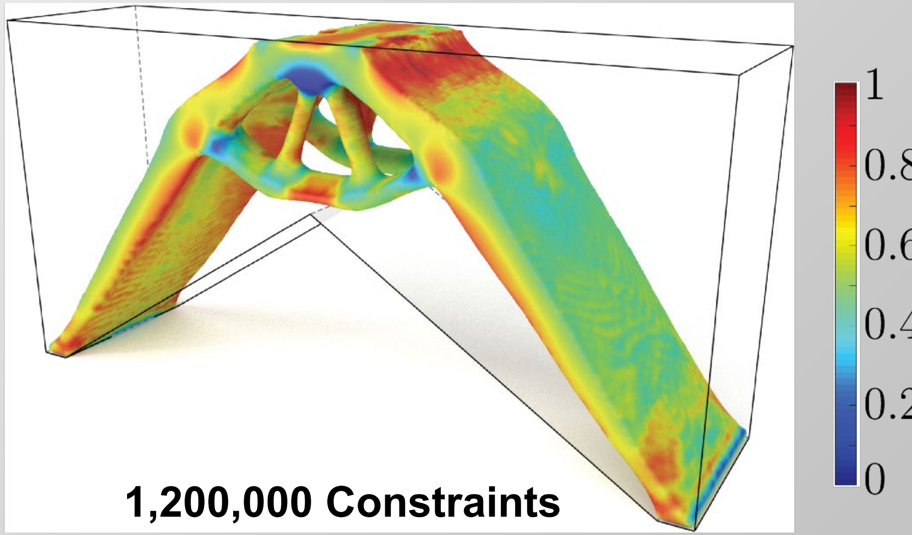
Topology optimization with LOCAL stress constraints: *A clustering-free approach*

Glaucio H. Paulino
Raymond Allen Jones Chair
School of Civil & Env. Eng.
Georgia Institute of Technology



Collaborators:

Fernando V. da Senhora (GATech)

Oliver Giraldo-Londoño (GATech)

Ivan F. M. Menezes (PUC-Rio, Brazil)

Acknowledgment: Martin P. Bendsoe (WCSMO 2013)

May 22, 2019

WCSMO13 World Congress of Structural and Multidisciplinary Optimization
May 20-24, 2019, Beijing, China

Aggregated vs. non-aggregated optimization problems

Non-aggregated constrained optimization

$$\begin{aligned} \min_{\mathbf{z}} \quad & f(\mathbf{z}) \\ \text{s.t.} \quad & g_j(\mathbf{z}) \leq 0, \quad j = 1, \ldots, N_c \\ & z \in Z \end{aligned}$$

We aim to solve topology optimization problems with ***local stress constraints*** as a non-aggregated constrained optimization problem

Aggregated constrained optimization

$$\begin{aligned} \min_{\mathbf{z}} \quad & f(\mathbf{z}) \\ \text{s.t.} \quad & G^{(k)}(\mathbf{z}) \leq 0 \\ & z \in Z \\ \text{with:} \quad & G^{(k)}(\mathbf{z}) = \sum_{j=1}^{N_c} s_j^{(k)} g_j(\mathbf{z}) \end{aligned}$$

Surrogate inequality

$s_j^{(k)} \geq 0$ are aggregation coefficients

Ermoliev, Y. M., Kryazhimskii, A. V., & Ruszczyński, A. (1997). Constraint aggregation principle in convex optimization. *Mathematical Programming*, 76(3), 353-372.

Typical stress-constrained TopOpt

$$\min_{\mathbf{z}} \quad m(\mathbf{z}) = \sum_{e=1}^{N_e} \rho_e(\mathbf{z}) v_e$$ Objective function

$$\text{s.t.} \quad g_j(\mathbf{z}) = \frac{\sigma_j^{VM}(\mathbf{z})}{\sigma_{\lim}} - 1 \leq 0, \quad j = 1, \ldots, N_c$$ Stress constraints

$$0 < z_{\min} \leq z_e \leq 1, \quad e = 1, \ldots, N_e$$ Box constraints

$$\text{with:} \quad \mathbf{K}(\mathbf{z})\mathbf{u} = \mathbf{f}$$ Equilibrium

$$\boldsymbol{\rho}(\mathbf{z}) = \mathbf{P}\mathbf{z}$$ Regularization filter

Clustering techniques are typically used to solve stress-constrained topology optimization problems

The problem is reformulated by taking a stress measure in each of various clusters:

$$g_j(\mathbf{z}) = \frac{\sigma_j^{VM}(\mathbf{z})}{\sigma_{\lim}} - 1 \leq 0, \quad j = 1, \ldots, N_c$$

Local constraint

$$G_j(\boldsymbol{\sigma}^{VM}(\mathbf{z})) \approx \max_{i \in \Omega_j} \left\{ \frac{\sigma_i^{VM}}{\sigma_{\lim}} \right\} - 1 \leq 0, \quad j = 1, \ldots, m, \quad m \ll N_c$$

Clustered constraint

Different norms used to estimate the maximum stress in cluster j, $j = 1, \ldots, m$

$$G_j = \left[\sum_{i \in \Omega_j} \left(\frac{\sigma_i^{VM}(\mathbf{z})}{\sigma_{\lim}} \right)^p \right]^{1/p} - 1 \leq 0 \qquad p - \text{norm}$$

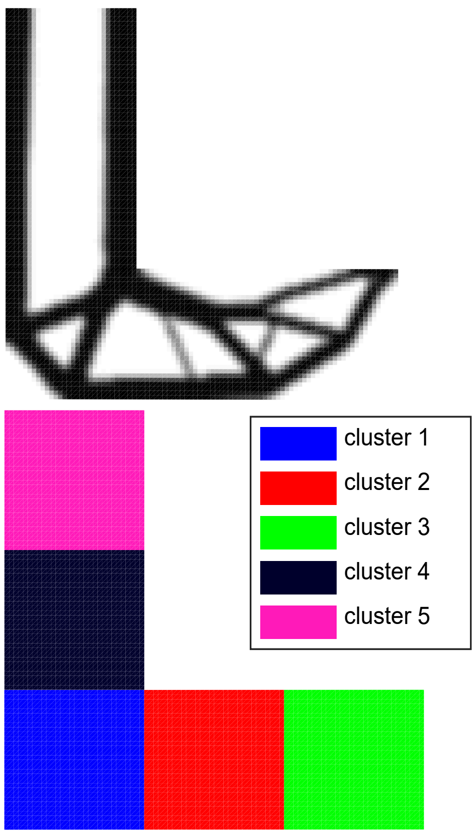
$$G_j = \frac{1}{N_j} \ln \left[\sum_{i \in \Omega_j} \exp \left[\frac{\sigma_i^{VM}(\mathbf{z})}{\sigma_{\lim}} p \right] \right] - 1 \leq 0 \quad KS - \text{function}$$

Q: How should the clusters be defined?

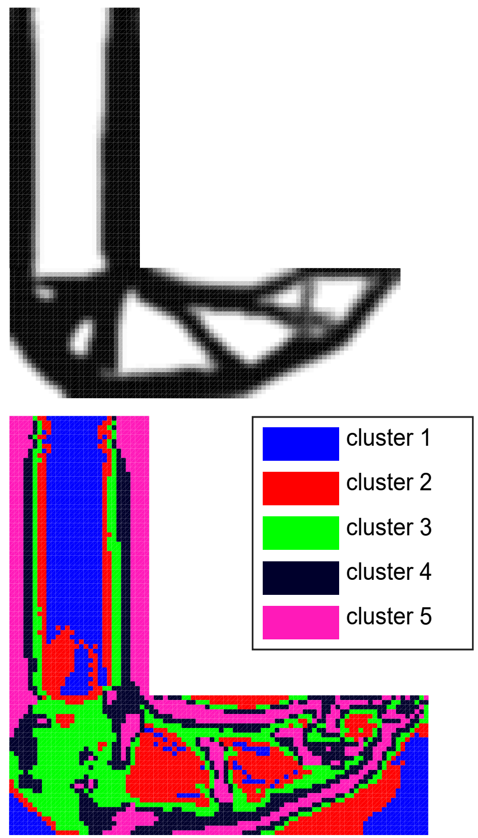
Topology optimization results highly depend on the type of clustering technique

- **Aggregation function:** $p - norm$
- **Parameter** $p = 12$
- **Number of clusters,** $m = 5$

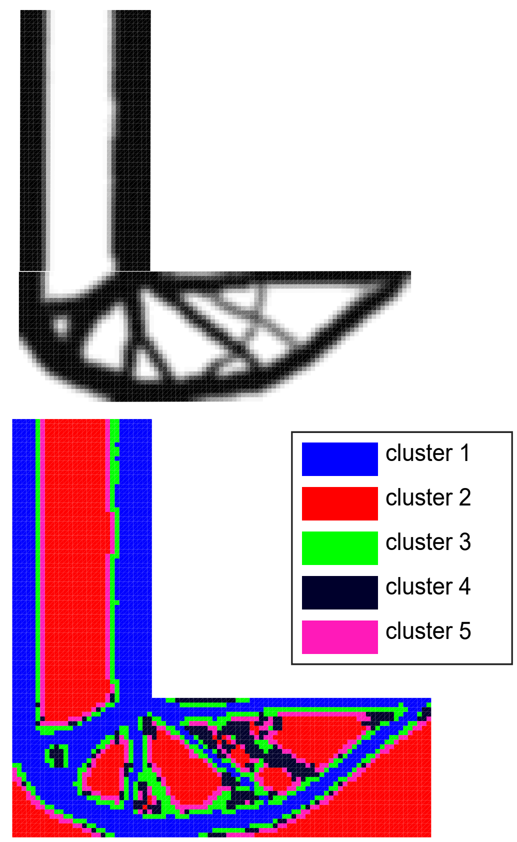
Element proximity



Stress level



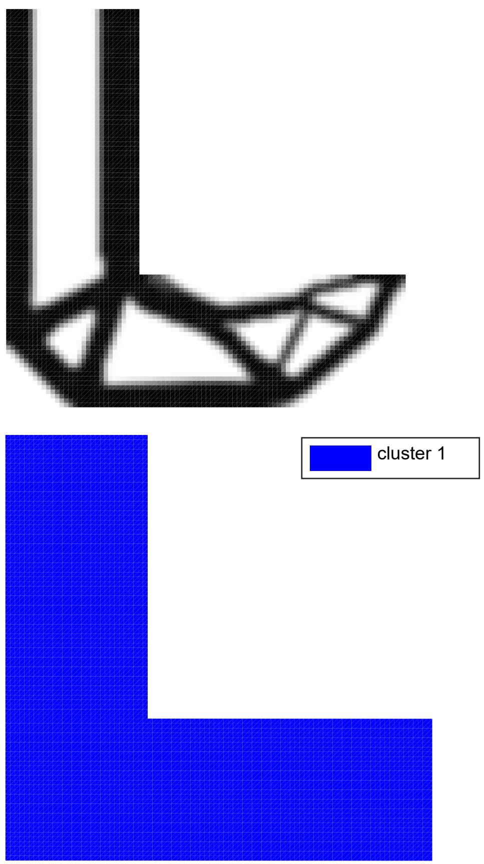
Hierarchical clustering



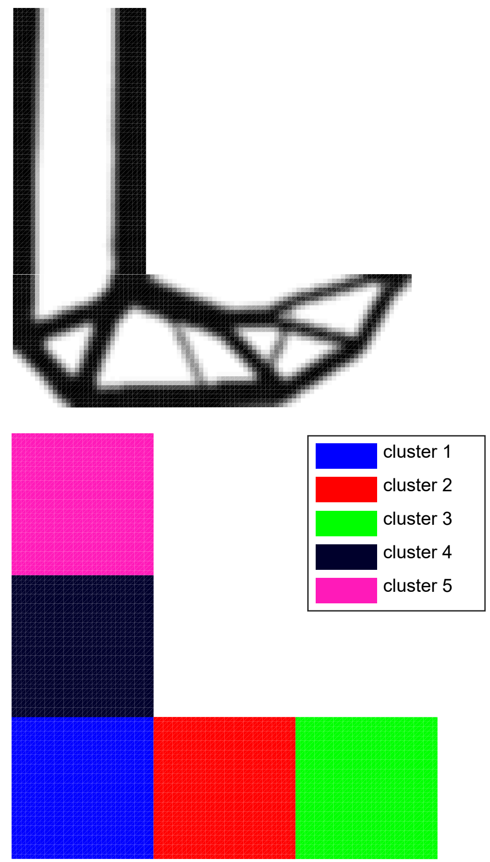
Topology optimization results highly depend on the number of clusters

- **Aggregation function:** $p - norm$
- **Parameter** $p = 12$
- **Number of clusters,** $m = \text{variable}$

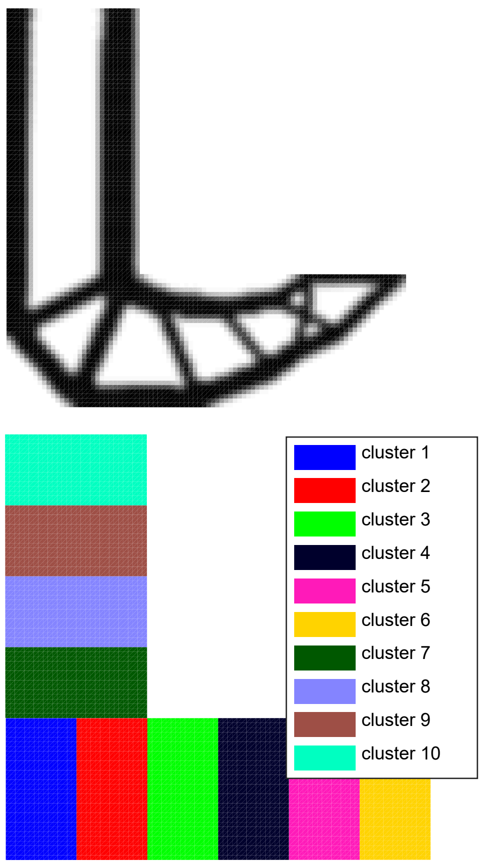
$\boldsymbol{m = 1}$



$\boldsymbol{m = 5}$



$\boldsymbol{m = 10}$



Our formulation aims to find the lightest structure satisfying von Mises stress limits locally

$$\min_{\mathbf{z}} \quad m(\mathbf{z}) = \sum_{e=1}^{N_e} \tilde{\rho}_e v_e$$ **Objective function**

$$\text{s.t.} \quad g_j(\mathbf{z}) \leq 0, \quad j = 1, \ldots, N_c$$ **Stress constraints**

$$0 < z_{\min} \leq z_e \leq 1, \quad e = 1, \ldots, N_e$$ **Box constraints**

$$\text{with:} \quad \mathbf{K}(\mathbf{z})\mathbf{u} = \mathbf{f}$$ **Equilibrium**

$$\tilde{\rho}_e = 1 - e^{-\beta \rho_e(\mathbf{z})} + \rho_e(\mathbf{z}) e^{-\beta}$$ **Volume fraction**

$$\boldsymbol{\rho}(\mathbf{z}) = \mathbf{P}\mathbf{z}$$ **Regularization filter**

Piecewise vanishing constraint

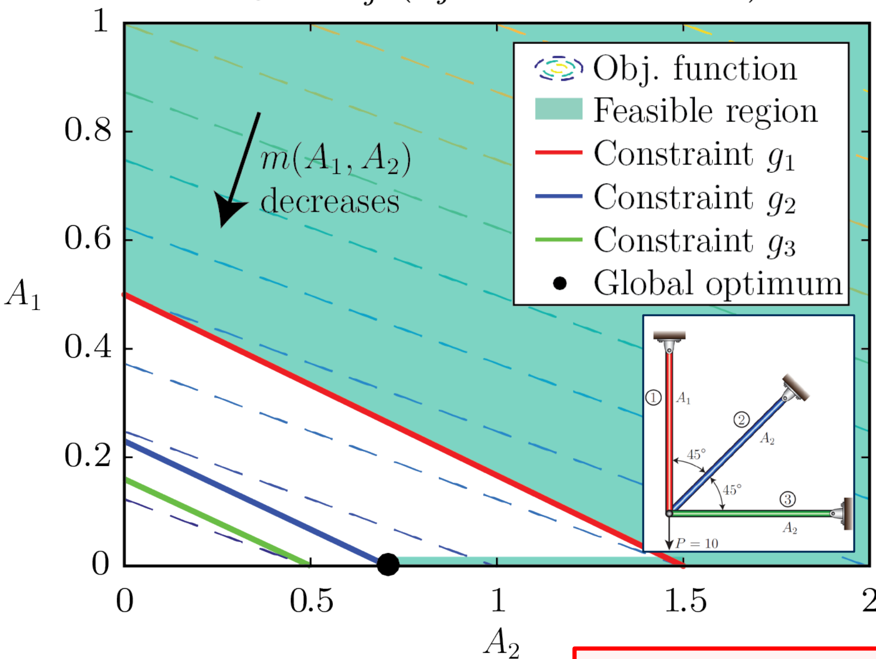
$$g_j = \begin{cases} \rho_j^3 (\sigma_j^{VM}/\sigma_{\lim} - 1)^2 & \text{if} \ \ \sigma_j^{VM}/\sigma_{\lim} > 1 \\ 0 & \text{otherwise} \end{cases}$$

Relaxation is used to reach inside singular regions, but we solve the problem using unrelaxed constraints

Feasible design space

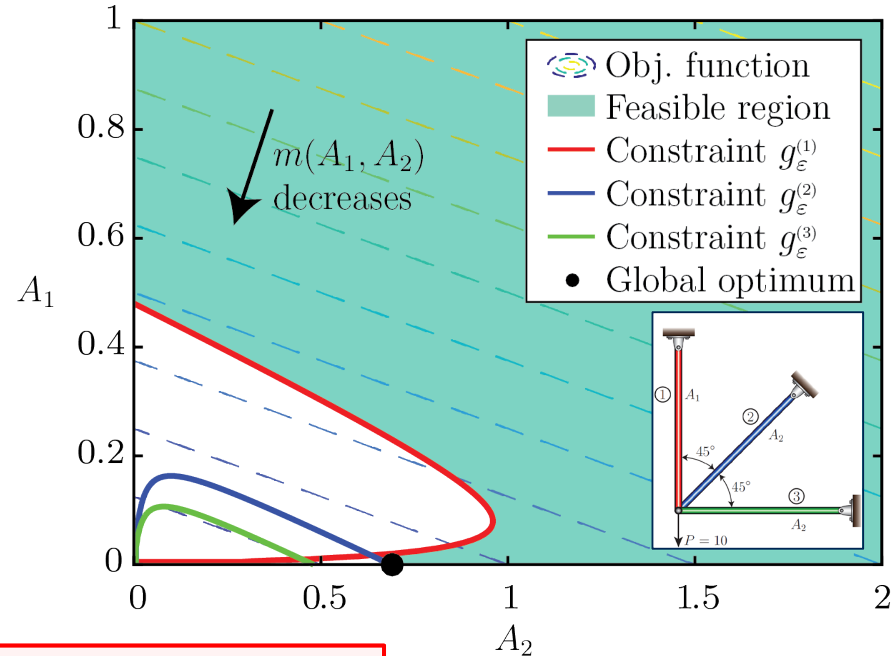
Unrelaxed

$$g_j = A_j^{\eta}\left(\sigma_j^{VM}(\mathbf{A})/\sigma_{\lim} - 1\right)$$



Relaxed

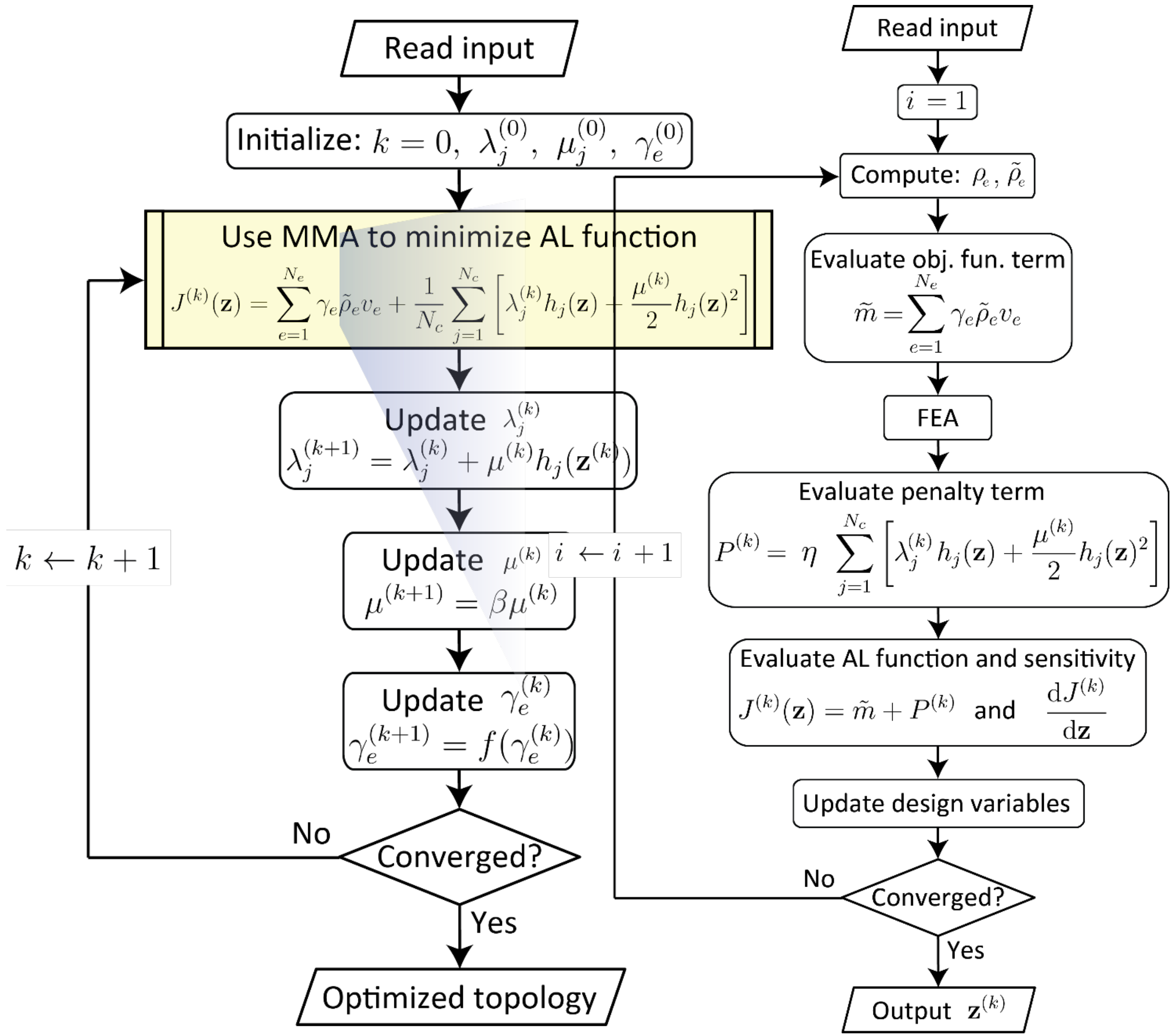
$$g_j = A_j\left(\sigma_j^{VM}(\mathbf{A})/\sigma_{\lim} - 1\right) - \varepsilon$$



We solve the optimization problem using ***unrelaxed*** constraints

G.D. Cheng and X. Guo (1997) ε-relaxed approach in Structural optimization. *Structural Optimization* 13(4), 258-266
Kirsch, U. (1990). On singular topologies in optimum structural design. *Structural Optimization*, 2(3),133-142
Y. K. Park. (1995). Extensions of optimal layout design using the homogenization method. Ph.D. thesis, University of Michigan, Ann Arbor.

Our approach:
A modified Augmented Lagrangian method



We modify the traditional Augmented Lagrangian function using two factors

Traditional AL function

$$J^{(k)}(\mathbf{z}) = \sum_{e=1}^{N_e} \tilde{\rho}_e v_e + \sum_{j=1}^{N_c} \left[\lambda_j^{(k)} h_j(\mathbf{z}) + \frac{\mu^{(k)}}{2} h_j(\mathbf{z})^2 \right]$$

Modified AL function

Scale factor

$$J^{(k)}(\mathbf{z}) = \sum_{e=1}^{N_e} \gamma_e \tilde{\rho}_e v_e + \eta \sum_{j=1}^{N_c} \left[\lambda_j^{(k)} h_j(\mathbf{z}) + \frac{\mu^{(k)}}{2} h_j(\mathbf{z})^2 \right]$$

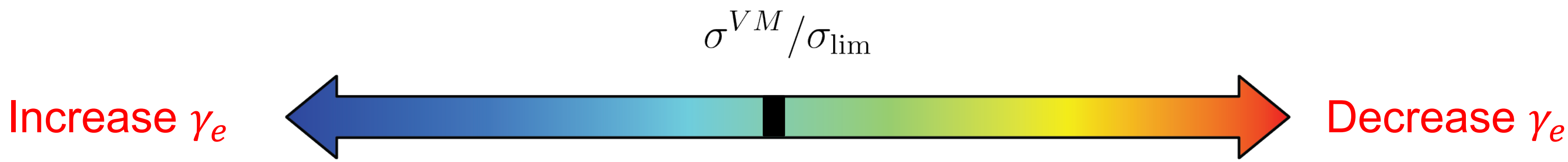
Weight factor

Remarks:

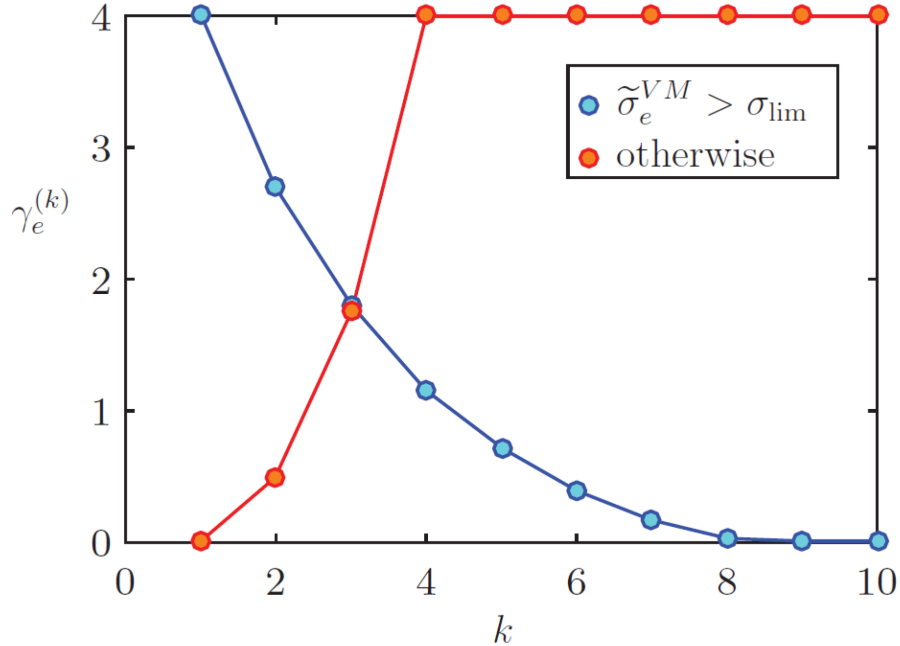
- Scale factor, $\eta = \frac{1}{N_c}$, normalizes the penalization factor of the AL function, which allows us to solve problems with a large number of constraints.
- Weight factors, γ_e, lead to better optimization results.

Weight factors, γ_e, control the relevance of the objective function term w.r.t. the penalty term

$$J^{(k)}(\mathbf{z}) = \boxed{\sum_{e=1}^{N_e} \gamma_e \tilde{\rho}_e v_e} + \boxed{\eta \sum_{j=1}^{N_c} \left[\lambda_j^{(k)} h_j(\mathbf{z}) + \frac{\mu^{(k)}}{2} h_j(\mathbf{z})^2 \right]}$$

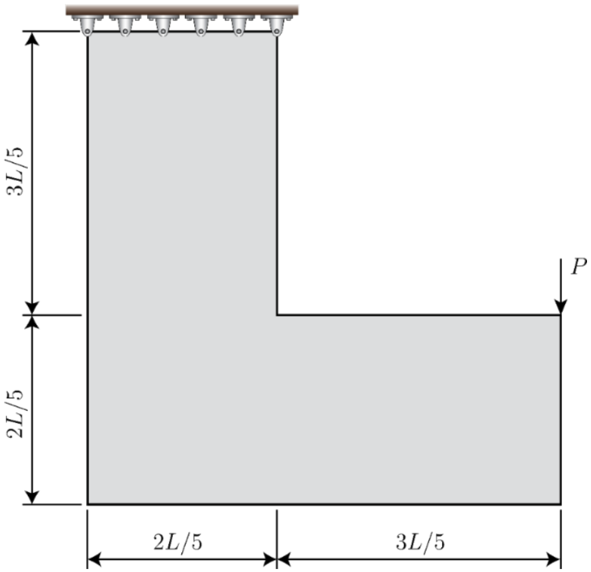


Heuristic evolution of weight factors, γ_e

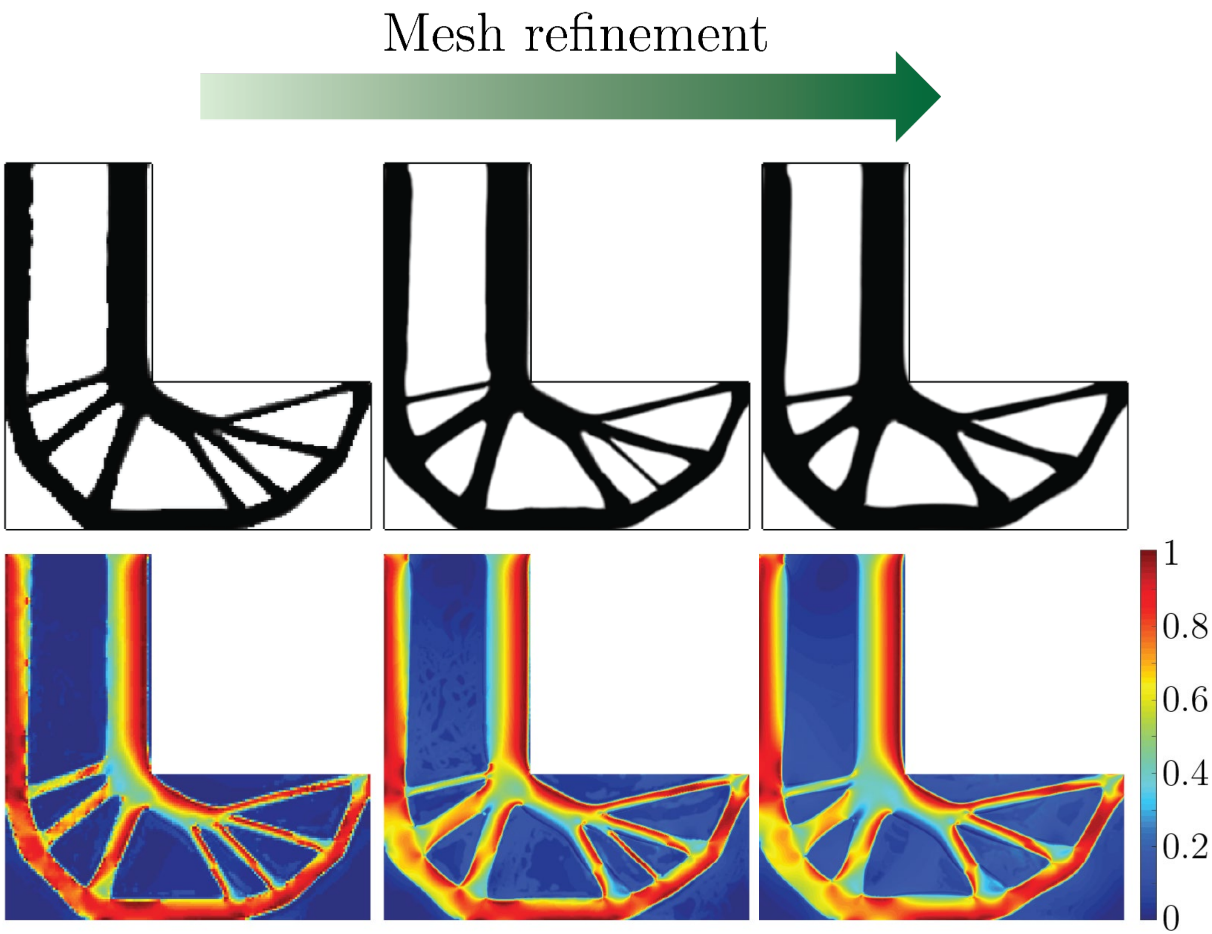


2D L-bracket results

Geometry and loading



Mesh refinement

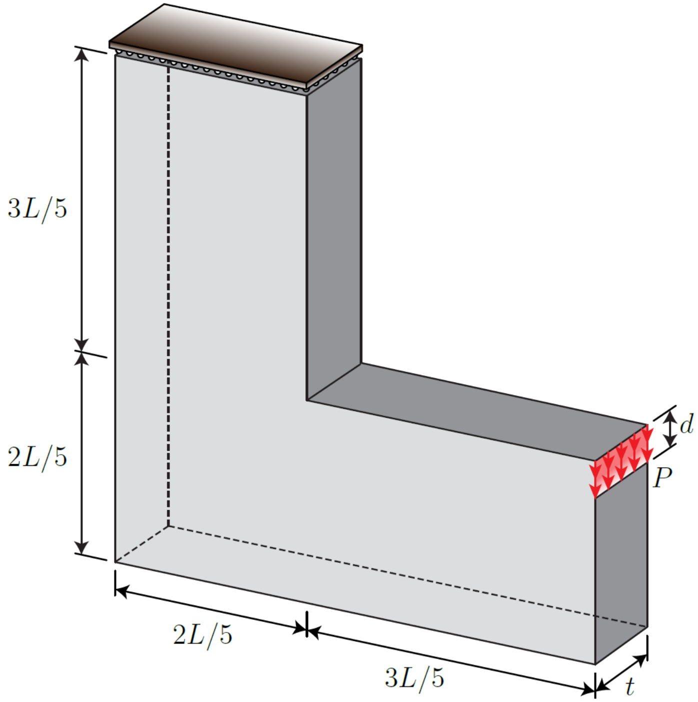


16,834 Constraints | **160,000 Constraints** | **500,000 Constraints**

F. V. da Senhora, O. Giraldo-Londoño, and G.H. Paulino (2019). Topology optimization with local stress constraints: An aggregation-free approach. *(under review)*

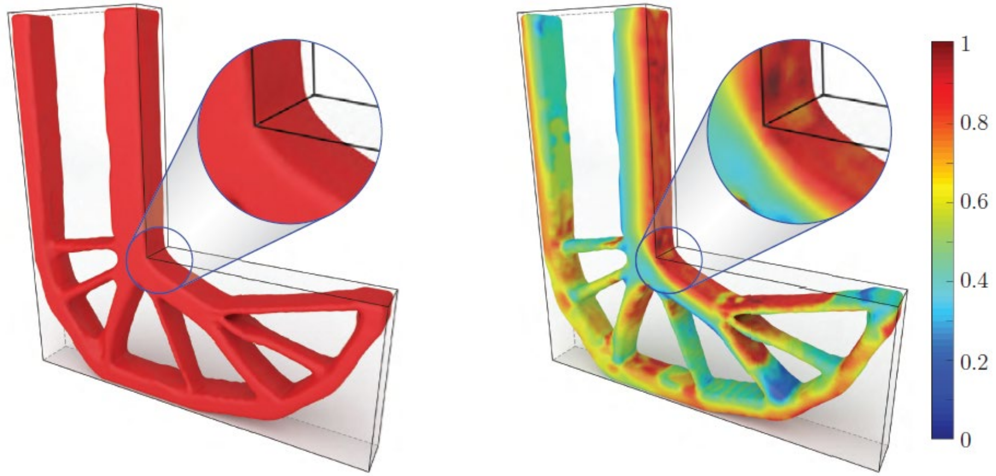
3D L-bracket results

Geometry and loading

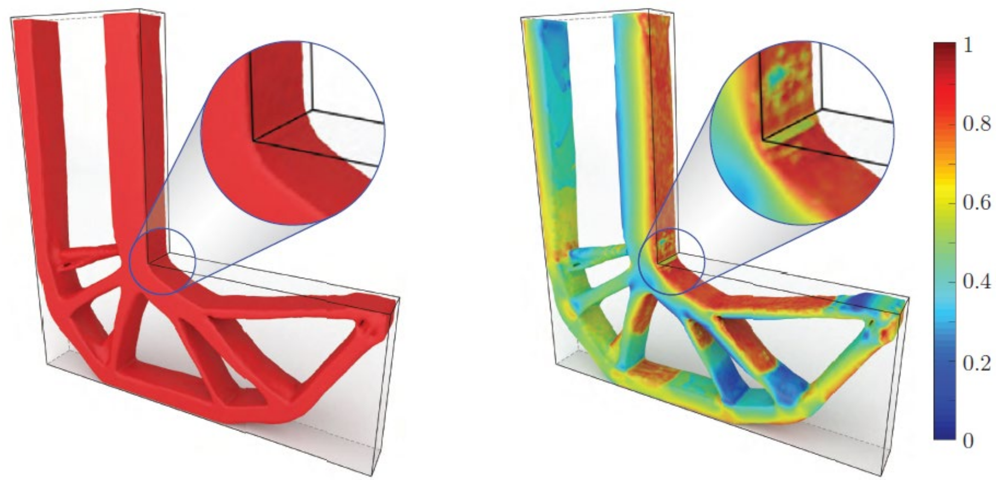


Optimized topology

Normalized von Mises stress



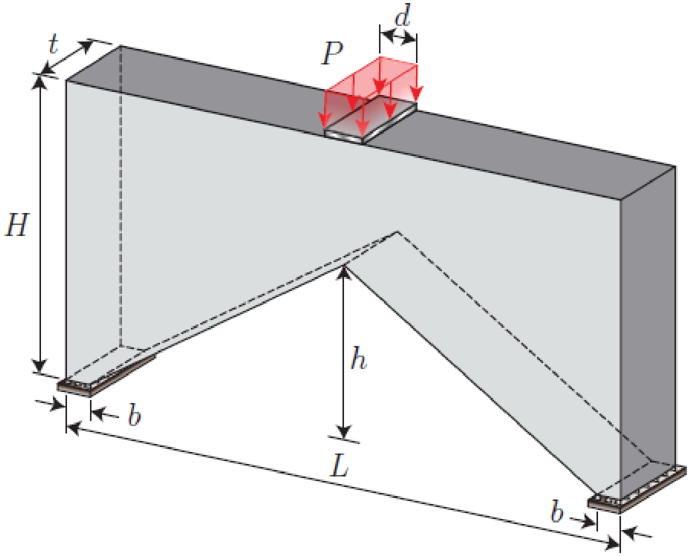
265,000 Constraints

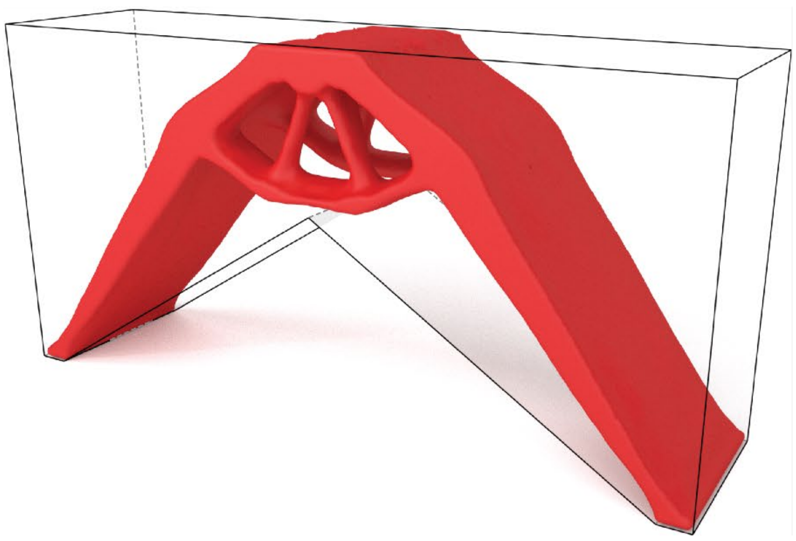


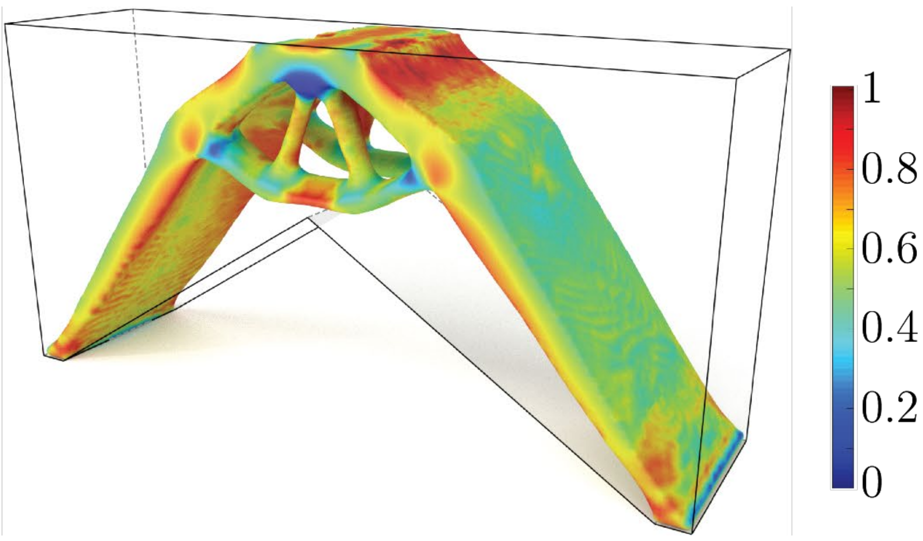
1,728,000 Constraints

F. V. da Senhora, O. Giraldo-Londoño, and G.H. Paulino (2019). Topology optimization with local stress constraints: An aggregation-free approach. *(under review)*

3D portal frame results







1,200,000 Constraints

Concluding remarks

- We have developed an *AL-based framework* to solve topology optimization problems with local stress constraints

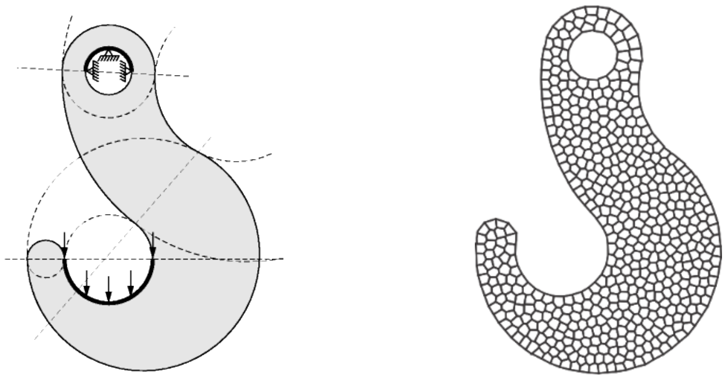
- The method can handle a *large number of stress constraints* while solving the original optimization problem.

- AL-based framework is a rational alternative to widely used aggregation approaches.

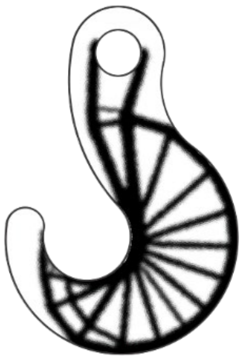
WCSMO 2021: **PolyStress**

PolyTech Family of MATLAB codes: SMO Educational papers

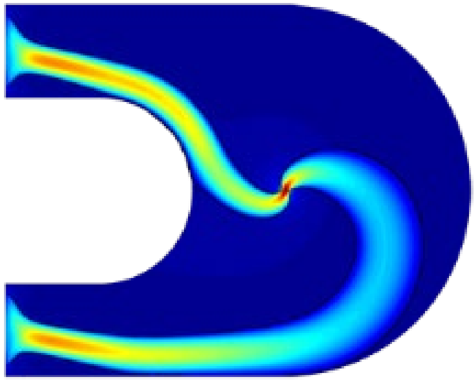
PolyMesher



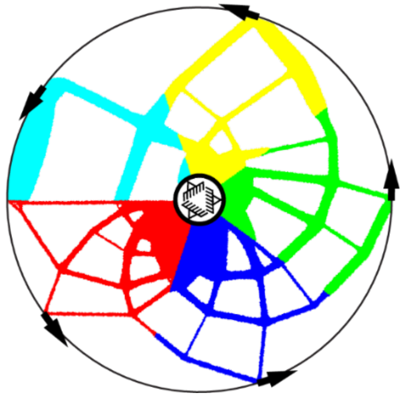
PolyTop



PolyFluid



PolyMat



Thank you!

Acknowledgments

Georgia Tech

Raymond Allen Jones Chair

PUC RIO