Dissertation Defense

System Reliability-Based Design and Multiresolution Topology Optimization

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Vietnam Education Foundation

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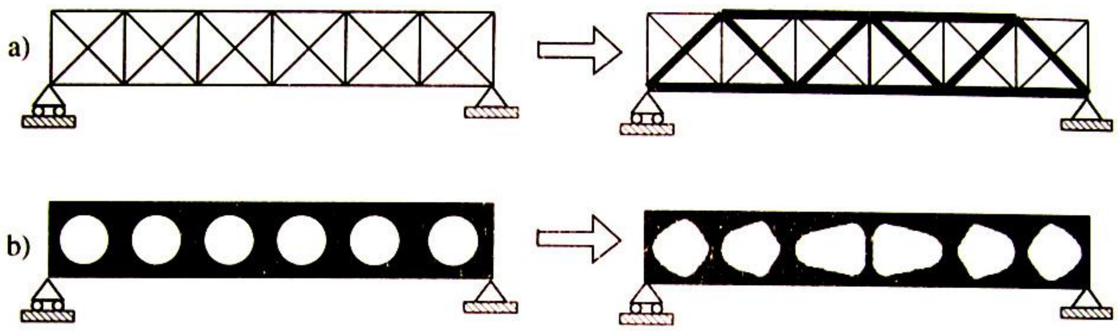
Contents

Topology Optimization

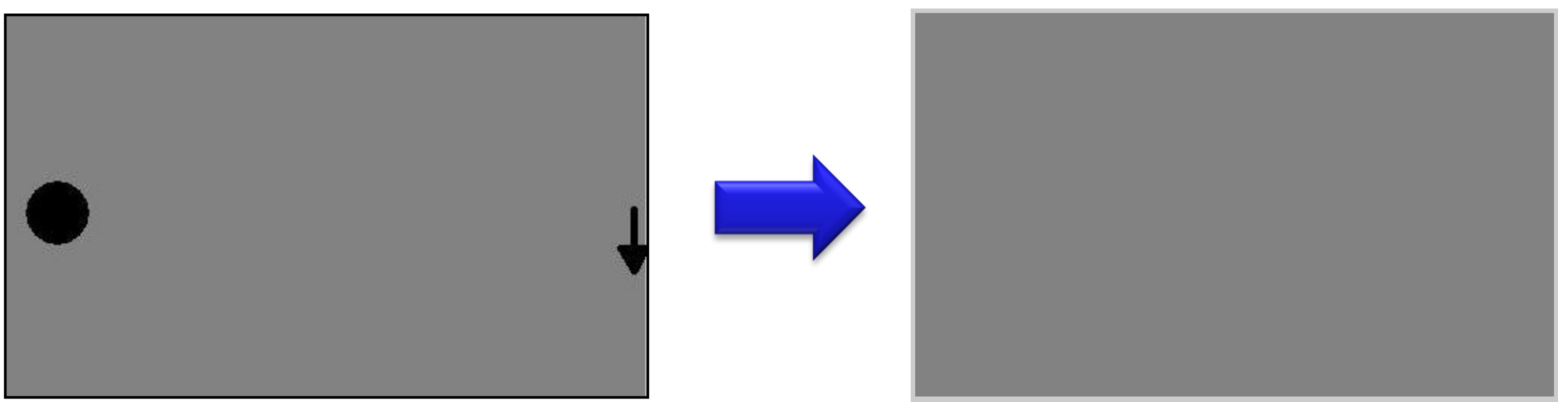
- **Classical structural design optimization: the optimal sizes or shapes for a given layout and connectivity**



size optimization

shape optimization

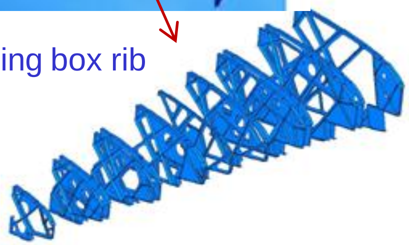
- **Topology optimization: the best topology, shape, size under a given domain and boundary conditions**

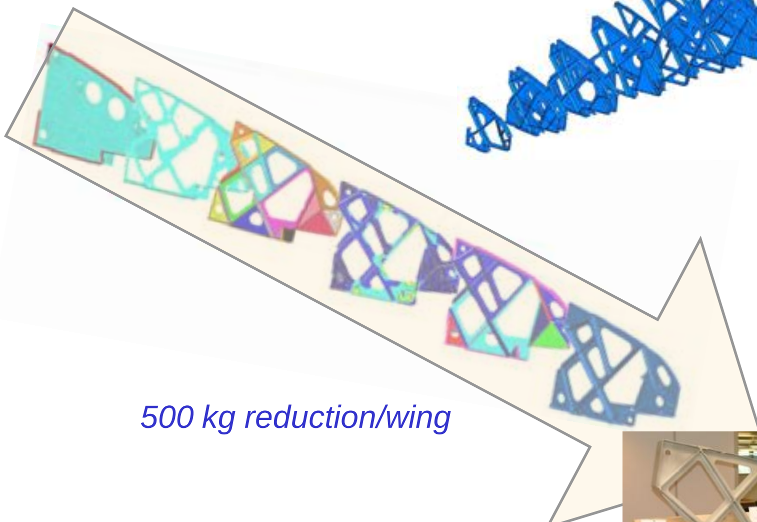


Topology Optimization Applications

Airbus

Wing box rib





500 kg reduction/wing

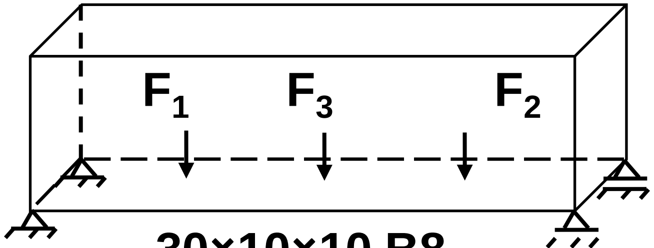
(www.altair.com)

Skidmore, Owings & Merrill, LLP (SOM)

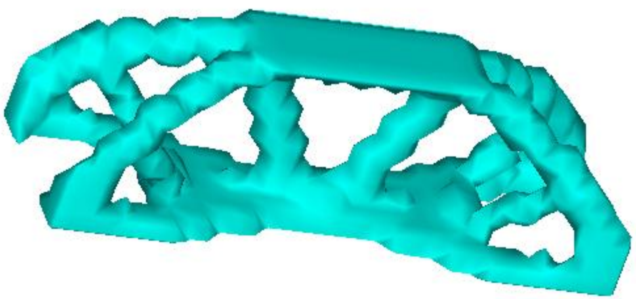
Large-scale Topology Optimization

Coarse mesh

Low resolution



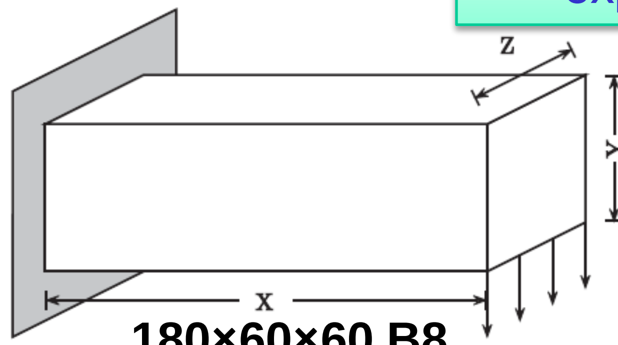
30×10×10 B8



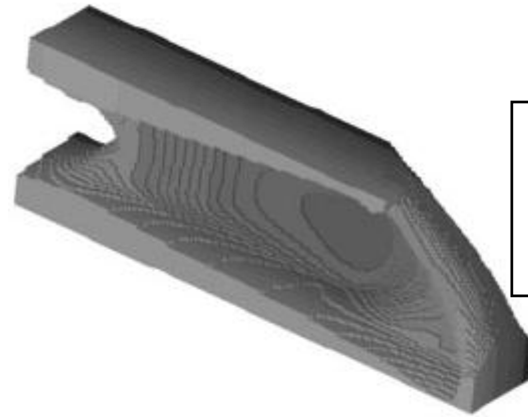
An example using Matlab code

Fine mesh

Computationally expensive



180×60×60 B8



~ 1.0 mil. unknowns

Fast solver, PC, C++

Run time: ~ 45.7 hours

Wang, de Stuler, and Paulino, (2007), *IJNME*

Question 1: How to obtain high resolution with affordable computational cost?

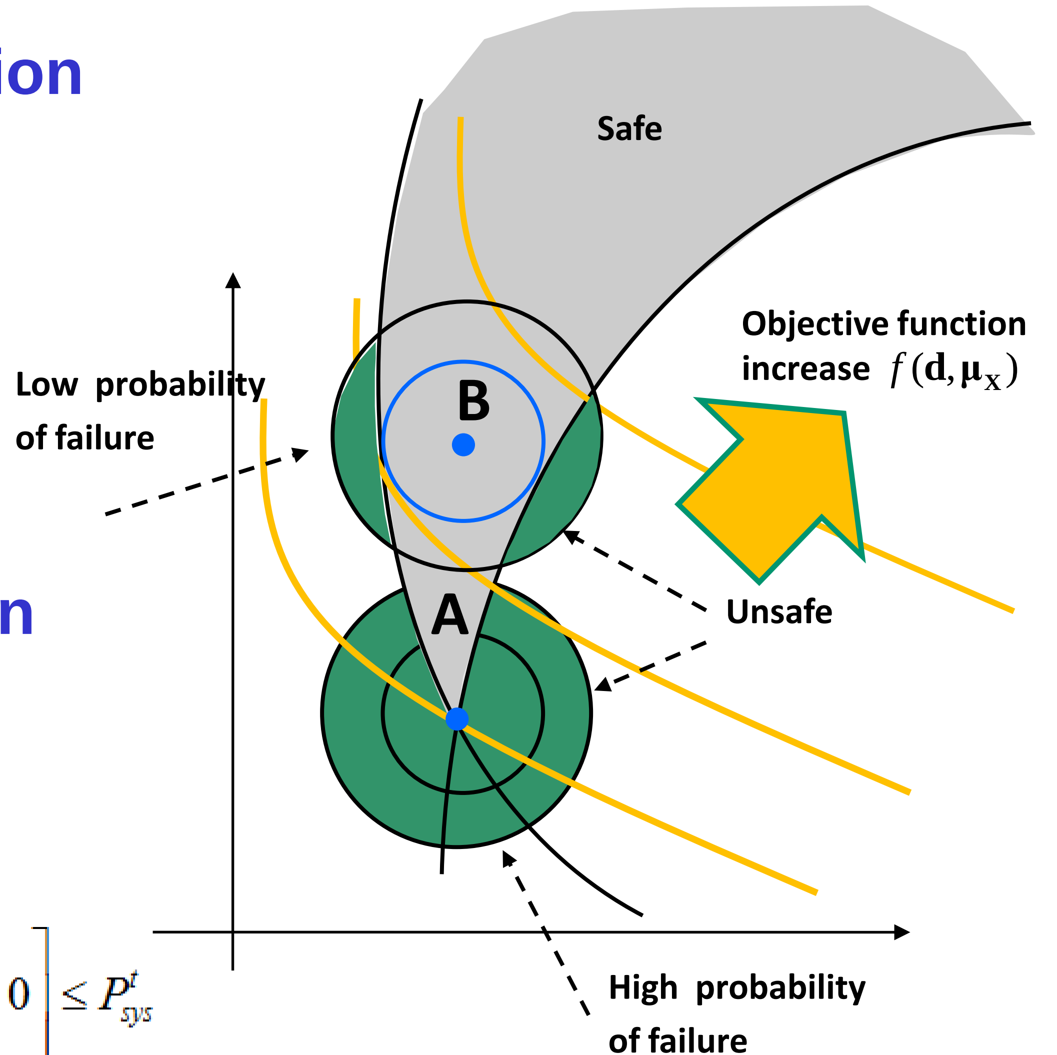
Reliability-Based Design Optimization

- **Deterministic Optimization**

$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X}} \quad f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$
$$s.t. \quad g_i(\mathbf{d},\mathbf{X}) > 0$$
$$\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U$$

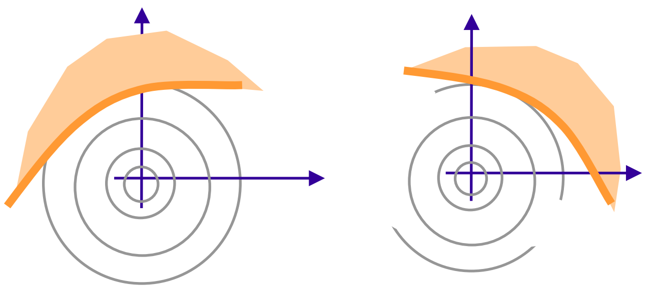
- **Reliability-Based Design Optimization (RBDO)**

$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X}} \quad f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$
$$s.t. \quad P_{sys} = P(E_{sys}) = P\left[\bigcup_k \bigcap_{i\in C_k} g_i(\mathbf{d},\mathbf{X}) \le 0\right] \le P_{sys}^t$$



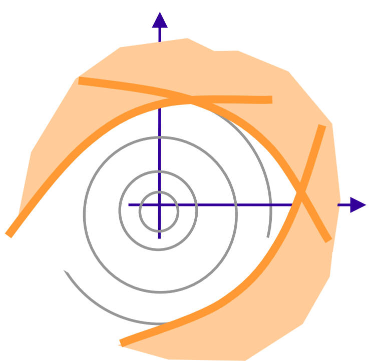
System Reliability-Based Design Optimization

Component RBDO



$$
\begin{aligned}
\min_{\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}}} \quad & f(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}}) \\
s.t. \quad & P\left[g_i(\mathbf{d},\mathbf{X}) \le 0\right] \le P_i^t \quad i=1,\ldots,n \\
& \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \qquad \boldsymbol{\mu}_{\mathbf{X}}^L \le \boldsymbol{\mu}_{\mathbf{X}} \le \boldsymbol{\mu}_{\mathbf{X}}^U
\end{aligned}
$$

System RBDO



$$
\begin{aligned}
\min_{\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}}} \quad & f(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}}) \\
s.t. \quad & P(E_{sys}) = P\left[\bigcup_{k} \bigcap_{i \in C_k} g_i(\mathbf{d},\mathbf{X}) \le 0\right] \le P_{sys}^t \\
& \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \qquad \boldsymbol{\mu}_{\mathbf{X}}^L \le \boldsymbol{\mu}_{\mathbf{X}} \le \boldsymbol{\mu}_{\mathbf{X}}^U
\end{aligned}
$$

?

Question 2: How to handle system probability in RBDO?

Objectives

1. To obtain **high resolution** with **affordable** computational cost in **topology optimization.**

2. To handle **system probability** in Reliability-Based Design Optimization (RBDO).

3. To apply RBDO framework in **topology** optimization (RBTO).

Multiresolution Topology Optimization

Topology Optimization Procedure

Problem formulation

$$\min_{\rho} \quad C(\rho,\mathbf{u}_d) = \mathbf{f}^{\mathrm{T}}\mathbf{u}_d$$

$$s.t.: \quad \mathbf{K}(\rho)\mathbf{u}_d = \mathbf{f}$$

$$V(\rho) = \int_{\Omega} \rho(\psi)\, dV \le V_s$$

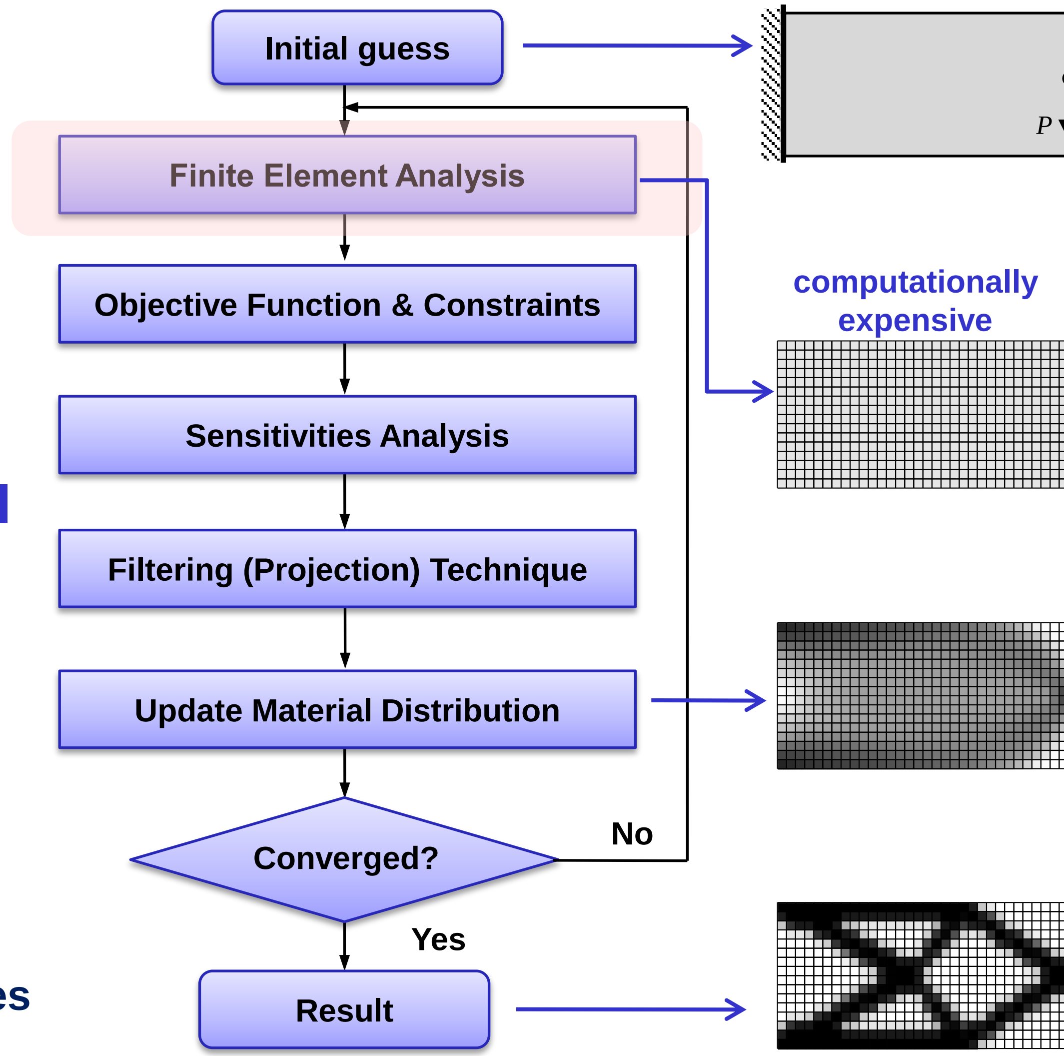
$$0 < \rho_{\min} \le \rho(\psi) \le 1$$

Solid and Isotropic Material with Penalization (SIMP)

$$E(\psi) = \rho(\psi)^{p} E^{0}$$

Optimizers

- Optimality Criteria (OC)
- Method of Moving Asymptotes (MMA)



High Resolution Topology Optimization

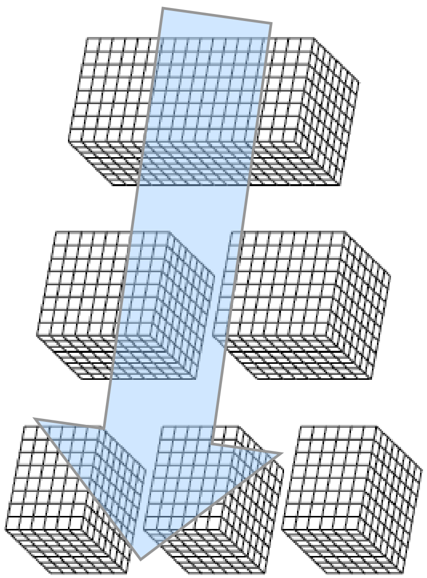
- **Large-scale (high resolution) TOP**
  - **Large number of finite elements**
  - **Computationally expensive**
- **Existing high resolution TOP**
  - **Parallel computing (Borrvall and Petersson, 2000)**
  - **Fast solvers (Wang et al. 2007)**
  - **Approximate reanalysis (Amir et al. 2009)**
  - **Adaptive mesh refinement (de Stuler et al. 2008)**

TOP (1): Parallel Computing

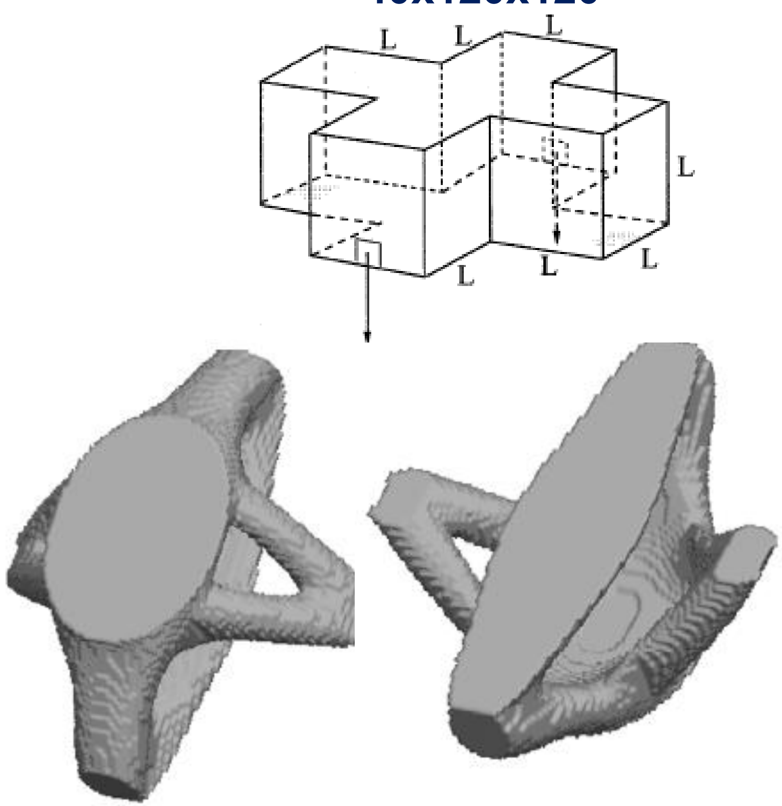
- **Parallel computing:**

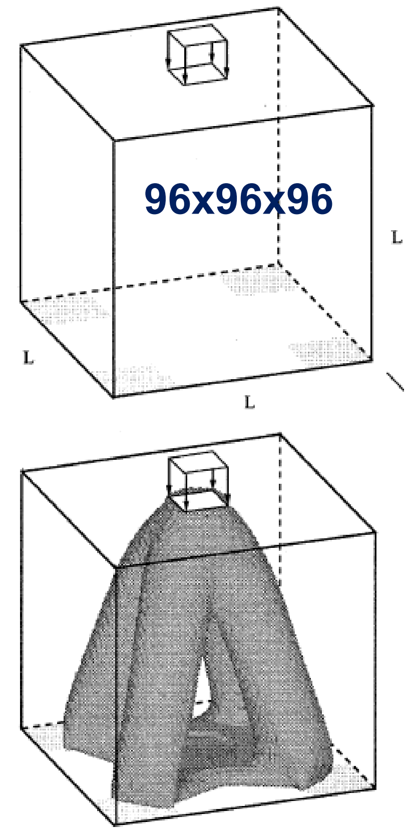
Borrvall and Petersson, (2000), *IJNME*

  - **Domain decomposition**



40x120x120





A cross-shaped section (320,000 B8/U)

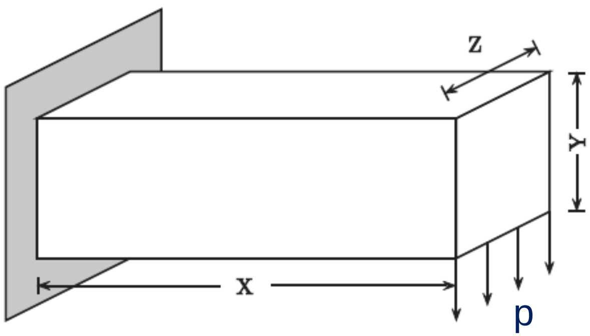
A stool (884,736 B8/U)

TOP (2): Fast Solvers

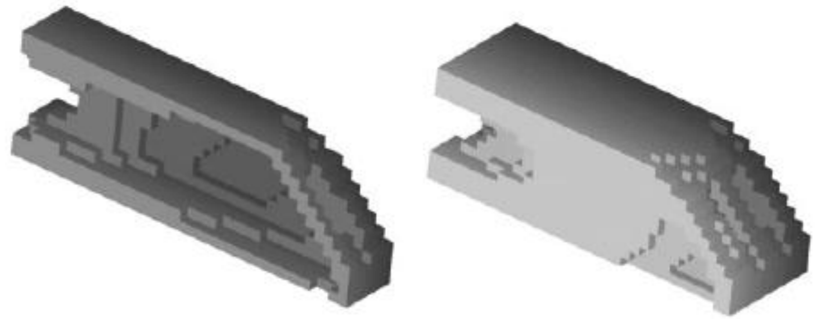
Fast iterative solvers

Wang, de Stuler, and Paulino, (2007), *IJNME*

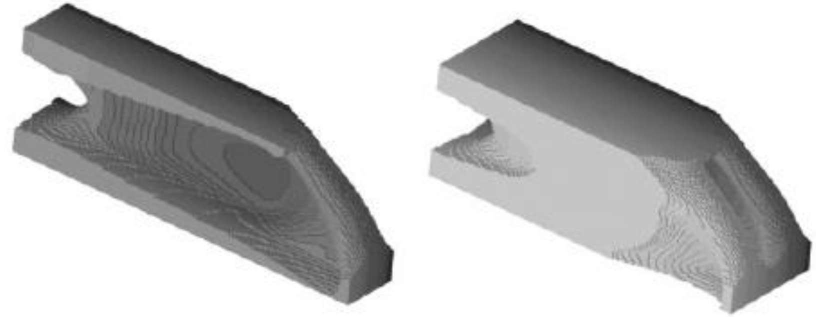
- Use precondition Krylov subspace methods with recycling
- Reduce computational time for FEA



Configuration



Coarse mesh: 32x12x12

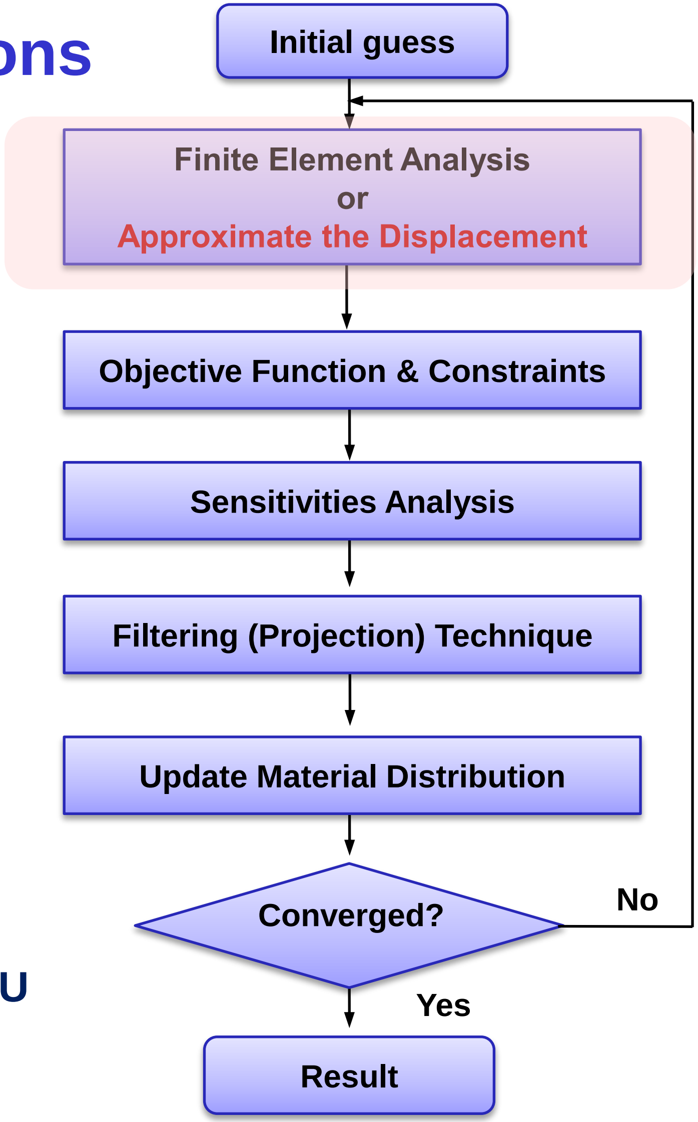


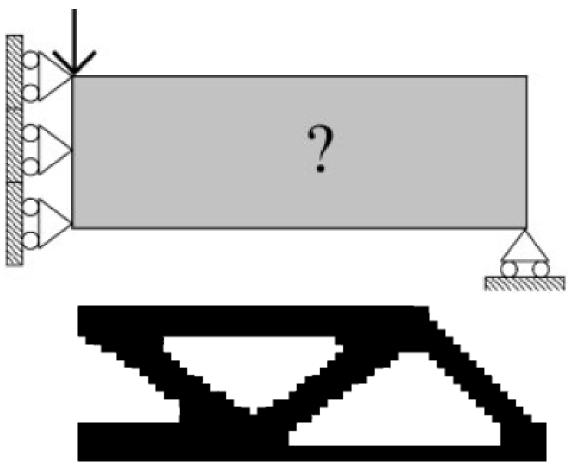
Fine mesh: 180x60x60

Solution on a PC with approx. 1 million unknowns

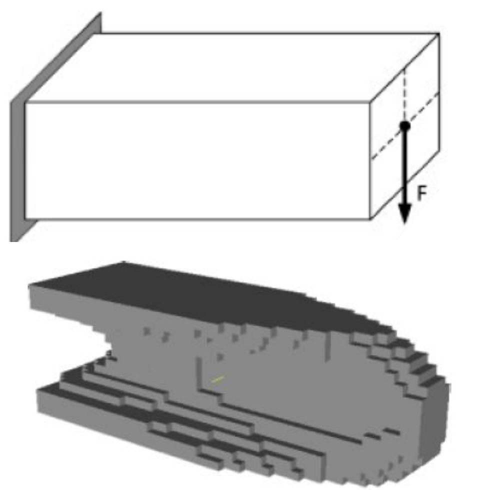
TOP (3): Approximate Reanalysis

- **Reduce the number of FEA solutions**
  - **FEA at an interval of iterations**
  - **Approximate at other iterations**
  - **Efficiency factor: 1 ~ 5 times**





MBB: 60x20 Q4/U



Cantilever: 48x16x16 B8/U

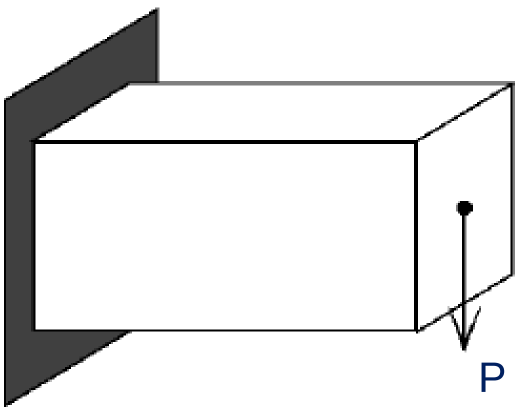
Amir, Bendsoe, and Sigmund, (2009), *IJNME*

TOP (4): Adaptive Mesh Refinement

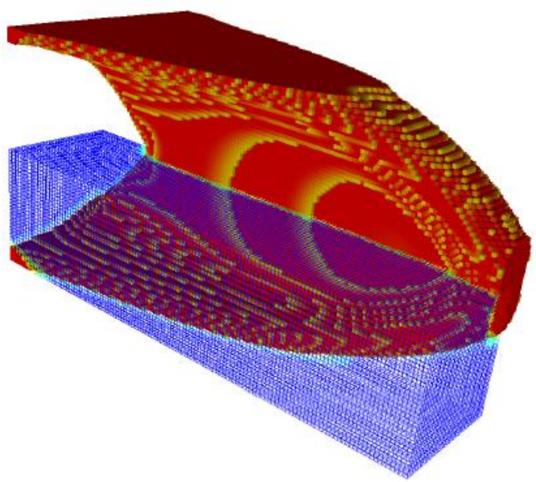
AMR TOP

de Stuler, Wang, and Paulino, (2008), IASS-IACM

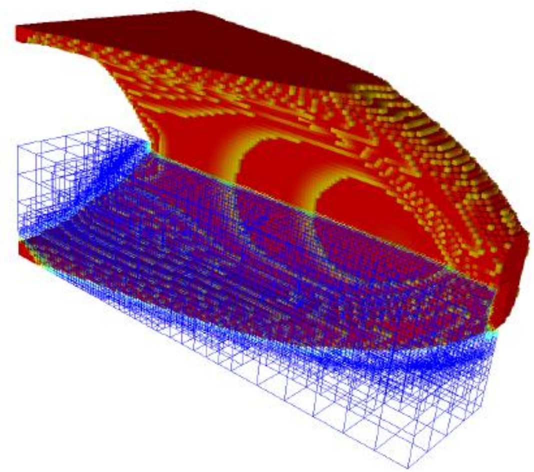
- **Refine the solid and surface regions**
- **Reduce the total number of FEs**
- **Obtain resolution as fine uniform mesh (efficiency factor 3)**



Configuration: 2:1:1



Uniform mesh: 128x64x64
524,288 B8/U



AMR: initial mesh 64x32x32
initial 65,520 B8/U
final 228,692 B8/U

Remarks on High Resolution TOP

Large-scale TOP

- Fine mesh: → Large number of finite elements
- FEA cost increases

Existing approaches:

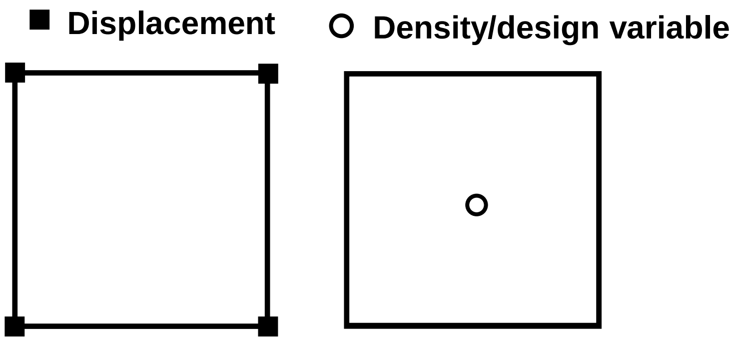
- Powerful computing resources: many processors
- Reduce cost associated with FEA:
  - Fast solvers
  - Approximate reanalysis
  - Adaptive mesh refinement

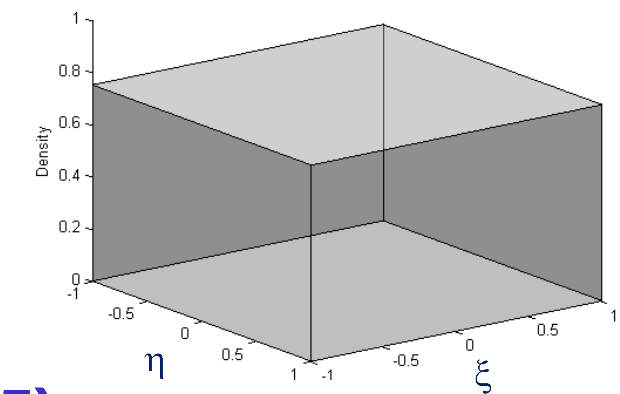
Same discretization for analysis and design

Proposed Multiresolution TOP (MTOP)

Conventional element-based approach (Q4/U)

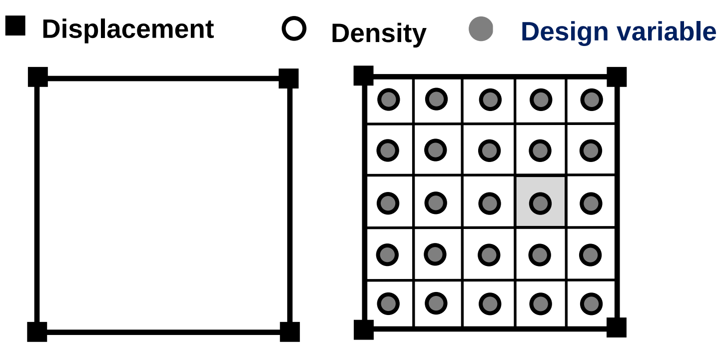
- **Same discretization for displacement and density**

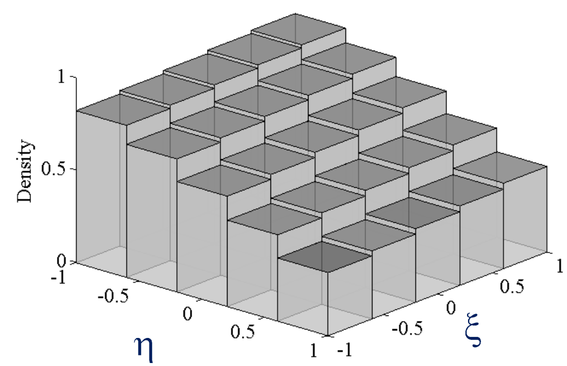




Proposed MTOP approach (Q4/n25)

- **Different discretizations for displacement and density/design variables**





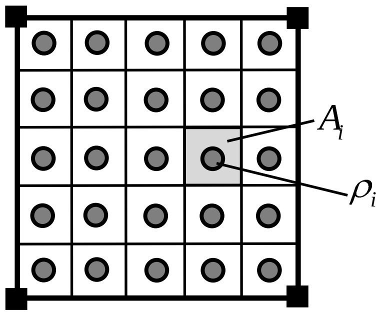
MTOP: Integration of Stiffness Matrix

Stiffness matrix

$$\mathbf{K}_e = \int_{\Omega_e} \mathbf{B}^T \mathbf{D} \mathbf{B} \, d\Omega$$

Numerical integration

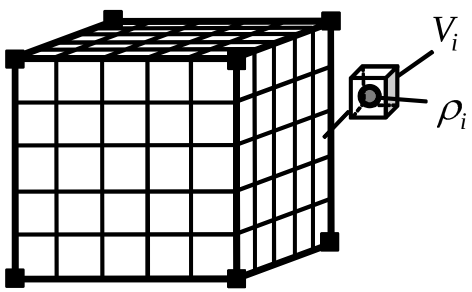
$$\mathbf{K}_e \simeq \sum_{i=1}^{n} \mathbf{B}^T \mathbf{D} \mathbf{B} \Big|_i A_i$$



Q4/n25

SIMP model

$$\mathbf{K}_e \simeq \sum_{i=1}^{N_n} (\rho_i)^p \left[\mathbf{B}_e^{\mathrm{T}}\Big|_i \mathbf{D}_0 \mathbf{B}_e\Big|_i \; A_i \right] = \sum_{i=1}^{N_n} (\rho_i)^p \mathbf{I}_i$$



B8/n125

Sensitivity

$$\frac{\partial \mathbf{K}_e}{\partial d_n} = \frac{\partial \mathbf{K}_e}{\partial \rho_i} \frac{\partial \rho_i}{\partial d_n} = \frac{\partial \left(\sum_{j=1}^{N_n} (\rho_j)^p \mathbf{I}_j \right)}{\partial \rho_i} \frac{\partial \rho_i}{\partial d_n} = p(\rho_i)^{p-1} \mathbf{I}_i \frac{\partial \rho_i}{\partial d_n}$$

MTOP: Projection (filtering)

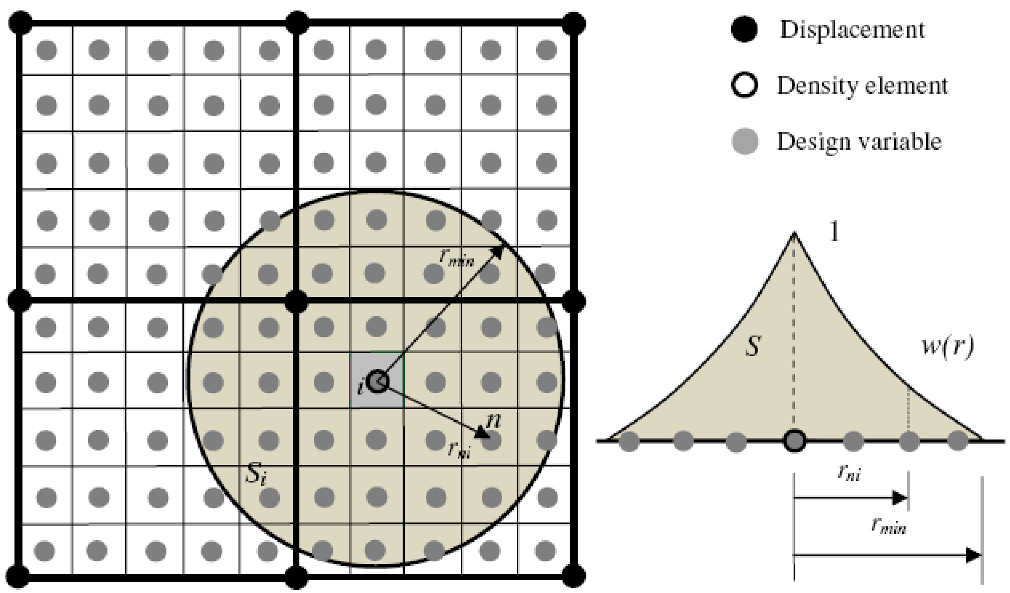
- **Compute density from design variables**
  - **Minimum length-scale (Guest *et al*. 2004, Almeida *et al*. 2009)**

$$\rho_i = f(d_n)$$

$$\rho_i = \frac{\sum_{n \in S_i} d_n w(r_{ni})}{\sum_{m \in S_i} w(r_{mi})}$$

$$w(r_{mi}) = \begin{cases} \dfrac{r_{\min} - r_{mi}}{r_{\min}} & \text{if } r_{mi} \le r_{\min} \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial \rho_i}{\partial d_n} = \frac{w(r_{ni})}{\sum_{m \in S_i} w(r_{mi})}$$



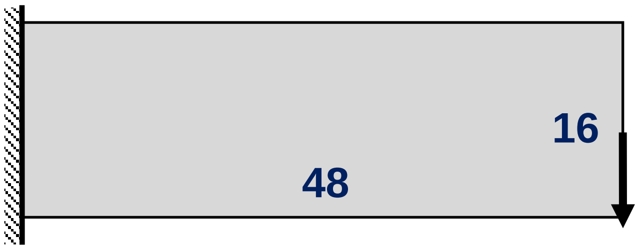
MTOP Examples: 2D Cantilever Beam

Objective: minimum compliance

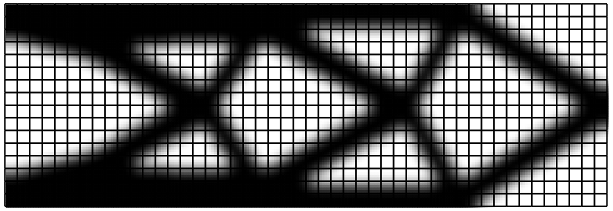
Constraint: *volfrac* = 0.5

Length scale: $r_{min} = 1.2$

Nguyen, Paulino, Song, and Le, (2010), *JSMO*

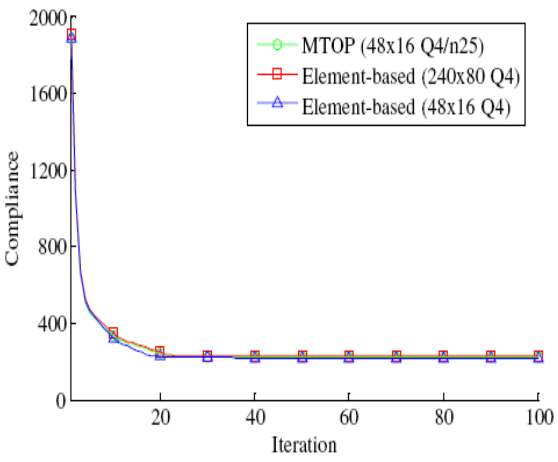


Configuration

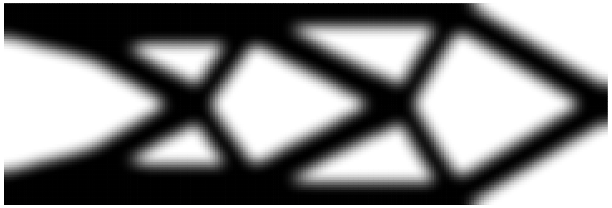


MTOP Q4/n25 FE mesh 48x16

(C=208.23)

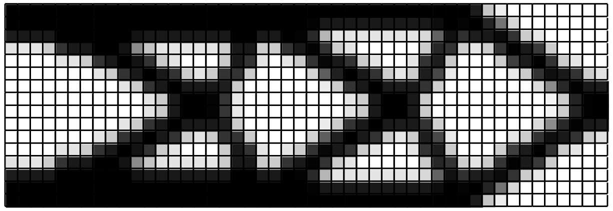


Convergence history



Q4/U FE mesh 240x80

(C=210.68)

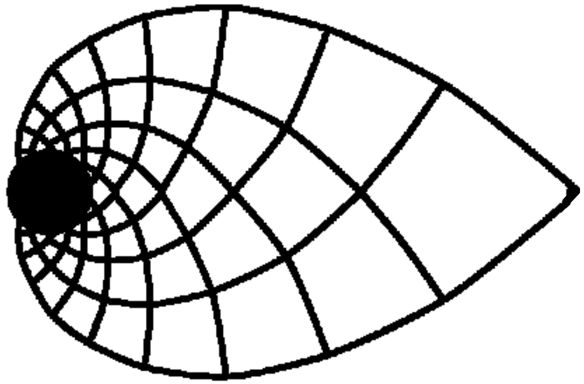


Q4/U FE mesh 48x16

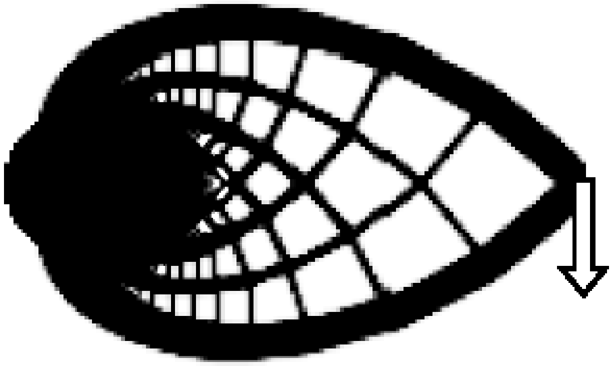
(C=205.57)

MTOP Examples: 2D Michell Truss

Domain (3:2)



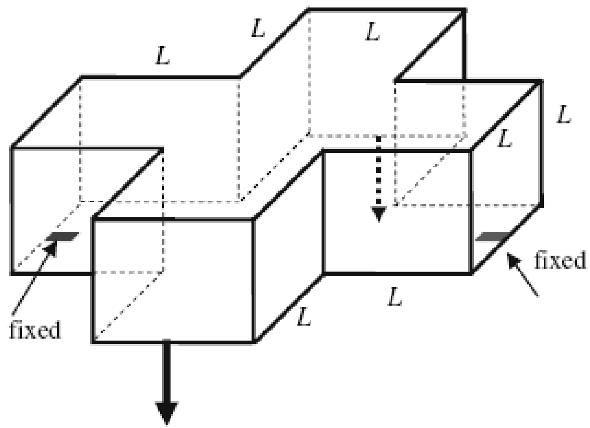
Analytical solution



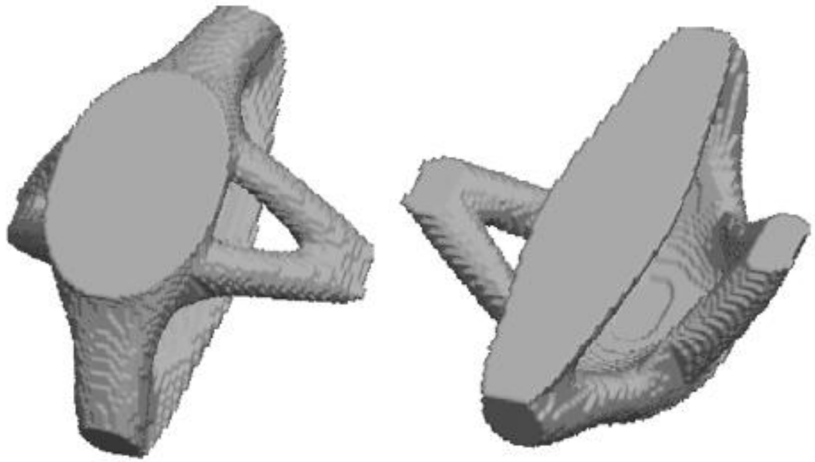
Sigmund solution (Sigmund, 2000)

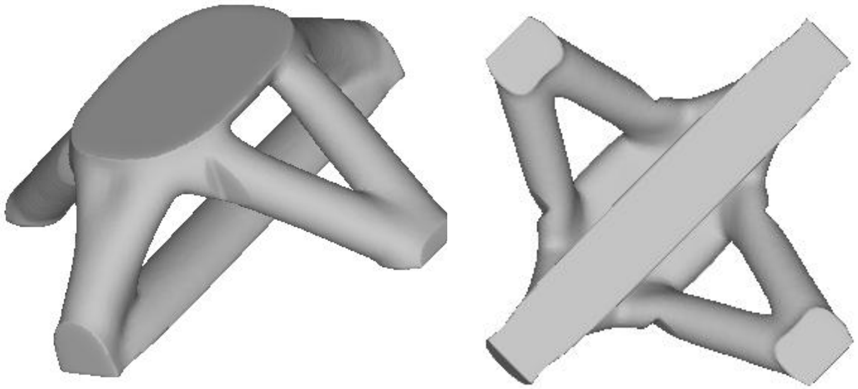
MTOP: 180x120 Q4/n25 elements

MTOP: 3D Cross-shaped Section



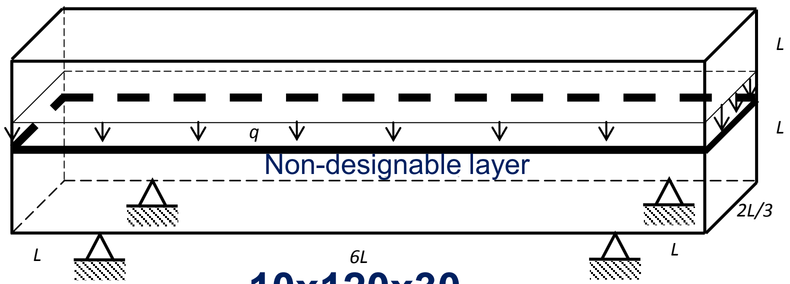
- **Borrvall & Petersson (2000)**
  - **320,000 B8/U elements**
- **MTOP**
  - **5,000 B8/n125 elements**



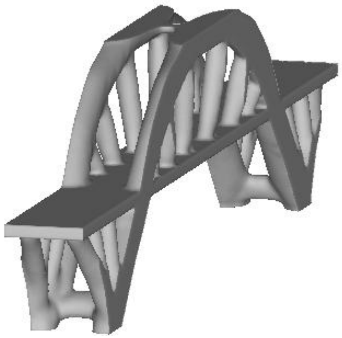


MTOP: 3D Bridge Design

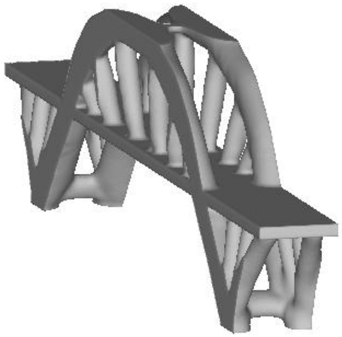
Configuration



10x120x30



MTOP B8/n125
36,000 elements



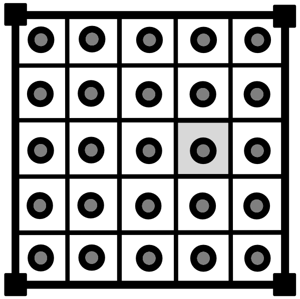
Existing design

(http://www.sellwoodbridge.org)

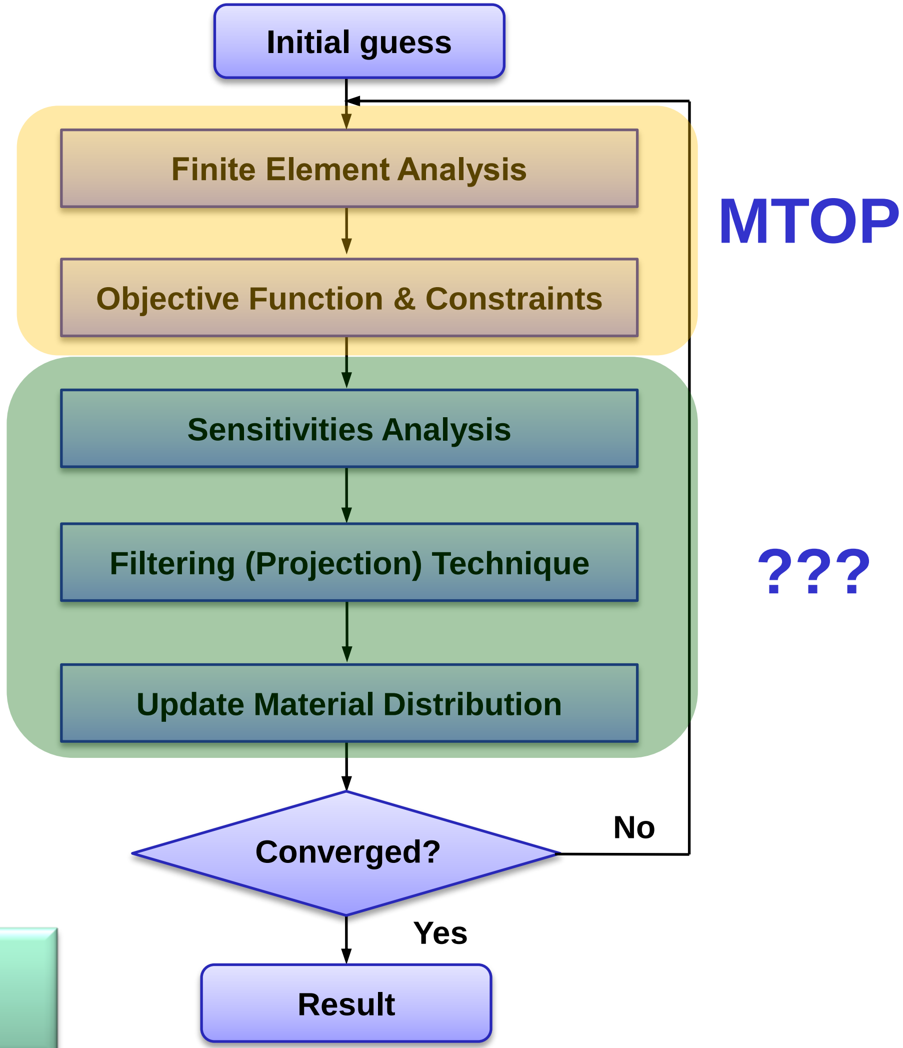
Can MTOP's efficiency be improved?

- **MTOP approach**
  - Density/design variable: same fine mesh
  - FE mesh: coarse
  - Reduce cost $\mathbf{K}(\rho)\mathbf{u}_d = \mathbf{f}$
- **Improving MTOP efficiency?**

Q4/n25

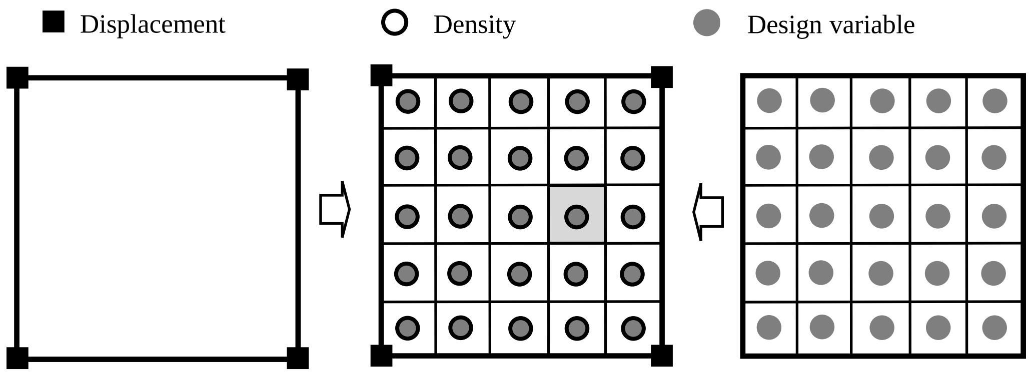


➢ Different discretizations for density & design variable?

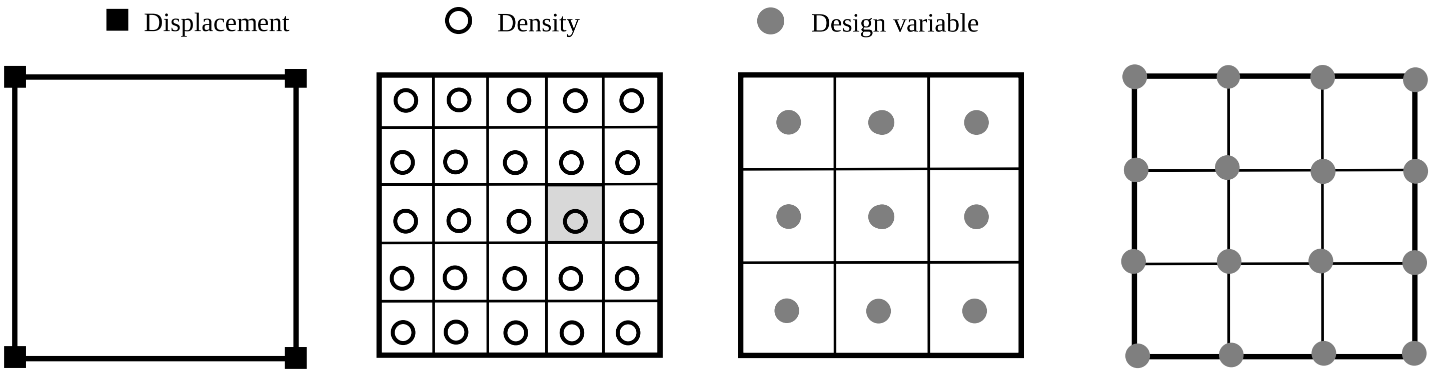


Improving Multiresolution Topology Optimization (iMTOP)

MTOP approach (Q4/n25) or (Q4/n25/d25)



Proposed iMTOP approach (Q4/n25/d9) and (Q4/n25/d16)

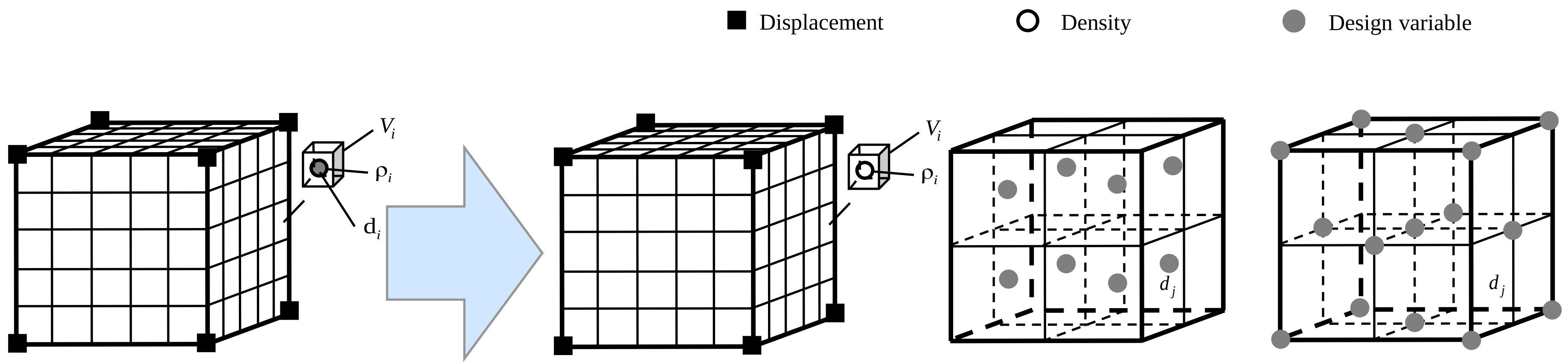


Q4/n25/d9 Q4/n25/d16

Improving Multiresolution Topology Optimization (iMTOP)

MTOP

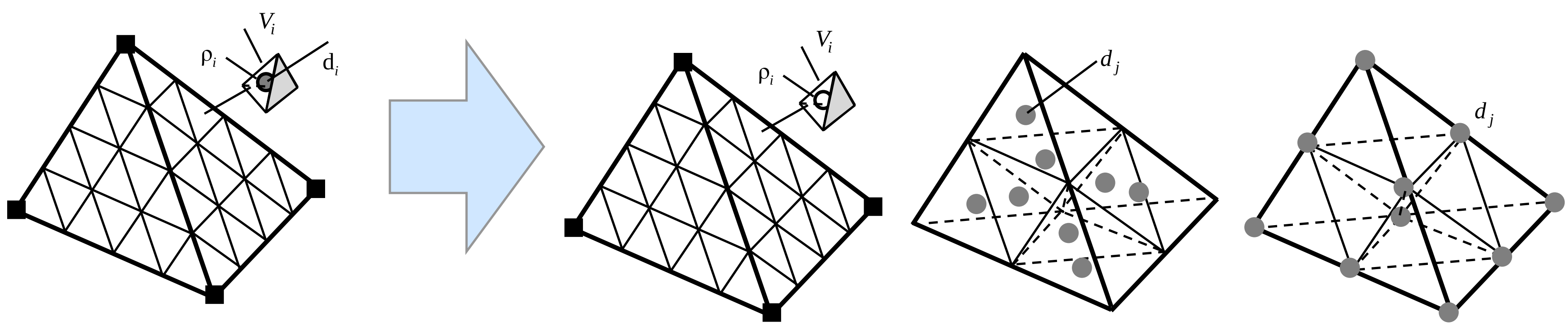
iMTOP



B8/n125

B8/n125/d8

B8/n125/d15



TET4/n64

TET4/n64/d8

TET4/n64/d10

iMTOP: Projection (Q4/n25/d9)

Compute density from design variables

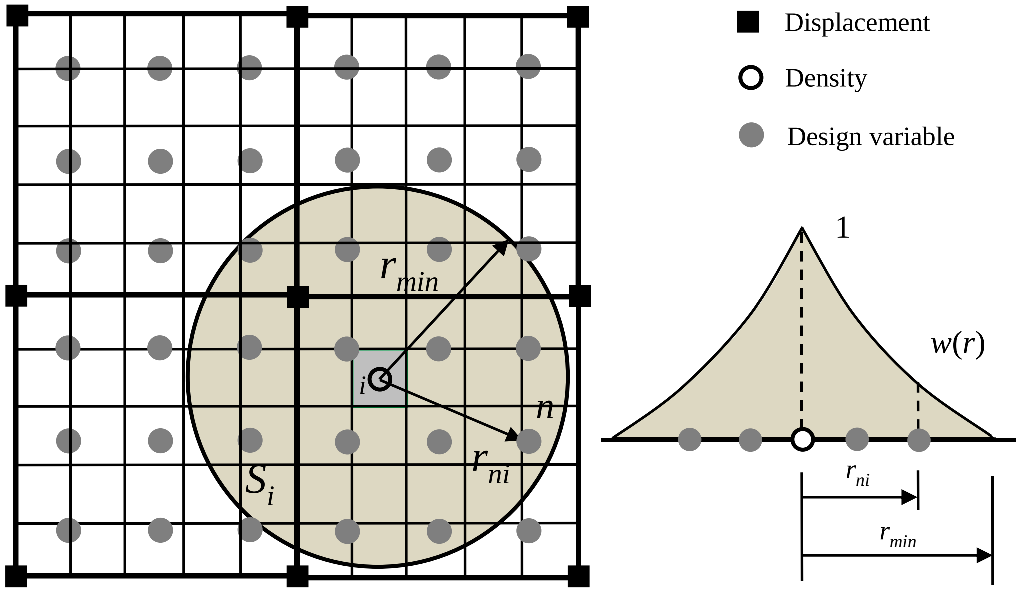
- **Minimum length-scale (Guest *et al*. 2004, Almeida *et al*. 2009)**

$$\rho_i = f(d_n)$$

$$\rho_i = \frac{\sum_{n \in S_i} d_n w(r_{ni})}{\sum_{m \in S_i} w(r_{mi})}$$

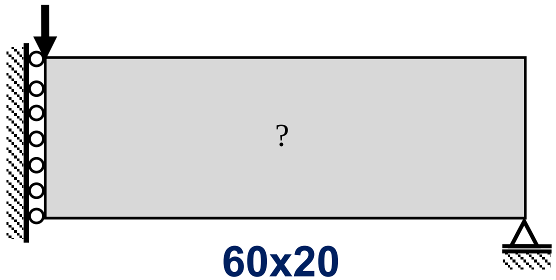
$$w(r_{mi}) = \begin{cases} \dfrac{r_{\min} - r_{mi}}{r_{\min}} & \text{if } r_{mi} \le r_{\min} \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial \rho_i}{\partial d_n} = \frac{w(r_{ni})}{\sum_{m \in S_i} w(r_{mi})}$$

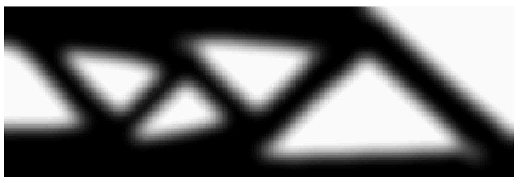


iMTOP: MBB Beam

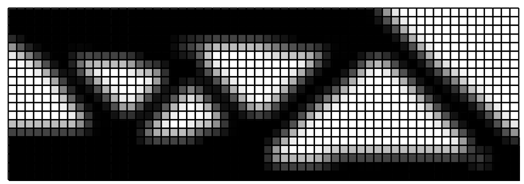
Messerschmitt-Bolkow-Blohm (MBB) beam



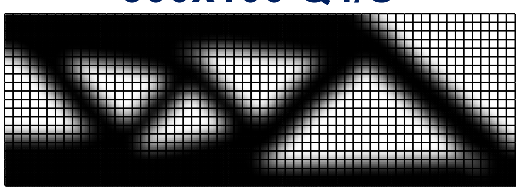
60x20



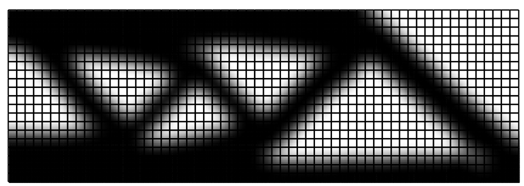
300x100 Q4/U



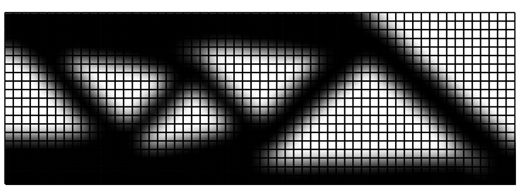
60x20 Q4/U



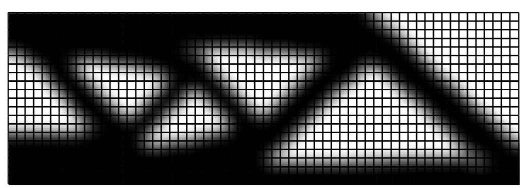
60x20 Q4/n25/d25



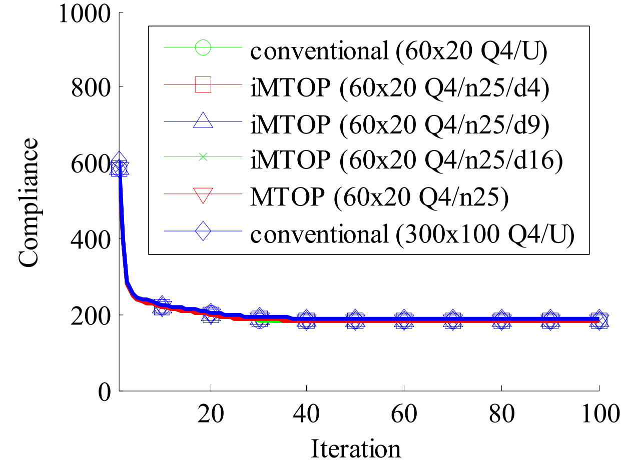
60x20 Q4/n25/d16



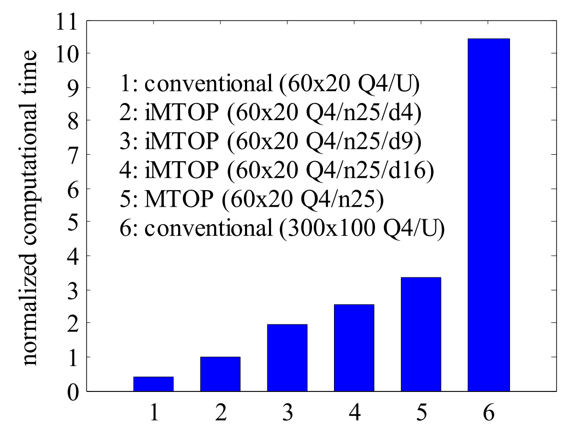
60x20 Q4/n25/d9



60x20 Q4/n25/d4



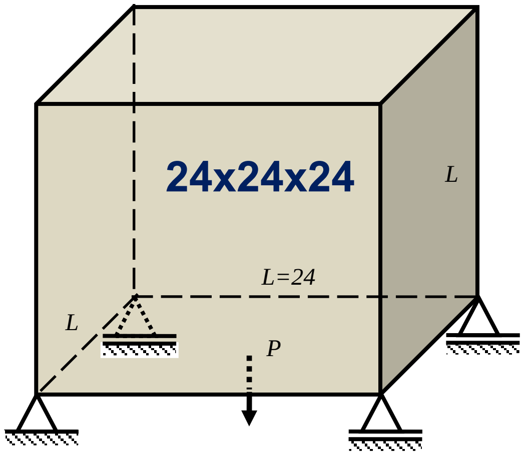
convergence

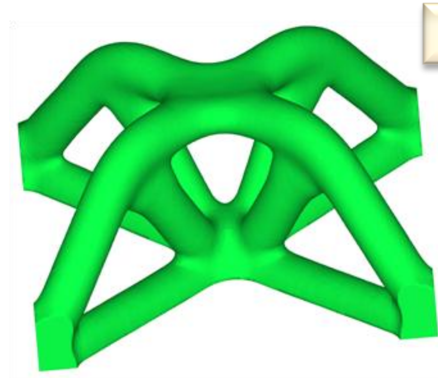


Efficiency

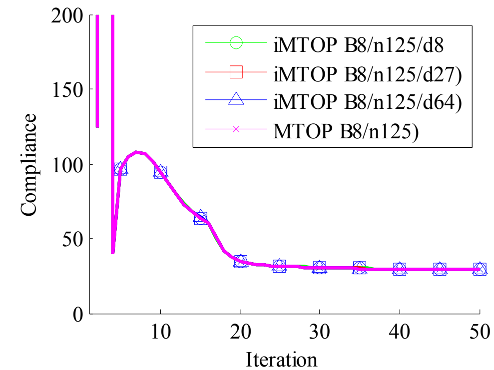
iMTOP: A Cube with Concentrated Load

Nguyen, Paulino, Song, and Le, (submitted), *IJNME*

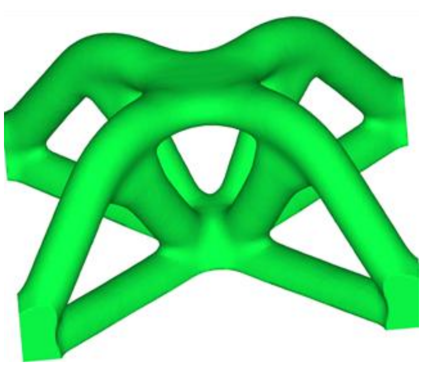




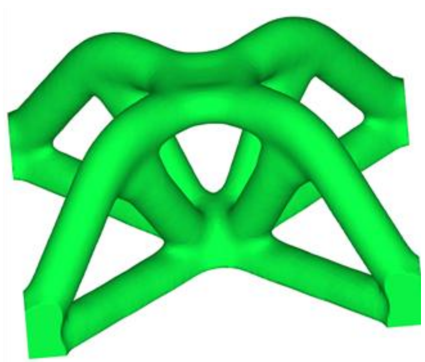
MTOP B8/n125/d125
(C=29.04)



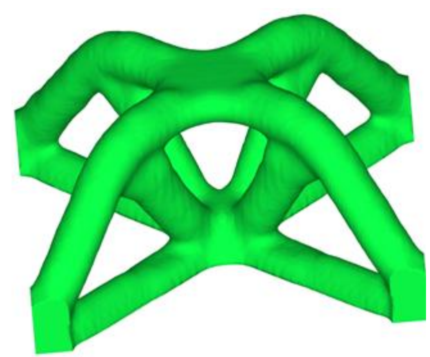
convergence



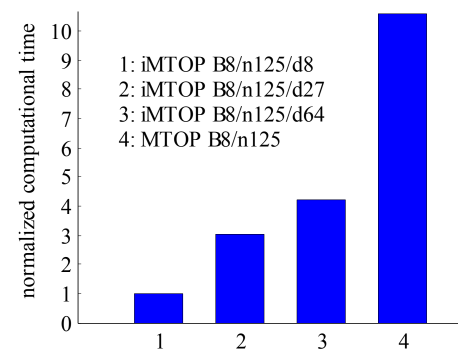
iMTOP B8/n125/d64
(C=29.06)



iMTOP B8/n125/d27
(C=29.08)



iMTOP B8/n125/d8
(C=29.33)

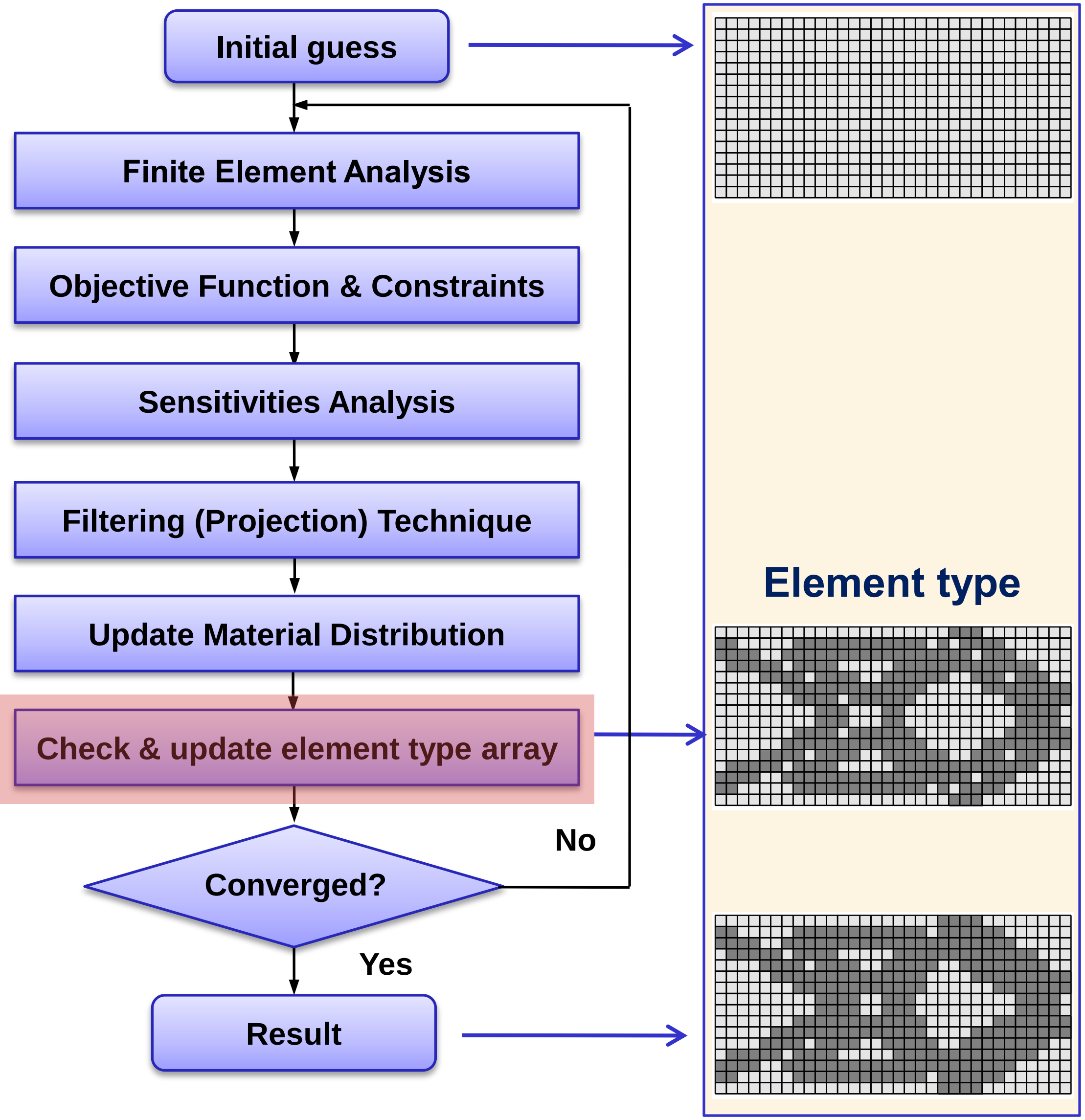


Efficiency

Adaptive MTOP

- **Why Adaptive MTOP?**
  - **Further improve the efficiency**
  - **Reduce the number of density elements and design variables during optimization process?**
- **Adaptive MTOP (e.g. Q4/U & Q4/n25/d4)**
  - **Q4/n25/d4 requires more computational cost than Q4/U**
  - **Q4/n25/d4 provides higher resolution**
  - **Use Q4/n25/d4 where and when needed only, otherwise Q4/U**
  - **Unchanged the Finite Element Mesh during optimization process**

Adaptive MTOP Procedure



Element type array
A(i,j) = 1
(all Q4/U elements)

□ Q4/U A(i,j) = 1

■ Q4/n25/d4 A(i,j) = 25

□ Q4/U → ■ Q4/n25/d4

if $\rho^L < \rho < \rho^U$

A(i,j) = 1 → A(i,j) = 25

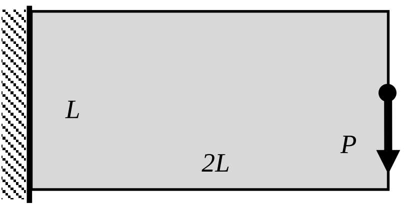
■ Q4/n25/d4 → □ Q4/U

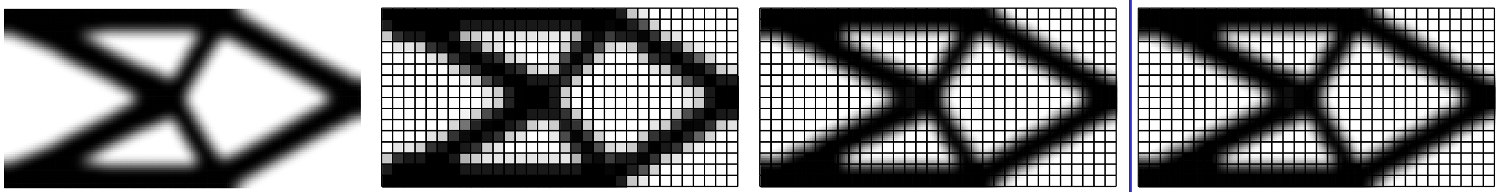
if $\rho_k < \rho^L$ or $\rho^U < \rho_k$, k=1,25

A(i,j) = 25 → A(i,j) = 1

Adaptive MTOP: 2D Cantilever

Optimal topologies by iMTOP and adaptive MTOP



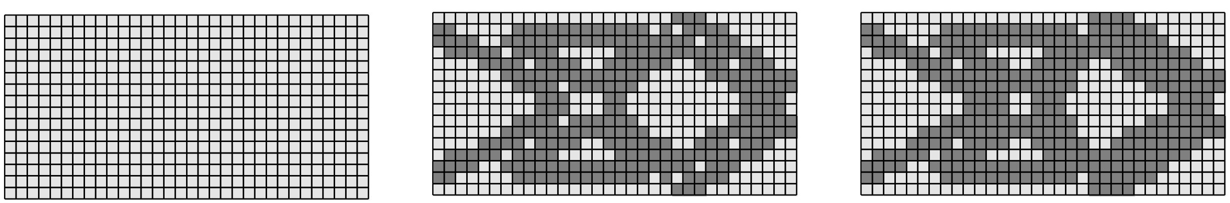


Configuration | **160x80 Q4/U** | **32x16 Q4/U** | **32x16 Q4/n25/d4** | **32x16 - adaptive** Q4/n25/d4 & Q4/U

Adaptive MTOP optimization process

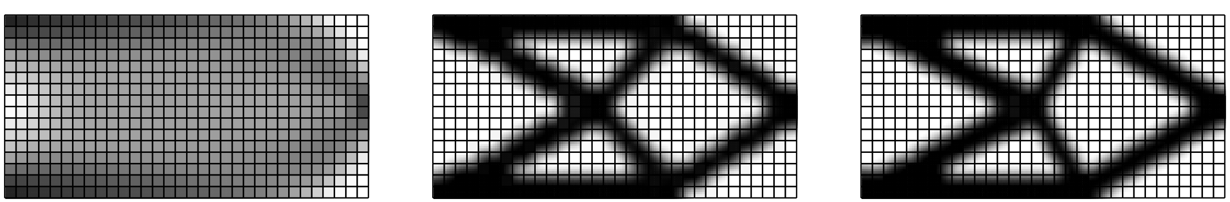
Mesh (element types)

□ Q4/U ■ Q4/n25/d4

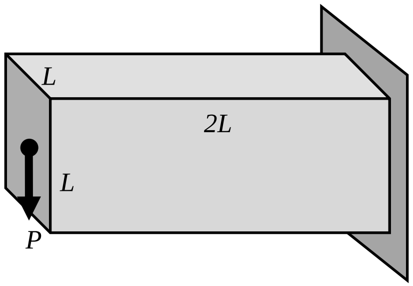


initial iteration | **intermediate iteration** | **final iteration**

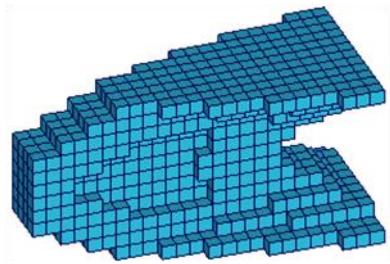
Topology



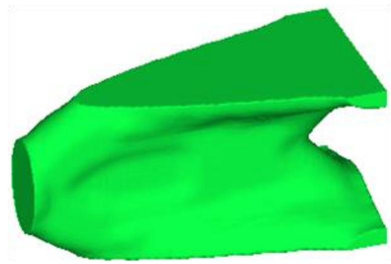
Adaptive MTOP: 3D Cantilever Beam



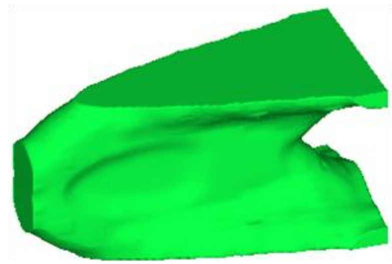
Configuration



24x12x12
B8/U

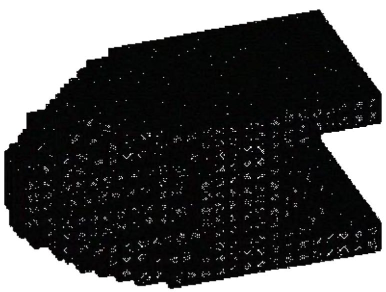


24x12x12
B8/n125/d8

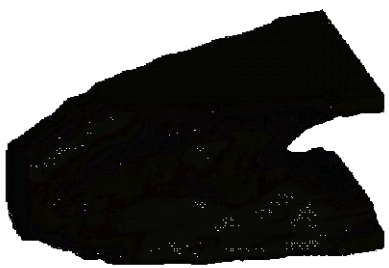


24x12x12
B8/n125/d8 & B8/U

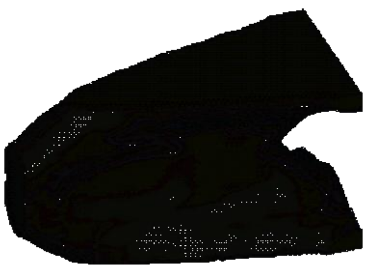
Adaptive MTOP optimization process



Initial iteration
(3,456 B8/U)



Intermediate iteration
(2,288 B8/U & 1,168 B8/n125/d8)



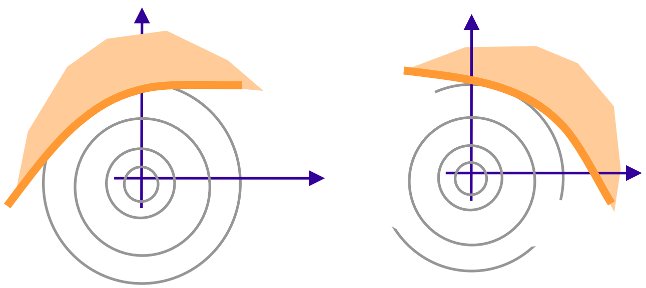
Final iteration
(2,072 B8/U & 1,384 B8/n125/d8)

(FE mesh : 24x12x12 unchanged)

System Reliability-Based Design/Topology Optimization

RBDO Formulation

- **Component RBDO**

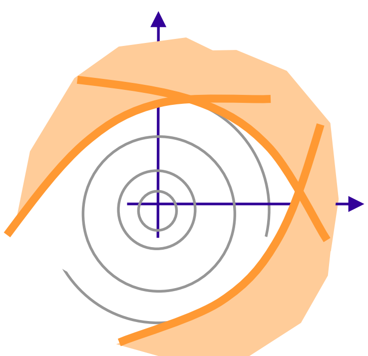


$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X}} \quad f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$

$$s.t. \qquad P\;\; g_i(\mathbf{d},\mathbf{X}) \le 0 \;\; \le P_i^t \qquad i=1,\ldots,n$$

$$\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \qquad \boldsymbol{\mu}_\mathbf{X}^L \le \boldsymbol{\mu}_\mathbf{X} \le \boldsymbol{\mu}_\mathbf{X}^U$$

- **System RBDO**



$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X}} \quad f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$

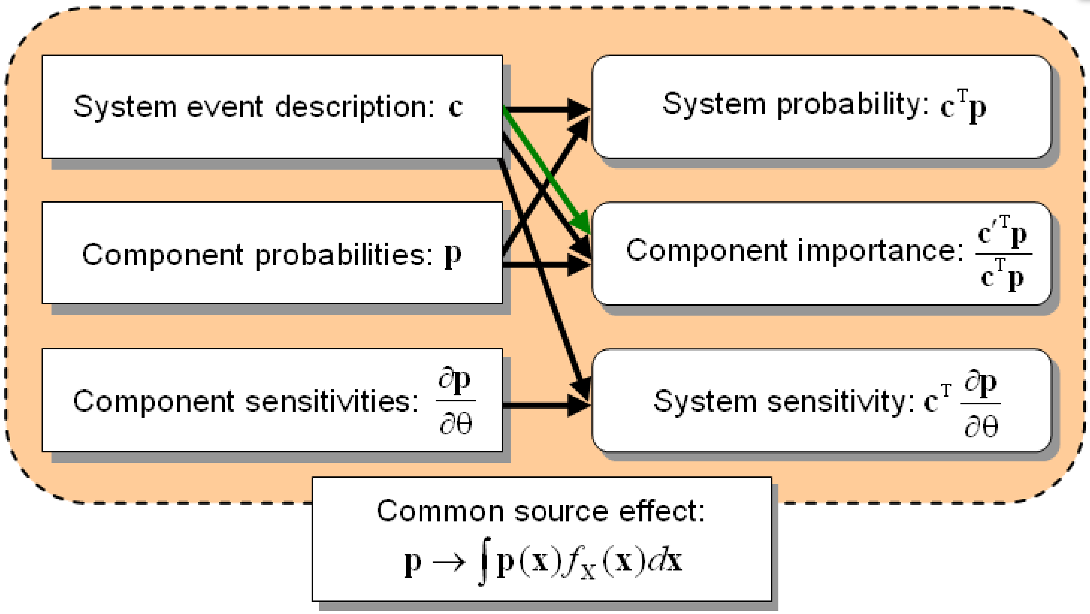
$$s.t. \qquad P(E_{sys}) = P\left[\bigcup_k \bigcap_{i\in C_k} g_i(\mathbf{d},\mathbf{X}) \le 0\right] \le P_{sys}^t$$

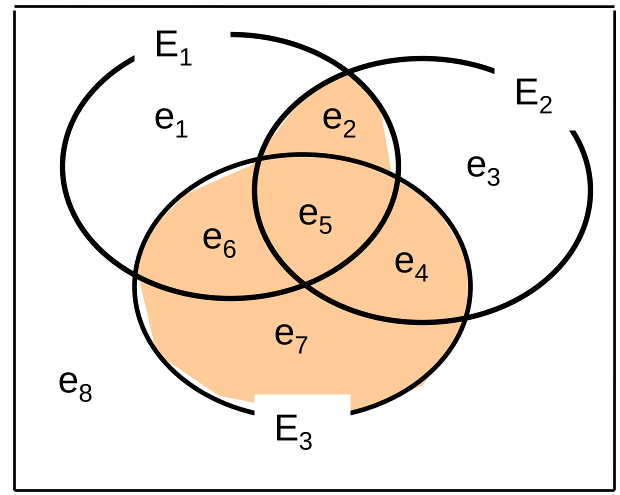
?

$$\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \qquad \boldsymbol{\mu}_\mathbf{X}^L \le \boldsymbol{\mu}_\mathbf{X} \le \boldsymbol{\mu}_\mathbf{X}^U$$

Matrix-based System Reliability (MSR) Method

Song and Kang, (2009); *Structural Safety*



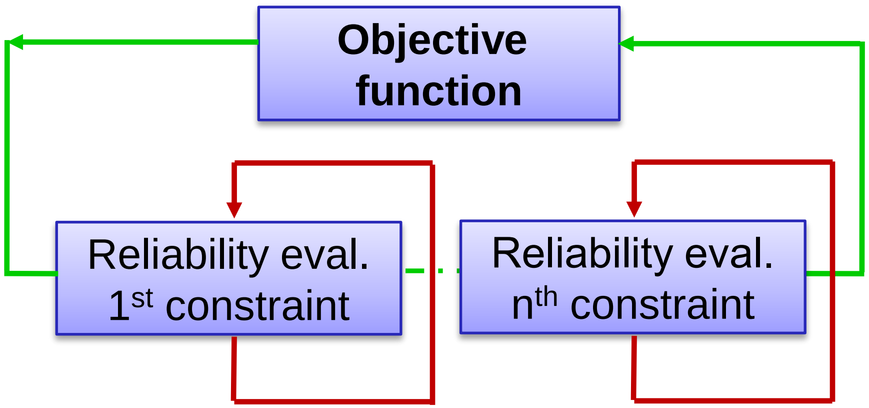


mutually exclusive and collectively exhaustive events (MECE)

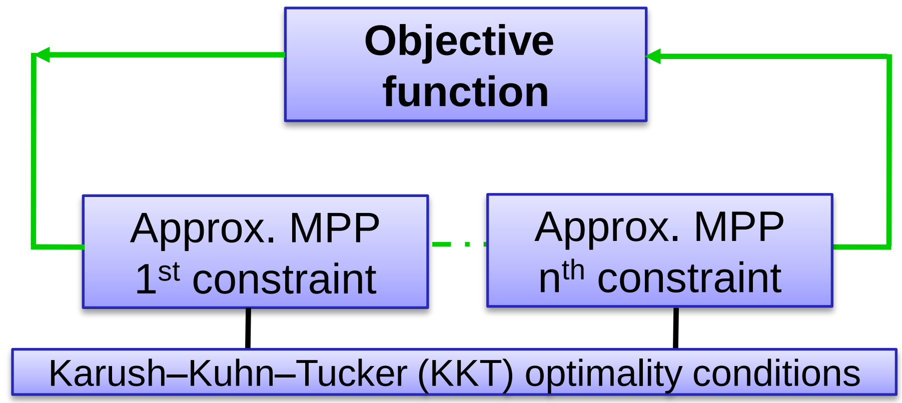
- **Convenient**: matrix-based procedures for c and p; easy SRA calculation (inner product)
- **General**: uniform application to series, parallel, and any general systems
- **Flexible**: inequality-type information; incomplete information ("LP bounds" method)
- **Efficient**: no need to re-compute "p"; replace "c" for SRA of a new event
- **Common Source Effect**: can account for statistical dependence between components
- **Decision Support**: parameter sensitivities, component importance measure; inferences

Proposed approach: SRBDO using MSR

- **Adopt a single-loop RBDO (Liang et al. 2007)**
  - **Double-loop RBDO**



  - **Single-loop RBDO**



- **Use MSR method to compute P_{sys} and its gradients**

$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{X},P_1^t,\ldots,P_n^t} \quad f(\mathbf{d},\boldsymbol{\mu}_\mathbf{X})$$

$$s.t. \quad g_i(\mathbf{d},\mathbf{X}(\mathbf{U}_i^t)) \geq 0 \quad i=1,\ldots,n$$

Single-loop PMA

$$P_{sys} = \begin{cases} \int_\mathbf{s} \mathbf{c}^\mathrm{T}\mathbf{p}(\mathbf{s}) f_\mathbf{S}(\mathbf{s}) ds \leq P_{sys}^t & \text{dependent} \\ \mathbf{c}^\mathrm{T}\mathbf{p} \leq P_{sys}^t & \text{indepdendent} \end{cases}$$

MSR method

Proposed approach: SRBDO using MSR (contd.)

- **Sensitivity w.r.t. design variables $\theta = \{\mathbf{d}, \boldsymbol{\mu}_{\mathbf{x}}\}$**

$$\frac{\partial P_{\text{sys}}}{\partial \theta} = \int_{\mathbf{s}} \mathbf{c}^{\mathrm{T}} \frac{\partial \mathbf{p}(\mathbf{s})}{\partial \theta} f_{\mathbf{S}}(\mathbf{s}) d\mathbf{s}$$

$$\frac{\partial \mathbf{p}(\mathbf{s})}{\partial \theta} = \left[\mathbf{p}(\mathbf{s})^{\langle 1 \rangle} \ \mathbf{p}(\mathbf{s})^{\langle 2 \rangle} \ldots \mathbf{p}(\mathbf{s})^{\langle n \rangle} \right] \frac{\partial \mathbf{P}(\mathbf{s})}{\partial \theta} = \hat{\mathbf{P}}(\mathbf{s}) \frac{\partial \mathbf{P}(\mathbf{s})}{\partial \theta}$$

$$\mathbf{P}(\mathbf{s}) = [P_1(\mathbf{s}) \ P_2(\mathbf{s}) \ \cdots P_n(\mathbf{s})]^{\mathrm{T}}$$

→ Use probabilities and sensitivities by component reliability analysis (FORM)

- **Sensitivity w.r.t. component failure probability P_i^t**

$$\frac{\partial P_i(\mathbf{s})}{\partial P_i} = \frac{\partial P_i(\mathbf{s})}{\partial \beta_i} \cdot \frac{\partial \beta_i}{\partial P_i} = -\frac{\partial P_i(\mathbf{s})}{\partial \beta_i} \cdot \frac{1}{\varphi(-\beta_i)}$$

SRBDO of Truss System

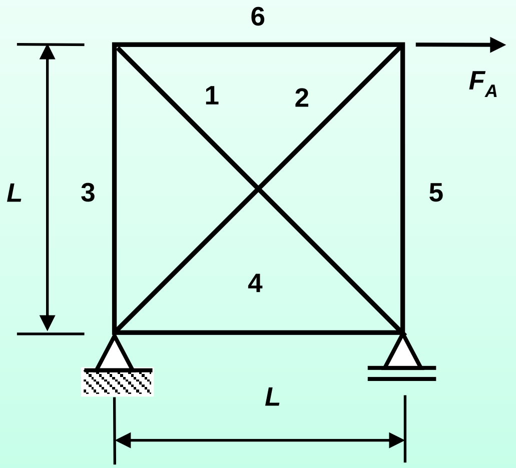
$$\min_{\mathbf{d}=\{A_1,\ldots,A_6\}} f(\mathbf{d}) = \sqrt{2}(A_1 + A_2) + A_3 + A_4 + A_5 + A_6$$

$$s.t. \quad P_{sys} = P\left[\bigcup_{k=1}^{15}\bigcap_{i\in C_k} g_i(\mathbf{d},\mathbf{X}) \le 0\right] \le P_{sys}^t = 0.001$$

$$g_i(\mathbf{d},\mathbf{X}) = \begin{cases} A_i F_i - 0.707 F_A & i = 1,2 \\ A_i F_i - 0.500 F_A & i = 3,\ldots,6 \end{cases}$$

$$A_1, A_2, A_3, A_4, A_5, A_6 \ge 0$$

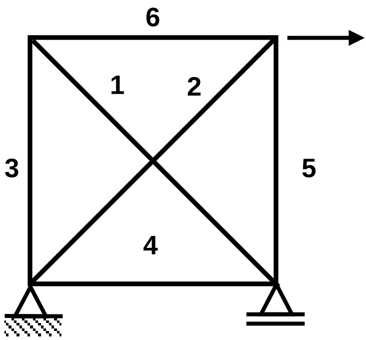
- → **Minimize total weight of the system**
- → **Definition of system failure: at least two members fail (cut-set systems): effects of load re-distributions NOT considered**



Random Variables (Gaussian distribution)	Mean	Std Dev
Member strength F_i , i=1,..6 (Mpa)	745	62
Applied load F_A (kN)	4450	45

SRBDO of Truss System (contd.)

Members	Area: A_i ($\times 10^{-3}$ mm^2)		Reliability Index: β_i	
	McDonald & Mahadevan	SRBDO/MSR	McDonald & Mahadevan	SRBDO/MSR
1	18.43	17.89	2.89	2.67
2	18.27	17.89	2.83	2.67
3	13.51	13.20	3.16	2.99
4	13.44	13.20	3.12	2.99
5	13.33	13.20	3.06	2.99
6	13.09	13.20	2.92	2.99
$f(x)$	105.24 **>**	103.36		



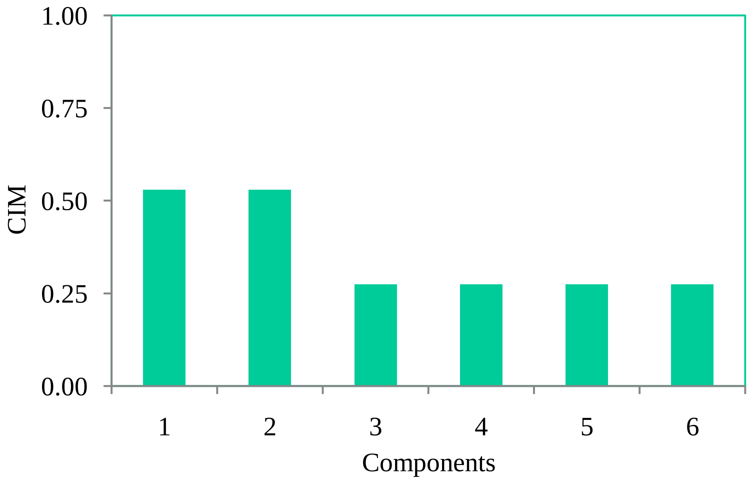
- Better optimal design (i.e. less total weight) and symmetric results
- Monte Carlo simulations (c.o.v. = 0.03, 10^6 times) on the system failure probability: P_{sys} = 0.00107 (cf. MSR gives 0.001)

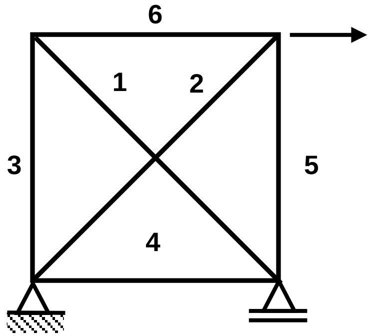
SRBDO of Truss System (contd.)

- **Conditional probability Importance Measure (CIM)**

$$\mathrm{CIM}_i = P(E_i | E_{sys}) = \frac{P(E_i E_{sys})}{P(E_{sys})} = \frac{\mathbf{c}'^{\mathrm{T}}\mathbf{p}}{\mathbf{c}^{\mathrm{T}}\mathbf{p}}$$

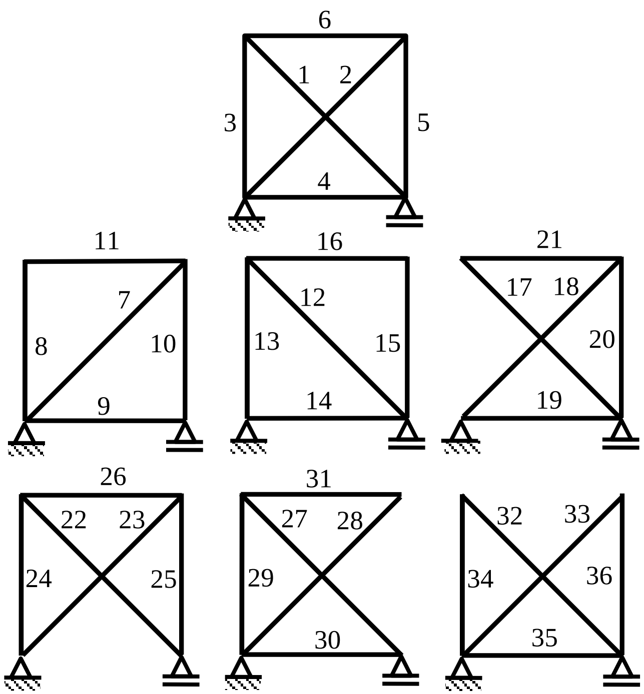
  - **Relative contribution of components to the system failure probability (can be computed efficiently by MSR method)**

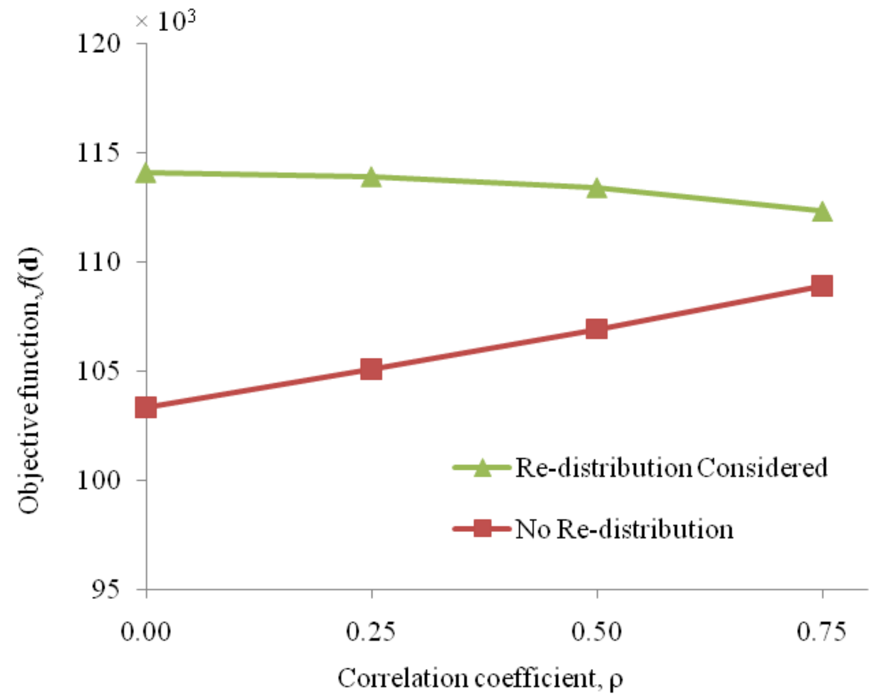




SRBDO of Truss System (contd.)

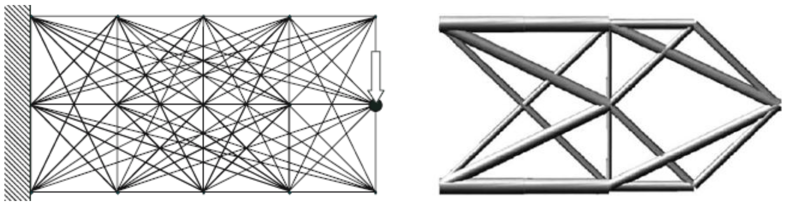
- **Effects of load re-distributions (sequential failures)**
- **Effects of correlation between random variables and between components**





Existing SRBTO Approaches

- **Discrete structures**
  - **Mogami *et al*. (2006)**
  - **Truss examples**

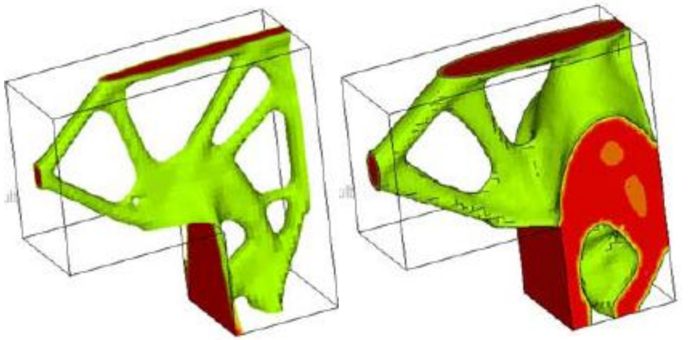


Mogami *et al*. (2006), *JSMO*

- **Continuum structures**
  - **Silvia *et al*. (2010)**
  - **Limit-states: statistically independent**

$$P(E_{sys}) = P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i) - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} P(E_i E_j) + \cdots + (-1)^{n-1} P(E_1 E_2 \cdots E_n)$$

$$P(E_1E_2E_3) = P(E_1)P(E_2)P(E_3)$$



Silvia *et al*. (2010), JSMO

Objective: SRBTO for continuum structures with dependent limit-states?

Proposed Approach: SORM-based RBTO

Enhance the accuracy in RBTO

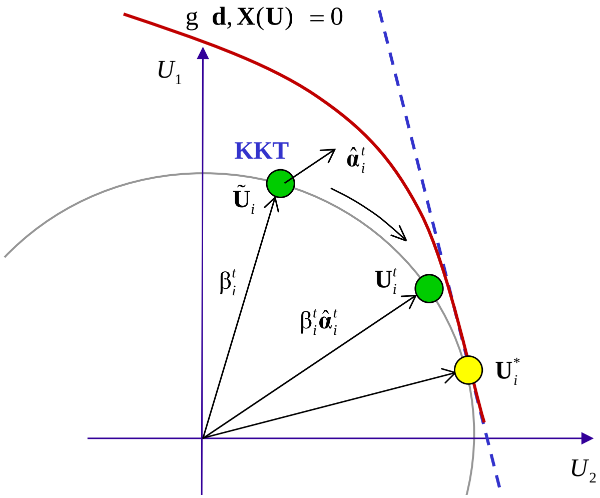
- First-Order Reliability Method (FORM) → inaccurate for nonlinear limit-states
- Propose to use Second-Order Reliability Method (SORM) to improve the accuracy

SORM-based CRBTO

$$\beta_i^{t(k)} = \beta_i^t$$

At the k-th step

$$\beta_i^{t(k)} = \frac{\beta_i^t}{\beta_i^{t(k-1)(SORM)}} \times \beta_i^{t(k-1)}$$



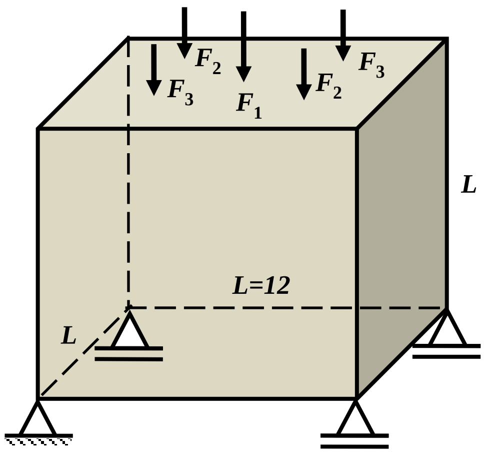
SORM-based SRBTO

$$P(E_{sys};\mathbf{P}^t) = \begin{cases} \int_{\mathbf{s}} \mathbf{c}^{\mathrm{T}}\mathbf{p}^t(\mathbf{s}) f_{\mathbf{S}}(\mathbf{s}) d\mathbf{s} \le P_{sys}^t \\ \mathbf{c}^{\mathrm{T}}\mathbf{p}^t \le P_{sys}^t \end{cases}$$

At the k-th step

$$P(E_{sys};\mathbf{P}^{t(SORM)}) = \begin{cases} \int_{\mathbf{s}} \mathbf{c}^{\mathrm{T}}\mathbf{p}^{t(SORM)}(\mathbf{s}) f_{\mathbf{S}}(\mathbf{s}) d\mathbf{s} \le P_{sys}^t \\ \mathbf{c}^{\mathrm{T}}\mathbf{p}^{t(SORM)} \le P_{sys}^t \end{cases}$$

SRBTO of a Stool

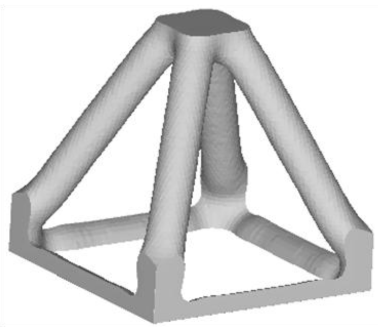
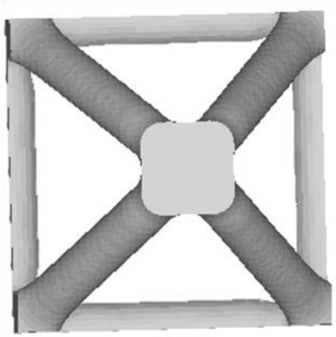


- **Objective: minimize volume $V(\rho)$**
- **Limit-states:** $g_i(\boldsymbol{\rho}, \bar{\mathbf{F}}_i) = 120 - C_i(\boldsymbol{\rho}, \bar{\mathbf{F}}_i), \quad i = 1,2$
- **Random loads: : F ~ (F_1,F_2,F_3) ~**

  N(100,10), N(0,30), N(0,40)
- **Load cases:** $\bar{\mathbf{F}}_1 = (F_1, F_2), \quad \bar{\mathbf{F}}_2 = (F_1, F_3)$

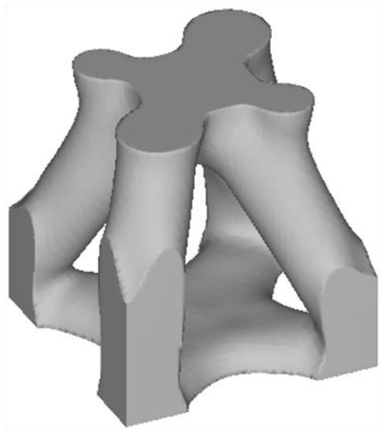
- **Constraints**
  - **Deterministic TO (DTO):** $g_i(\boldsymbol{\rho}, \mathbf{f}) > 0, \quad i = 1,2$
  - **Component RBTO (CRBTO):** $P(g_i(\boldsymbol{\rho}, \bar{\mathbf{F}}_i) \leq 0) \leq P_i^t, \quad i = 1,2$
  - **System RBTO (SRBTO):** $P(\cup \cap g_i(\boldsymbol{\rho}, \bar{\mathbf{F}}_i) \leq 0) \leq P_{sys}^t$

Optimal Topologies

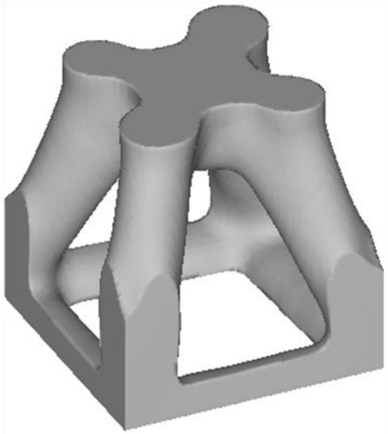
volfrac = 6.3%

DTO



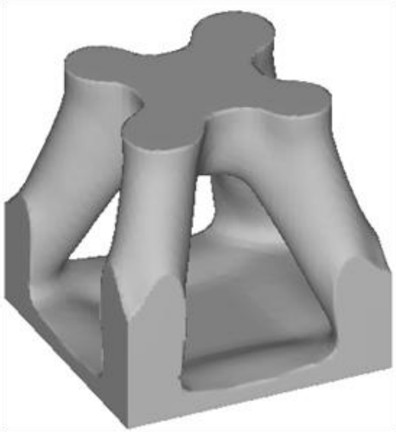
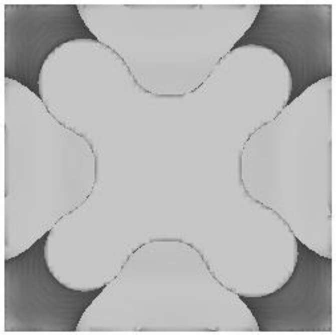

volfrac = 24.4%
(σ_F=10)

CRBTO




volfrac =22.3%
(σ_{F1}=10)

SRBTO

volfrac =23.9%
(σ_{F1}=20)

SRBTO

Improve Accuracy by Second-Order Reliability Method

Component RBTO

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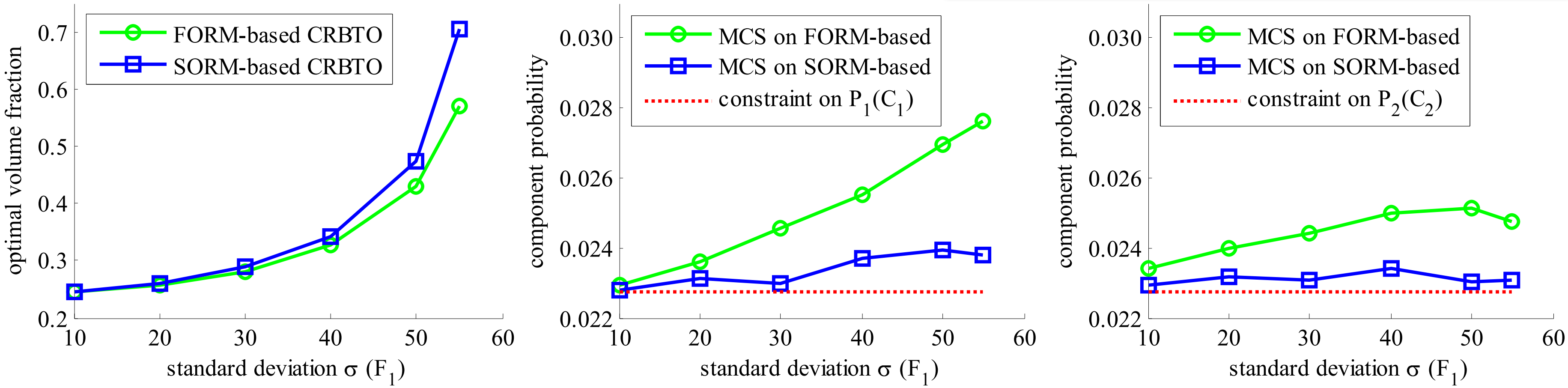
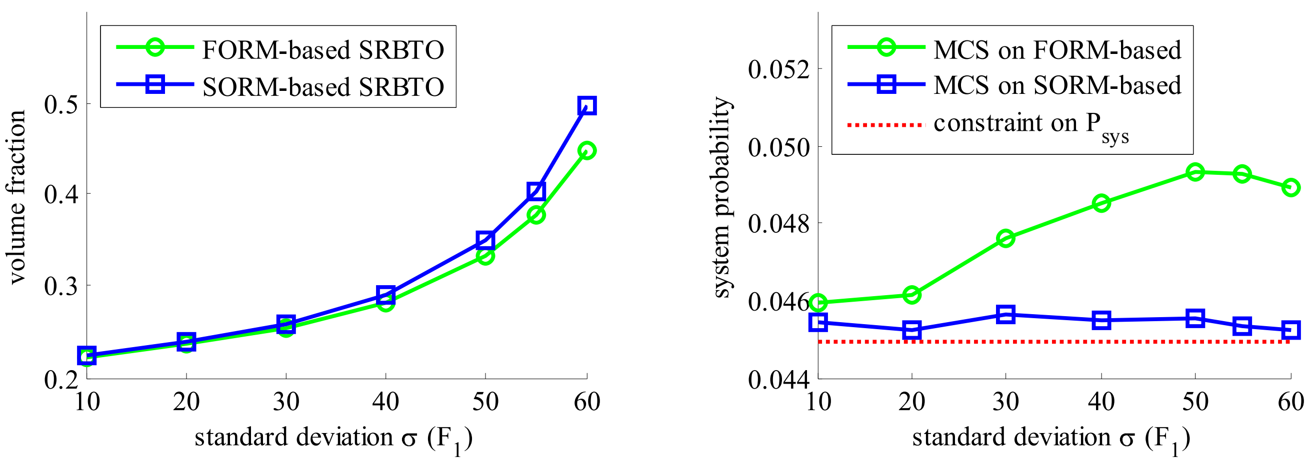
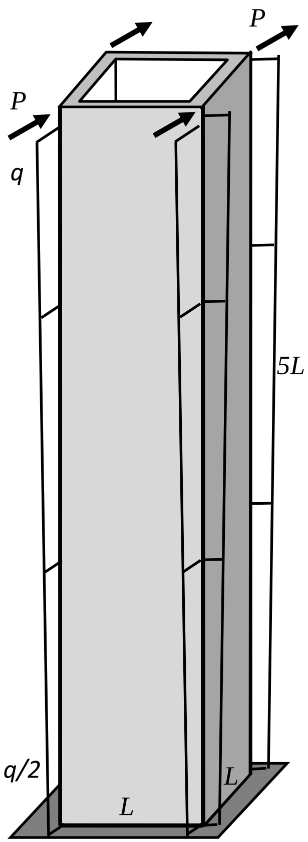
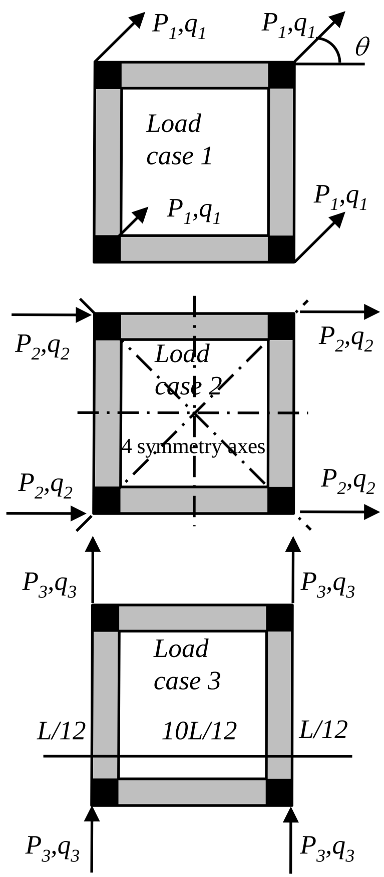


System RBTO



SRBTO of a Building Core



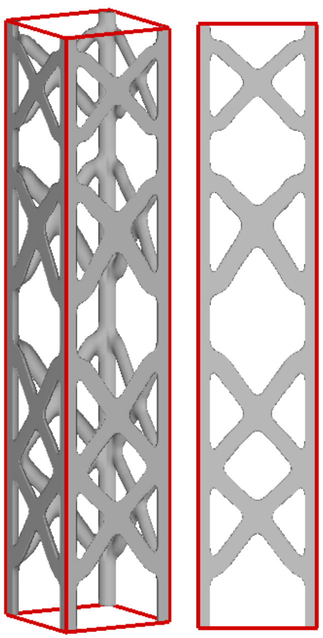


- **Objective: minimize volume $V(\rho)$**
- **Limit-states:** $g_i(\boldsymbol{\rho}, \bar{\mathbf{F}}_i) = C_i^0 - C_i(\boldsymbol{\rho}, \bar{\mathbf{F}}_i)$
- **Random loads: F ~ $(P_1, P_2, P_3, q_1, q_2, q_3)$**
- **Load cases:** $\bar{\mathbf{F}}_i = (P_i, q_i)$

Load Cases	*P* mean	*P* c.o.v	*q* (at top) mean	*q* (at top) c.o.v	C_i^t
Case 1	70.71	0.30	2.82	0.15	250
Case 2	50.00	0.15	2.00	0.30	125
Case 3	50.00	0.20	2.00	0.15	125

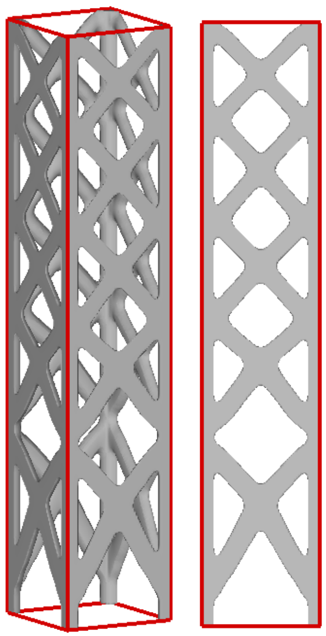
Optimal Topologies of the Building Core

Nguyen, Song, and Paulino, (submitted), *JSMO*



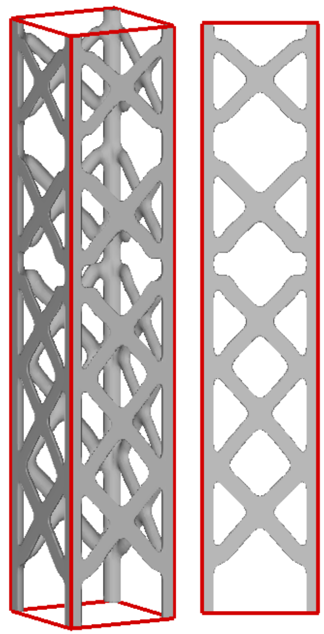
volfrac **= 21.93%**

DTO



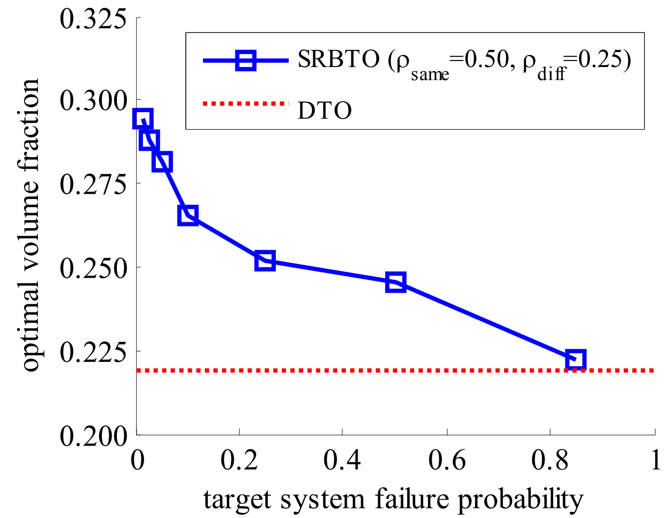
volfrac **=28.15%**
(P_{sys}=0.05)

SRBTO



volfrac **=22.25%**
(P_{sys}=0.85)

SRBTO



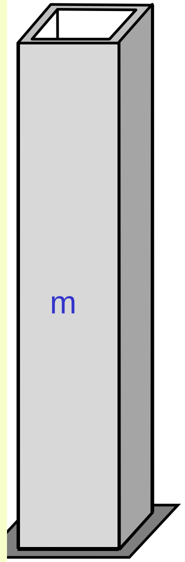
volfrac **v.s** P_{sys}

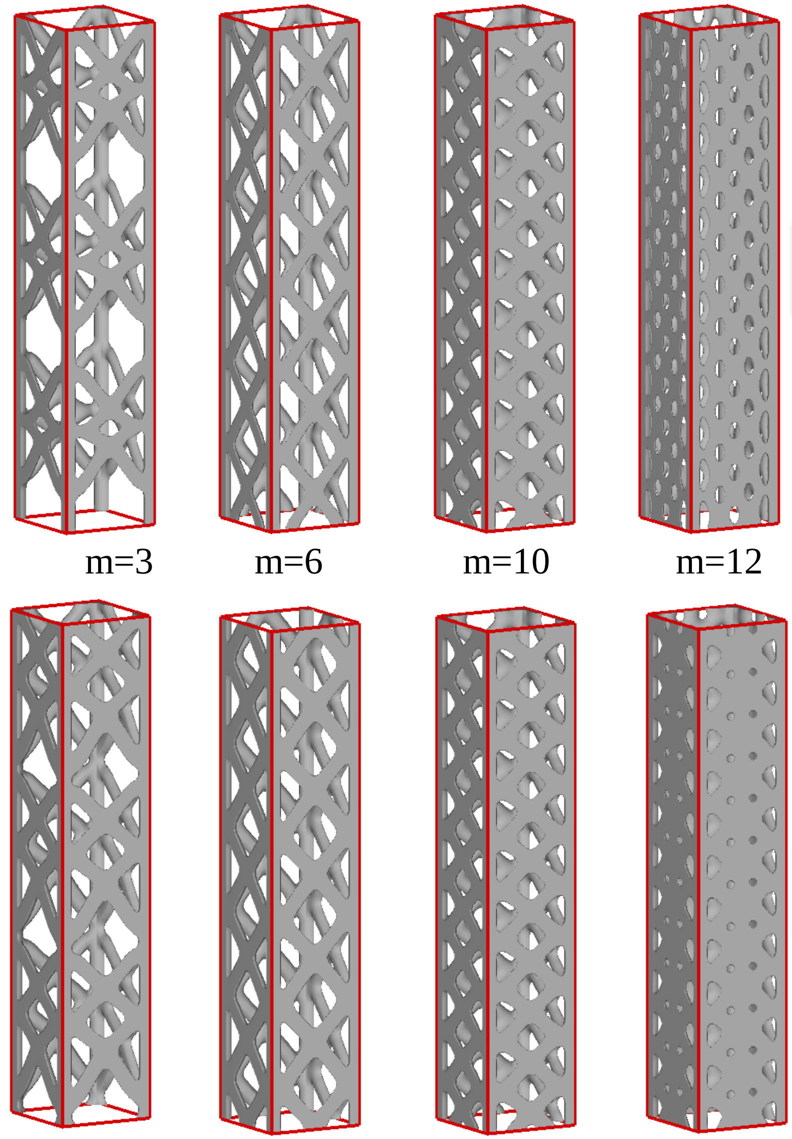
Probabilities: SRBTO/MSR v.s MCS

		P_1	P_2	P_3	P_{sys}
ρ_{same} = 0.5, ρ_{diff} = 0.25	SRBTO	0.02731	0.02088	0.00539	0.05000
	MCS	0.02747	0.02101	0.00542	0.05023
ρ_{same} = 0.5, ρ_{diff} = 0.25	SRBTO	0.26940	0.25973	0.20818	**0.50000**
	MCS	0.26977	0.26006	0.20800	0.50008
ρ_{same} = **0.9**, ρ_{diff} = **0.45**	SRBTO	0.02812	0.02227	0.00625	0.05000
	MCS	0.02816	0.02242	0.00638	0.05017

Building Core with Pattern Repetition

http://www.som.com





DTO

SRBTO

Summary & Conclusions

- **MTOP & iMTOP:**
  - Use three distinct displacement, density, and design variable fields
  - Improve efficiency, apply to large-scale problems
- **Adaptive MTOP:**
  - Use MTOP and iMTOP elements where and when needed
  - Reduce the number of density elements and design variables
- **SRBDO/MSR:**
  - Apply to general system including link-set, cut-set
  - Address dependence between limit-states, provide sensitivity
- **SRBTO**
  - Propose accurate single-loop SORM-based CRBTO & SRBTO approaches
  - Include pattern repetition constraints

Future Research Topics

- Optimal locations of design variables in MTOP
- MTOP approach in nonlinear and stress-based problems
- MTOP using Krylov subspace methods and recycling
- SRBDO with multi-scale MSR approach
- SRBDO with mixed continuous-discrete random variables

Contributions

- Nguyen, T. H., Paulino, G. H., Song, J., Le, C. H., (2010). "A computational paradigm for multiresolution topology optimization (MTOP)." *Structural and Multidisciplinary Optimization* 41(4): 525-539.

- Nguyen, T. H., Song, J., Paulino, G. H., (2010). "Single-loop system reliability-based design optimization using matrix-based system reliability method: theory and applications." *Journal of Mechanical Design* 132(1): 011005.

- Sutradhar, A., Paulino, G. H., Miller, M. J., Nguyen, T. H., (2010). "Topological optimization for designing patient-specific large craniofacial segmental bone replacements." *Proceedings of the National Academy of Sciences* 107(30) 13222-13227.

- Nguyen, T. H., Paulino, G. H., Song, J., Le, C. H., "Improving multiresolution topology optimization via multiple discretizations." *International Journal for Numerical Methods in Engineering* (submitted).

- Nguyen, T. H., Song, J., Paulino, G. H., "Single-loop system reliability-based topology optimization considering statistical dependence between limit states." *Structural and Multidisciplinary Optimization* (submitted).

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