PolyMat

An efficient Matlab code for multi-material topology optimization

Emily D. Sanders[a], Miguel A. Aguiló[b], Anderson Pereira[c], and Glaucio H. Paulino[a]

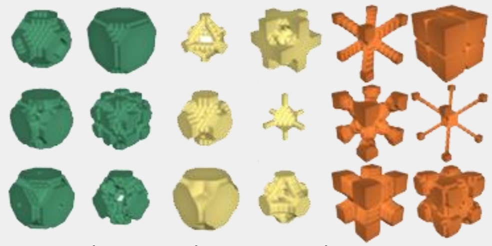
[a]Georgia Institute of Technology
[b]Sandia National Laboratories
[c]Pontifical Catholic University of Rio de Janeiro

May 21, 2019

WCSMO13

Outline

Motivation

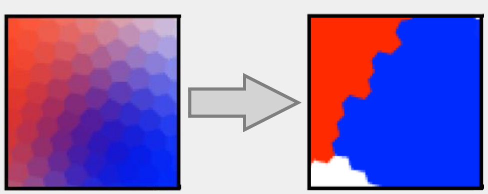


Schumacher, et. al., 2015

Formulation

$$\min \quad f(\underset{\sim}{x})$$
$$\text{s.t.} \quad g_j(\underset{\sim}{x}) \le 0$$
$$j = 1, \ldots, K$$

Material interpolation and continuation



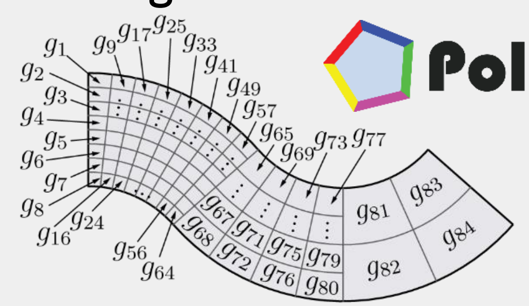
Design variable update

ZPR

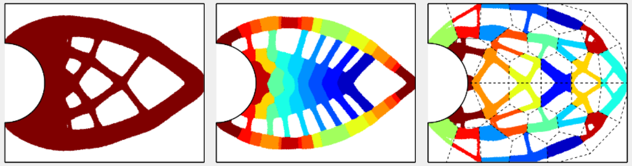
PolyMat

Defining the constraints

PolyMat

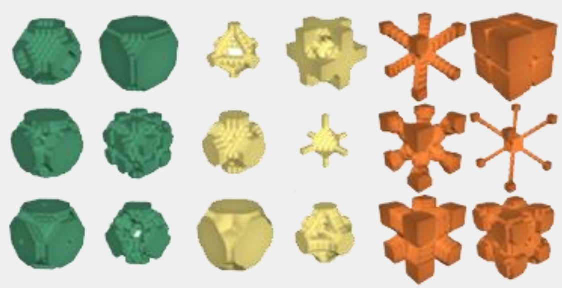


Examples



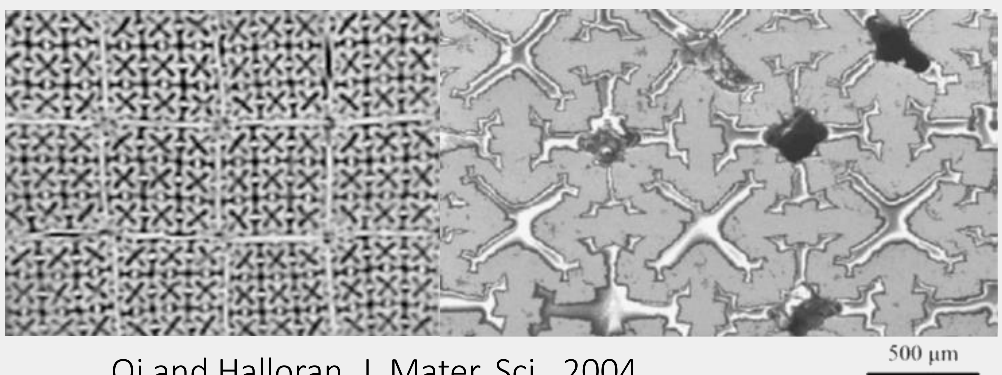
Motivation

Multiple materials ⟹ Functional designs



Schumacher, Bickel, Rys, Marschner, Daraio, Gross, ACM Trans. Graph., 2015

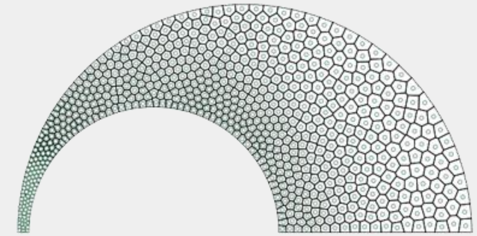
Disney Research



Qi and Halloran, J. Mater. Sci., 2004

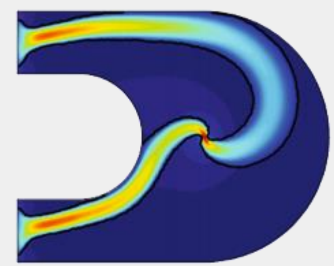
Education ⟹ Research

PolyMesher



Talischi, Paulino, Pereira, Menezes, SMO, 2012

PolyTop Fluid



Pereira, Talischi, Paulino, Menezes, Carvalho, SMO, 2016

PolyMat

Discrete formulation

$$\min_{\mathbf{Z}\in[0,1]^{N\times m}} \quad f = \mathbf{F}^T\mathbf{U} \quad \text{subject to}$$

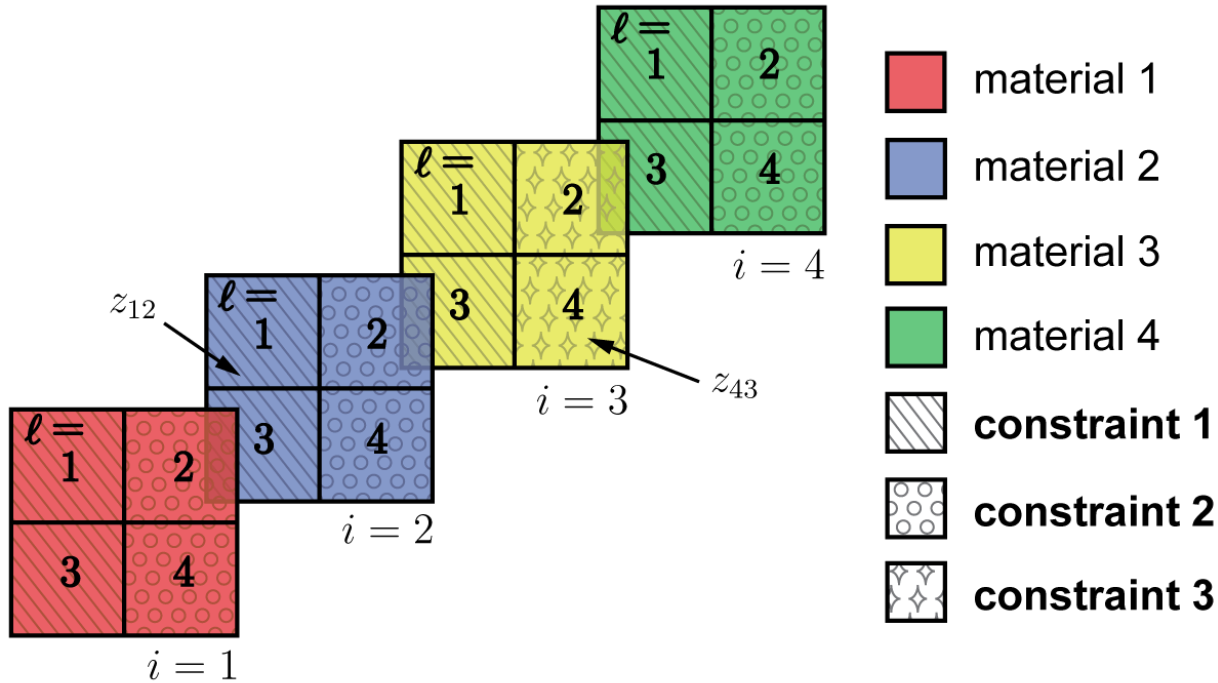
$$g_j = \frac{\sum_{i\in\mathcal{G}_j}\sum_{\ell\in\mathcal{E}_j} A_\ell m_V(y_{i\ell})}{\sum_{\ell\in\mathcal{E}_j} A_\ell} - \overline{v}_j \leq 0, \quad j = 1,\ldots,K$$

where:

$\mathbf{Z} = \{z_{\ell 1},\ldots,z_{\ell m}\}_{\ell=1}^N$	Design variables
$\mathbf{Y} = \{y_{\ell 1},\ldots,y_{\ell m}\}_{\ell=1}^N$	Element values ($\mathbf{y}_i = \mathbf{P}\mathbf{z}_i$)
$\mathcal{G}_j$	Set of material indices assoc. with constraint j
$\mathcal{E}_j$	Set of element indices assoc. with constraint j
$\mathbf{U}$	Solves $\mathbf{KU} = \mathbf{F}$

1. Zhang, Paulino, and Ramos Jr. Multi-material topology optimization with multiple volume constraints: A general approach applied to ground structures with material nonlinearity. *SMO*. 57:161-182, 2018.
2. Sanders, Aguiló, and Paulino. Multi-material continuum topology optimization with arbitrary volume and mass constraints. *CMAME*. 340:798-823, 2018.
3. Sanders, Pereira, Aguiló, Paulino. PolyMat: a Matlab implementation of multi-material topology optimization using unstructured polygonal finite element meshes. *SMO*. 58:2727-2759, 2018.

Problem setup

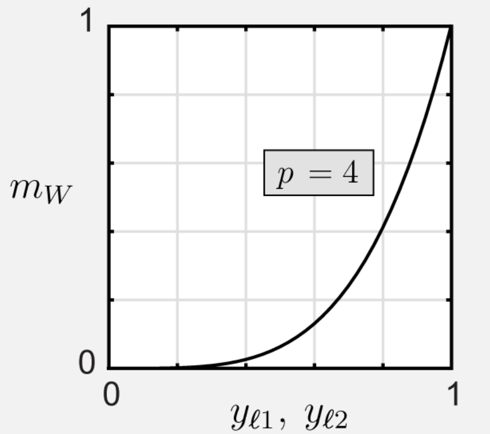


$$\mathcal{G}_1 = \{1,2,3,4\} \quad \mathcal{E}_1 = \{1,3\}$$
$$\mathcal{G}_2 = \{1,2,4\} \quad \mathcal{E}_2 = \{2,4\}$$
$$\mathcal{G}_3 = \{3\} \quad \mathcal{E}_3 = \{2,4\}$$

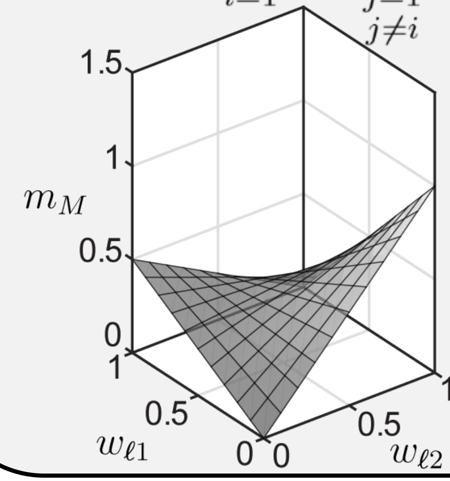
1. Zhang, Paulino, and Ramos Jr. Multi-material topology optimization with multiple volume constraints: A general approach applied to ground structures with material nonlinearity. *SMO*. 57:161-182, 2018.
2. Sanders, Aguiló, and Paulino. Multi-material continuum topology optimization with arbitrary volume and mass constraints. *CMAME*. 340:798-823, 2018.
3. Sanders, Pereira, Aguiló, Paulino. PolyMat: a Matlab implementation of multi-material topology optimization using unstructured polygonal finite element meshes. *SMO*. 58:2727-2759, 2018.

Material interpolation

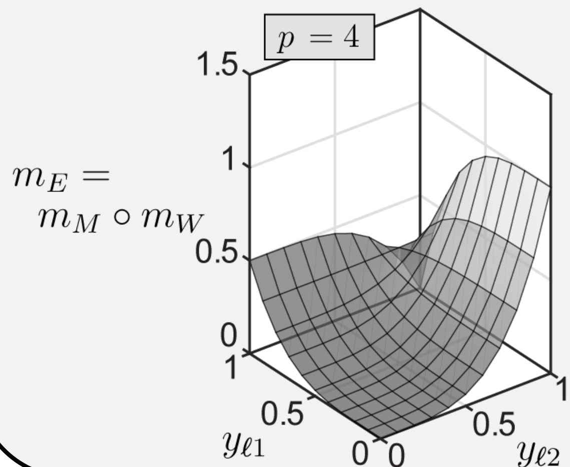
$$m_W\left(y_{\ell i}\right) = y_{\ell i}^p \quad (:= w_{\ell i})\ (\mathrm{SIMP})$$



$$m_M\left(w_{\ell i}\right) = \sum_{i=1}^{m} w_{\ell i} \prod_{\substack{j=1 \\ j \neq i}}^{m} \left(1 - w_{\ell j}\right) E_i^0$$



$$m_E\left(\mathbf{Y}\right) = \left(m_M \circ m_W\right)\left(\mathbf{Y}\right) \quad (:= \mathbf{E})$$



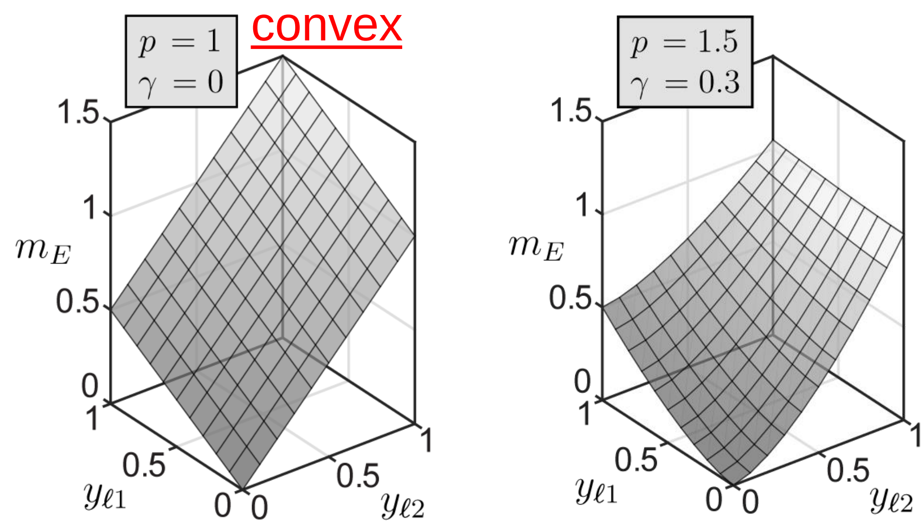
1. Zhou and Rozvany. “The COC algorithm, Part II: topological, geometrical and generalized shape optimization.” *CMAME*. 89:309-336. 1991.
2. Bendsoe. “Optimal shape design as a material distribution problem.” *Structural Optimization*. 1:193-202. 1989.
3. Stegmann and Lund. “Discrete material optimization of general composite shell structures.” *IJNME*. 62:2009-2027. 2005.

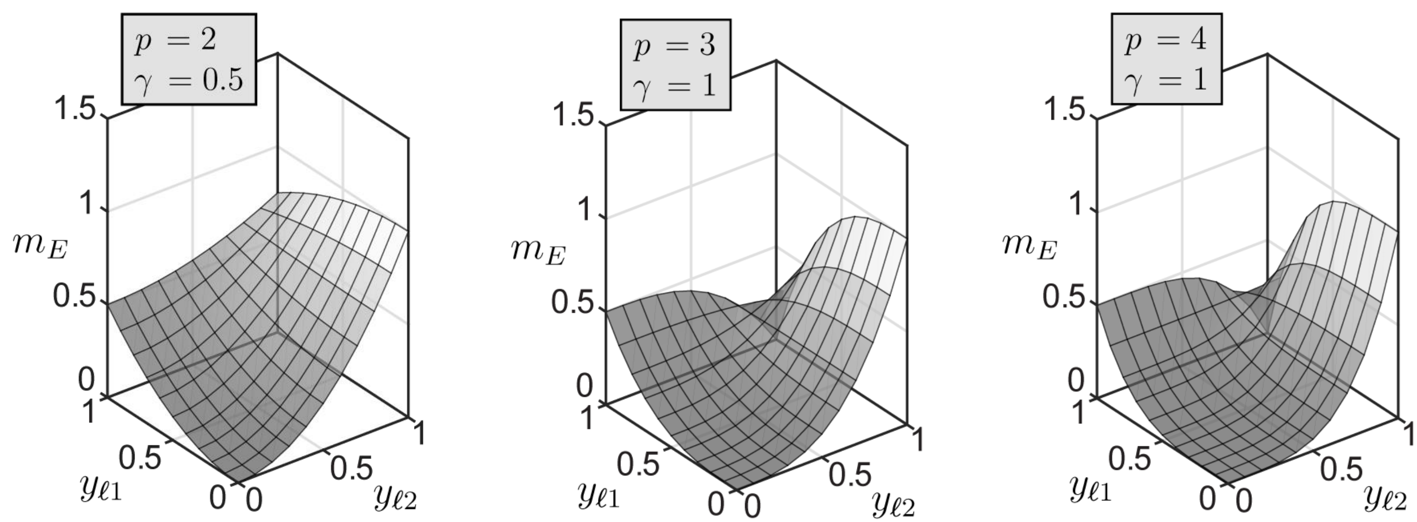
Continuation

$$m_W\left(y_{\ell i}\right) = y_{\ell i}^{p} \qquad \text{(SIMP)}$$

$$m_M\left(w_{\ell i}\right) = \sum_{i=1}^{m} w_{\ell i} \prod_{\substack{j=1 \\ j \neq i}}^{m} \left(1 - \gamma w_{\ell j}\right) E_i^0$$

$$m_E\left(\mathbf{Y}\right) = \left(m_M \circ m_W\right)\left(\mathbf{Y}\right)$$





Sanders, Pereira, Aguiló, Paulino. PolyMat: a Matlab implementation of multi-material topology optimization using unstructured polygonal finite element meshes. *SMO*. 58:2727-2759, 2018.

ZPR design variable update

(Primal) Linearized sub-problem at current design, $\mathbf{Z}^0$:

$$
\begin{aligned}
\min_{\mathbf{Z}\in[0,1]^{N\times m}} \quad & \tilde{f}(\mathbf{Z}) = \cancel{f(\mathbf{Z}^0)} + \sum_{i=1}^{m} \cancelto{\mathbf{b}_i}{\left(\left.\frac{\partial f}{\partial \xi_i}\right|_{\mathbf{Z}=\mathbf{Z}^0} \right)}^{T} \left(\xi_i(\mathbf{z}_i) - \cancel{\xi_i(\mathbf{z}_i^0)} \right) \\
\text{s.t.} \quad & \tilde{g}_j(\mathbf{Z}) = g_j(\mathbf{Z}^0) + \sum_{i=1}^{m} \left(\left.\frac{\partial g_j}{\partial \mathbf{z}_i}\right|_{\mathbf{Z}=\mathbf{Z}^0} \right)^{T} (\mathbf{z}_i - \mathbf{z}_i^0), \quad j = 1, \ldots, K
\end{aligned}
$$

exponential intermediate variables:
$\xi_{\ell i}(z_{\ell i}) = z_{\ell i}^{-\alpha}, \quad \alpha > 0$

linear intermediate variables

Lagrangian of primal sub-problem:

$$
\begin{aligned}
\mathcal{L}(\mathbf{Z}, \lambda_1, \ldots, \lambda_K) &= \sum_{i=1}^{m} \mathbf{b}_i^T \xi_i(\mathbf{z}_i) + \sum_{j=1}^{k} \lambda_j \tilde{g}_j(\mathbf{z}_i) \\
&= \sum_{j=1}^{K} \left[\sum_{i\in\mathcal{G}_j} \mathbf{b}_i^T \xi_i(\mathbf{z}_i) + \lambda_j \tilde{g}_j(\mathbf{z}_i) \right]
\end{aligned}
$$

Dual objective:

$$
\begin{aligned}
\phi(\lambda_1, \ldots, \lambda_K) &= \min_{\substack{z_{\ell i}^- \le z_{\ell i} \le z_{\ell i}^+, \\ i=1,\ldots,m}} \mathcal{L}(\mathbf{Z}, \lambda_1, \ldots, \lambda_K) \\
&= \sum_{j=1}^{K} \left[\min_{\substack{z_{\ell i}^- \le z_{\ell i} \le z_{\ell i}^+, \\ i=1,\ldots,m}} \sum_{i=\mathcal{G}_j} \mathbf{b}_i^T \xi_i(\mathbf{z}_i) + \lambda_j \tilde{g}_j(\mathbf{z}_i) \right]
\end{aligned}
$$

separable

$$
\phi(\lambda_1, \ldots, \lambda_K) = \sum_{j=1}^{K} \phi_j(\lambda_j)
$$

Zhang, Paulino, and Ramos Jr., *SMO*. 57:161-182, 2018.

ZPR design variable update

Stationary conditions of the separated Lagrangian…

$$\frac{\partial \mathcal{L}}{\partial z_{\ell i}} = 0 = \frac{\partial}{\partial z_{\ell i}} \left(\sum_{i \in \mathcal{G}_j} \mathbf{b}_i^T \xi \left(\mathbf{z}_i \right) \right) + \lambda_j \frac{\partial \tilde{g}_j}{\partial z_{\ell i}}$$

…lead to an explicit design variable update:

$$z_{\ell i}^{\text{new}} = \begin{cases} z_{\ell i}^{+}, & z_{\ell i}^{*} \geq z_{\ell i}^{+} \\ z_{\ell i}^{-}, & z_{\ell i}^{*} \leq z_{\ell i}^{-} \\ z_{\ell i}^{*}, & \text{otherwise} \end{cases} \qquad \text{where: } z_{\ell i}^{*} \left(\lambda_j \right) = z_{\ell i}^{0} \left(- \frac{ \left. \frac{\partial f}{\partial z_{\ell i}} \right|_{\mathbf{Z} = \mathbf{Z}^0} }{ \lambda_j \left. \frac{\partial g_j}{\partial z_{\ell i}} \right|_{\mathbf{Z} = \mathbf{Z}^0} } \right)^{\frac{1}{1+\alpha}}$$

Dependent on a single Lagrange multiplier

Stationary conditions of the dual:

$$\begin{aligned} \frac{\partial \phi}{\partial \lambda_j} = 0 &= \frac{\partial \phi}{\partial z_{\ell i}} \frac{\partial z_{\ell i}}{\partial \lambda_j} + \frac{\partial \phi}{\partial \lambda_j} \\ &= g_j \left(\mathbf{Z}^0 \right) + \sum_{i=1}^{m} \left(\left. \frac{\partial g_j}{\partial \mathbf{z}_i} \right|_{\mathbf{Z} = \mathbf{Z}^0} \right)^T \left(\mathbf{z}_i \left(\lambda_j \right) - \mathbf{z}_i^0 \right) \end{aligned}$$

monotonic in λ_j,
solve for λ_j using bisection

Zhang, Paulino, and Ramos Jr., *SMO*. 57:161-182, 2018.

ZPR design variable update

Lagrangian duality leads to the following explicit design variable update:

$$z_{\ell i}^{*}\left(\lambda_j\right) = z_{\ell i}^{0}\left(-\frac{\left.\frac{\partial f}{\partial z_{\ell i}}\right|_{\mathbf{Z}=\mathbf{Z}_0}}{\lambda_j \left.\frac{\partial g_j}{\partial z_{\ell i}}\right|_{\mathbf{Z}=\mathbf{Z}_0}}\right)^{\eta} \qquad \forall i \in \mathcal{G}_j,\ \ \forall \ell \in \mathcal{E}_j,\ \ j = 1,\dots,K$$

Design is updated for each volume constraint, independently

```
%Update design variable and analysis parameters
for c=1:opt.NConstr
  ElemInd = cell2mat(opt.ElemInd(c));
  MatInd = cell2mat(opt.MatInd(c));
  [z(ElemInd,MatInd),Change(c)] = UpdateScheme(...
      dfdz(ElemInd,MatInd),g(c),...
      dgdz(ElemInd,c,MatInd),z(ElemInd,MatInd),opt,V(ElemInd,MatInd));
end
```

PolyMat

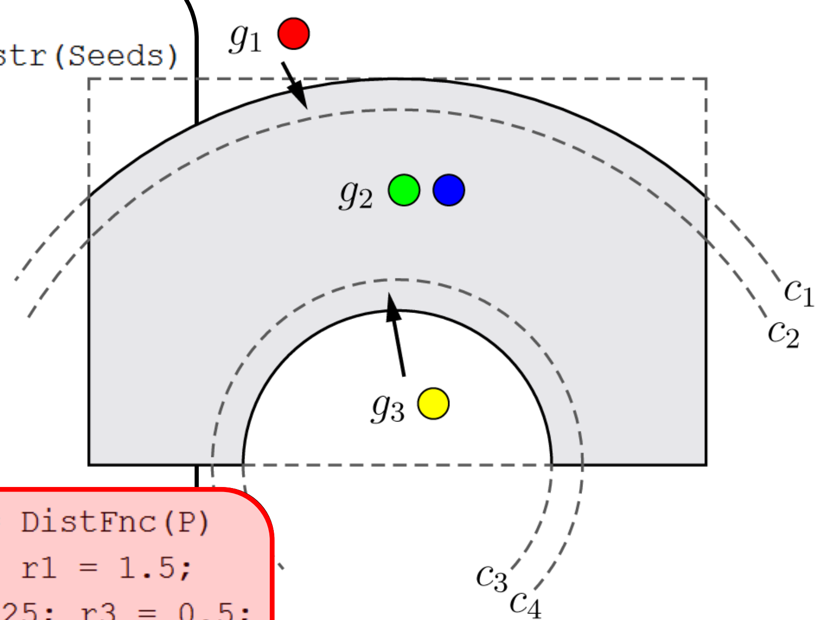
1. Zhang, Paulino, and Ramos Jr. Multi-material topology optimization with multiple volume constraints: A general approach applied to ground structures with material nonlinearity. *SMO*. 57:161-182, 2018.
2. Sanders, Aguiló, and Paulino. Multi-material continuum topology optimization with arbitrary volume and mass constraints. *CMAME*. 340:798-823, 2018.
3. Sanders, Pereira, Aguiló, Paulino. PolyMat: a Matlab implementation of multi-material topology optimization using unstructured polygonal finite element meshes. *SMO*. 58:2727-2759, 2018.

Defining the constraints

PolyMat

```
[Node,Element,Supp,Load,Seeds] = PolyMesher(@CurvedBeamDomain,8000,30);
[VolFrac,ElemInd,MatInd,SElemInd,SMatInd,mat,color] = CurvedBeamConstr(Seeds);
```

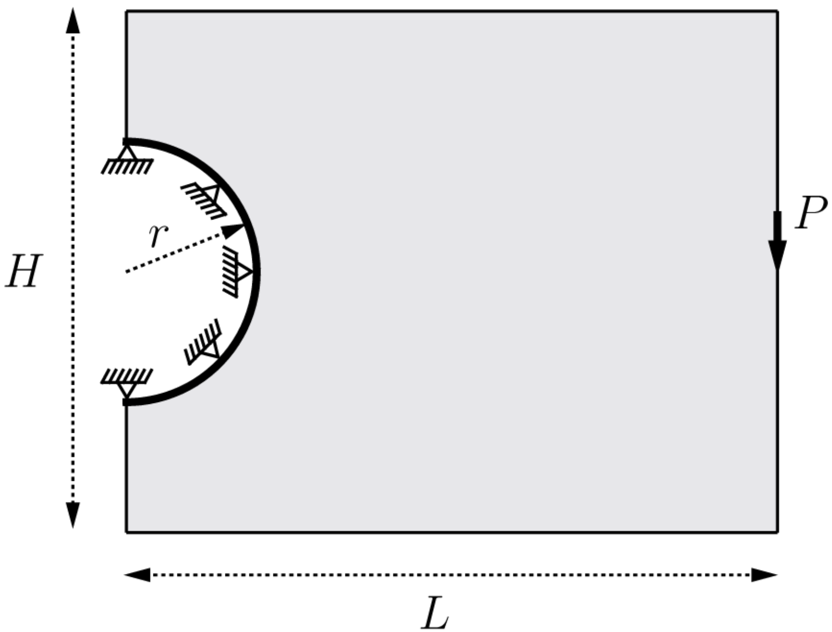
```
function [VolFrac,ElemInd,MatInd,...
          SElemInd,SMatInd,mat,color] = CurvedBeamConstr(Seeds)
  NConstr = 3;
  VolFrac = [0.5; 0.5; 0.5];
  mat = [1; 0.8; 0.6; 0.4];
  color = [1 0 0;  %red
           0 1 0;  %green
           0 0 1;  %blue
           1 1 0]; %yellow
  MatInd{1} = 1;
  MatInd{2} = [2 3];
  MatInd{3} = 4;
  % Constrained regions
  [Dist] = DistFnc(Seeds);
  ElemInd = cell(NConstr,1);
  for i = 1:NConstr
    ElemInd{i} = find(Dist{i}<=0);
  end
  % Passive regions
  SElemInd = []; SMatInd = [];
```




```
function [Dist] = DistFnc(P)
  x1 = 0; y1 = 0; r1 = 1.5;
  x3 = 0; y3 = 0.25; r3 = 0.5;
  t = 0.1;
  c2 = dCircle(P,x1,y1,r1-t);
  c4 = dCircle(P,x3,y3,r3+t);
  Dist{1} = -c2(:,end);
  dist2 = dIntersect(c2,-c4);
  Dist{2} = dist2(:,end);
  Dist{3} = c4(:,end);
```

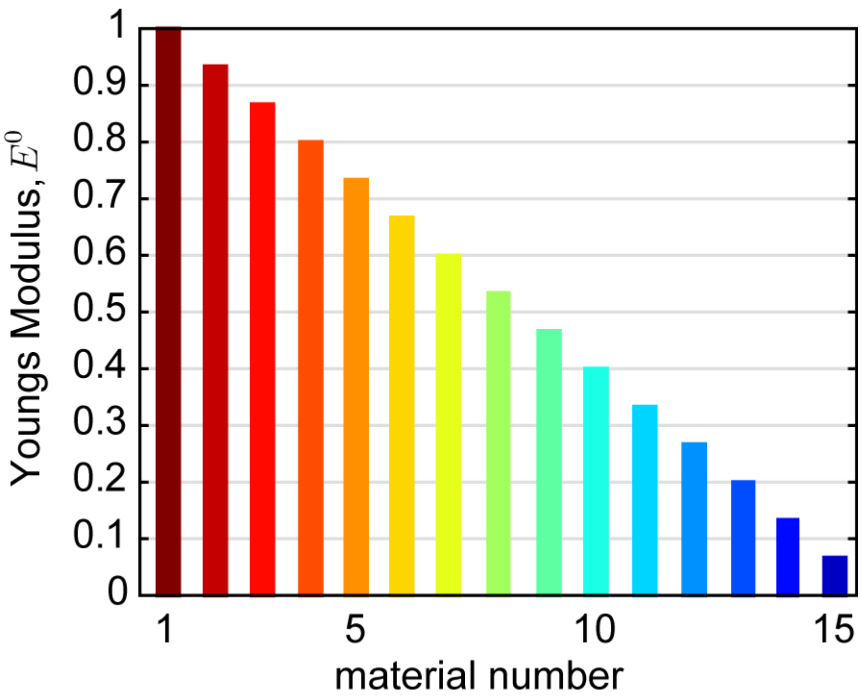
Talischi, Paulino, Pereira, Menezes. "PolyMesher: a general-purpose mesh generator for polygonal elements written in Matlab." *SMO*. 45:309-328. 2012.

Michell cantilever



$H = 4\,,\, L = 5\,,\, r = 1\,,\, P = 1$

** mesh has horizontal line of symmetry

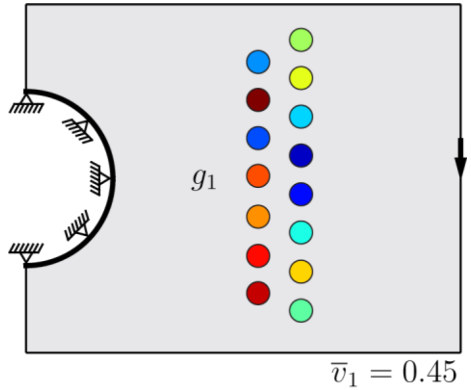


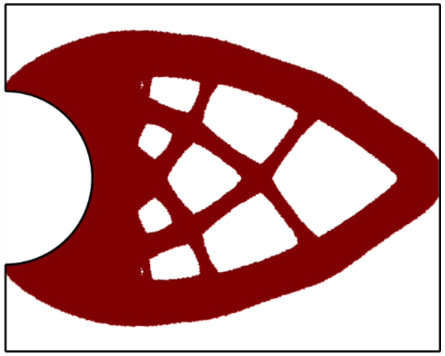
number of elements	90,000
SIMP penalty parameter, p	$[1, 1.5, 2, 3, 4]$
Material interpolation factor, γ	$[0, 0.3, 0.5, 1, 1]$
filter radius, R	$[0.05, 0.05, 0.05, 0.05, -1]$
max. number of iterations (per continuation step)	200

Talischi, Paulino, Pereira, Menezes. "PolyTop: a Matlab implementation of a general topology optimization framework using unstructured polygonal finite element meshes." *SMO*. 45:329-357. 2012.

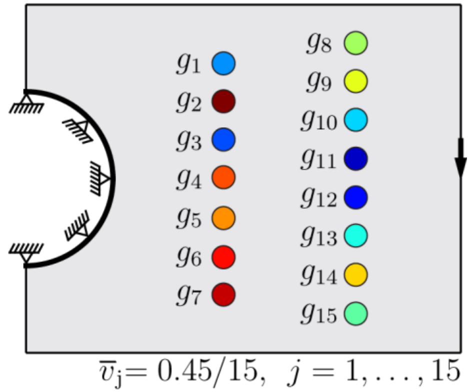
Michell cantilever

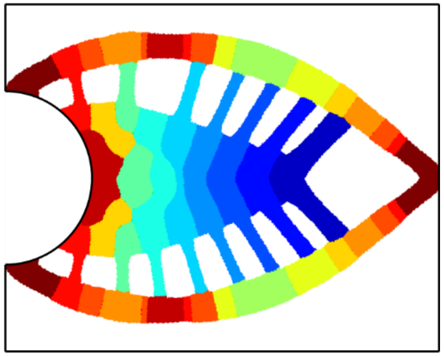
1 global constraint



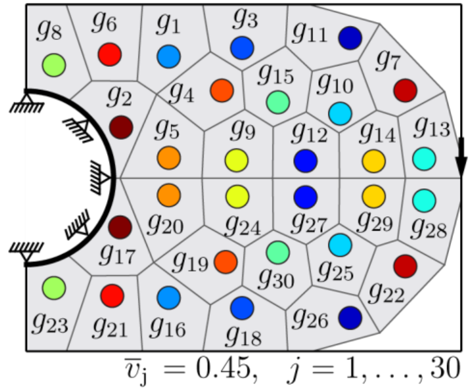


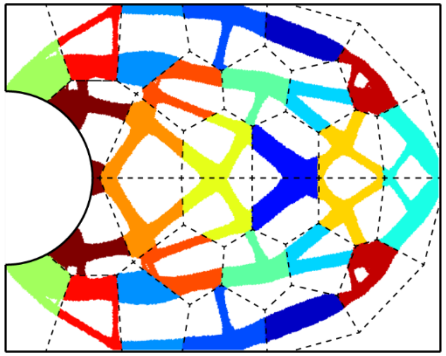
15 global constraints
(one for each material)



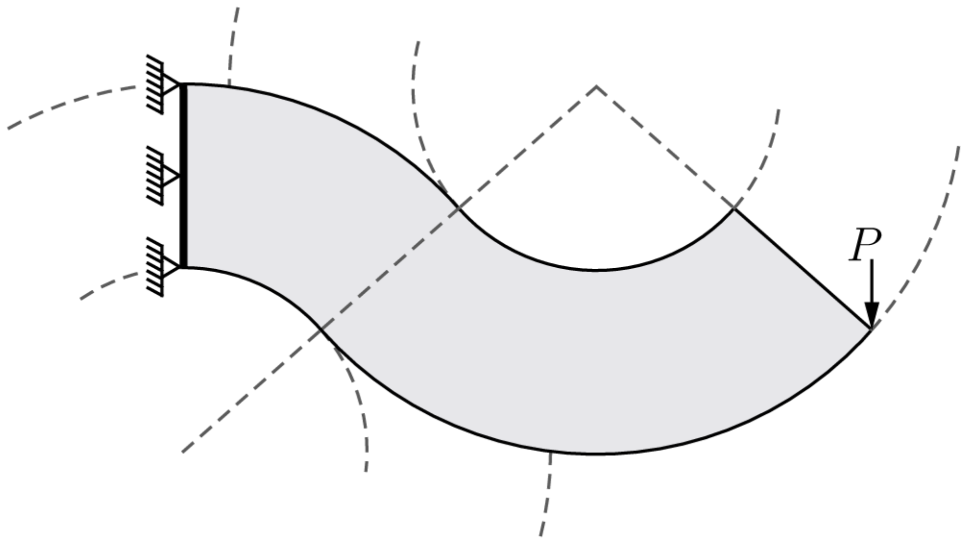


30 local constraints





Controlling geometric gradation: Serpentine



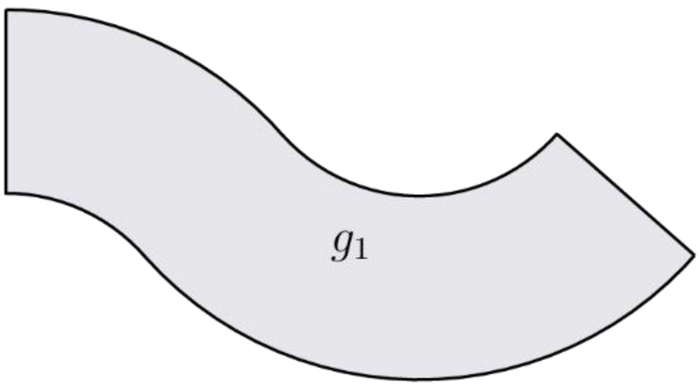
** single material

number of elements	75,000
SIMP penalty parameter, p	$[3, 3]$
Material interpolation factor, γ	$[1, 1]$
filter radius, R	$[0.1, -1]$
max. number of iterations (per continuation step)	500

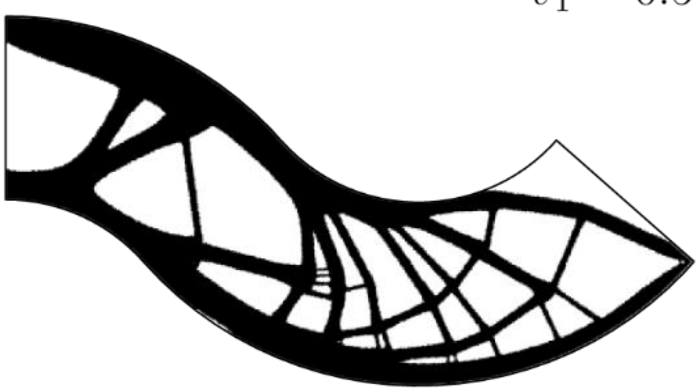
Talischi, Paulino, Pereira, Menezes. “PolyTop: a Matlab implementation of a general topology optimization framework using unstructured polygonal finite element meshes.” *SMO*. 45:329-357. 2012.

Controlling geometric gradation: Serpentine

1 global constraint

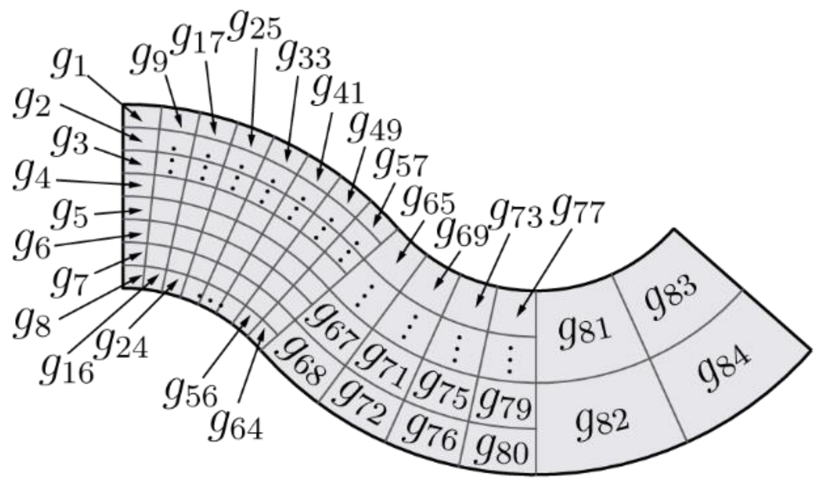


$\overline{v}_1 = 0.5$



$f = 384.32$

84 local constraints



$\overline{v}_j = 0.5, \quad j = 1, \ldots, 84$

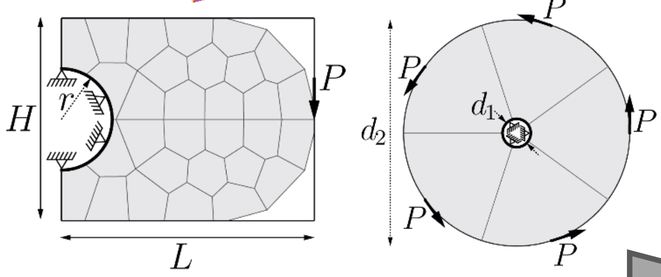
$f = 505.75$

Summary

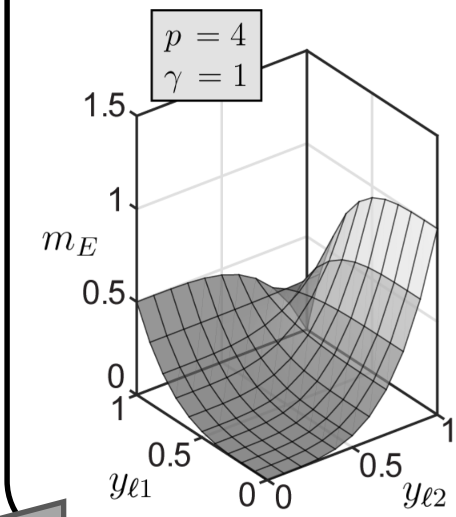
ZPR design variable update scheme

$$z^*_{\ell i}\left(\lambda_j\right) = z^0_{\ell i}\left(-\frac{\frac{\partial f}{\partial z_{\ell i}}\Big|_{\mathbf{Z}=\mathbf{Z}_0}}{\lambda_j \frac{\partial g_j}{\partial z_{\ell i}}\Big|_{\mathbf{Z}=\mathbf{Z}_0}}\right)^{\eta}$$

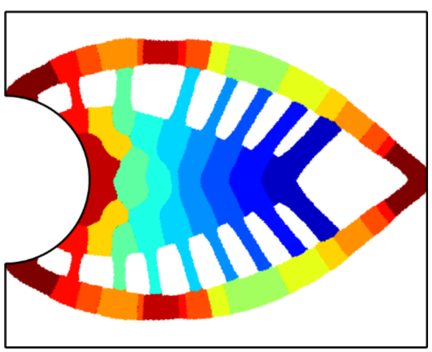
PolyMat

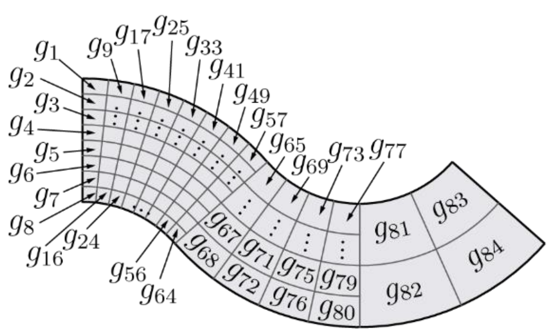


Mixing and intermediate density penalization

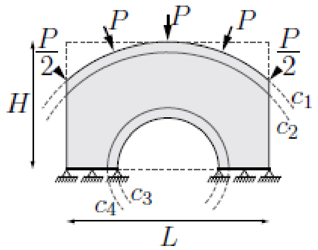
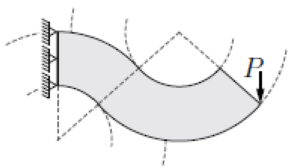
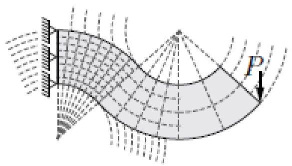
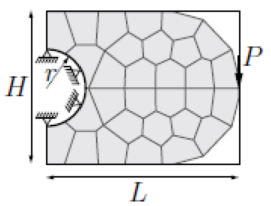
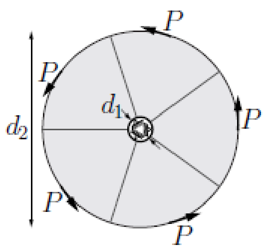


Many materials, Many constraints





PolyMat Library

Domain	PolyMesher domain file	Constraint file	Description
	`@CurvedBeamDomain`	`CurvedBeamConstraints`	• Defaults: $H = 1.25$, $L = 2$, $c_1 = (x_{c1}, y_{c1}, r_{c1}) = (0, 0, 1.5)$, $c_2 = (0, 0, 1.4)$, $c_3 = (0, 0.25, 0.5)$, $c_4 = (0, 0.25, 0.6)$, $P = 1$ • 2 passive regions • 3 volume constraints, each controlling 1 of 3 candidate materials in the optimizable region
	`@SerpentineDomain`	`SerpentineConstraints1Constr`	• Defaults: $P = 1$ • 1 global volume constraint controlling 1 candidate material
	`@SerpentineDomain`	`SerpentineConstraints84Constr`	• Defaults: $P = 1$ • 84 local volume constraints, each controlling 1 of 1 candidate material in each of 84 sub-regions
	`@MichellDomain`	`MichellConstraints3`	• Defaults: $H = 4$, $L = 5$, $r = 1$, $P = 1$ • 30 local volume constraints, each controlling a sub-region of the domain and 1 of 15 candidate materials
	`@FlowerDomain`	`FlowerConstraints5Mat`	• Defaults: $d_1 = 0.25$, $d_2 = 2$, $P = 1$ • 5 local volume constraints, each controlling 1 of 1 candidate material in each of 5 sub-regions

Sanders, Pereira, Aguiló, Paulino. *SMO*. 58:2727-2759, 2018.

Thank You