Dissertation Defense

Potential-based Fracture Mechanics Using Cohesive Zone and Virtual Internal Bond Modeling

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7/21/2009

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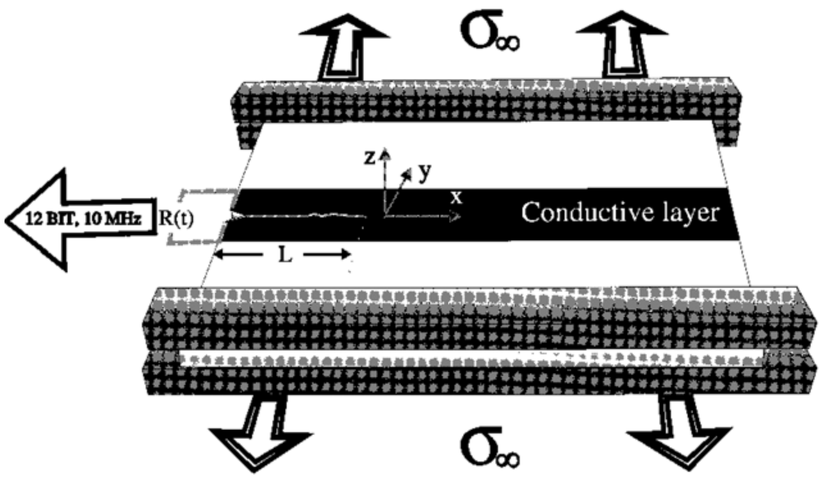
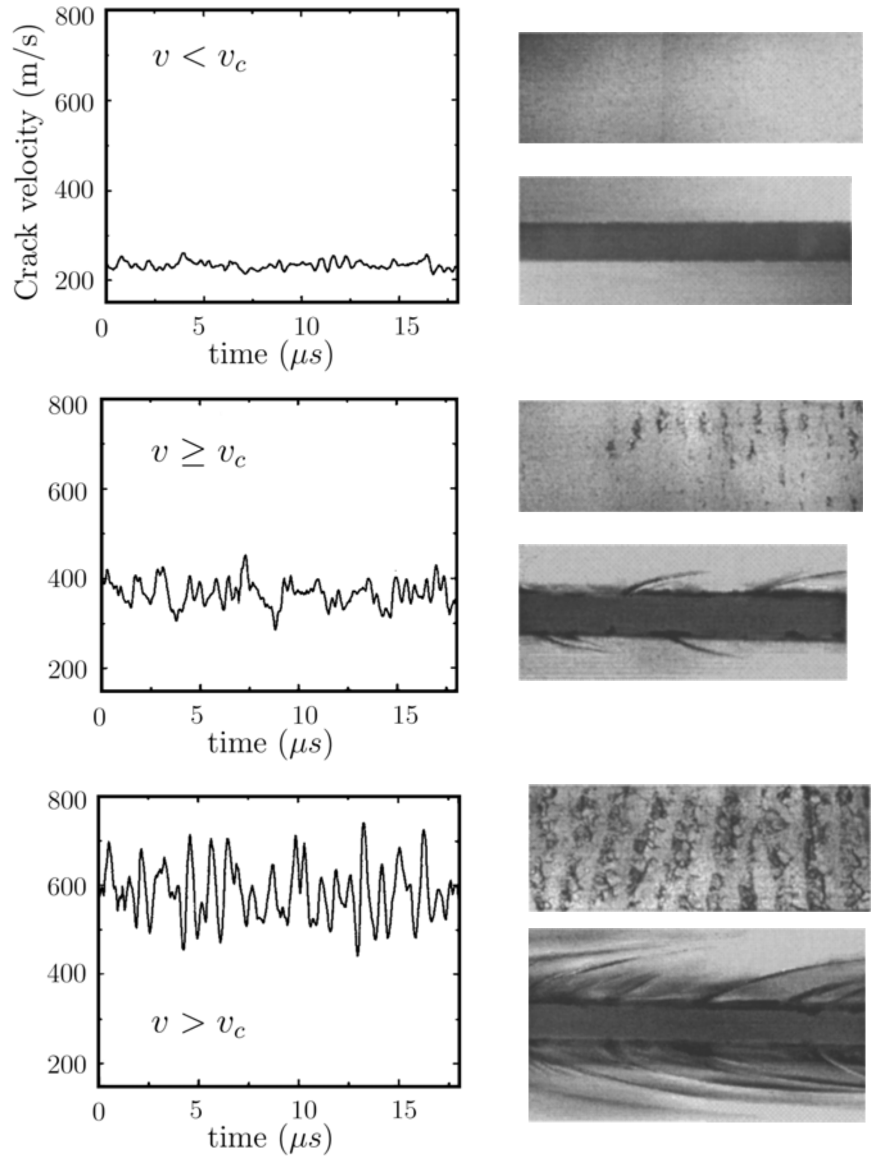
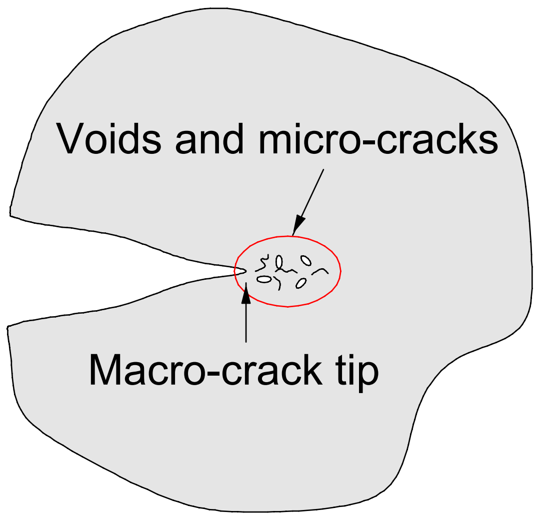
Micro-Branching Experiment

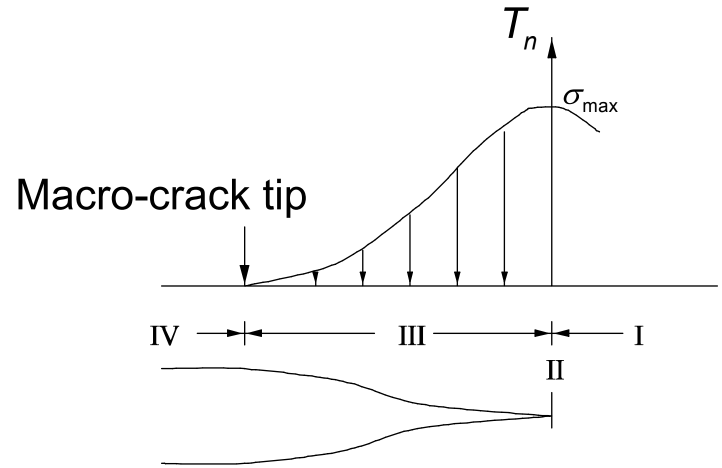


Sharon E, Fineberg J. Microbranching instability and the dynamic fracture of brittle materials. Physical Review B 1996; 54(10):7128–7139.



Cohesive Zone Modeling





- **Constitutive Relationship of Cohesive Fracture**
  - Non-potential based-model vs. Potential based-model
- **Computational Methods**
  - Cohesive surface elements, enrichment functions, embedded discontinuities

● Wells, G. N., Sluys, L. J., 2001. A new method for modelling cohesive cracks using finite elements. International Journal for Numerical Methods in Engineering 50 (12), 2667–2682.

● Jirasek, M., 2000. Comparative study on finite elements with embedded discontinuities. Computer Methods in Applied Mechanics and Engineering 188 (1-3), 307–330.

● Linder, C., Armero, F., 2009. Finite elements with embedded branching. Finite Elements in Analysis and Design 45 (4), 280–293.

Contents

- Introduction (Ch. 1)
- **Potential-based Cohesive Model (Ch. 3)**
- **Quasi-Static Fracture (Ch. 5)**
  - Particle/matrix debonding
- **Dynamic Fracture Problems (Ch. 4, 6, 7)**
  - Computational framework
  - Micro-branching and fragmentation
  - Mode I predefined crack, mixed-mode and branching
- **Virtual Internal Pair-Bond (VIPB) Model (Ch. 2)**
- **Summary (Ch. 8)**

Potentials for Cohesive Fracture

- **Needleman, A. (1987)**
  - Polynomial potential / linear shear interaction
- **Needleman, A. (1990)**
  - Exponential potential / periodic dependence
- **Beltz, G.E. and Rice, J.R. (1991)**
  - Generalized the potential (Exponential + Sinusoid)
- **Xu, X.P. and Needleman, A. (1993)**
  - Exponential potential (Exponential + Exponential)
- **Park, K., Paulino, G.H. and Roesler, J.R. (2009) – PPR**
  - Polynomial potential (Polynomial + Polynomial)

- Needleman A. 1987, A continuum model for void nucleation by inclusion debonding, Journal of Applied Mechanics, 54, 525-531
- Needleman A. 1990, An analysis of tensile decohesion along an interface, Journal of the Mechanics and Physics of Solid, 3, 289-324
- Beltz GE and Rice JR, 1991, Dislocation nucleation versus cleavage decohesion at crack tip, Modeling the Deformation of Crystalline Solids, 457-480.
- Xu XP and Needleman, 1993, Void nucleation by inclusion debonding in a crystal matrix, Modeling Simulation Material Science Engineering, 1, 111-132.
- Park K, Paulino GH, Roesler JR, 2009, A unified potential-based cohesive model of mixed-mode fracture, Journal of the Mechanics and Physics of Solids 57, 891-908.

1. Needleman A. (1987)

Needleman A. 1987, A continuum model for void nucleation by inclusion debonding, Journal of Applied Mechanics, 54, 525-531

Polynomial Potential

$$\Psi(\Delta_n, \Delta_t) = \frac{27}{4}\sigma_{\max}\delta_n \left\{ \frac{1}{2}\left(\frac{\Delta_n}{\delta_n}\right)^2 \left[1 - \frac{4}{3}\left(\frac{\Delta_n}{\delta_n}\right) + \frac{1}{2}\left(\frac{\Delta_n}{\delta_n}\right)^2\right] + \frac{1}{2}\alpha_s \left(\frac{\Delta_t}{\delta_n}\right)^2 \left[1 - 2\left(\frac{\Delta_n}{\delta_n}\right) + \left(\frac{\Delta_n}{\delta_n}\right)^2\right] \right\}$$

Cohesive Relationship

$\Delta_n \le \delta_n$:

$$T_n = \frac{\partial \Psi}{\partial \Delta_n} = \frac{27}{4}\sigma_{\max} \left\{ \left(\frac{\Delta_n}{\delta_n}\right) \left[1 - 2\left(\frac{\Delta_n}{\delta_n}\right) + \left(\frac{\Delta_n}{\delta_n}\right)^2\right] + \alpha_s \left(\frac{\Delta_t}{\delta_n}\right)^2 \left[\left(\frac{\Delta_n}{\delta_n}\right) - 1\right] \right\}$$

$$T_t = \frac{\partial \Psi}{\partial \Delta_t} = \frac{27}{4}\sigma_{\max} \left\{ \alpha_s \left(\frac{\Delta_t}{\delta_n}\right) \left[1 - 2\left(\frac{\Delta_n}{\delta_n}\right) + \left(\frac{\Delta_n}{\delta_n}\right)^2\right] \right\}$$

$\Delta_n > \delta_n$: $\quad T_n = T_t \equiv 0$

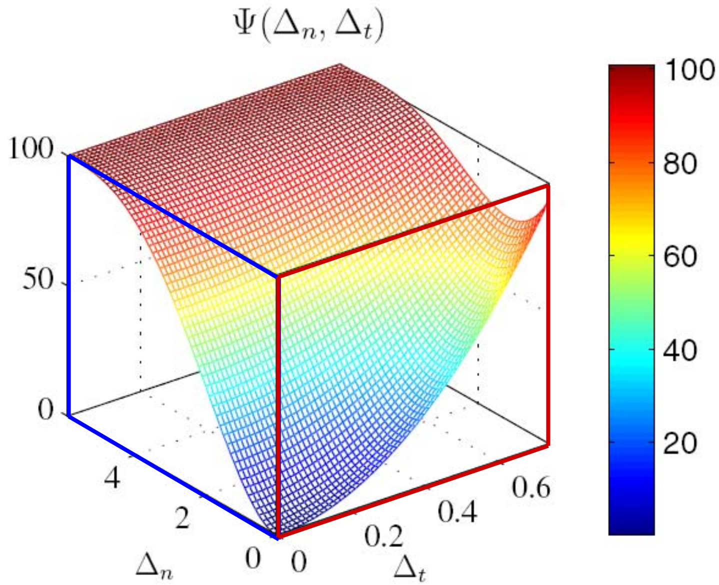
$$\phi_n = 9\sigma_{\max}\delta_n / 16$$

$$T_n(0, \delta_n/3) = \sigma_{\max}$$

α_s: shear stiffness parameter

Shear dependence → linear

Displacement jump → small



$$T_n = \frac{27}{4}\sigma_{\max}\left\{\left(\frac{\Delta_n}{\delta_n}\right)\left[1 - 2\left(\frac{\Delta_n}{\delta_n}\right) + \left(\frac{\Delta_n}{\delta_n}\right)^2\right] + \alpha_s\left(\frac{\Delta_t}{\delta_n}\right)^2\left[\left(\frac{\Delta_n}{\delta_n}\right) - 1\right]\right\}$$

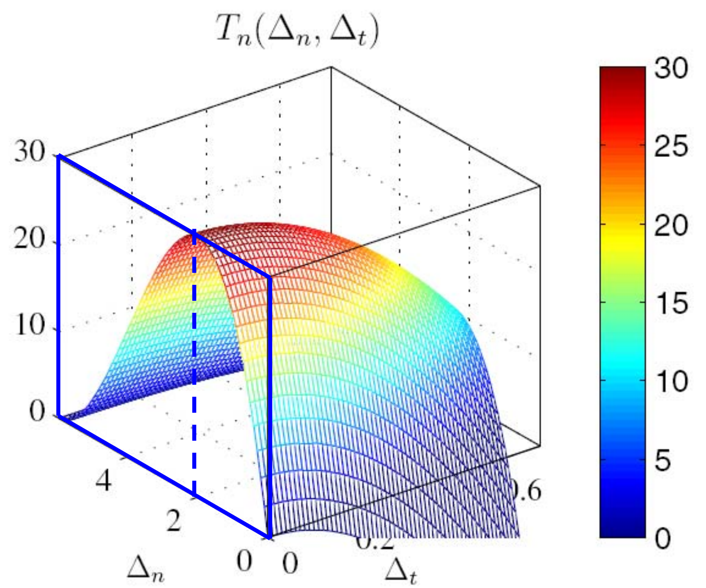
$$T_t = \frac{27}{4}\sigma_{\max}\left\{\alpha_s\left(\frac{\Delta_t}{\delta_n}\right)\left[1 - 2\left(\frac{\Delta_n}{\delta_n}\right) + \left(\frac{\Delta_n}{\delta_n}\right)^2\right]\right\}$$

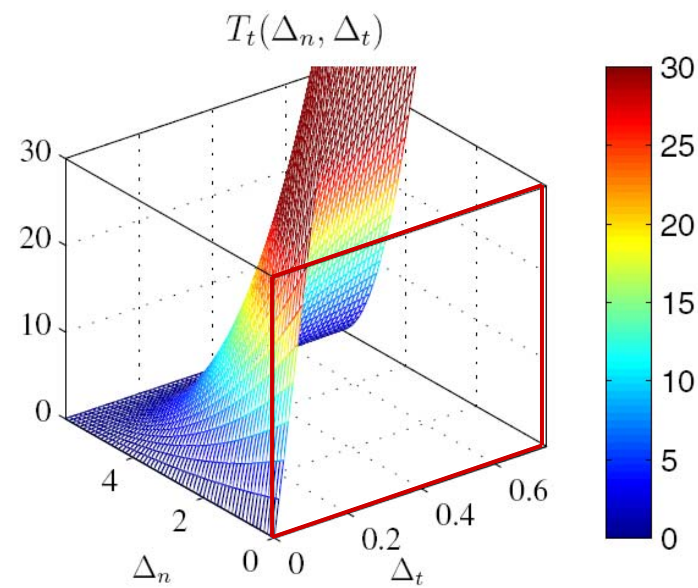
$\phi_n = 100\,N/m$

$\sigma_{\max} = 30\,MPa$

$\alpha_s = 10$

Axis	Units
Ψ	N/m
T_n, T_t	MPa
Δ_n, Δ_t	μm





Mode I

Mode II

2. Needleman A. (1990)

Needleman A. 1990, An analysis of tensile decohesion along an interface, Journal of the Mechanics and Physics of Solid, 3, 289-324

Exponential Potential with Periodic Dependence

$$\Psi(\Delta_n, \Delta_t) = \frac{\sigma_{\max} e \delta_n}{z} \left\{ 1 - \left[1 + \frac{z\Delta_n}{\delta_n} - \beta_s z^2 \left[1 - \cos\left(\frac{2\pi\Delta_t}{\delta_t} \right) \right] \right] \exp\left(-\frac{z\Delta_n}{\delta_n} \right) \right\}$$

Motivation

- Universal binding energy (Normal direction)

$$\boxed{E(a) = -(1+\beta a)\exp(-\beta a)}$$

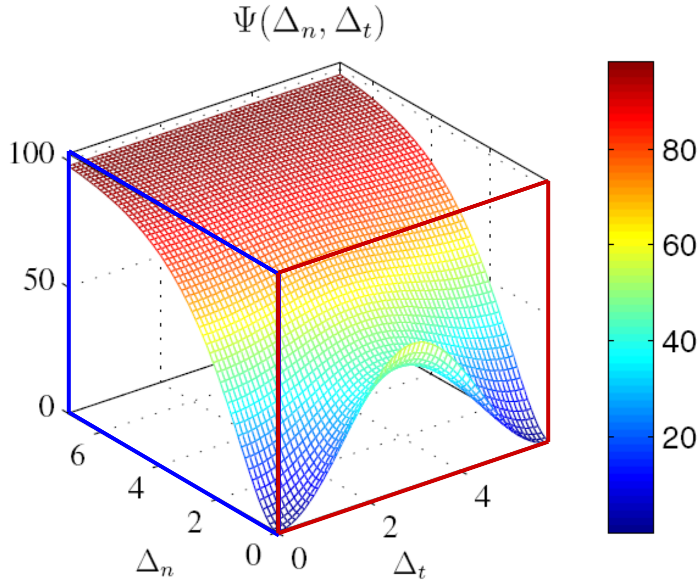
Rose JH, Rerrante J and Smith JR, 1981, Universal binding energy curves for metals and bimetallic interfaces, Physical Review Letters, 47, 675-678

- Periodicity of the underlying lattice (Tangential direction)

Cohesive Relationship

$$T_n = \sigma_{\max} e \left\{ \frac{z\Delta_n}{\delta_n} - \beta_s z^2 \left[1 - \cos\left(\frac{2\pi\Delta_t}{\delta_t} \right) \right] \right\} \exp\left(-\frac{z\Delta_n}{\delta_n} \right)$$

$$T_t = \sigma_{\max} e \left\{ 2\pi\beta_s z \left(\frac{\delta_n}{\delta_t} \right) \sin\left(\frac{2\pi\Delta_t}{\delta_t} \right) \right\} \exp\left(-\frac{z\Delta_n}{\delta_n} \right)$$

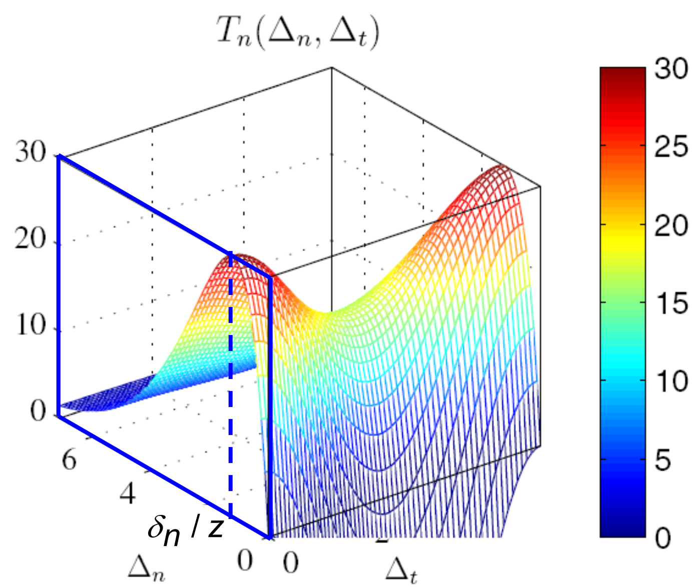


$$T_n = \sigma_{\max} \mathrm{e} \left\{ \frac{z\Delta_n}{\delta_n} - \beta_s z^2 \left[1 - \cos\left(\frac{2\pi\Delta_t}{\delta_t} \right) \right] \right\} \exp\left(-\frac{z\Delta_n}{\delta_n} \right)$$

$$T_t = \sigma_{\max} \mathrm{e} \left\{ 2\pi\beta_s z \left(\frac{\delta_n}{\delta_t} \right) \sin\left(\frac{2\pi\Delta_t}{\delta_t} \right) \right\} \exp\left(-\frac{z\Delta_n}{\delta_n} \right)$$

$$\phi_n = 100\, N/m$$
$$\sigma_{\max} = 30\, MPa$$
$$z = 16e/9$$

$$\delta_n = \frac{z\phi_n}{e\sigma_{\max}}$$
$$\delta_n = \delta_t$$
$$\beta_s = 1/2\pi e z$$



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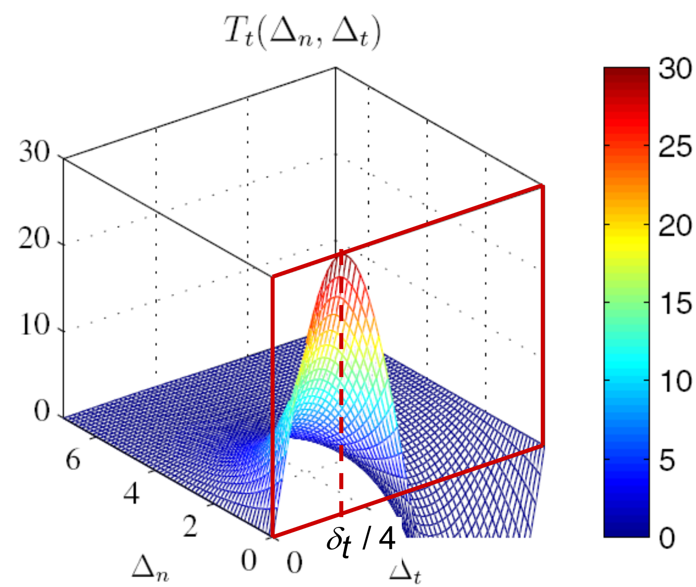

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3. Beltz G.E. and Rice J.R. (1991)

Beltz GE and Rice JR, 1991, Dislocation nucleation versus cleavage decohesion at crack tip, Modeling the Deformation of Crystalline Solids, 457-480.

▫ Cohesive Relationship

- Normal direction: Exponential → $T_n = [B(\Delta_t)\Delta_n - C(\Delta_t)]\exp(-\Delta_n/\delta_n)$
- Tangential direction: Sinusoid → $T_t = A(\Delta_n)\sin\left(\frac{2\pi\Delta_t}{\delta_t}\right)$
- Boundary Condition + Exact differential (Potential) $\frac{\partial T_n}{\partial \Delta_t} = \frac{\partial T_t}{\partial \Delta_n}$

$$\int_0^\infty T_n(\Delta_n, 0)\, d\Delta_n = 2\gamma_s = \phi_n$$

$$\int_0^{\delta_t/2} T_t(0, \Delta_t) d\Delta_t = \gamma_{us} = \phi_t$$

$$C(0) = 0 \longleftarrow \boxed{T_n = \frac{\phi_n}{\delta_n^2}\Delta_n \exp\left(-\frac{\Delta_n}{\delta_n}\right)}$$

Universal bonding correlation.

Rose JH, Rerrante J and Smith JR, 1981, Universal binding energy curves for metals and bimetallic interfaces, *Physical Review Letters*, 47, 675-678

$$\lim_{\Delta n \to \infty} T_n(\Delta_n, \Delta_t) = 0$$

$$\lim_{\Delta n \to \infty} T_t(\Delta_n, \Delta_t) = 0$$

$$\lim_{\Delta t \to b/2} T_t(\Delta_n, \Delta_t) = 0$$

$$\boxed{\lim_{\Delta t \to b/2} T_n(\Delta_n, \Delta_t) \neq 0}$$

Introduce additional condition Δ_n^* instead of $\lim_{\Delta t \to b/2} T_n(\Delta_n, \Delta_t) = 0$

$$T_n(\Delta_n^*, b/2) = \left[B(b/2)\Delta_n^* - C(b/2)\right]e^{-\Delta_n^*/\delta_n} = 0$$

3. Beltz G.E. and Rice J.R. (1991)

Solve PDE with BCs

$$\frac{\partial T_n}{\partial \Delta_t} = \frac{\partial T_t}{\partial \Delta_n}$$

$$T_n = [B(\Delta_t)\Delta_n - C(\Delta_t)]\exp(-\Delta_n/\delta_n)$$
$$T_t = A(\Delta_n)\sin\left(\frac{2\pi\Delta_t}{\delta_t}\right)$$

$$\int_0^\infty T_n(\Delta_n, 0)\,d\Delta_n = 2\gamma_s = \phi_n$$
$$\int_0^{\delta_t/2} T_t(0, \Delta_t)d\Delta_t = \gamma_{us} = \phi_t$$
$$\lim_{\Delta n\to\infty} T_n(\Delta_n, \Delta_t) = 0 \quad \lim_{\Delta n\to\infty} T_t(\Delta_n, \Delta_t) = 0$$
$$C(0) = 0$$

$$A(\Delta_n) = \frac{\pi\gamma_{us}}{\delta_t} - \frac{2\pi\gamma_s}{\delta_t}\left\{q\left[1-\exp\left(-\frac{\Delta_n}{\delta_n}\right)\right] - \left(\frac{q-r}{1-r}\right)\frac{\Delta_n}{\delta_n}\exp\left(-\frac{\Delta_n}{\delta_n}\right)\right\}$$
$$B(\Delta_t) = \frac{2\gamma_s}{\delta_n^2}\left\{1-\left(\frac{q-r}{1-r}\right)\sin^2\left(\frac{2\pi\Delta_t}{\delta_t}\right)\right\}$$
$$C(\Delta_t) = \frac{2\gamma_s}{\delta_n}\frac{r(1-q)}{1-r}\sin^2\left(\frac{2\pi\Delta_t}{\delta_t}\right)$$

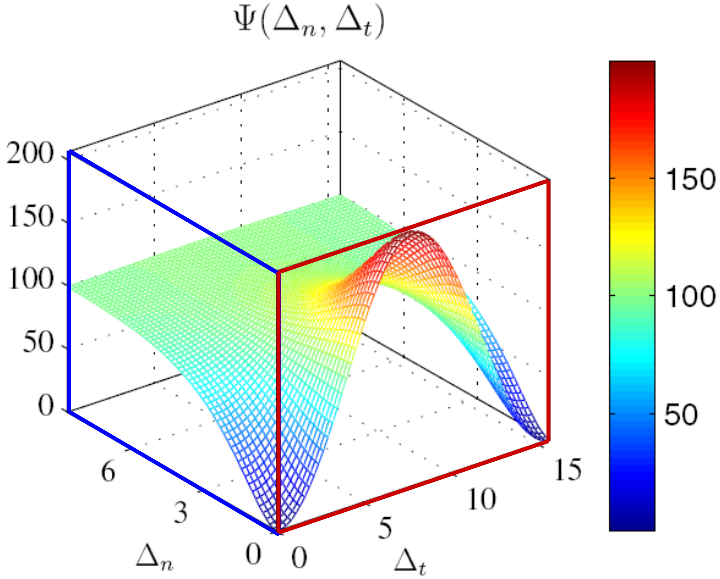
$$q = \frac{\gamma_{us}}{2\gamma_s}$$
$$r = \frac{\Delta_n^*}{\delta_n}$$

$$\Psi = 2\gamma_s + 2\gamma_s\exp\left(-\frac{\Delta_n}{\delta_n}\right)\left\{\left[q+\left(\frac{q-r}{1-r}\right)\frac{\Delta_n}{\delta_n}\right]\sin^2\left(\frac{2\pi\Delta_t}{\delta_t}\right) - \left[1+\frac{\Delta_n}{\delta_n}\right]\right\}$$

$$E(a) = -(1+\beta a)\exp(-\beta a)$$

Rose JH, Rerrante J and Smith JR, 1981, Universal binding energy curves for metals and bimetallic interfaces, *Physical Review Letters*, 47, 675-678

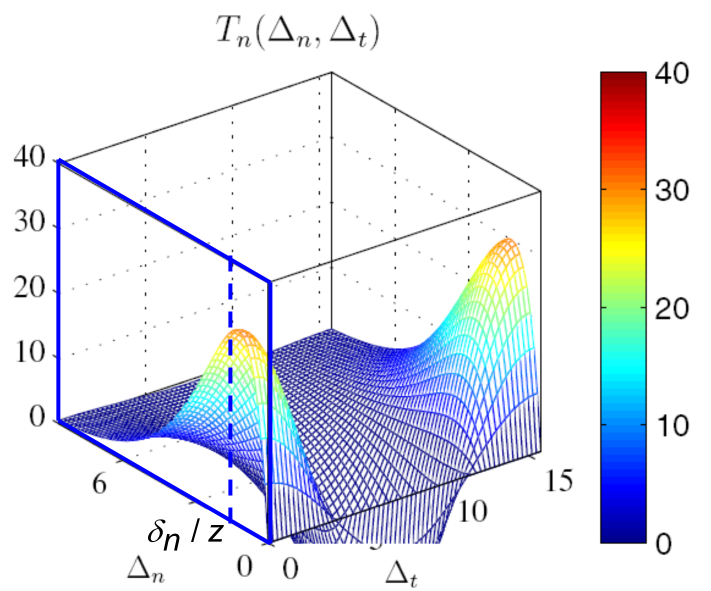
$$\phi_n = 100\,N/m \qquad \sigma_{\max} = 30\,MPa$$
$$\phi_t = 200\,N/m \qquad \tau_{\max} = 40\,MPa$$



$$\boxed{r = \frac{\Delta_n{}^*}{\delta_n} = 0}$$

"Δ_n is the value of Δ_n after shearing to the state Δ_t=b/2 under conditions of zero tension, T_n=0 (i.e. relaxed shearing)"*

$$\boxed{T_n\left(\Delta_n{}^*, b/2\right) = \left[B\left(b/2\right)\Delta_n{}^* - C\left(b/2\right)\right]e^{-\Delta_n{}^*/\delta_n} = 0}$$



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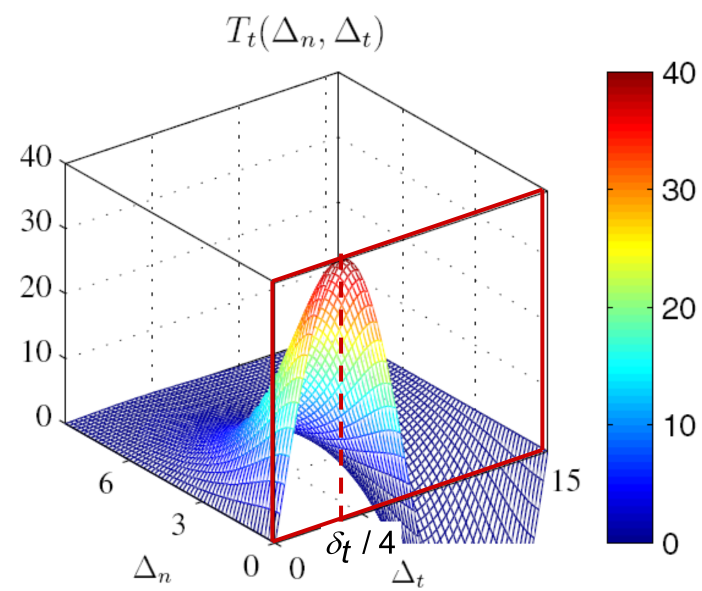

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4. Xu X.P. and Needleman A. (1993)

Xu XP and Needleman, 1993, Void nucleation by inclusion debonding in a crystal matrix, Modeling Simulation Material Science Engineering, 1, 111-132.

□ Cohesive Relationship

- Normal direction: Exponential → $T_n = \left[B(\Delta_t)\Delta_n - C(\Delta_t)\right]\exp(-\Delta_n/\delta_n)$
- Tangential direction: Exponential → $T_t = A(\Delta_n)\,\dfrac{\Delta_t}{\delta_t}\exp(-\Delta_t^2/\delta_t^2)$
- Boundary Condition

$$\phi_n = \int_0^\infty T_n(\Delta_n,0)\,d\Delta_n \qquad \phi_t = \int_0^\infty T_t(0,\Delta_t)\,d\Delta_t \qquad C(0)=0$$

$$\lim_{\Delta n\to\infty} T_n(\Delta_n,\Delta_t)=0 \qquad \lim_{\Delta n\to\infty} T_t(\Delta_n,\Delta_t)=0 \qquad \lim_{\Delta t\to\infty} T_t(\Delta_n,\Delta_t)=0 \qquad \boxed{\lim_{\Delta t\to\infty} T_n(\Delta_n,\Delta_t)\neq 0}$$

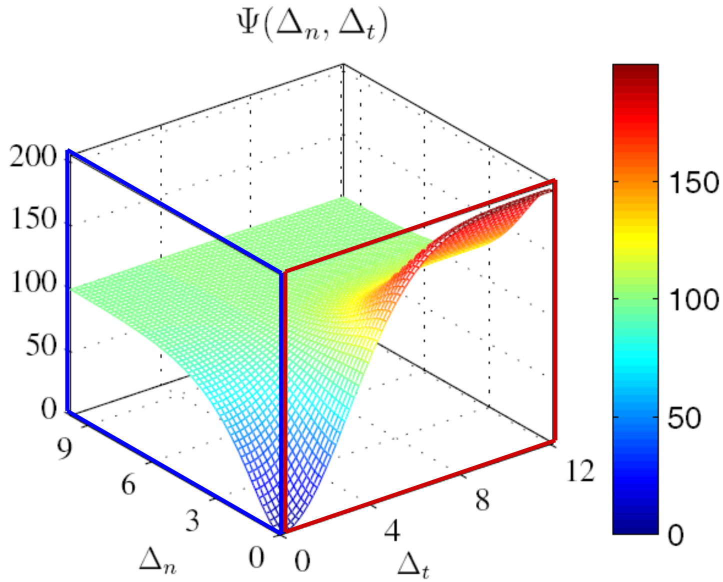
Introduce additional condition Δ_n^* instead of $\lim_{\Delta t\to\infty} T_n(\Delta_n,\Delta_t)=0$

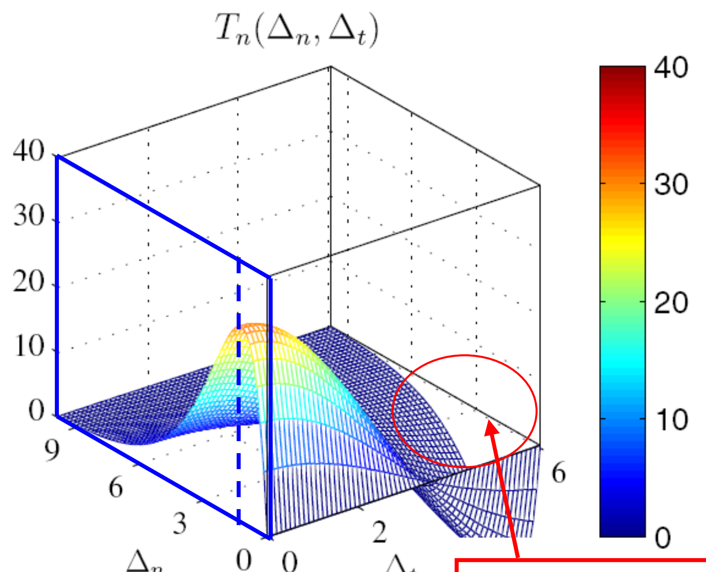
$$\lim_{\Delta_t\to\infty} T_n(\Delta_n^*,\Delta_t)=0$$

$$\Psi(\Delta_n,\Delta_t) = \phi_n + \phi_n \exp\left(\frac{-\Delta_n}{\delta_n}\right)\left\{\left[1-r+\frac{\Delta_n}{\delta_n}\right]\frac{(1-q)}{(r-1)} - \left[q+\frac{(r-q)}{(r-1)}\frac{\Delta_n}{\delta_n}\right]\exp\left(-\frac{\Delta_t^2}{\delta_t^2}\right)\right\}$$

$$\phi_n = 100\,N/m$$
$$\phi_t = 200\,N/m$$
$$\sigma_{\max} = 30\,MPa$$
$$\tau_{\max} = 40\,MPa$$

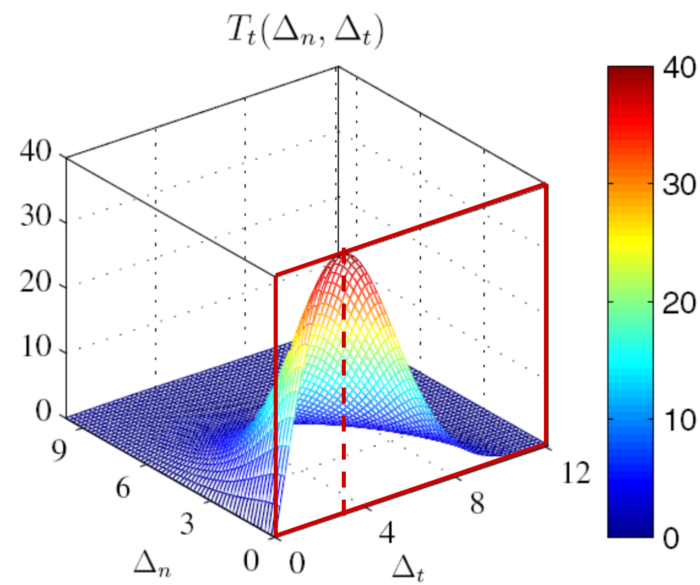
$$r = \frac{\Delta_n^{\,*}}{\delta_n} = 0$$





$$\lim_{\Delta t \to \infty} T_n(\Delta_n, \Delta_t) < 0$$

Mode I



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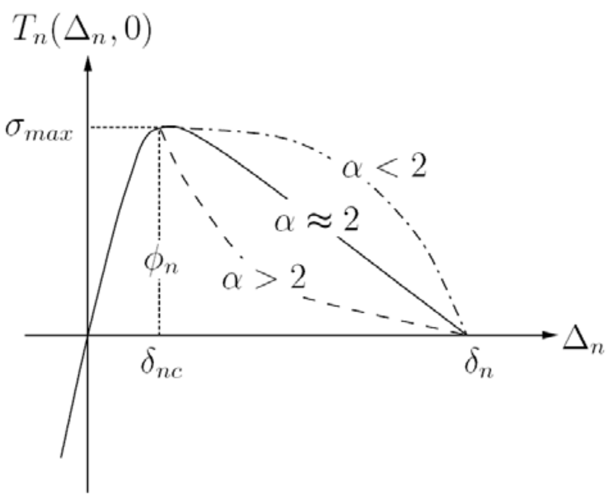
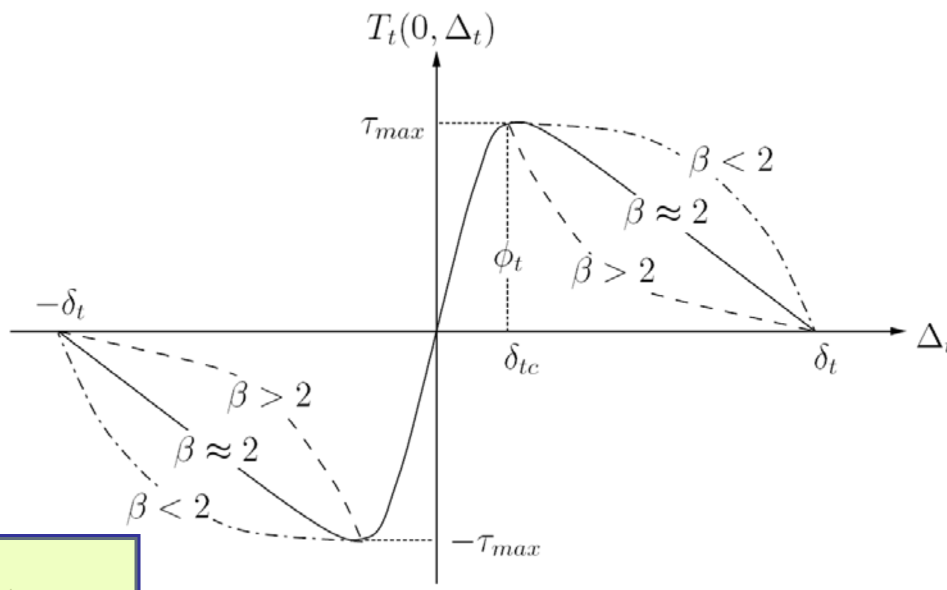
Remarks

- **Previous Potential**
  - Boundary condition is not symmetric → $\lim_{\Delta t \to \infty} T_n(\Delta_n, \Delta_t) \neq 0$
  - Vague fracture parameter, r (or Δ_n*) → Okay if fracture energies are the same
  - Complete separation at infinity
  - Could not control initial slope → Large compliance
- **Proposed Potential**
  - Expressed by a single function
  - Different fracture energy : ϕ_n, ϕ_t
  - Different cohesive strength : $\sigma_{\max}$, $\tau_{\max}$
  - Different cohesive law: α, β
  - Different initial slope : λ_n, λ_t

Boundary Conditions





$$\phi_n = \int_0^{\delta_n} T_n(\Delta_n, 0) \mathrm{d}\Delta_n$$

$$\frac{\partial T_n}{\partial \Delta_n}\bigg|_{\Delta_n = \delta_{nc}} = 0$$

$$T_n(\delta_{nc}, 0) = \sigma_{\max}$$

$$T_n(\delta_n, \Delta_t) = 0$$

$$T_n(\Delta_n, \bar{\delta}_t) = 0$$

$$\phi_t = \int_0^{\delta_t} T_t(0, \Delta_t) \mathrm{d}\Delta_t$$

$$\frac{\partial T_t}{\partial \Delta_t}\bigg|_{\Delta_t = \delta_{tc}} = 0$$

$$T_t(0, \delta_{tc}) = \tau_{\max}$$

$$T_t(\Delta_n, \delta_t) = 0$$

$$T_t(\bar{\delta}_n, \Delta_t) = 0$$

Various material behavior, i.e.

- **Plateau-type** $1 < \alpha, \beta << 2$
- **Brittle material** $\alpha, \beta \approx 2$
- **Quasi-brittle material** $\alpha, \beta >> 2$

PPR: Unified Mixed Mode Potential

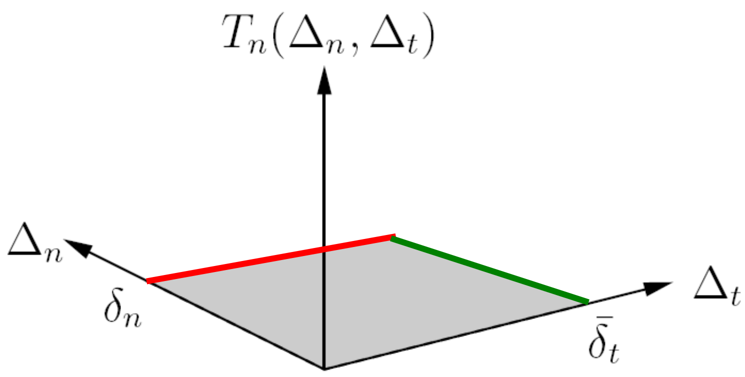
$$\Psi(\Delta_n, \Delta_t) = \min(\phi_n, \phi_t) + \left[\Gamma_n \left(1 - \frac{\Delta_n}{\delta_n}\right)^{\alpha} \left(\frac{m}{\alpha} + \frac{\Delta_n}{\delta_n}\right)^{m} + \langle \phi_n - \phi_t \rangle\right] \left[\Gamma_t \left(1 - \frac{|\Delta_t|}{\delta_t}\right)^{\beta} \left(\frac{n}{\beta} + \frac{|\Delta_t|}{\delta_t}\right)^{n} + \langle \phi_t - \phi_n \rangle\right]$$

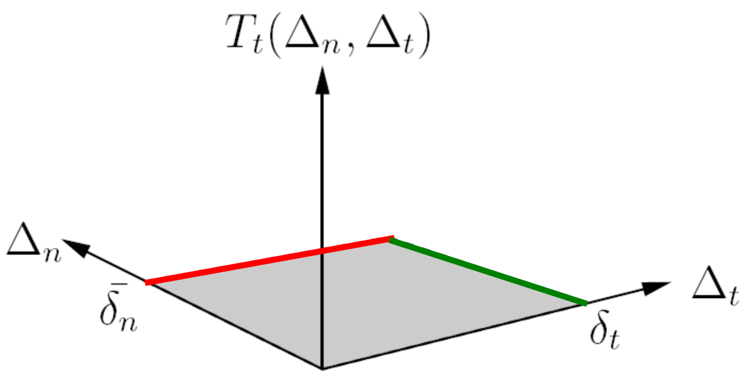
- **Energy Constants: Γ_n and Γ_t**
- **Exponents: *m* and *n***
- **Characteristic length scales: δ_n and δ_t**
- **Shape parameters : α and β**

- **Fracture energy**
- **Cohesive strength**
- **Cohesive interaction shape**
- **Initial slope**

K. Park, GH. Paulino, JR. Roesler, 2009, A unified potential-based cohesive model of mixed-mode fracture, *Journal of the Mechanics and Physics of Solids* 57, 891-908.

Softening Region





$$T_n(\delta_n, \Delta_t) = 0$$

$$\delta_n = \frac{\phi_n}{\sigma_{\max}} \alpha \lambda_n (1-\lambda_n)^{\alpha-1} \left(\frac{\alpha}{m}+1\right)\left(\frac{\alpha}{m}\lambda_n+1\right)^{m-1}$$

$$T_n(\Delta_n, \bar{\delta}_t) = 0$$

$$f_t(\Delta_t) = \Gamma_t \left(1-\frac{|\Delta_t|}{\delta_t}\right)^{\beta}\left(\frac{n}{\beta}+\frac{|\Delta_t|}{\delta_t}\right)^{n} + \langle \phi_t - \phi_n \rangle = 0$$

$$T_t(\bar{\delta}_n, \Delta_t) = 0$$

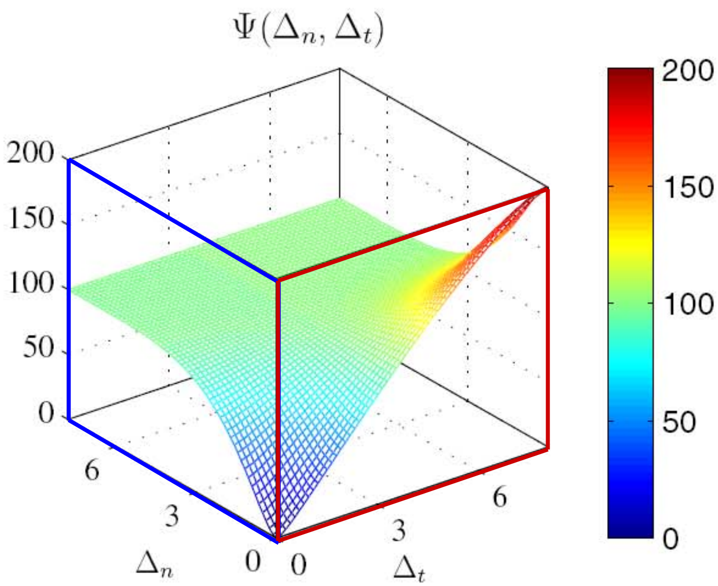
$$f_n(\Delta_n) = \Gamma_n \left(1-\frac{\Delta_n}{\delta_n}\right)^{\alpha}\left(\frac{m}{\alpha}+\frac{\Delta_n}{\delta_n}\right)^{m} + \langle \phi_n - \phi_t \rangle = 0$$

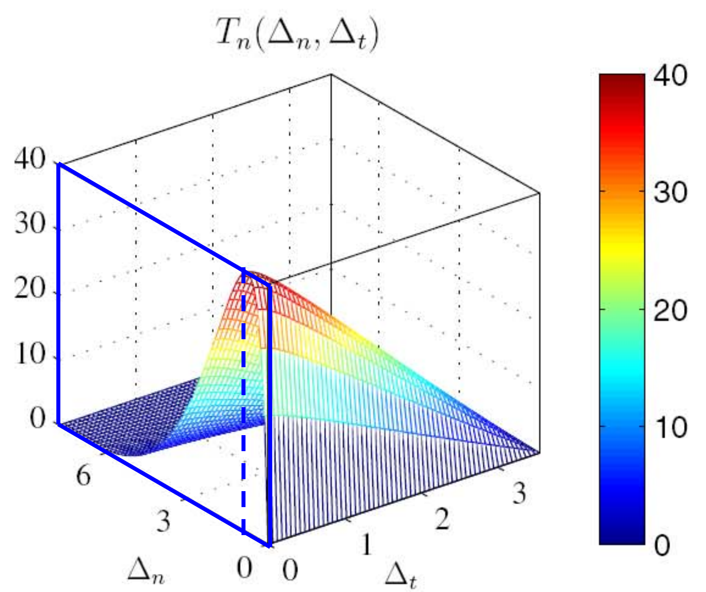
$$T_t(\Delta_n, \delta_t) = 0$$

$$\delta_t = \frac{\phi_t}{\tau_{\max}} \beta \lambda_t (1-\lambda_t)^{\beta-1} \left(\frac{\beta}{n}+1\right)\left(\frac{\beta}{n}\lambda_t+1\right)^{n-1}$$

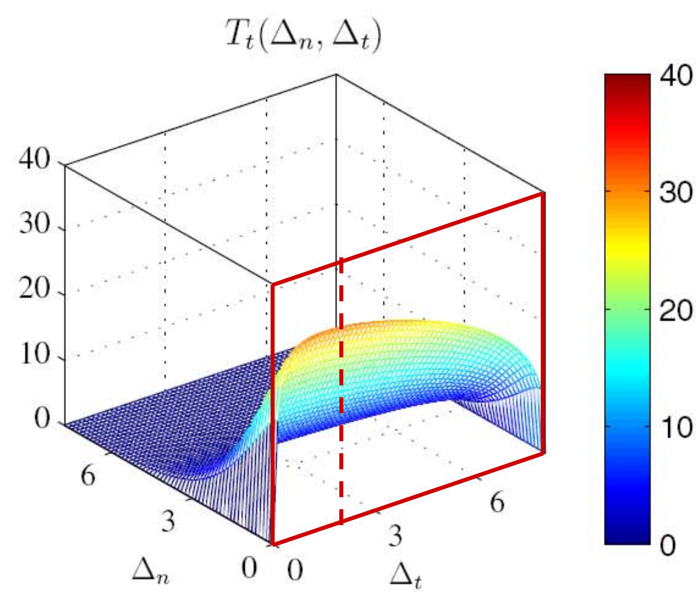
- **Fracture energy**
- **Cohesive strength**
- **Cohesive interaction shape**
- **Initial slope**

$\phi_n = 100\,N/m$	$\phi_t = 200\,N/m$
$\sigma_{\max} = 40\,MPa$	$\tau_{\max} = 30\,MPa$
$\alpha = 5$	$\beta = 1.3$
$\lambda_n = 0.1$	$\lambda_t = 0.2$





Mode I



Mode II

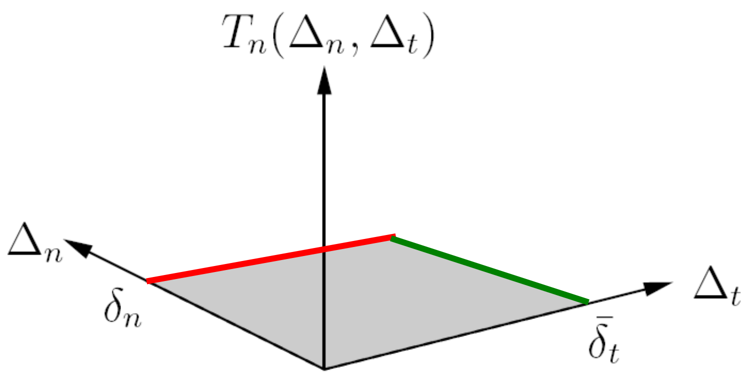
Extension for the *EXTRINSIC* Model

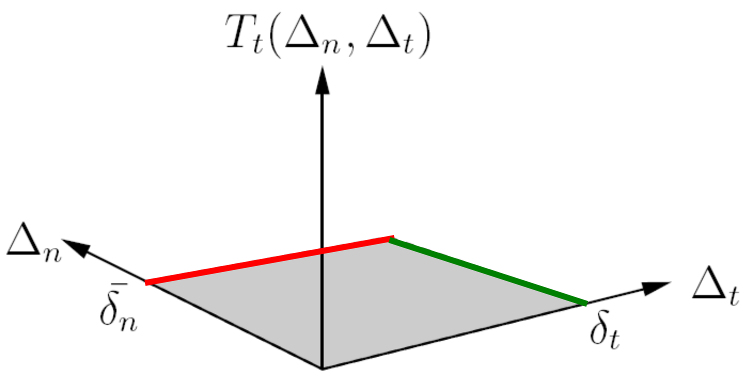
- **Correct Limit Procedure**
  - Limit of initial slope indicators in the potential

$$\Psi(\Delta_n, \Delta_t) = \min(\phi_n, \phi_t) + \left[\Gamma_n \left(1 - \frac{\Delta_n}{\delta_n}\right)^{\alpha} + \langle \phi_n - \phi_t \rangle\right] \left[\Gamma_t \left(1 - \frac{|\Delta_t|}{\delta_t}\right)^{\beta} + \langle \phi_t - \phi_n \rangle\right]$$

  - Energy constants: Γ_n and Γ_t
  - Characteristic length scales: δ_n and δ_t
  - Shape parameters: α and β
- **Exclude elastic behavior → Extrinsic model**
- **Consider different fracture energy:** ϕ_n, ϕ_t
- **Describe different cohesive strength:** σ_{max}, τ_{max}
- **Represent various cohesive shape:** α, β

Softening Region





$$T_n(\delta_n, \Delta_t) = 0$$

$$\delta_n = \alpha\phi_n / \sigma_{\max}$$

$$T_n(\Delta_n, \bar{\delta}_t) = 0$$

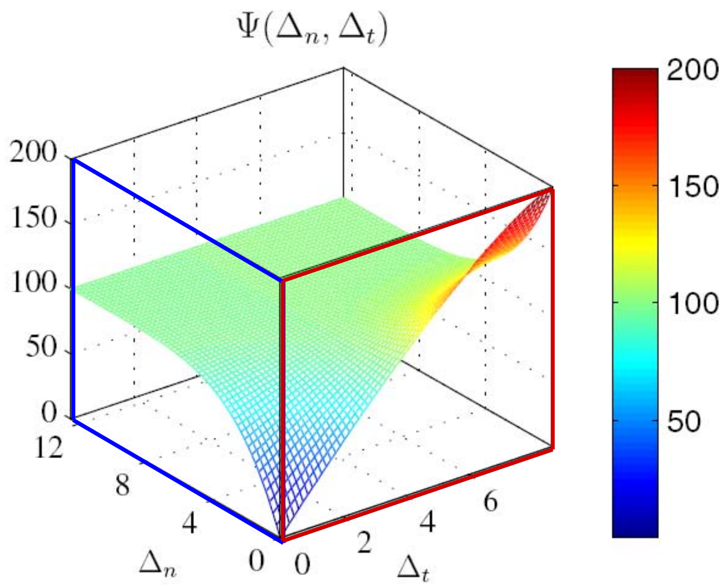
$$\bar{\delta}_t = \delta_t - \delta_t \left(\frac{\langle \phi_t - \phi_n \rangle}{\phi_t} \right)^{1/\beta}$$

$$T_t(\bar{\delta}_n, \Delta_t) = 0$$

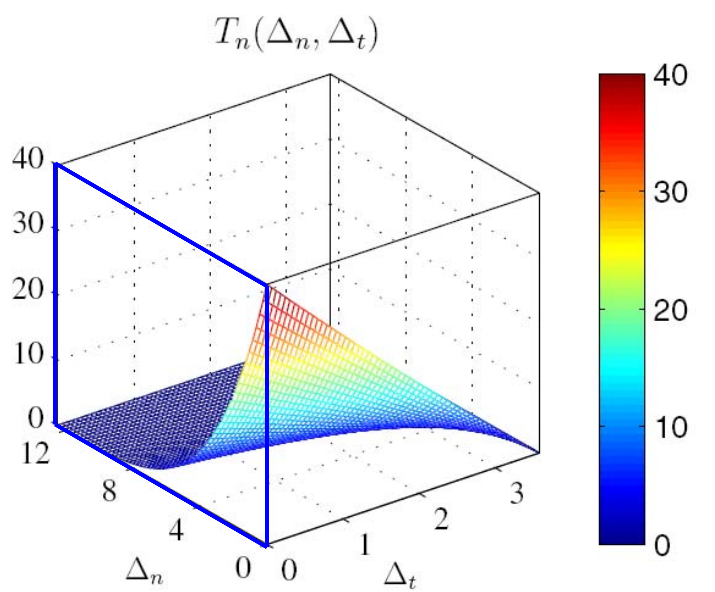
$$\bar{\delta}_n = \delta_n - \delta_n \left(\frac{\langle \phi_n - \phi_t \rangle}{\phi_n} \right)^{1/\alpha}$$

$$T_t(\Delta_n, \delta_t) = 0$$

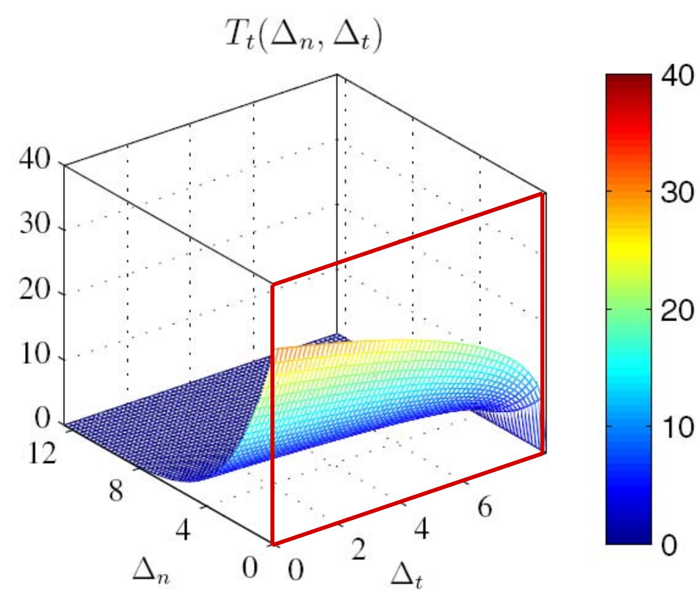
$$\delta_t = \beta\phi_t / \tau_{\max}$$



$\phi_n = 100\, N/m$	$\phi_t = 200\, N/m$
$\sigma_{\max} = 40\, MPa$	$\tau_{\max} = 30\, MPa$
$\alpha = 5$	$\beta = 1.3$



Mode I



Mode II

Remarks

- **Consistent boundary condition**
- **Fracture parameters**
  - Energy, strength, shape, slope
- **Intrinsic/Extrinsic cohesive zone modeling**
- **Path dependence of work-of-separation**
  - Proportional path
  - Non-proportional path
- **Unloading/reloading relationship is independent of the potential-based model**
  - Coupled unloading/reloading relationship
  - Uncoupled unloading/reloading relationship

Contents

- Introduction
- Potential-based Cohesive Model
- **Quasi-Static Fracture**
  - Particle/matrix debonding
- **Dynamic Fracture Problems**
  - Computational framework
  - Micro-branching and fragmentation
  - Mode I predefined crack, mixed-mode and branching
- **Virtual Internal Pair-Bond (VIPB) Model**
- **Summary**

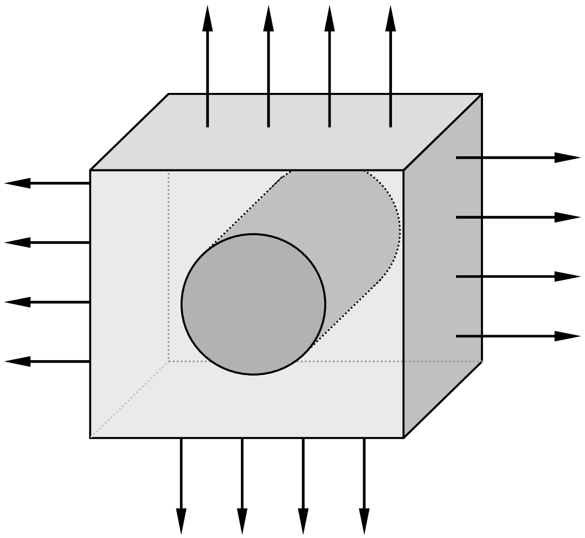
Particle/Matrix Debonding

- **Macroscopic Constitutive Relationship**
- **Micromechanics**
  - Extended Mori-Tanaka Method
    - Hydro-static state
    - Interface debonding of cohesive constitutive relationship

  Tan, H., Huang, Y., Liu, C., Ravichandran, G., Paulino, G. H., 2007. Constitutive behaviors of composites with interface debonding: The extended Mori-Tanaka method for uniaxial tension. *International Journal of Fracture* 146 (3), 139–148.
- **Computational Method**
  - Cohesive surface elements

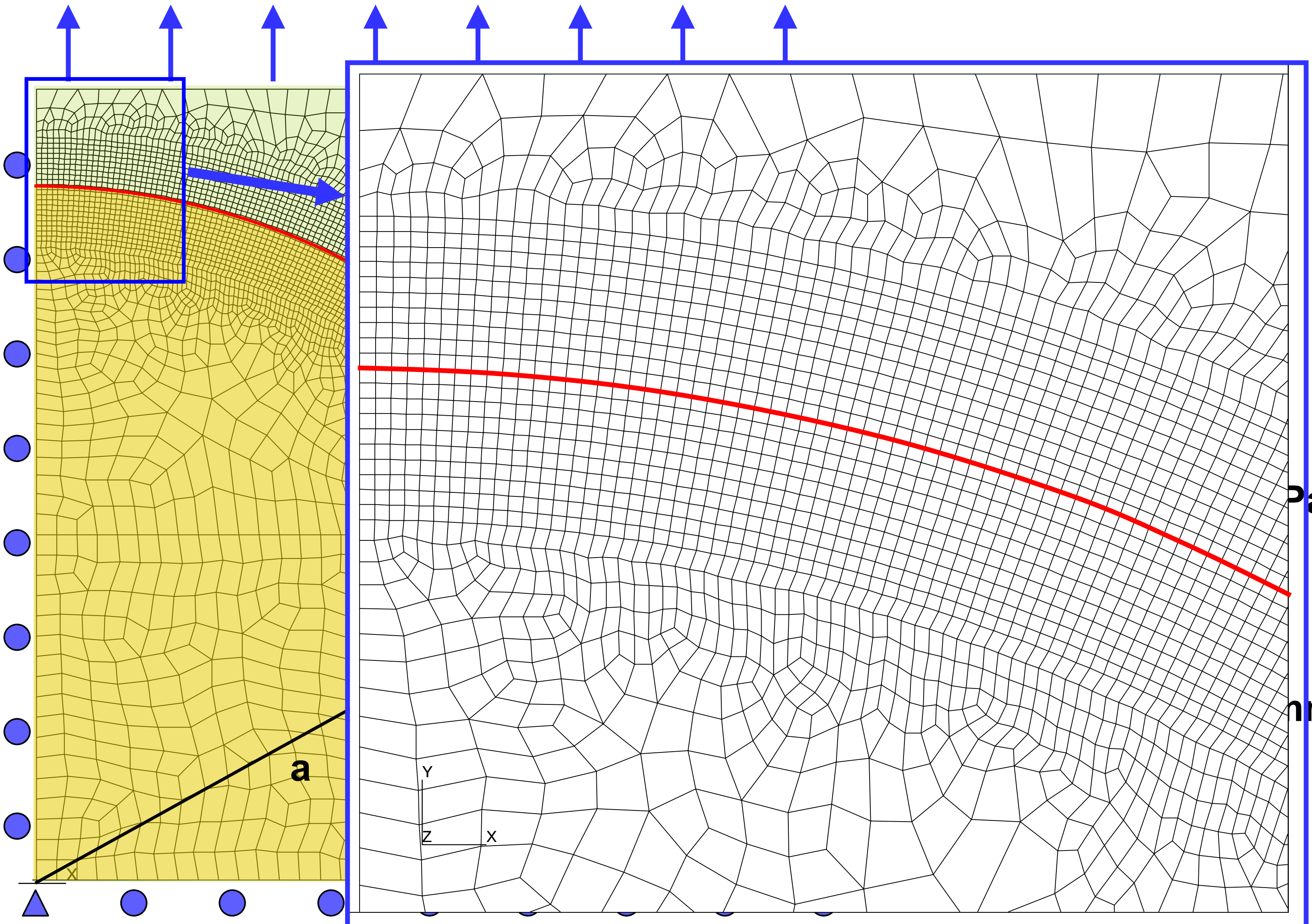
Finite Element Analysis

- **Equi-biaxial Tension**



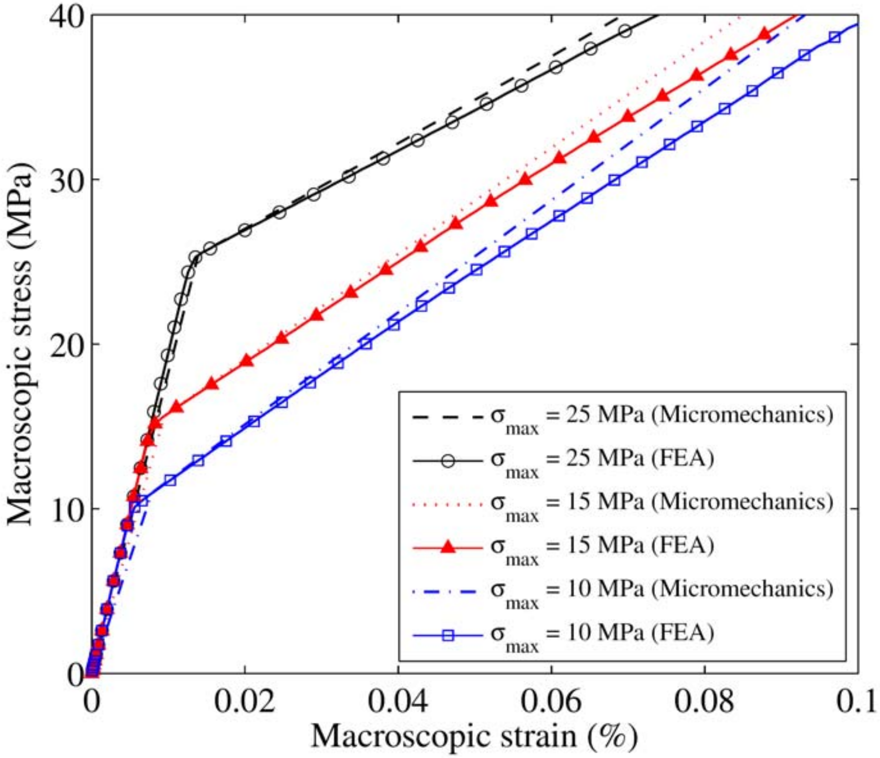
- **Assumptions**
  - Reduced to 2D Problem
    - Plane strain
  - Quarter domain
    - Symmetry boundary conditions of unit cell
  - Poisson's ratio = 0.25
  - Cohesive surface elements are inserted along the interface

Finite Element Mesh

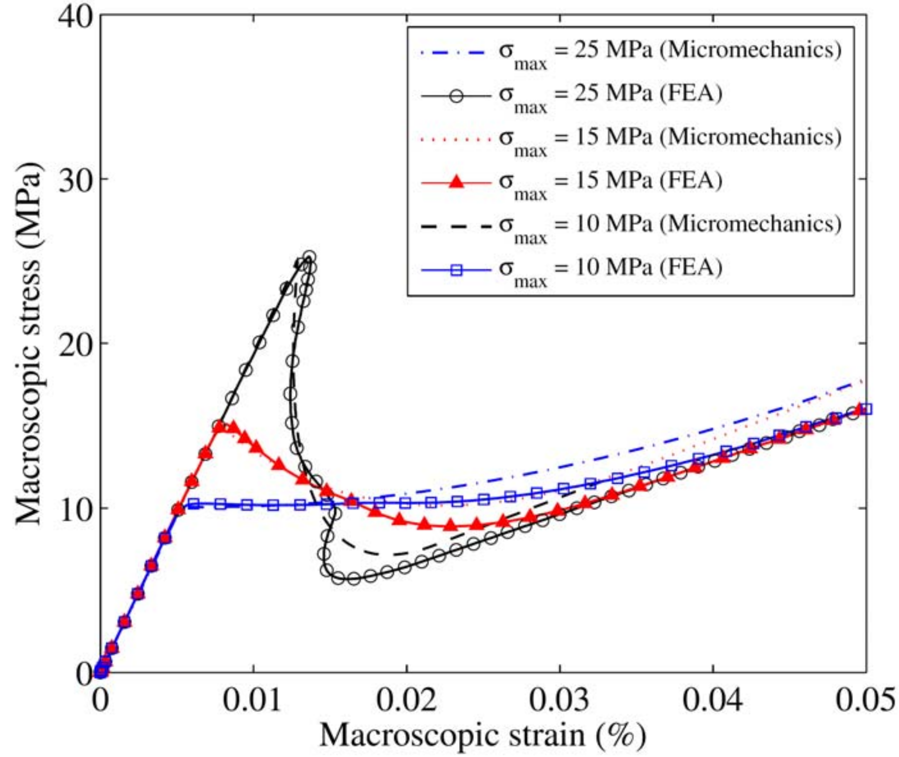


Effect of Cohesive Strength

- Particle size: 100 μm



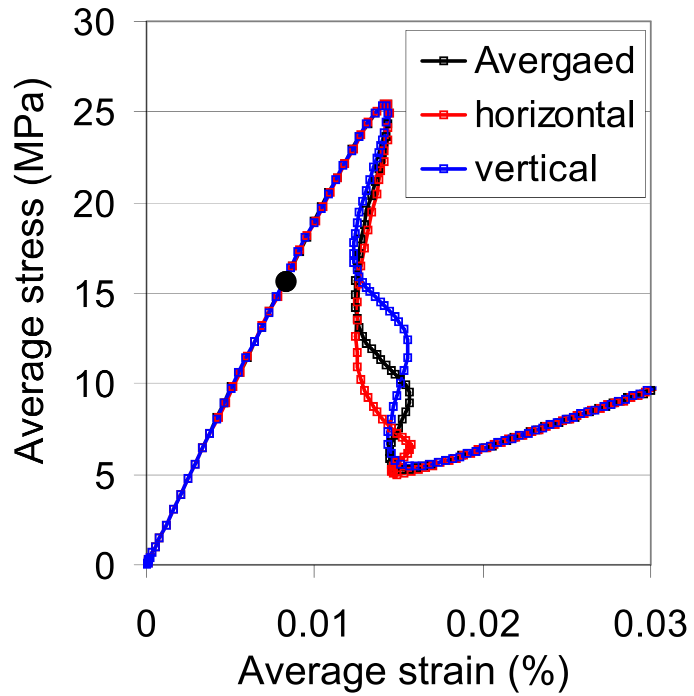
- Particle size: 2 mm

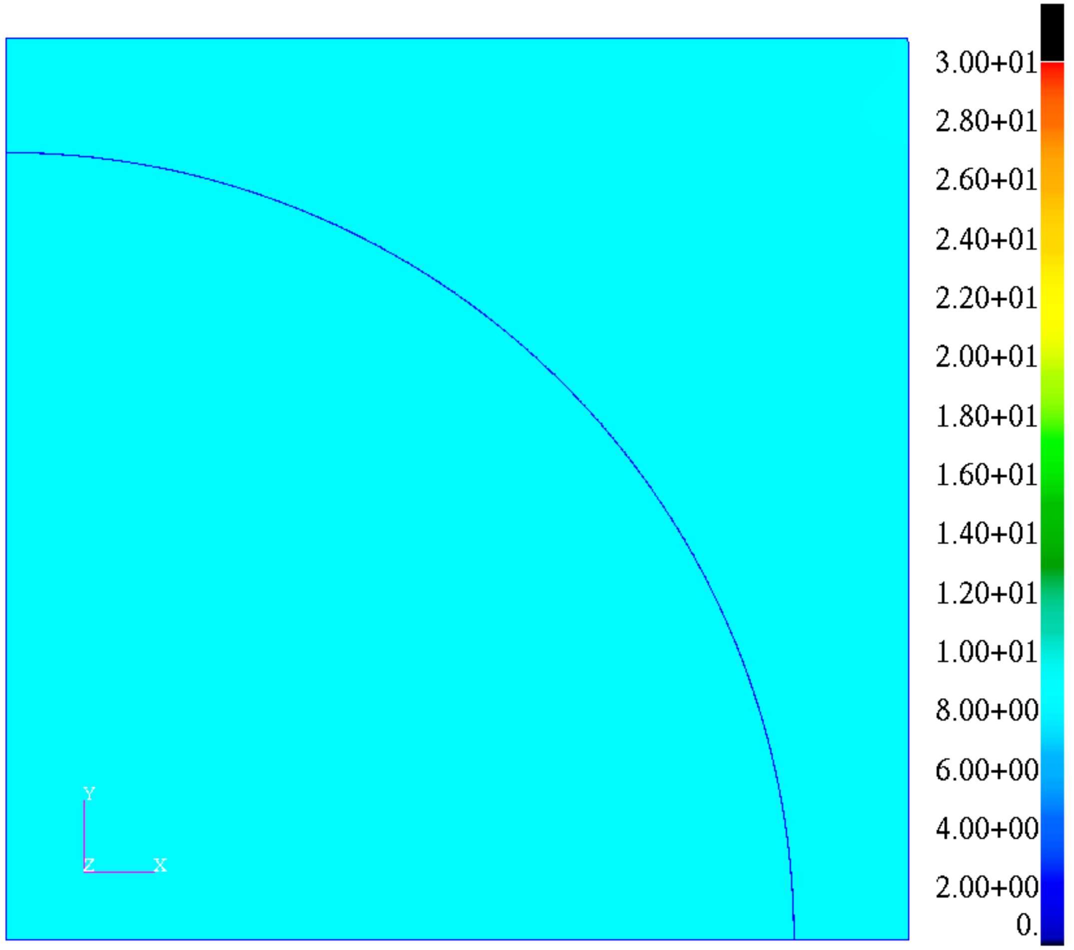


Micromechanics (Analytical)

D. Ngo, K. Park, G.H. Paulino, and Y. Huang, 2009, On the constitutive relation of materials with microstructure using the PPR potential-based cohesive model for interface interaction, Engineering Fracture Mechanics (submitted).

Debonding Process 1

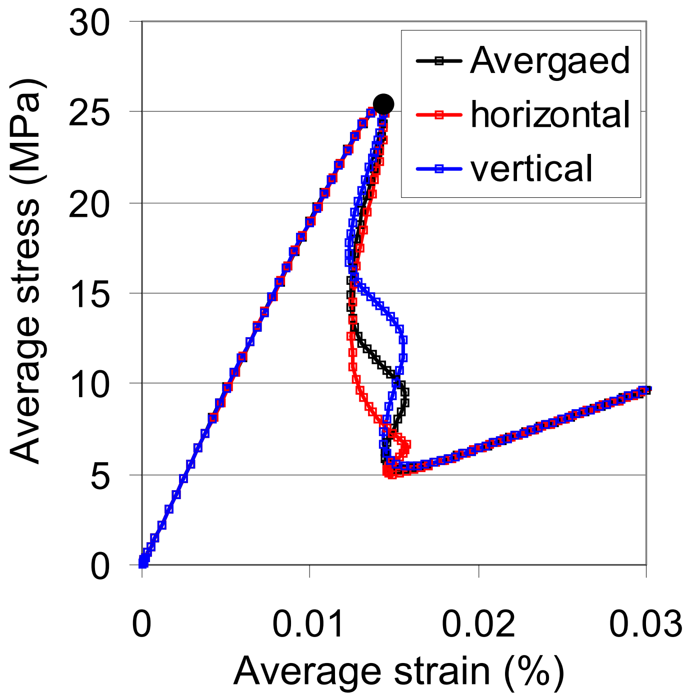


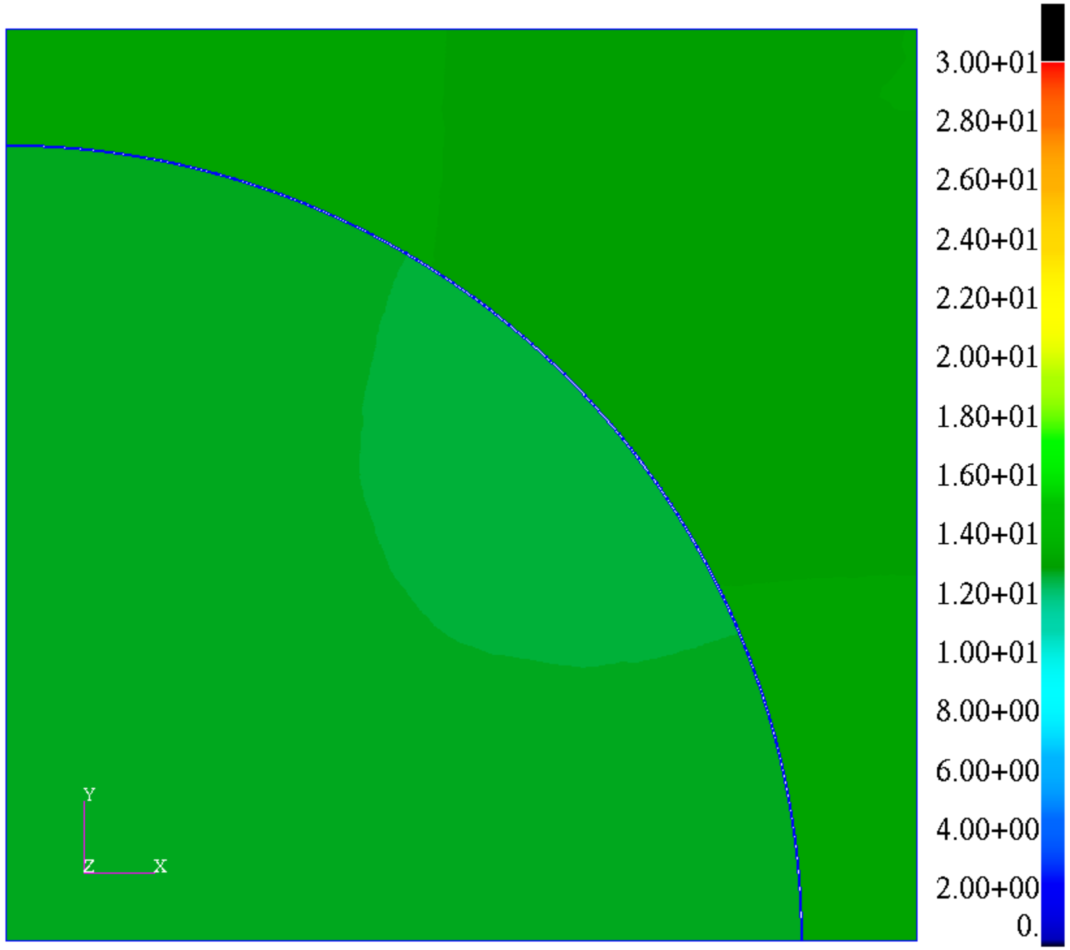


- **Fracture Parameters**

G (N/m)	T_{max} (MPa)	Shape	Slope
5	25	3	0.04

Debonding Process 2

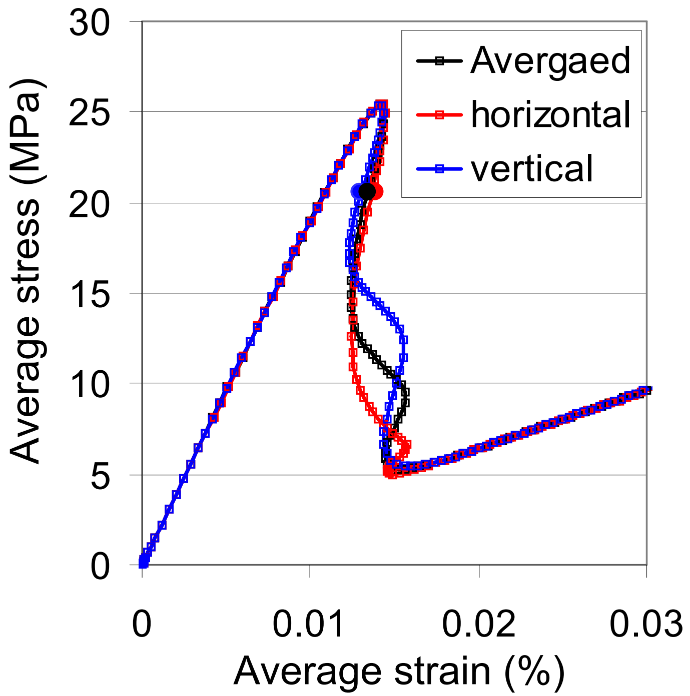


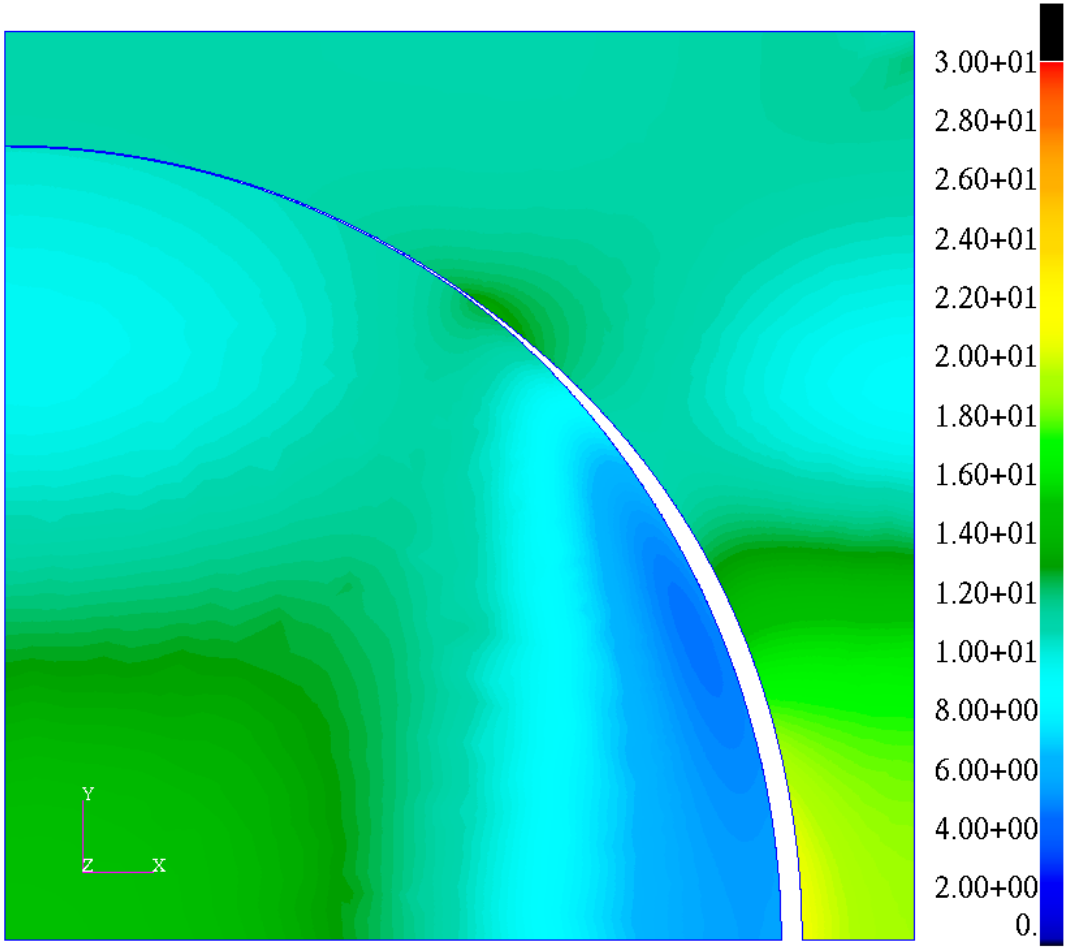


- **Fracture Parameters**

G (N/m)	**T_{max} (MPa)**	**Shape**	**Slope**
5	25	3	0.04

Debonding Process 3

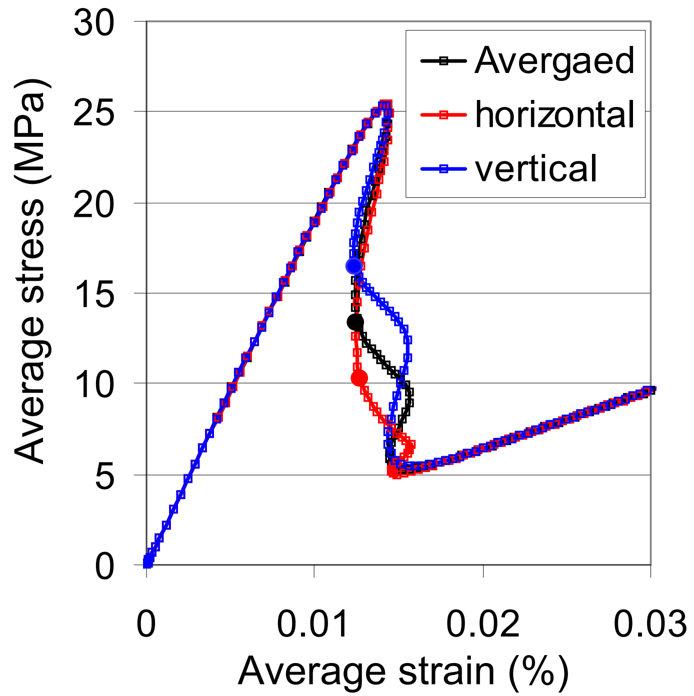


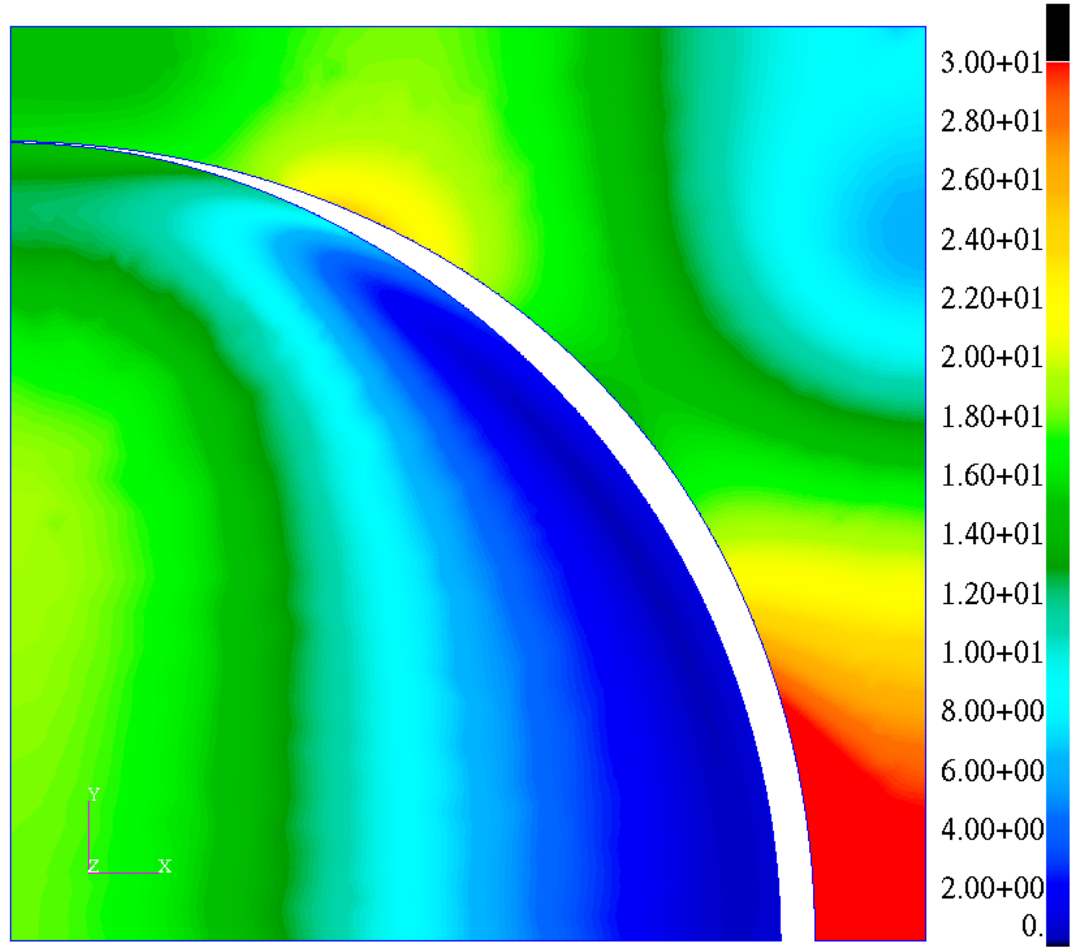


- **Fracture Parameters**

G (N/m)	T_{max} (MPa)	Shape	Slope
5	25	3	0.04

Debonding Process 4

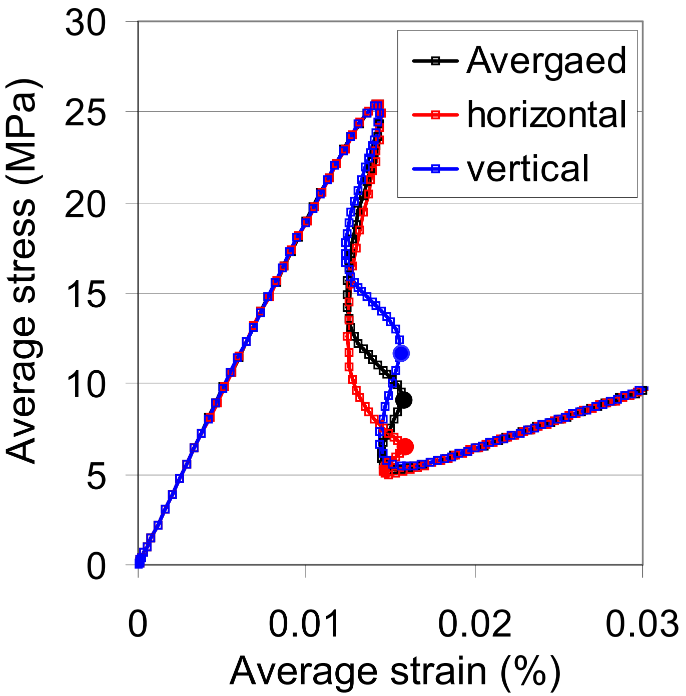


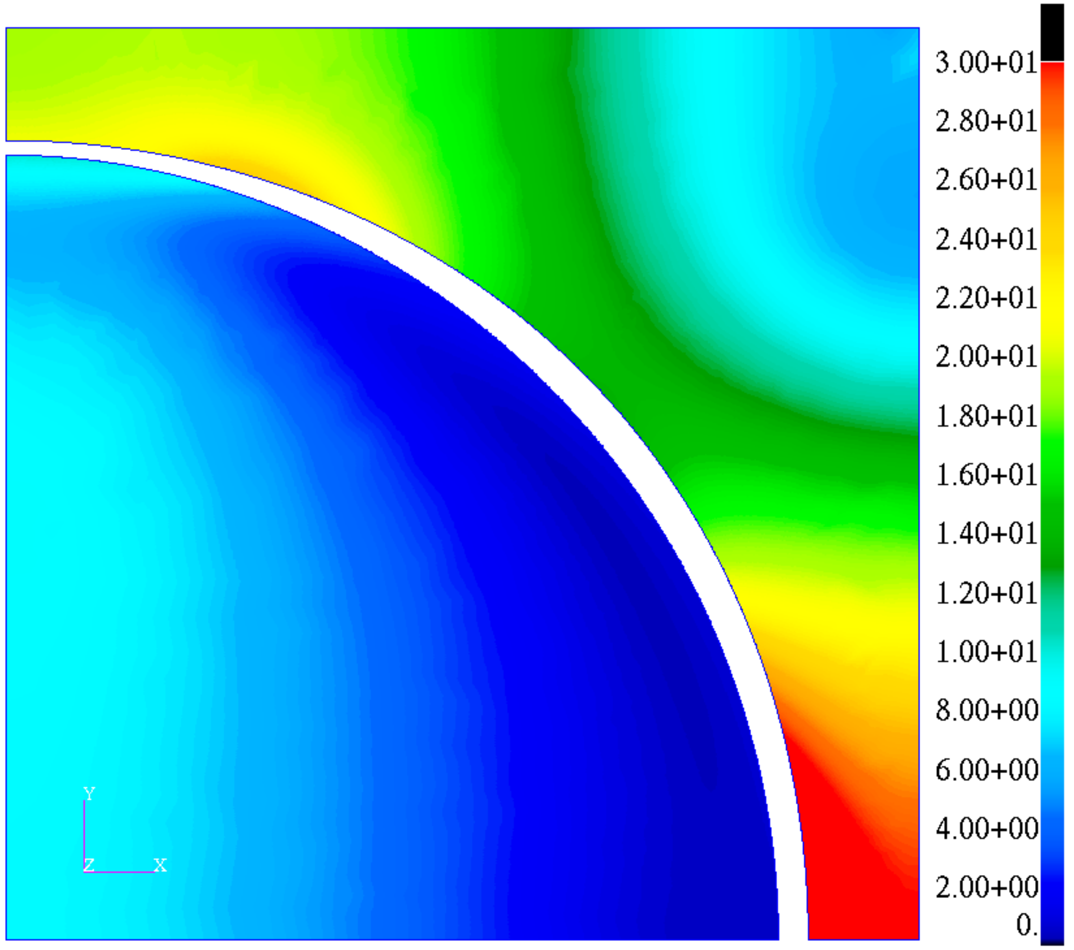


- **Fracture Parameters**

G (N/m)	**T_{max} (MPa)**	**Shape**	**Slope**
5	25	3	0.04

Debonding Process 5

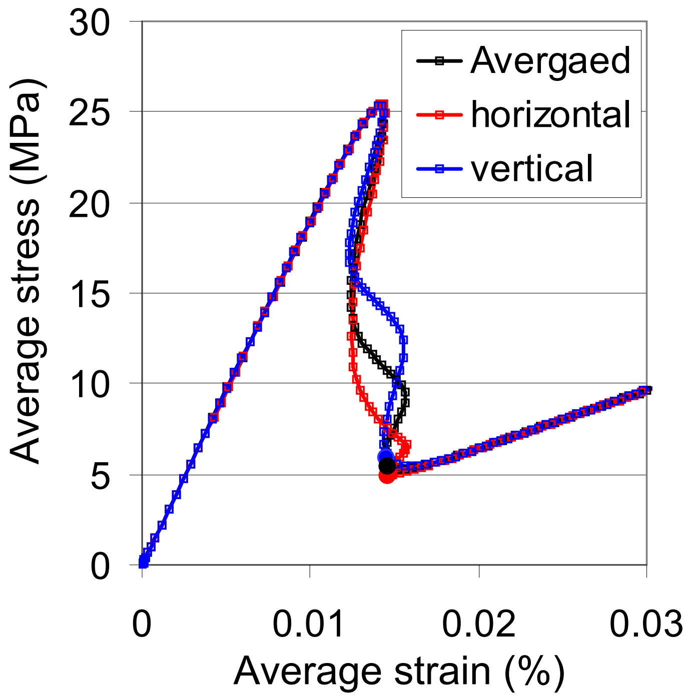


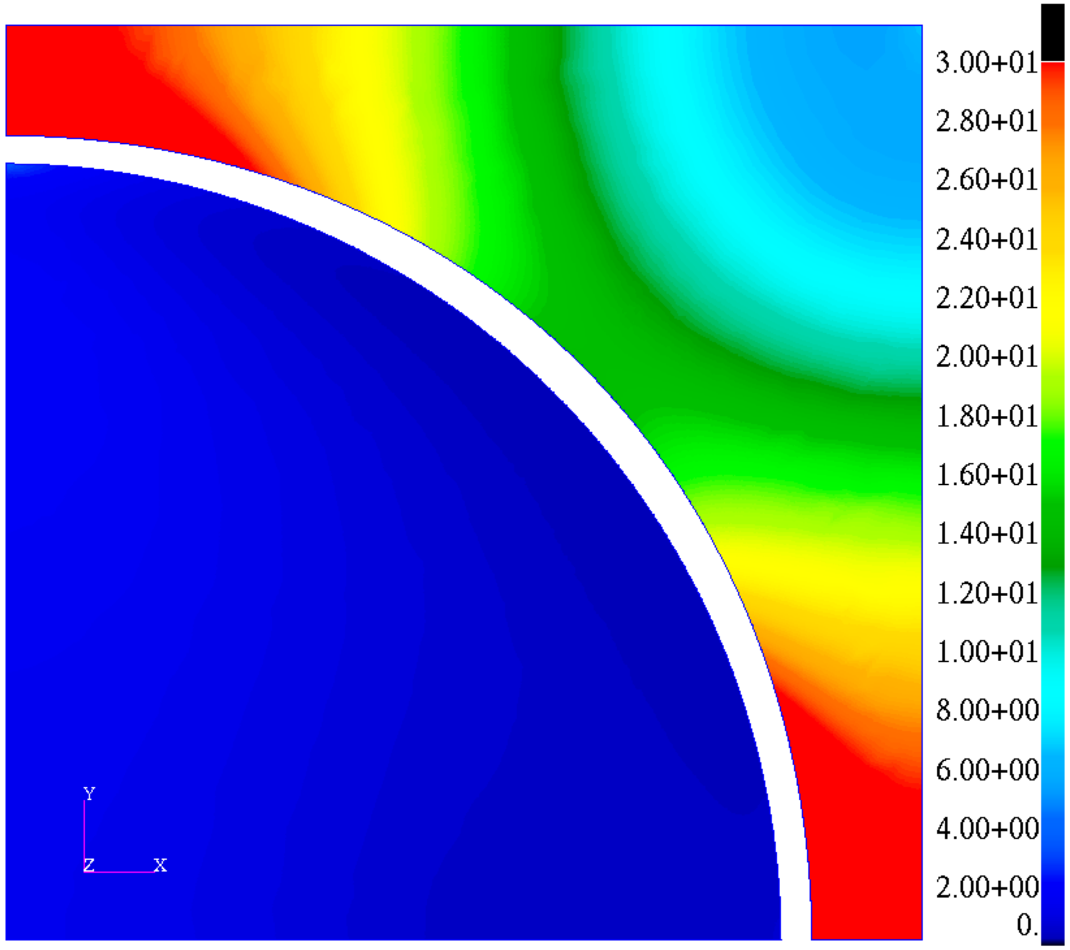


- **Fracture Parameters**

G (N/m)	T_{max} (MPa)	Shape	Slope
5	25	3	0.04

Debonding Process 6

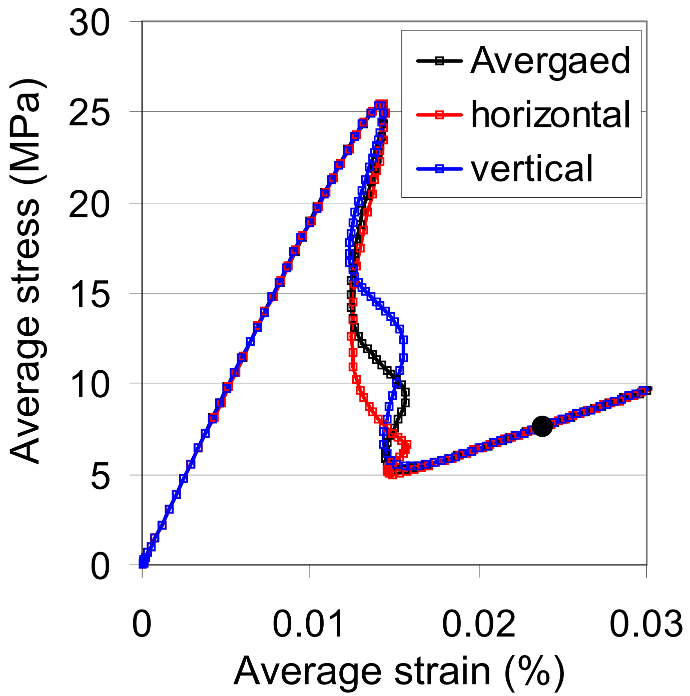


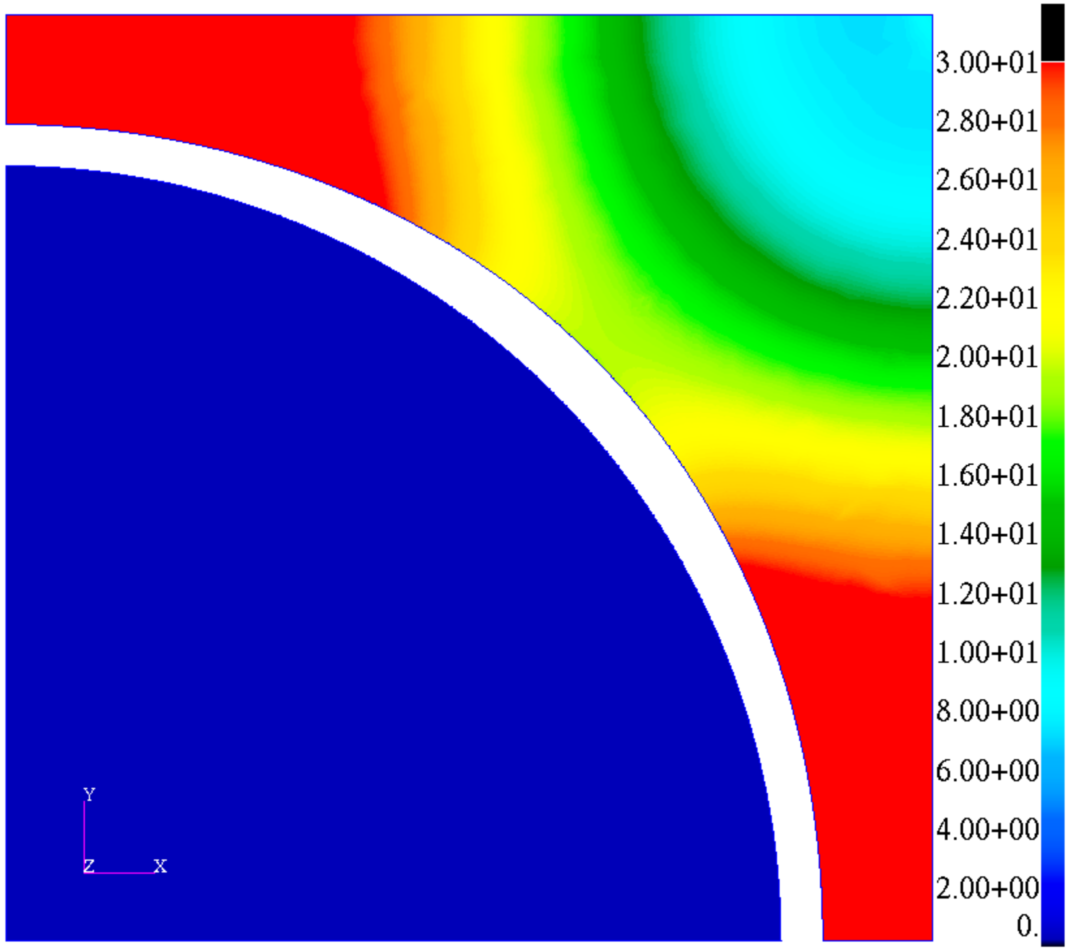


- **Fracture Parameters**

G (N/m)	T_{max} (MPa)	Shape	Slope
5	25	3	0.04

Debonding Process 7





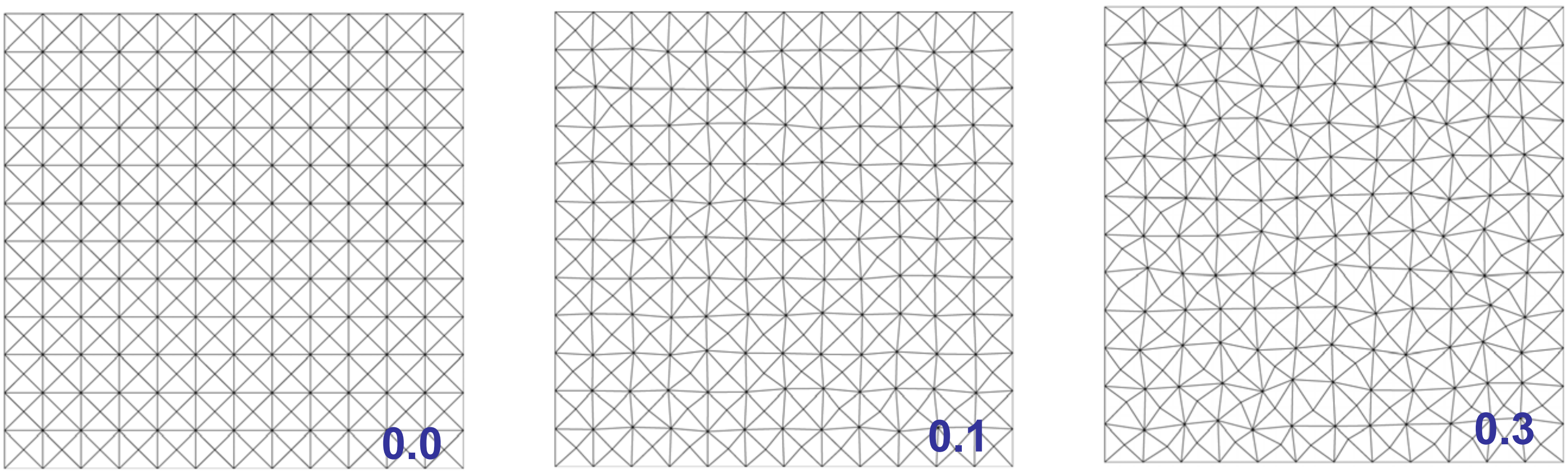
- **Fracture Parameters**

G (N/m)	T_{max} (MPa)	Shape	Slope
5	25	3	0.04

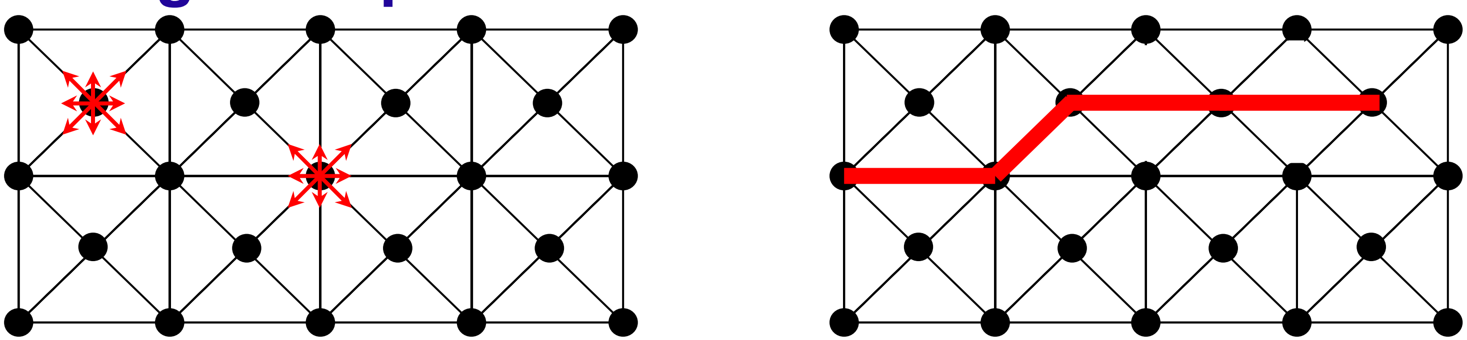
Contents

Topological Operators

Nodal Perturbation



Edge-Swap

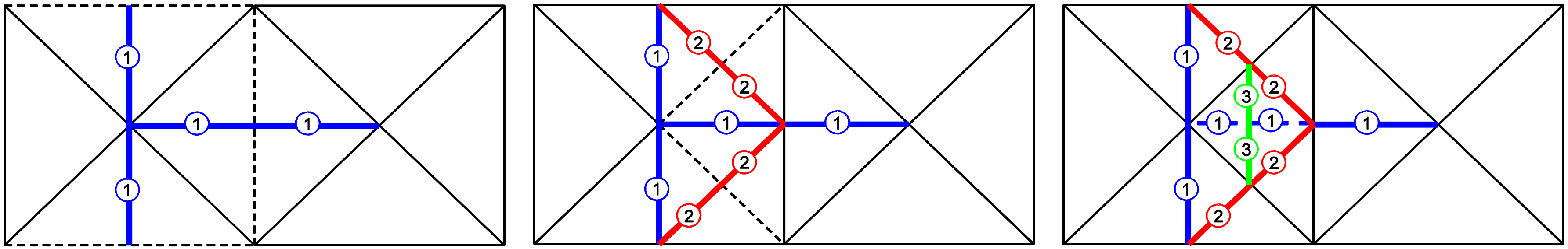


K. Park, G.H. Paulino, W. Celes, and R. Espinha, 2009, Adaptive dynamic cohesive fracture simulation using edge-swap and nodal perturbation operators, International Journal for Numerical Methods in Engineering (submitted).

Adaptive Mesh Refinement & Coarsening

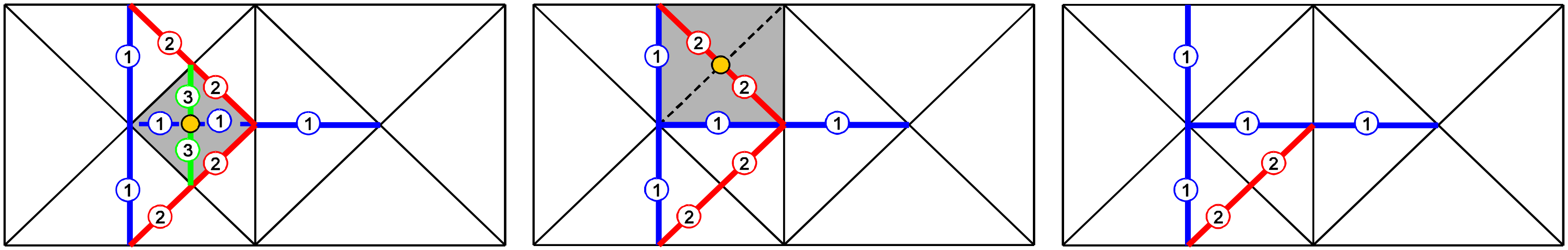
Edge-Split

- Adaptive mesh refinement based on *a priori* knowledge



Vertex-Removal (or Edge-Collapse)

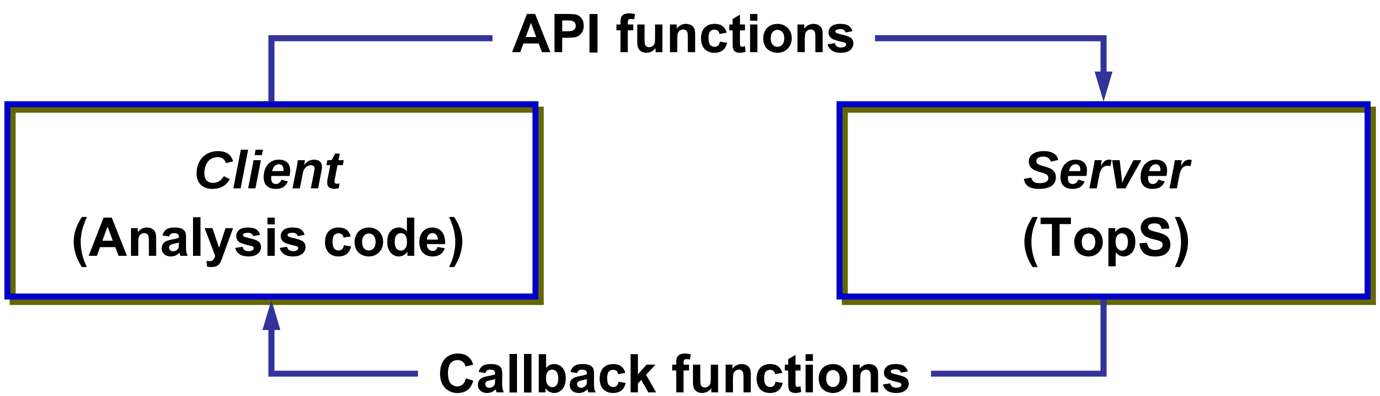
- Adaptive mesh coarsening based on *a posteriori* error estimation, i.e. root mean square of strain error



K. Park, G.H. Paulino, W. Celes, and R. Espinha, 2009, Adaptive mesh refinement and coarsening for cohesive dynamic fracture, (in preparation).

Topology-based Data Structure (TopS)

- **Complete Topological Data & Reduced Representation**
- **Support for Adaptive Analysis**
- **Client-Server Architecture**
  - Separate computational mechanics from data representation



• W. Celes, G.H. Paulino, R. Espinha, 2005, A compact adjacency-based topological data structure for finite element mesh representation, IJNME 64(11), 1529-1556

• G. H. Paulino, W. Celes, R. Espinha, Z. Zhang, 2008, A general topology-based framework for adaptive insertion of cohesive elements in finite element meshes, EWC 24, 59-78

• K. Park, G.H. Paulino, W. Celes, and R. Espinha, 2009, Adaptive dynamic cohesive fracture simulation using edge-swap and nodal perturbation operators, International Journal for Numerical Methods in Engineering (submitted).

Explicit Time Integration

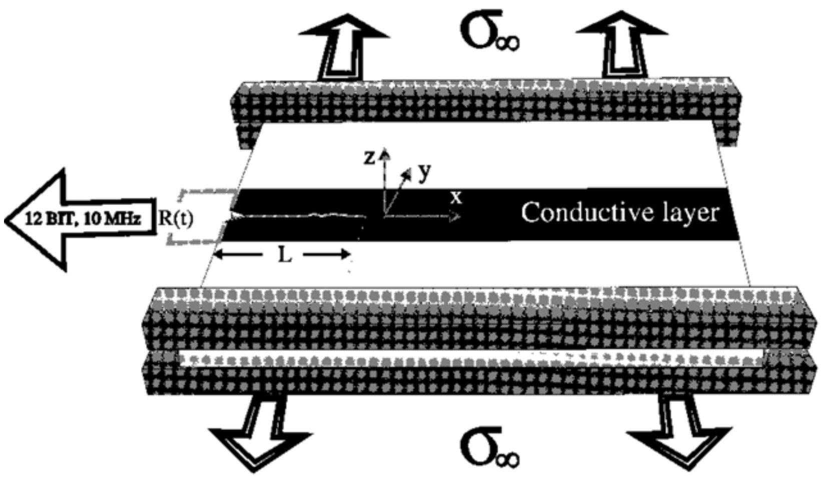
Initialization: displacement, velocity, acceleration

for *n* = 0 to n_{max} do

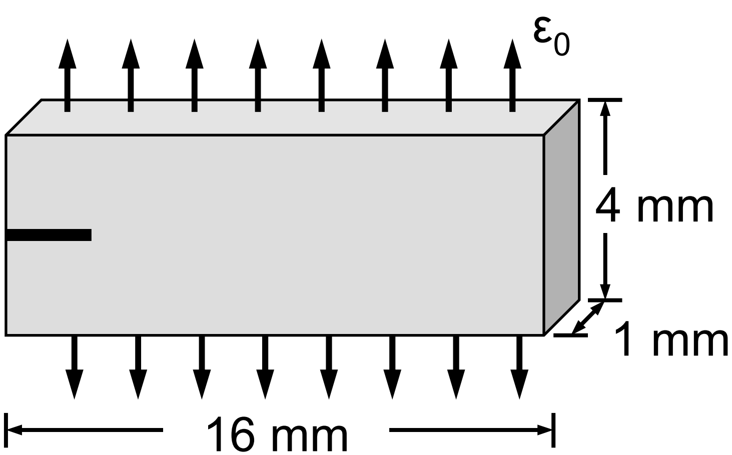
- Update displacement: $\mathbf{u}_{n+1} = \mathbf{u}_n + \Delta t \dot{\mathbf{u}}_n + \Delta t^2/2 \ddot{\mathbf{u}}_n$
- Adaptive mesh coarsening (vertex-removal)
- Check the insertion of cohesive element (edge-swap)
- Update acceleration: $\ddot{\mathbf{u}}_{n+1} = \mathbf{M}^{-1}(\mathbf{R}^{ext}_{n+1} + \mathbf{R}^{coh}_{n+1} - \mathbf{R}^{int}_{n+1})$
- Update velocity: $\dot{\mathbf{u}}_{n+1} = \dot{\mathbf{u}}_n + \Delta t/2(\ddot{\mathbf{u}}_n + \ddot{\mathbf{u}}_{n+1})$
- Update boundary conditions
- Adaptive mesh refinement (edge-split, nodal perturbation)

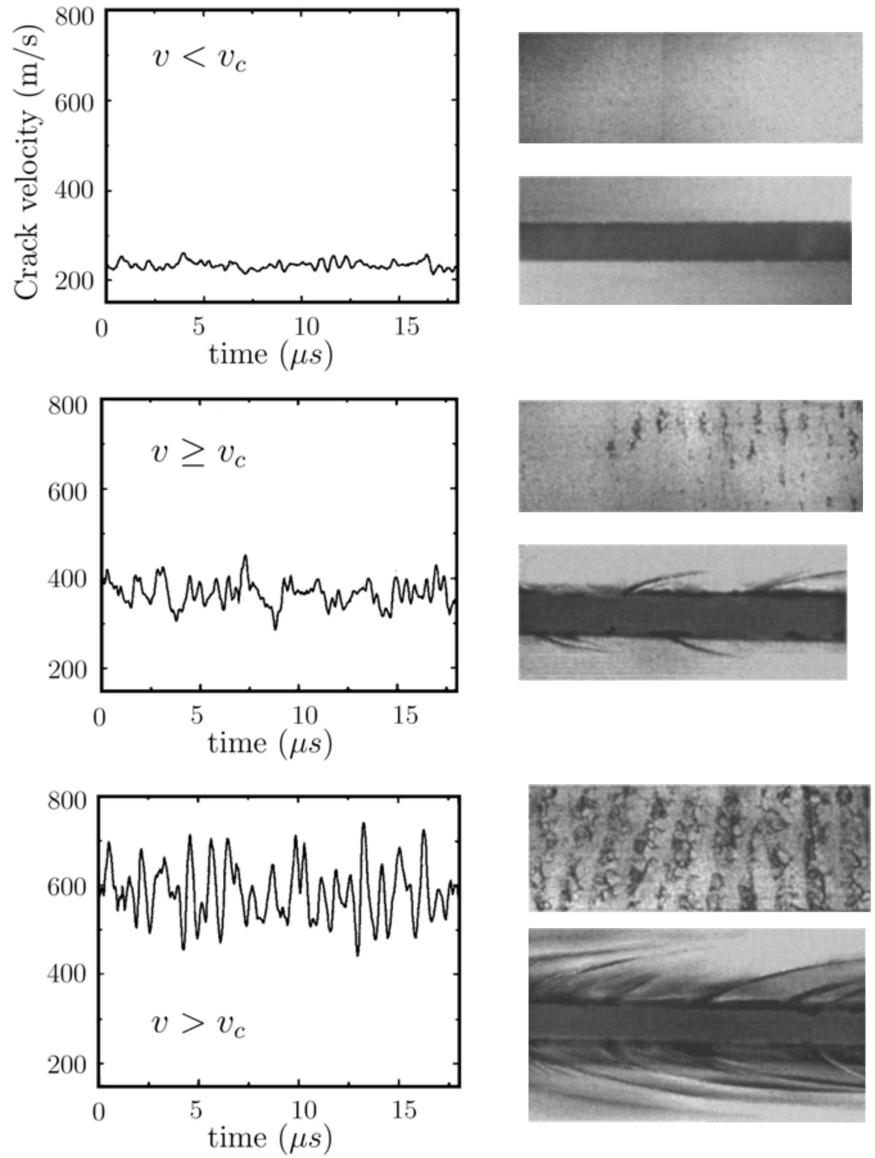
end

Micro-Branching Experiment

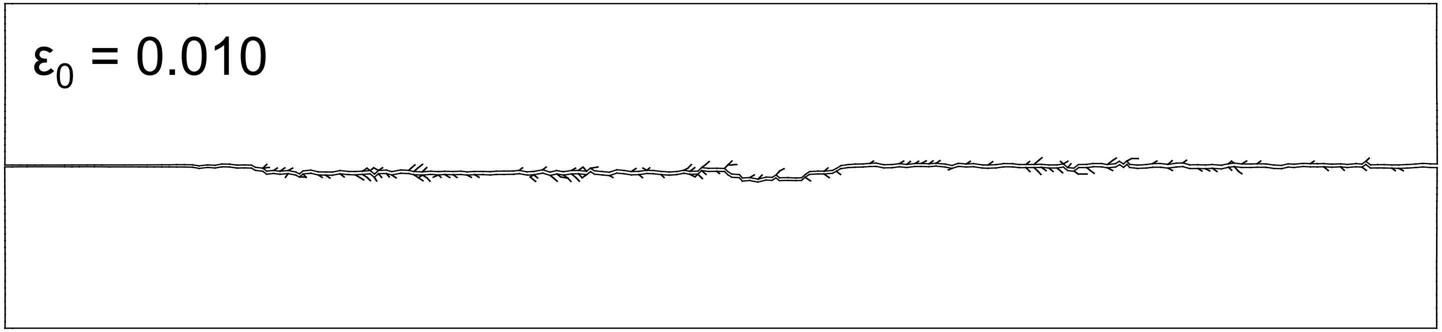


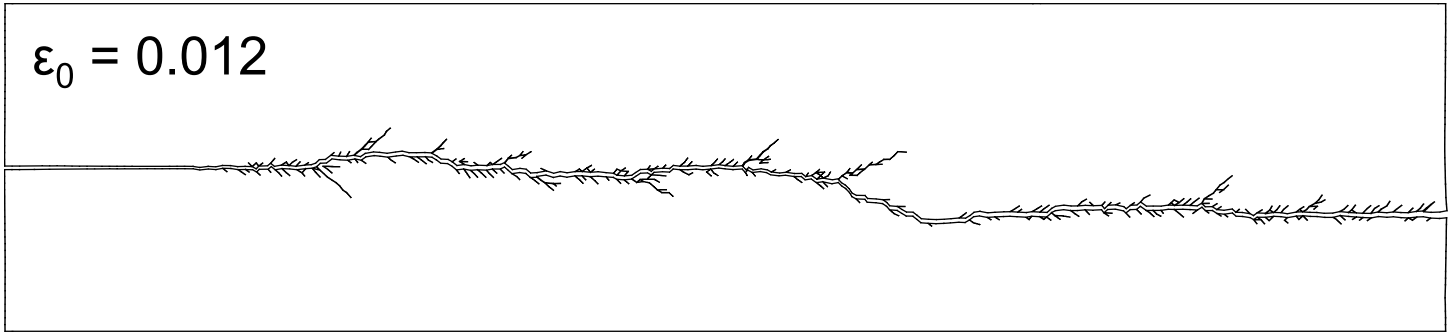
Sharon E, Fineberg J. Microbranching instability and the dynamic fracture of brittle materials. Physical Review B 1996; 54(10):7128–7139.

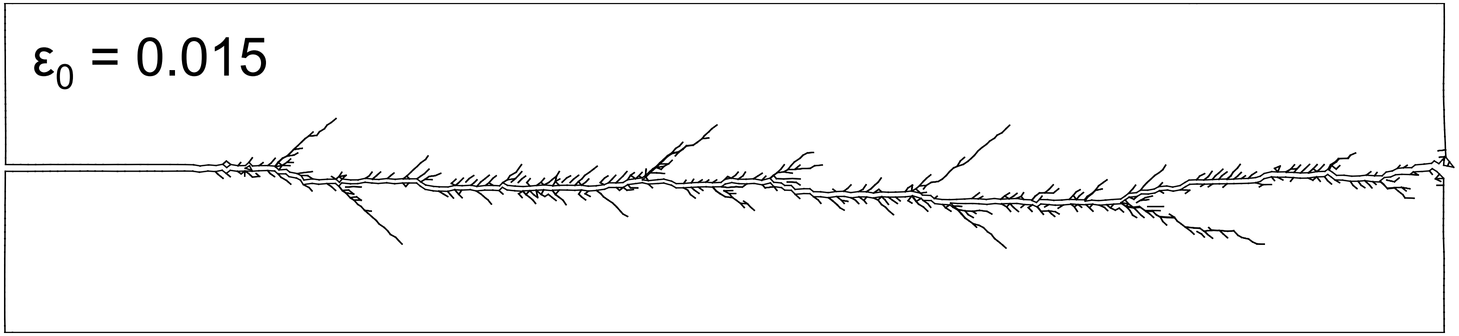




Computational Results

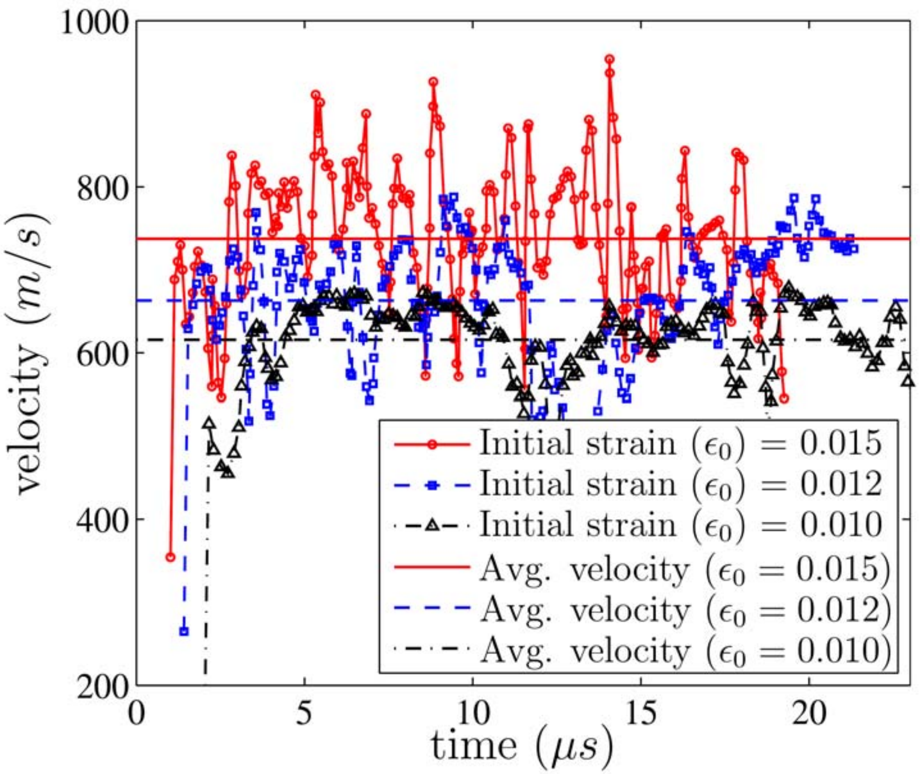




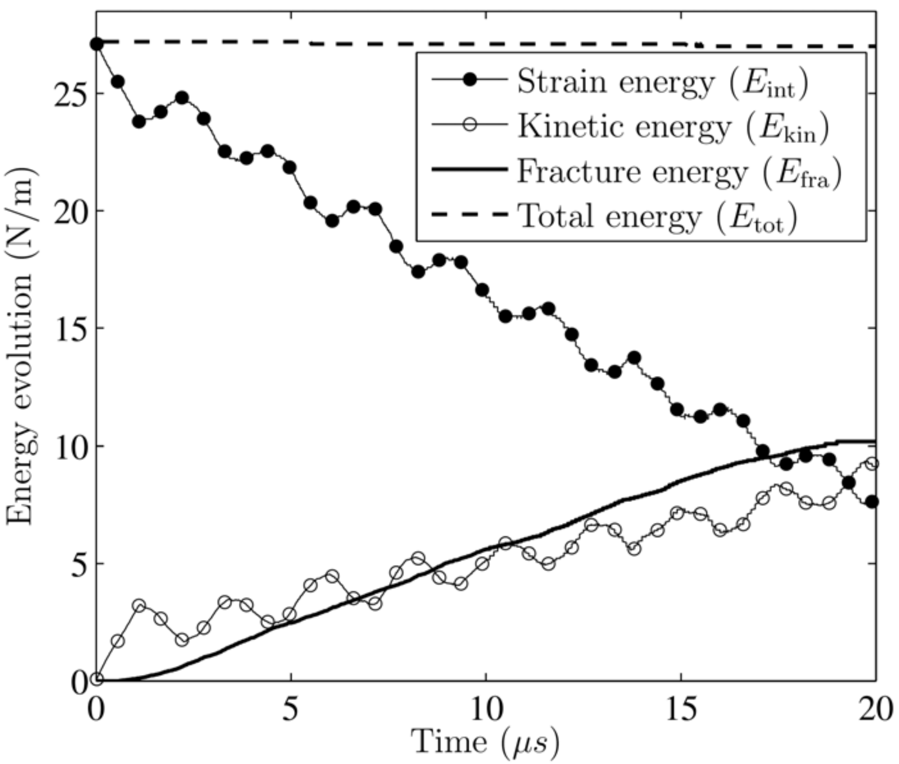


Computational Results

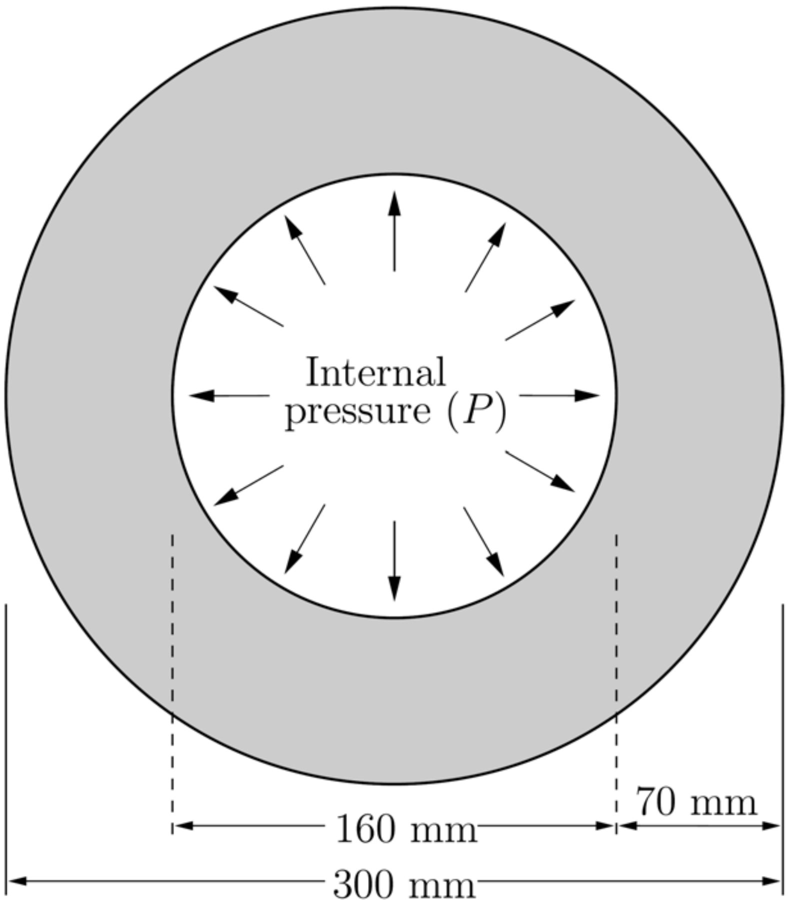
Crack Velocity

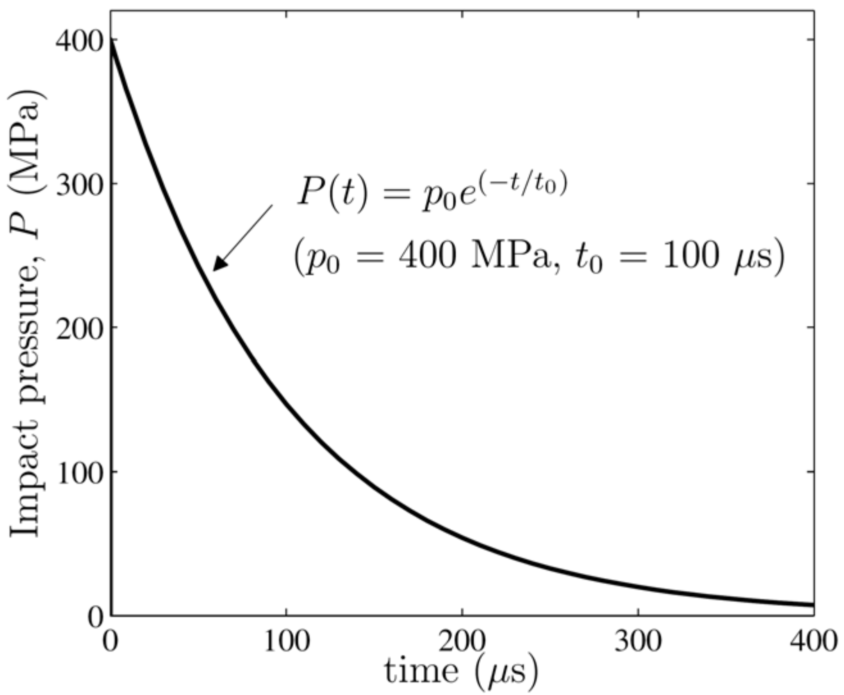


Energy Evolution (ε_0=0.015)

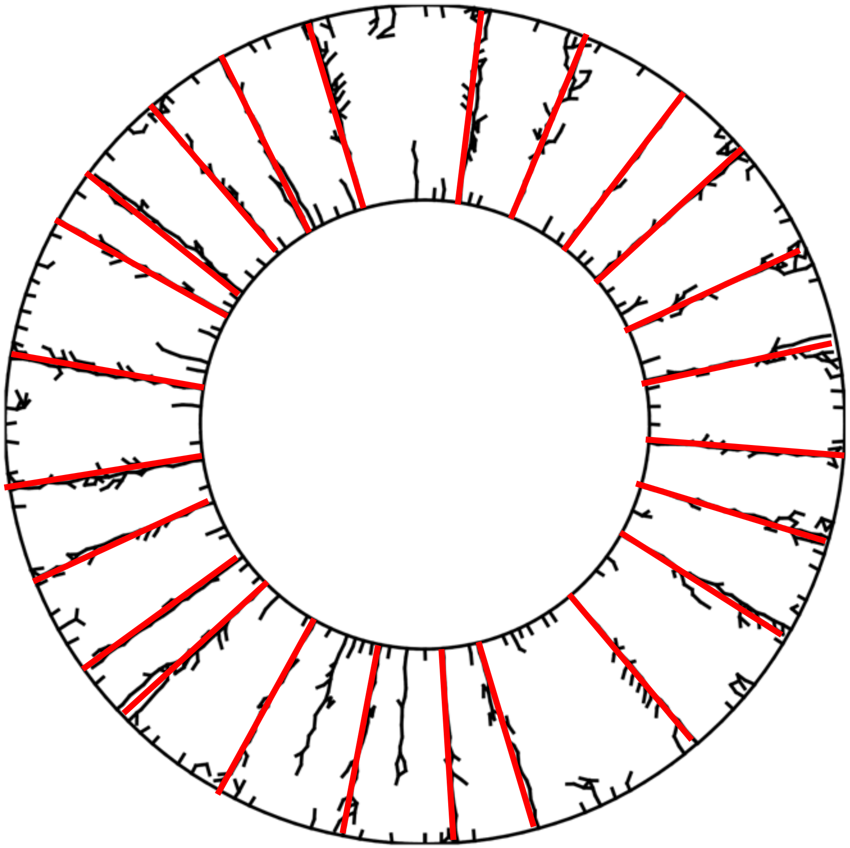


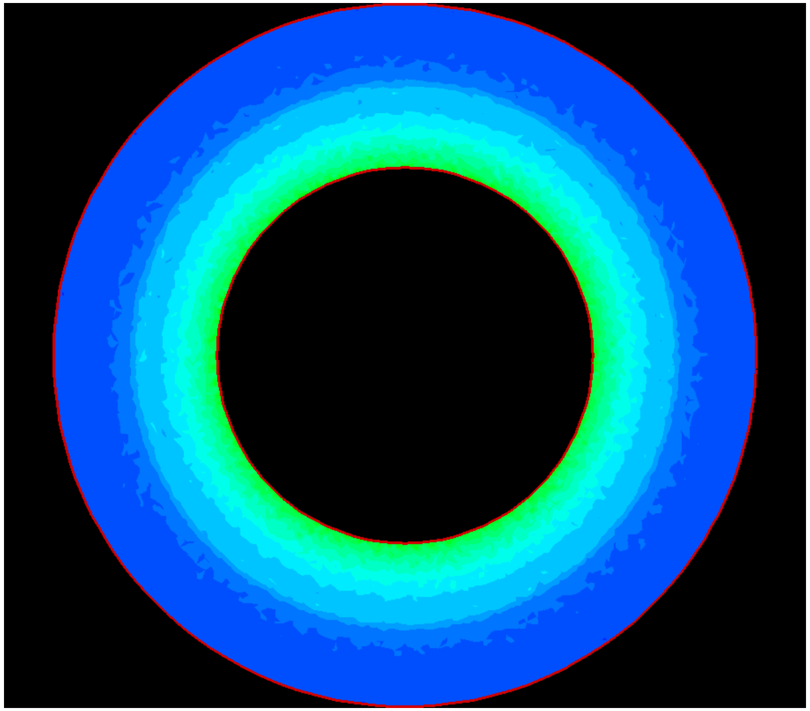
Fragmentation Problem



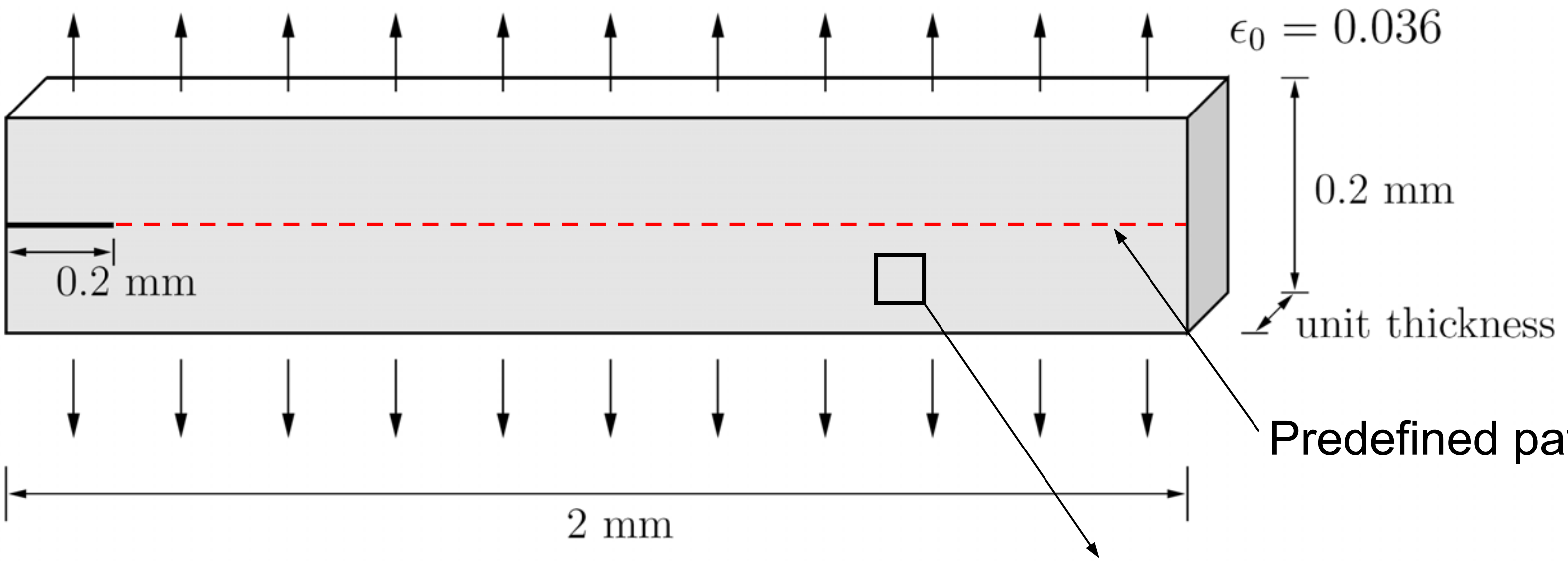


Computational Results

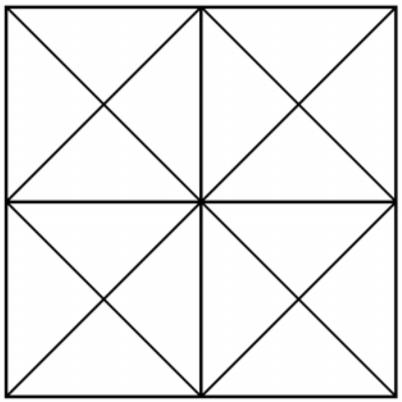




Mode I Pre-defined Crack Propagation

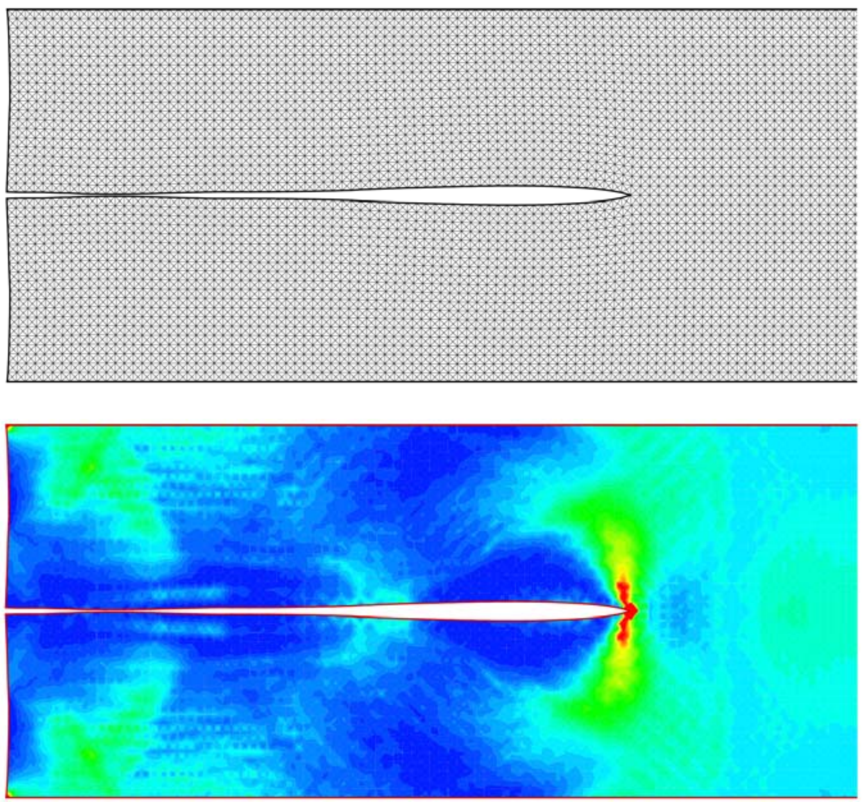


E	v	ρ	G_I	T_{max}
3.24 GPa	0.35	1190 kg/m^3	352 N/m	324 MPa

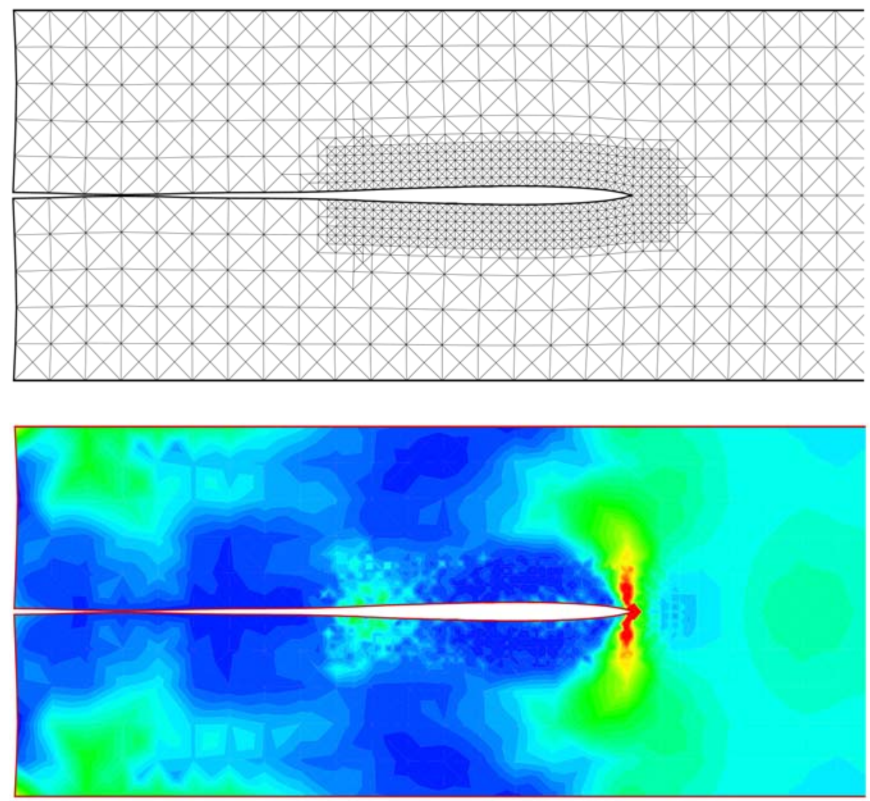


Computational Results

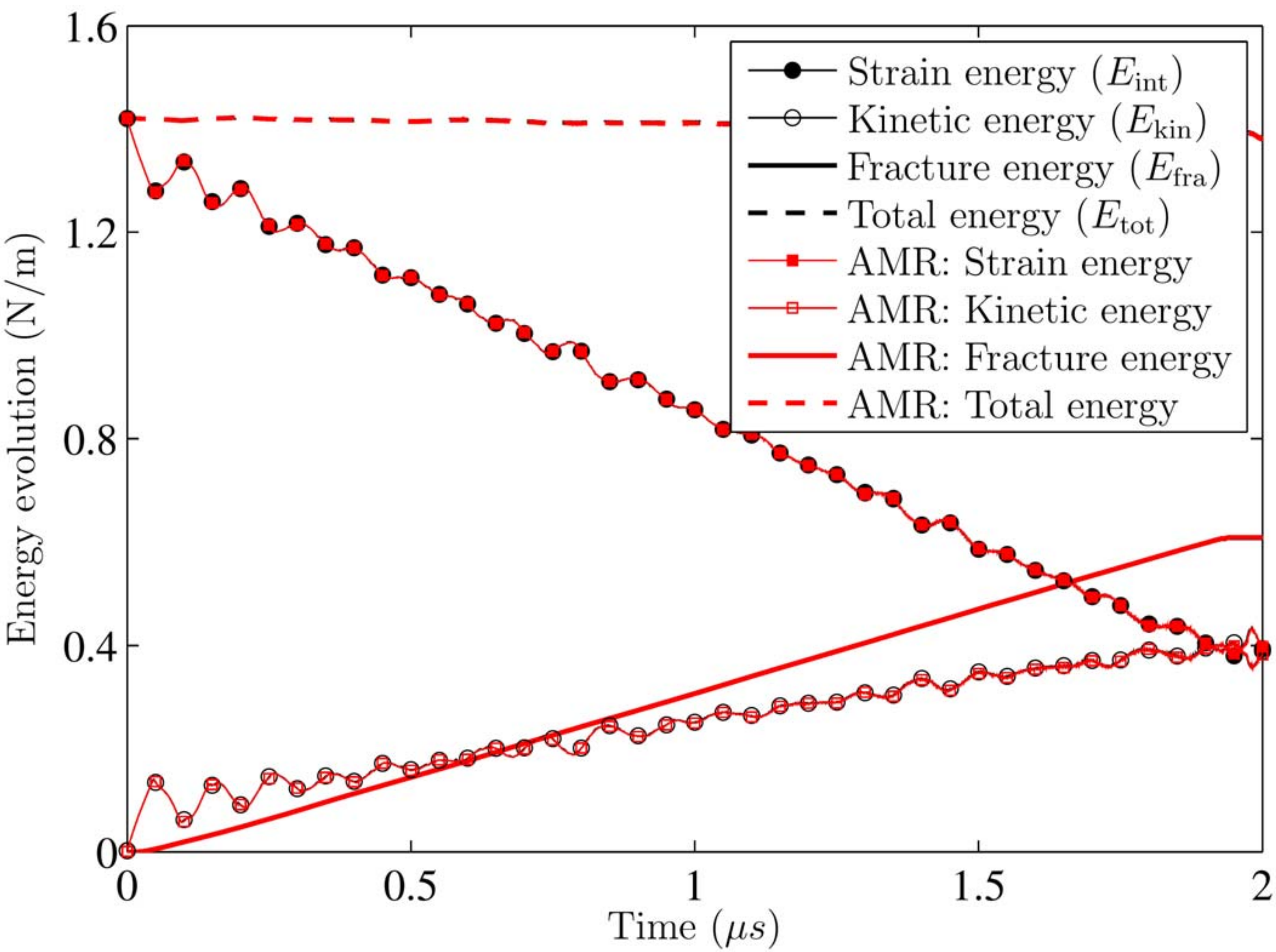
- **Uniform Mesh Refinement**
  - 400x40 mesh grid
  - Element size: 5μm
  - 64000 elements, 128881 nodes



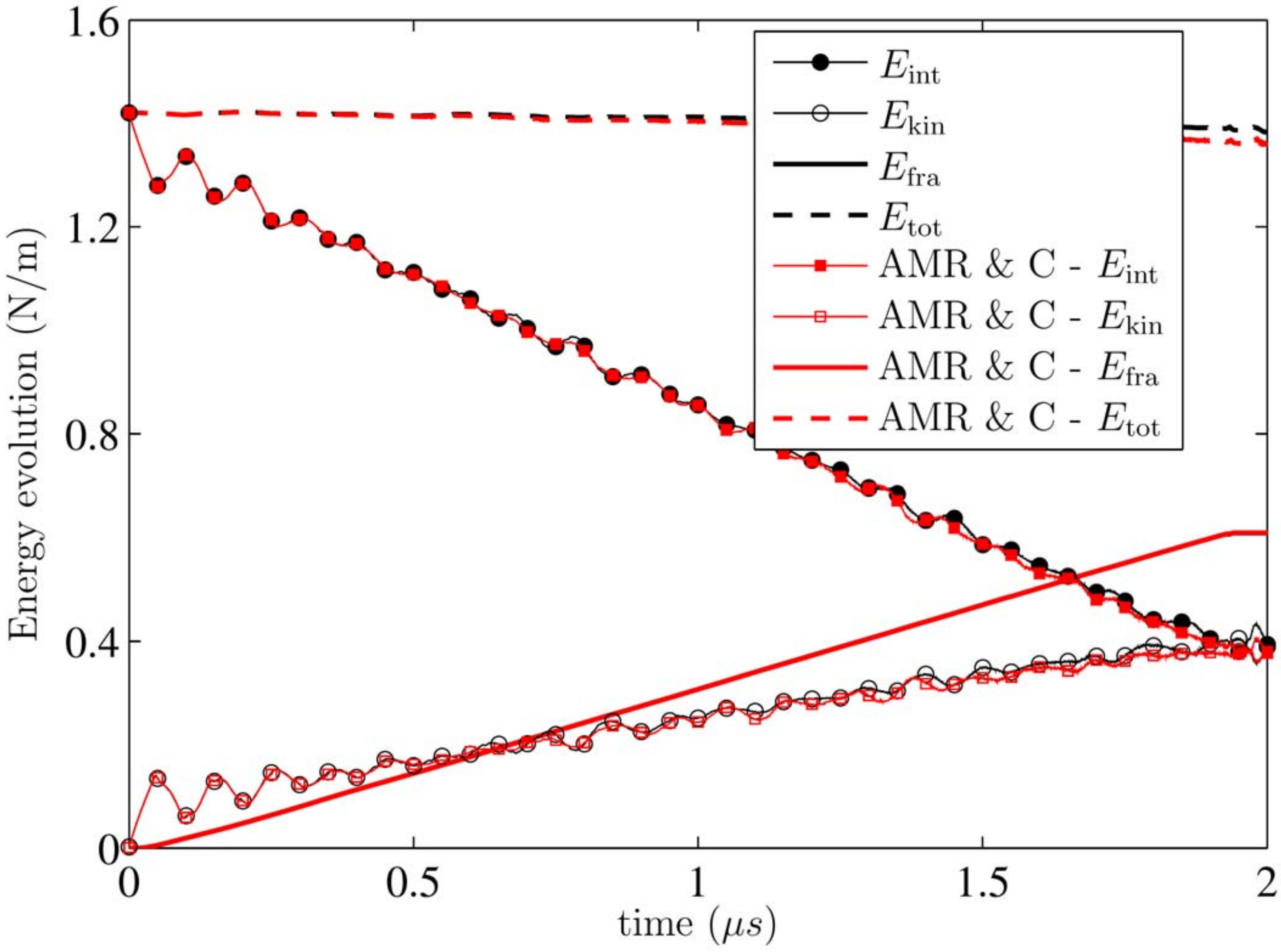
- **Adaptive Mesh Refinement**
  - 100x10 mesh grid
  - Element size: 20~5μm
  - 4448 elements, 9147 nodes



Computational Results (AMR)

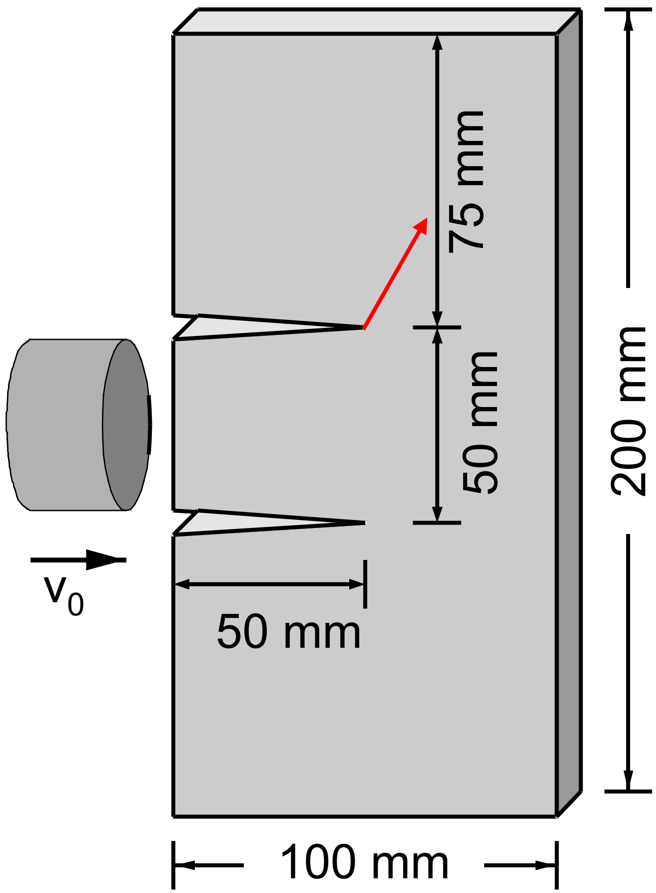


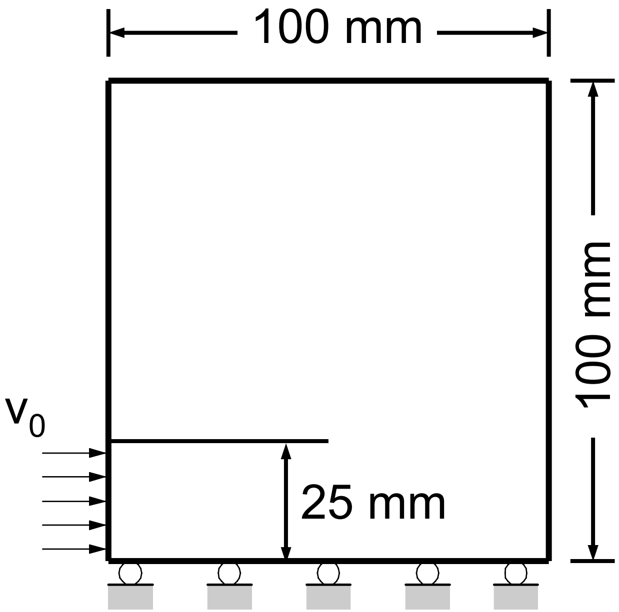
Computational Results (AMR+C)



Mixed-Mode Crack Propagation

Kalthoff-Winkler's Experiments

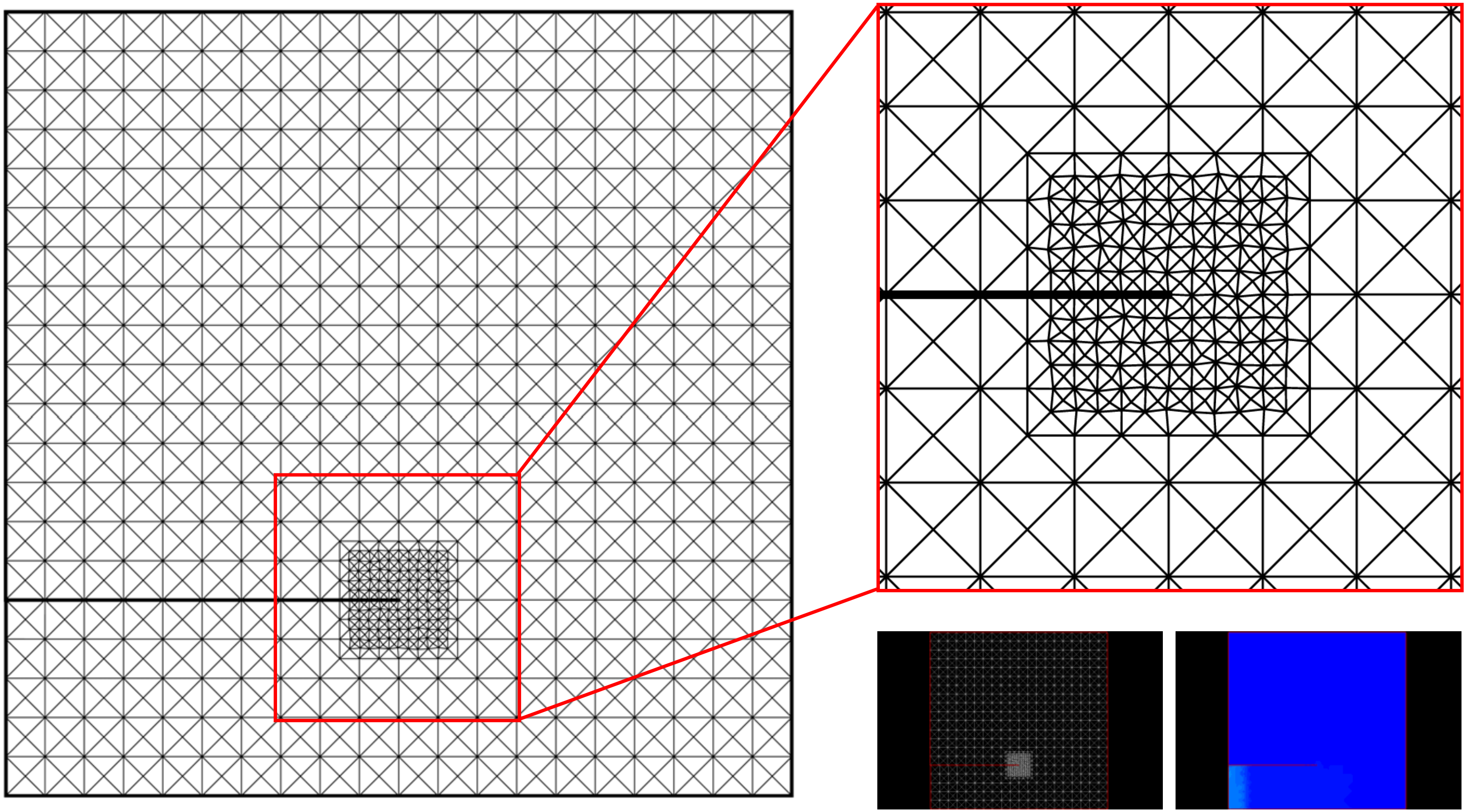




Kalthoff, J. F., Winkler, S., 1987. Failure mode transition at high rates of shear loading. International Conference on Impact Loading and Dynamic Behavior of Materials 1, 185–195.

Finite Element Mesh

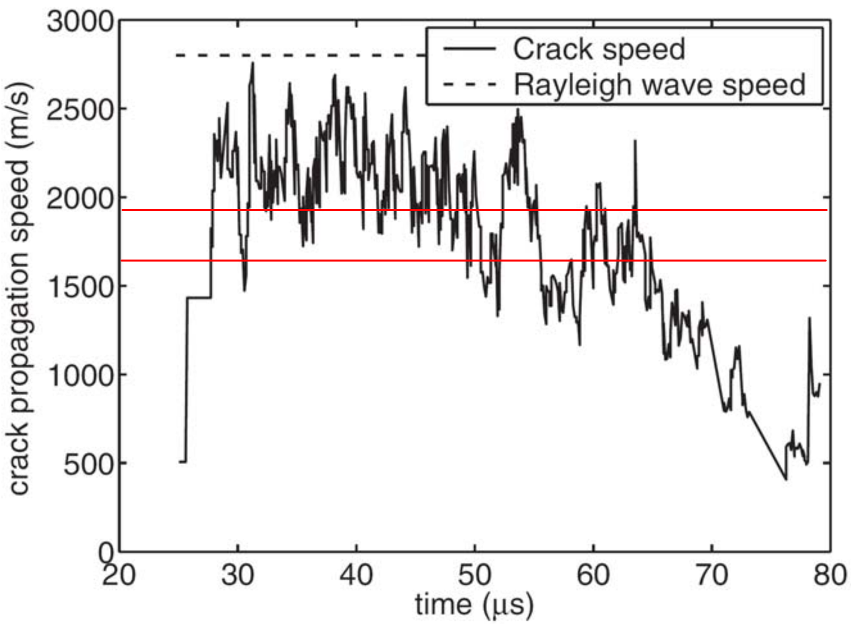
- Initial Discretization



Animations (FE Mesh & Strain energy)

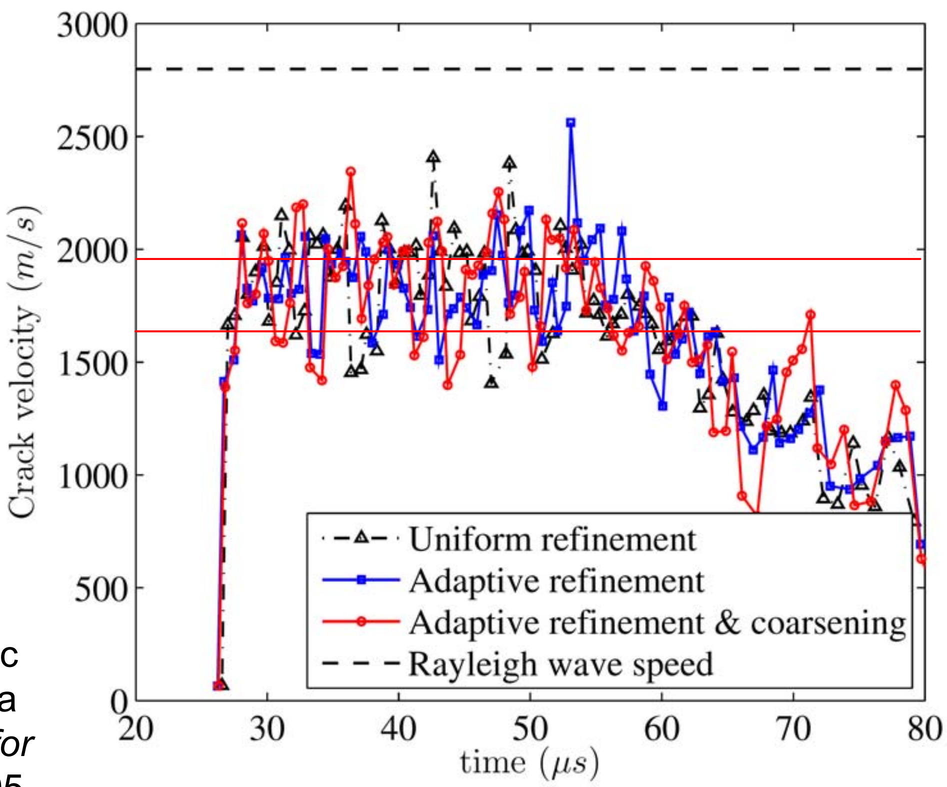
Computational Results

Previous results (X-FEM)

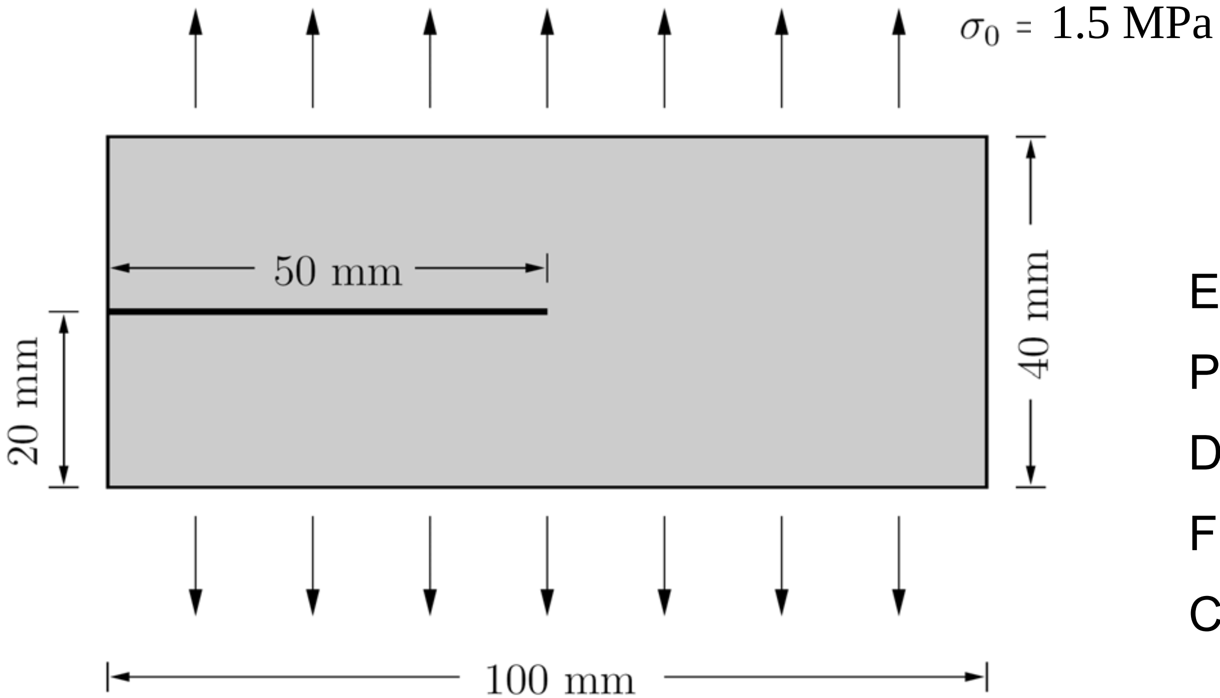


Belytschko, T., Chen, H., Xu, J., Zi, G., 2003. Dynamic crack propagation based on loss of hyperbolicity and a new discontinuous enrichment. *International Journal for Numerical Methods in Engineering* 58 (12), 1873–1905.

- Crack Velocity



Branching Problem



Elastic modulus: 32GPa

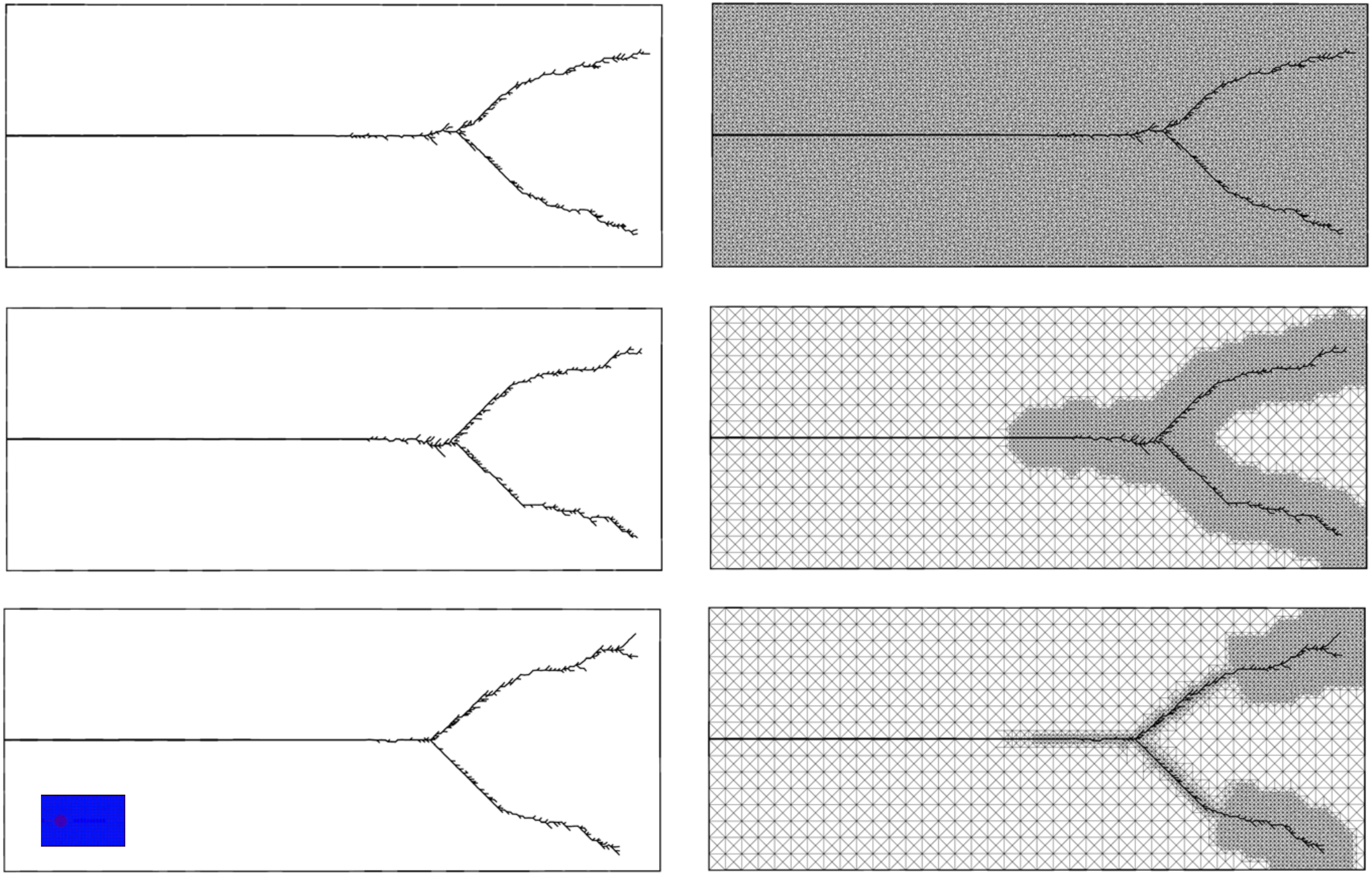
Poisson's ratio: 0.2

Density: 2450 kg/m^3

Fracture energy: 3N/m

Cohesive strength: 12 MPa

Computational Results



Contents

Summary

- **The potential-based constitutive model**
  - Consistent boundary conditions
  - Physical fracture parameters
- **Adaptive operators**
  - Insertion of cohesive elements (Extrinsic model)
  - Nodal perturbation, Edge-swap
  - Edge-split, Vertex-removal
- **Effective and efficient computational framework to simulate physical phenomena associated with quasi-static fracture, dynamic fracture, branching, and fragmentation problems**

Contributions

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