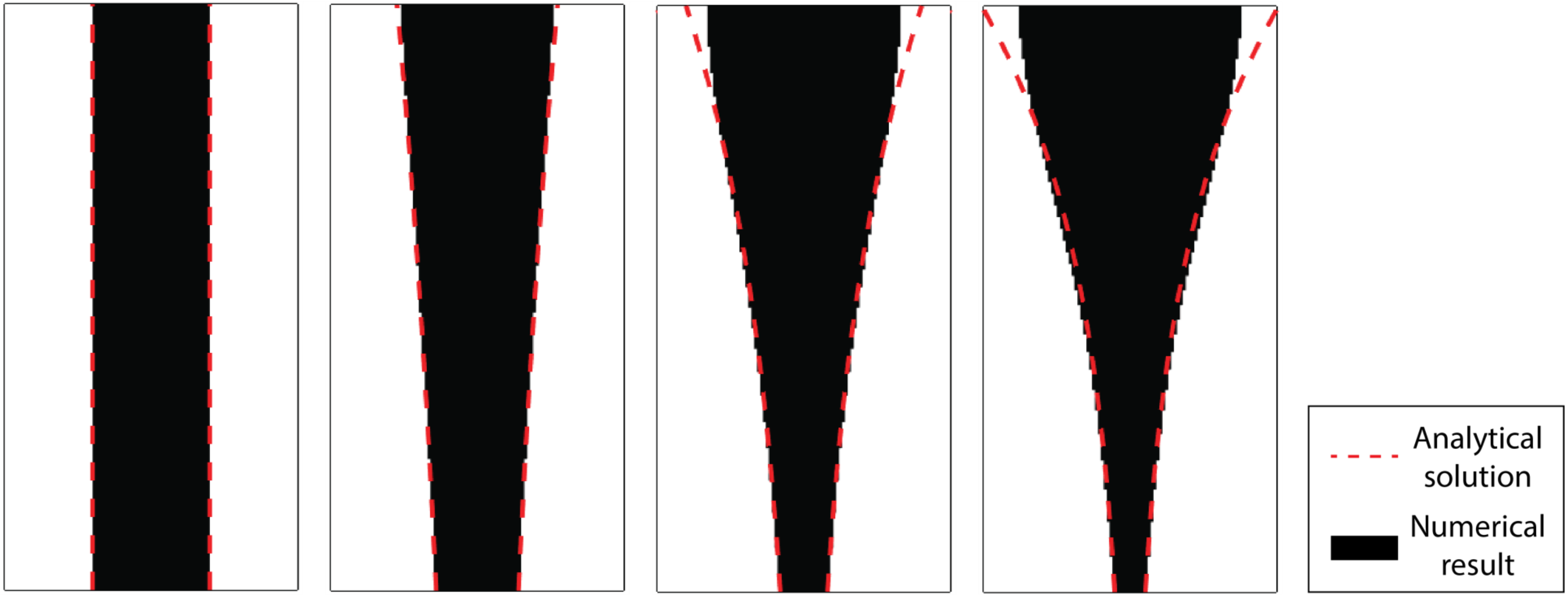
Objective: $C = \mathbf{f}^T\mathbf{u}$

B.L. Karihaloo, W.S. Hemp, Maximum strength/stiffness design of structural members in presence of self-weight, Proc. R. Soc. Lond. A 389, 119-132 (1983)

B. Zheng, C. Chang and H. C. Gea, Topology optimization considering body forces, Int. J. Simul. Multidisci. Des. Optim. 3, 316-320 (2009)

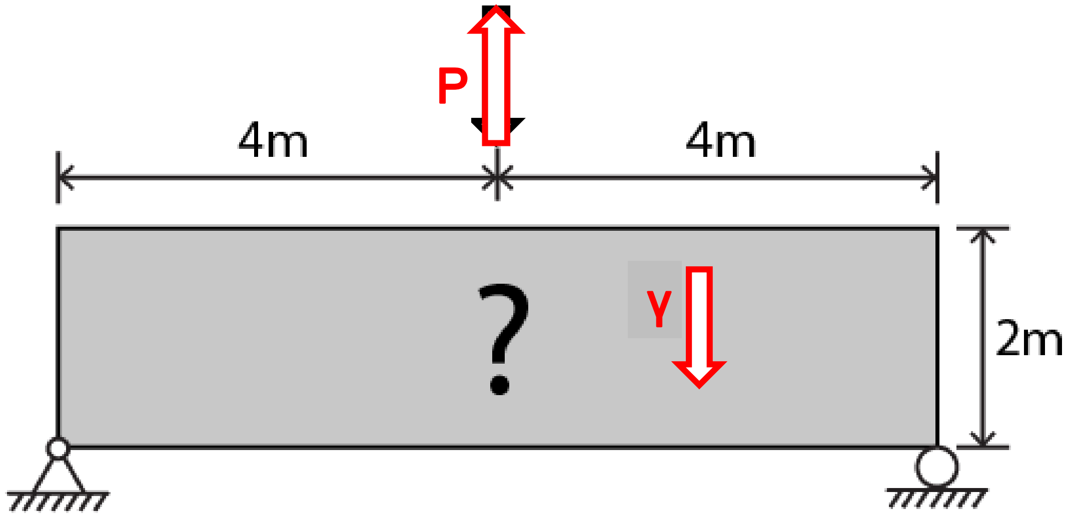
Hanging bar

- The numerical results match the analytical solution.



$T/\gamma = \infty$ $T/\gamma = 0.21$ $T/\gamma = 0.05$ $T/\gamma = 0.0023$

MBB beam



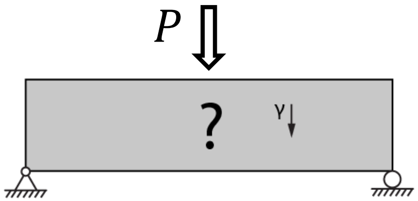
$E = 30000\ kN/m^2$

$\nu = 0.3$

$\gamma = 23.5\ kN/m^3$

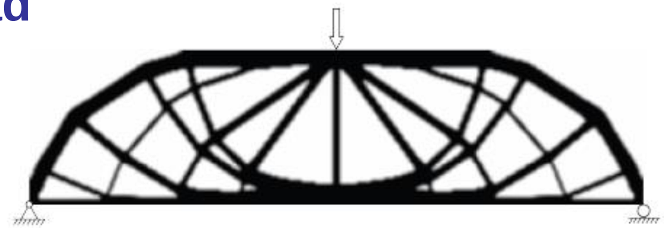
MBB beam under different loading scenarios

Downward external load

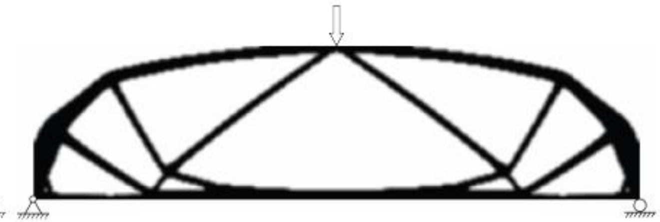


$\overline{V} = 0.38$

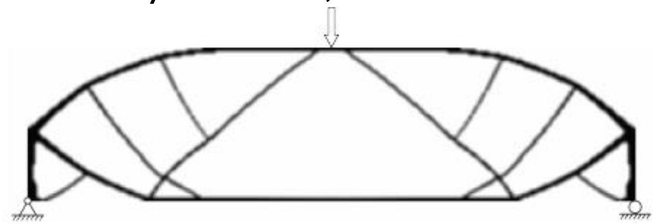
$W = \gamma * V_{total}$



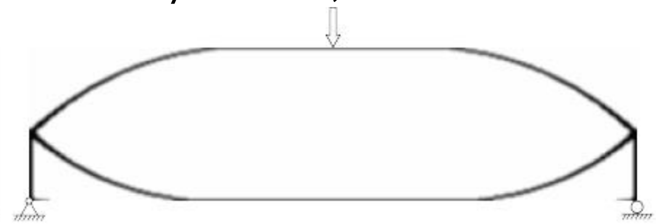
$P/W = \infty, V = 0.38$



$P/W = 1, V = 0.25$

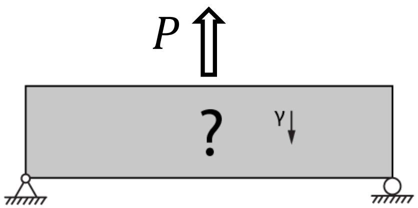


$P/W = 0.005, V = 0.12$



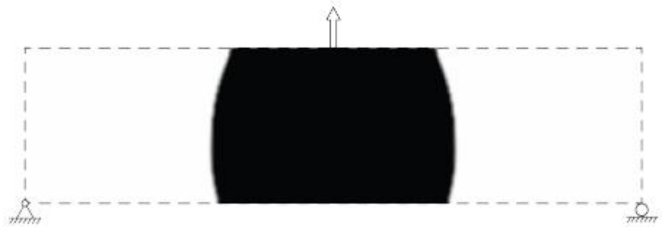
$P/W = 0.0015, V = 0.049$

Upward external load

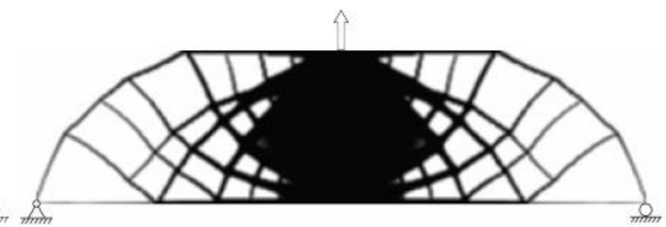


$\overline{V} = 0.38$

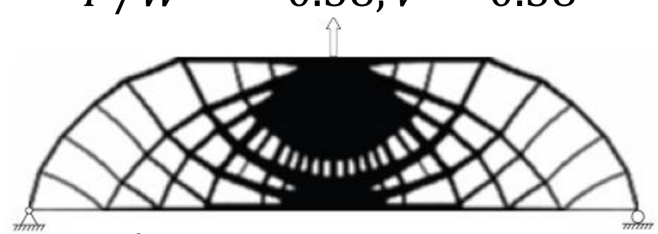
$W = \gamma * V_{total}$



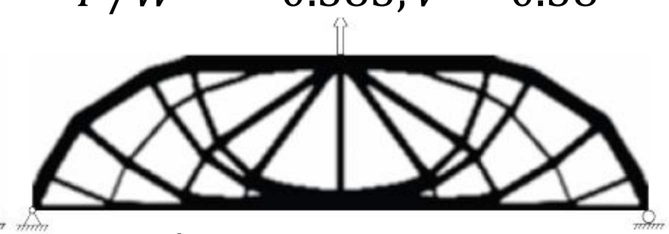
$P/W = -0.38, V = 0.38$



$P/W = -0.383, V = 0.38$



$P/W = -0.4, V = 0.38$

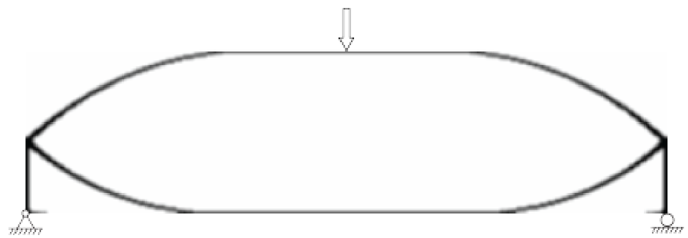


$P/W = -\infty, V = 0.38$

MBB beam design under large design-dependent loading

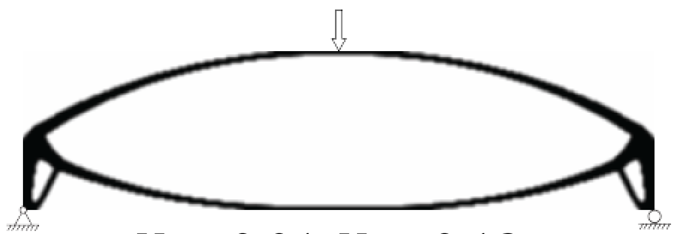
- In cases dominated by design-dependent loading, void formulation may provide feasible designs.

From solid formulation

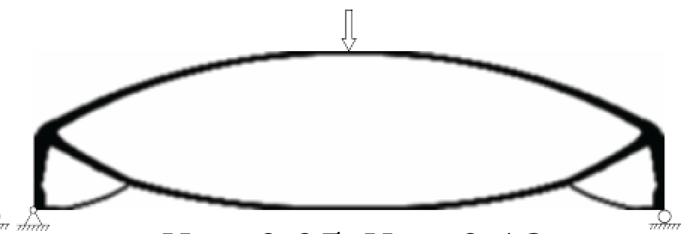


$P/W = 0.0015, V = 0.049$

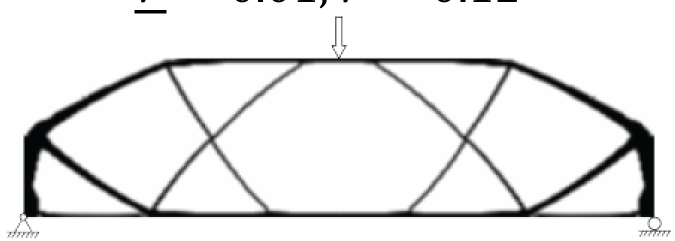
From void formulation



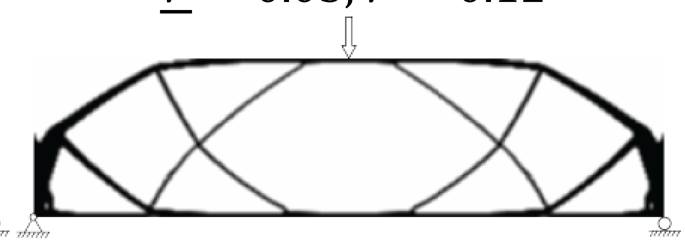
$\underline{V} = 0.01, V = 0.12$



$\underline{V} = 0.05, V = 0.12$

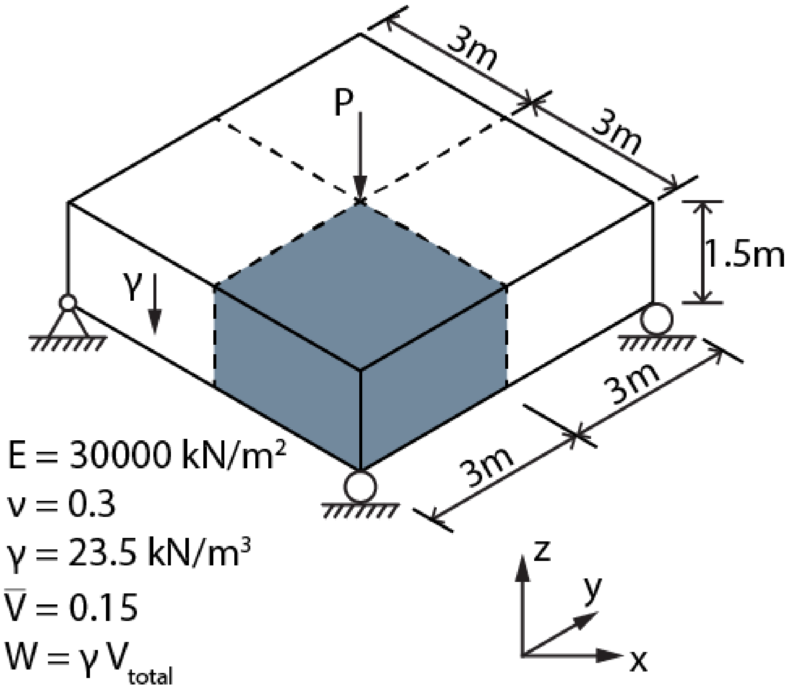


$\underline{V} = 0.06, V = 0.13$

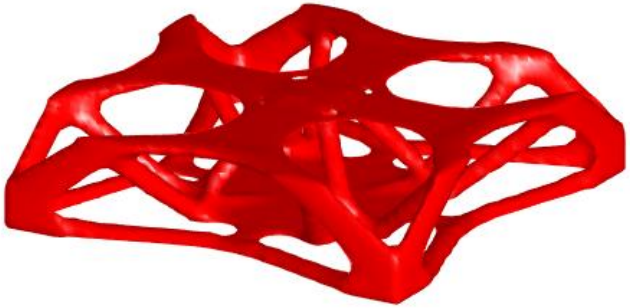


$\underline{V} = 0.15, V = 0.15$

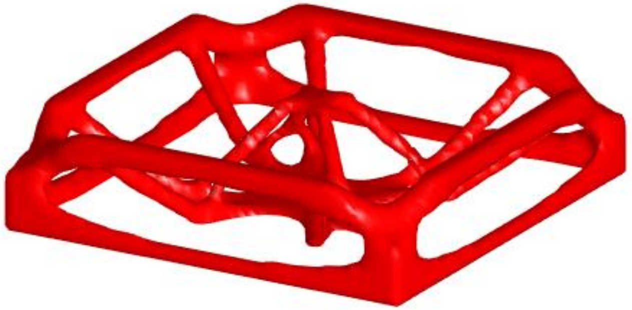
3D Dome



$P/W = 1, V = 0.15$

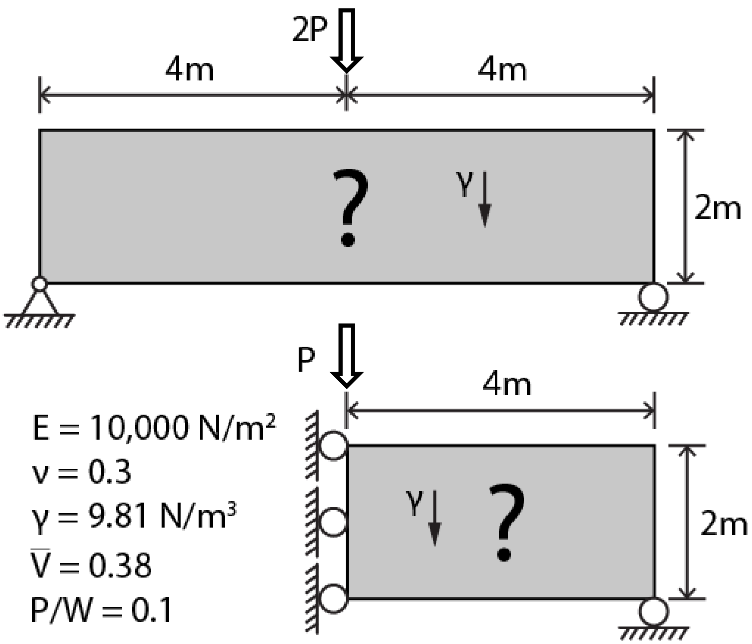


$P/W = 0.1, V = 0.15$



$P/W = 0.025, V = 0.11$

Computational efficiency comparison



		40X20 mesh	80X40 mesh	160X80 mesh
GCMMA	Objective value	7.161	6.049	5.475
	Number of iterations	65	258	500
	Update time (s)	5	75	1329
	Total time (s)	7	109	1895
Sensitivity Separation	Objective value	7.179	6.253	5.705
	Number of iterations	114	203	259
	Update time (s)	0.2	0.5	1
	Total time (s)	4	26	100

K. Svanberg, A class of globally convergent optimization methods based on conservative convex separable approximations, SIAM Journal of Optimization, 2002, 12, 555-573.
M. Bruyneel and P. Duysinx. “Note on topology optimization of continuum structures including self-weight”. Structural and Multidisciplinary Optimization 29.4 (2005), pp. 245–256.

Concluding remarks

- **To solve topology optimization problems with design-dependent loadings, we propose a novel design variable update scheme, which is based on sensitivity separation.**

- **This scheme solves problems with a material volume upper bound (solid formulation) or lower bound (void formulation).**

- **Numerical examples in 2D and 3D demonstrate that the update scheme is efficient and is able to handle different loading scenarios.**

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Thank you!