STRUCTURAL OPTIMIZATION:
FROM CONTINUUM AND GROUND STRUCTURES TO ADDITIVE MANUFACTURING

TOMAS ZEGARD

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PROF GLAUCIO H. PAULINO (CHAIR AND DIRECTOR)
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PROF PAOLO GARDONI (CEE UIUC)
ARKADIUSZ MAZUREK (SOM)
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ILLINOIS
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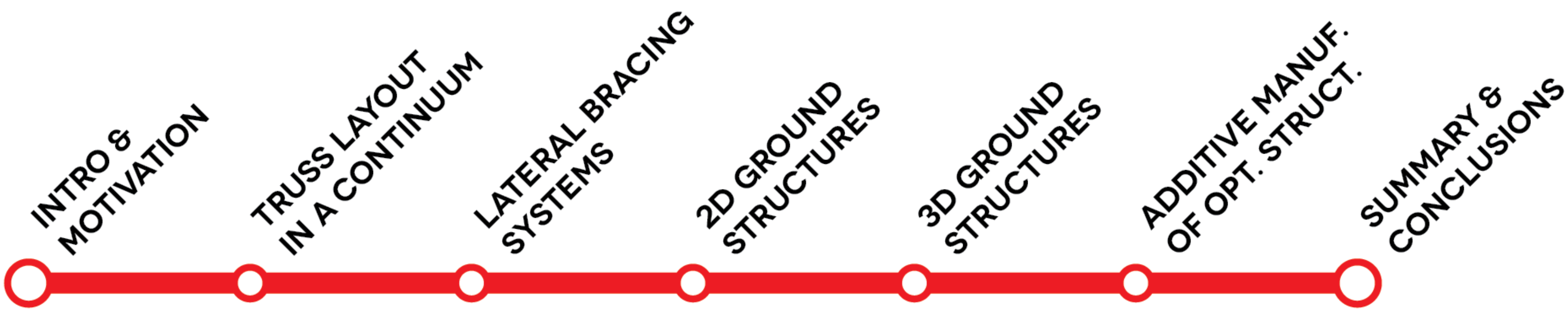
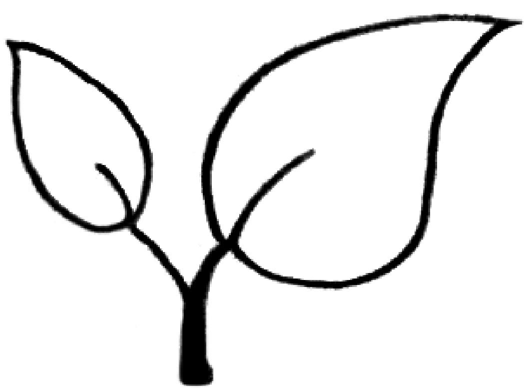

INTRO &
MOTIVATION
TRUSS LAYOUT
IN A CONTINUUM
LATERAL BRACING
SYSTEMS
2D GROUND
STRUCTURES
3D GROUND
STRUCTURES
ADDITIVE MANUF.
OF OPT. STRUCT.
SUMMARY &
CONCLUSIONS

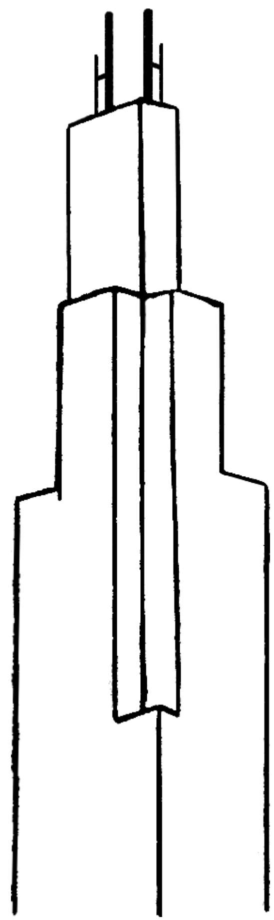
ROADMAP

1) INTRO & MOTIVATION

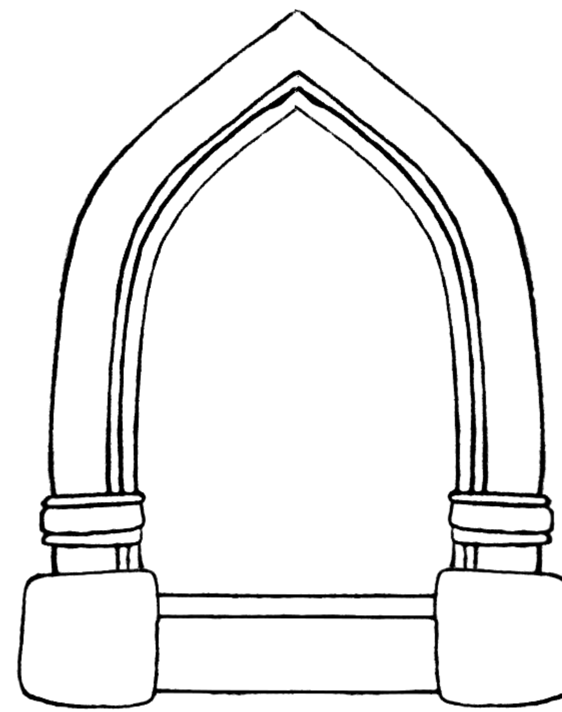
- WHY USE STRUCTURAL OPTIMIZATION?



LIMITED RESOURCES



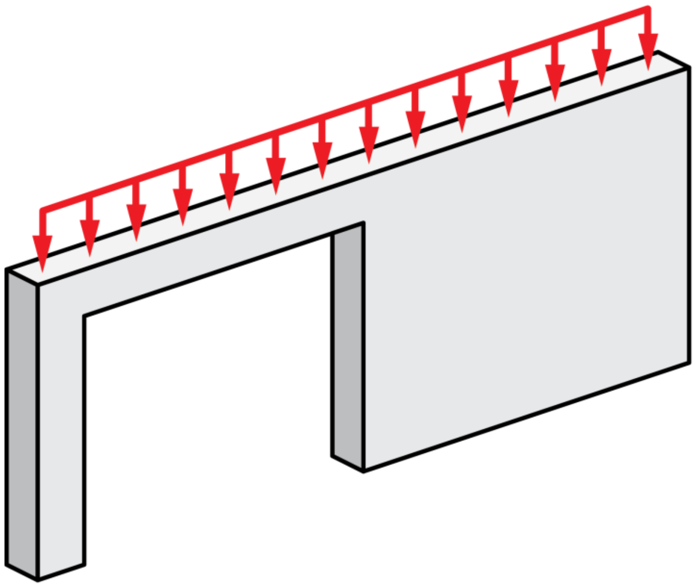
EXTREME STRUCTURES



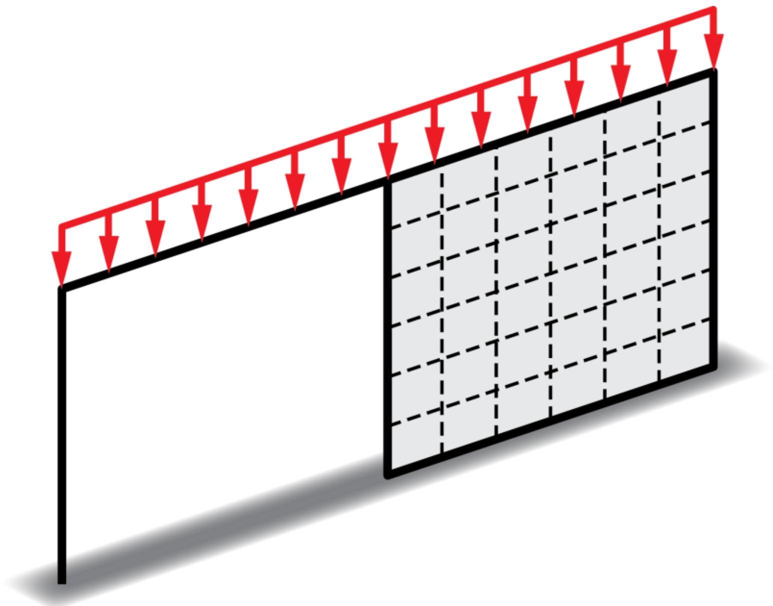
FUNCTIONAL

1) INTRO & MOTIVATION

- WHY DISCRETE—CONTINUUM?
  - LIMITED MODELING CAPABILITY
  - REASONABLE SIMPLIFICATIONS OF REALITY



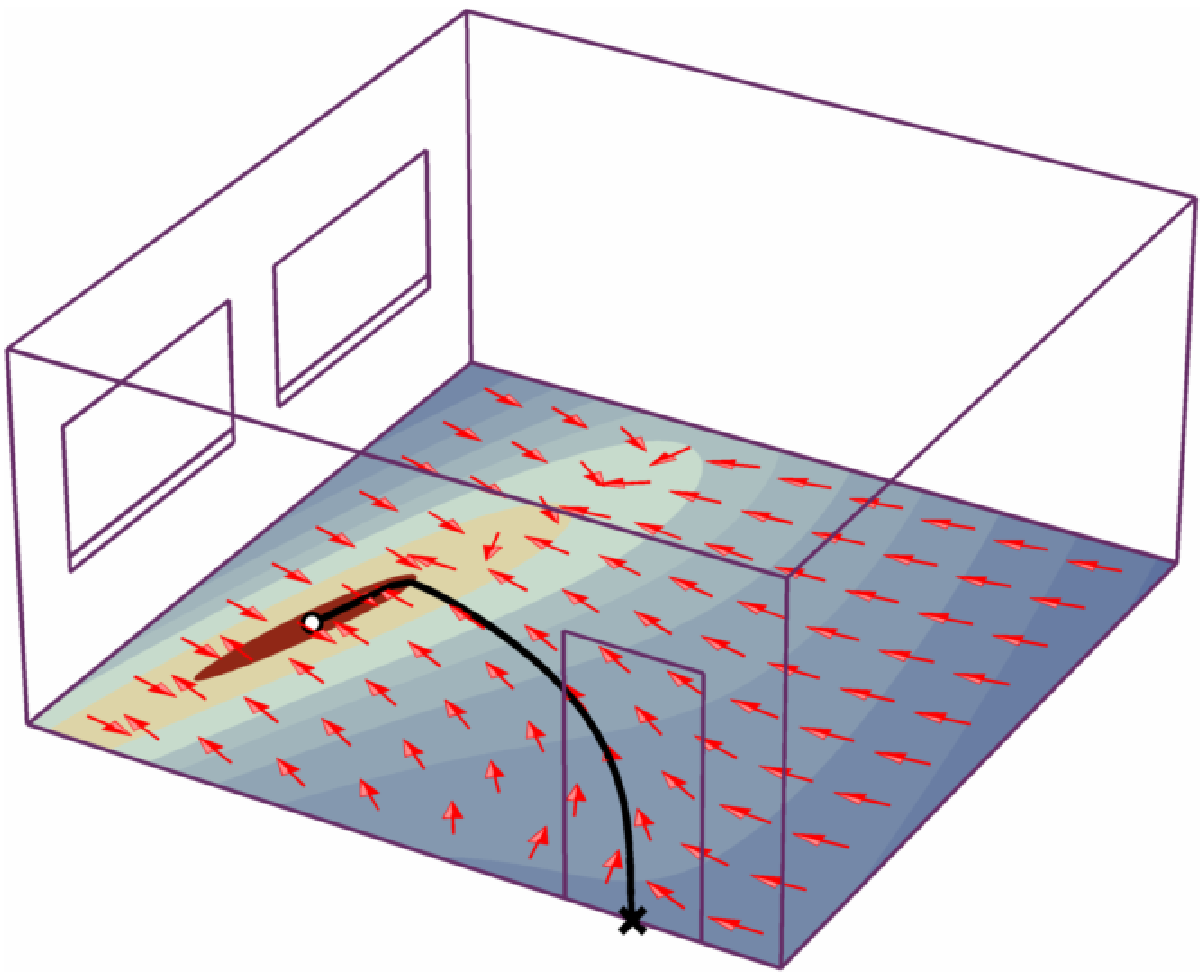
REAL FRAME



SIMPLIFIED FRAME MODEL

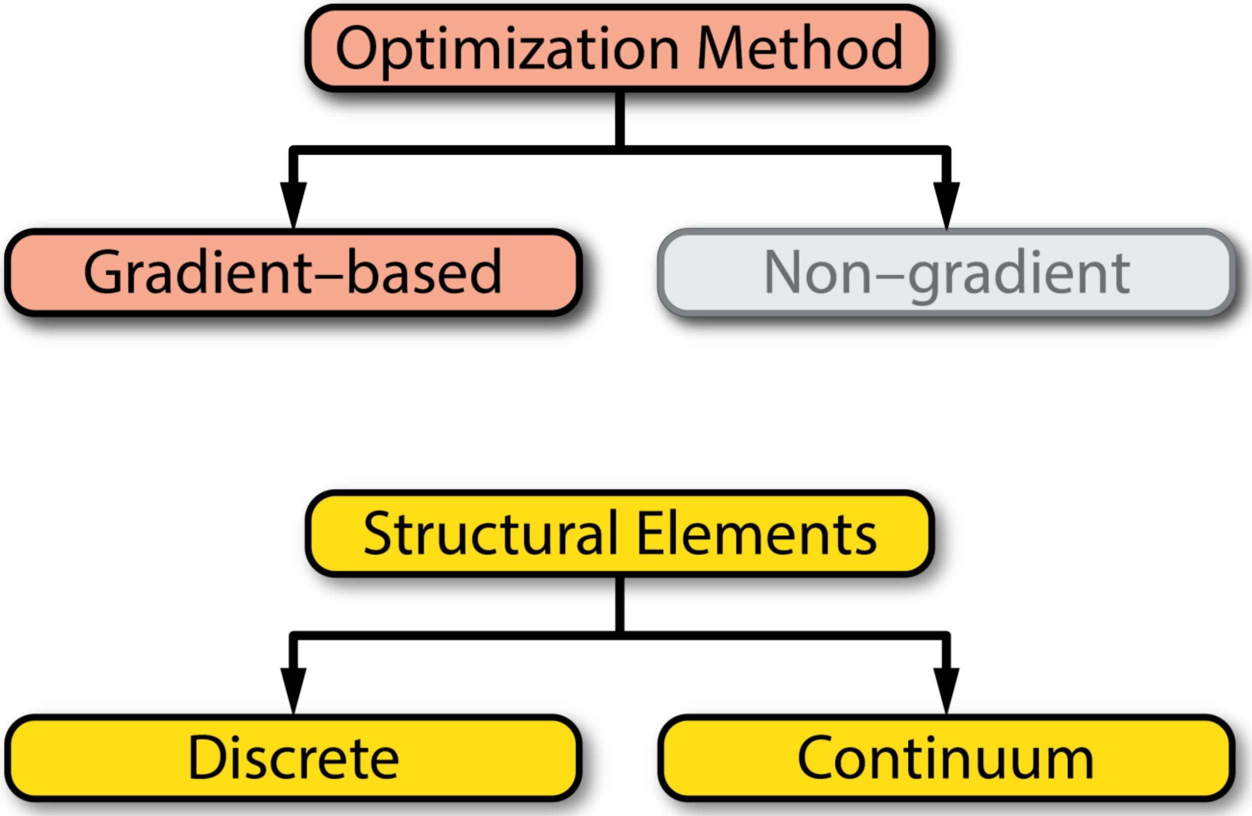
1) INTRO & MOTIVATION

- GRADIENT VS. NON-GRADIENT



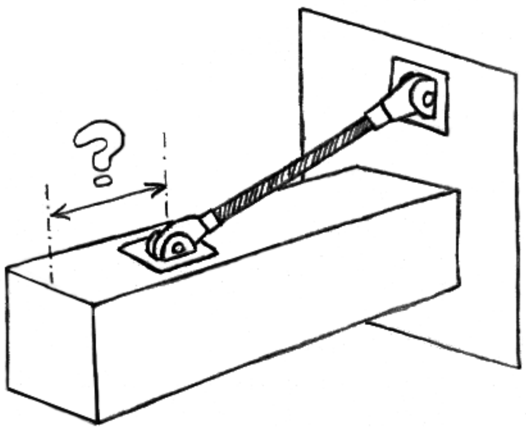
1) INTRO & MOTIVATION

- STRUCTURAL OPTIMIZATION METHODS

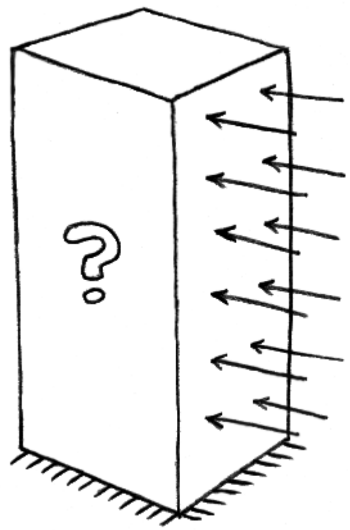


1) INTRO & MOTIVATION

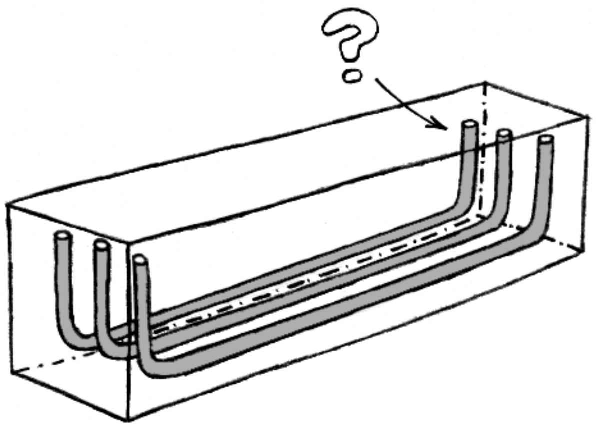
- POTENTIAL APPLICATIONS



ANCHOR POINT
LOCATION



LATERAL BRACING
SYSTEM



REINFORCEMENT
LAYOUT

ROADMAP

2) TRUSS LAYOUT WITHIN A CONTINUUM

- 3D BEAM WITH REINFORCEMENT

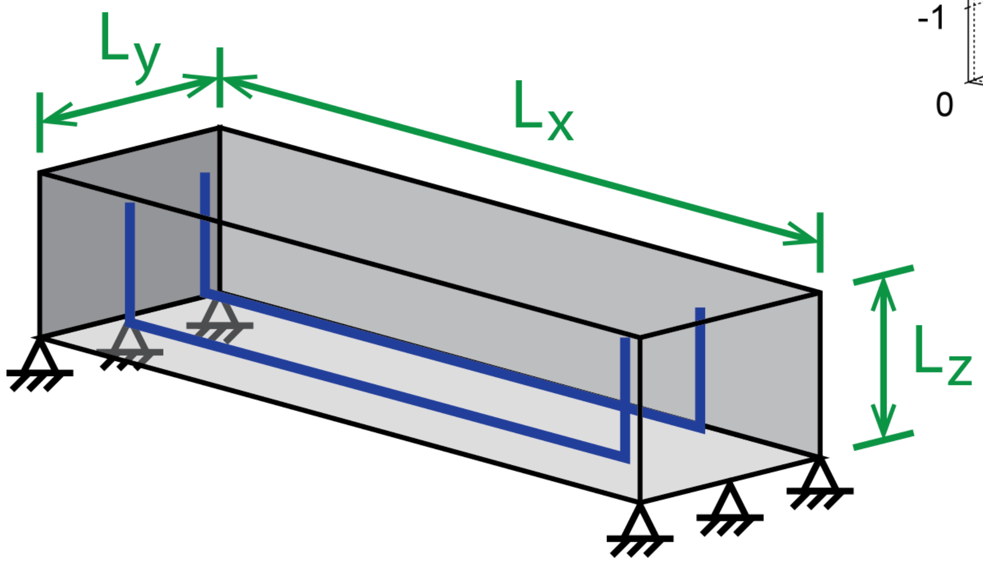
36X11X8 MESH (19008 TET10)

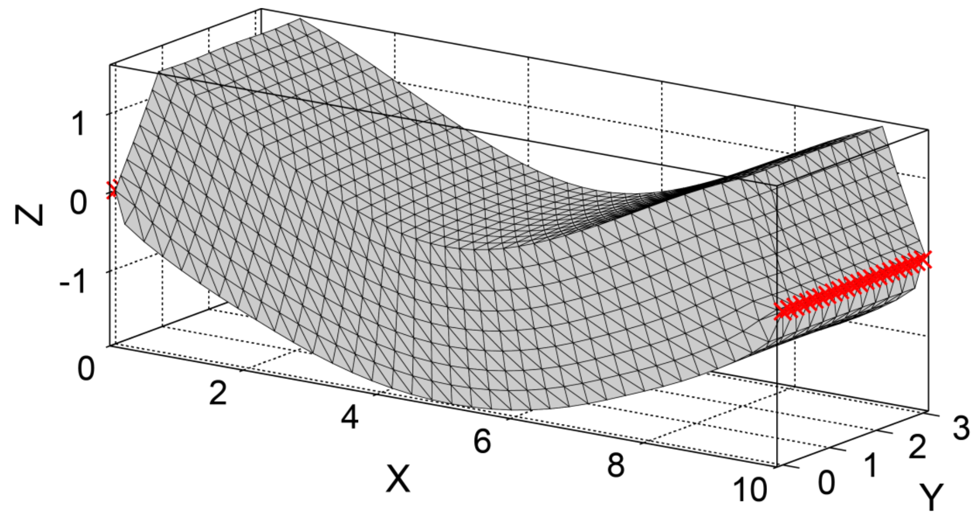
SLAB: LX=10 LY=3 LZ=2

E=100 ν =1/3

CABLE: AE=500

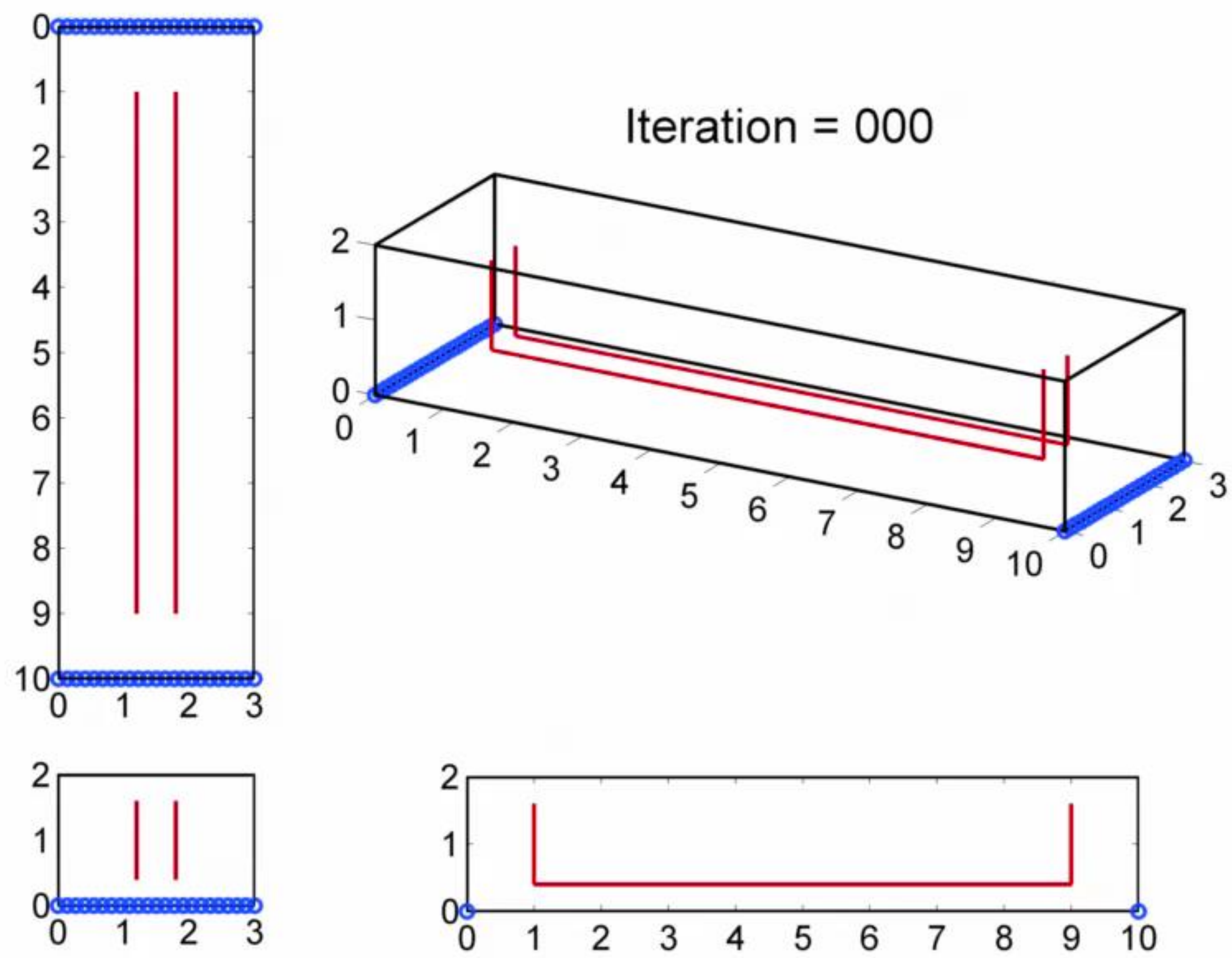
LOAD: NORMAL ON TOP FACET





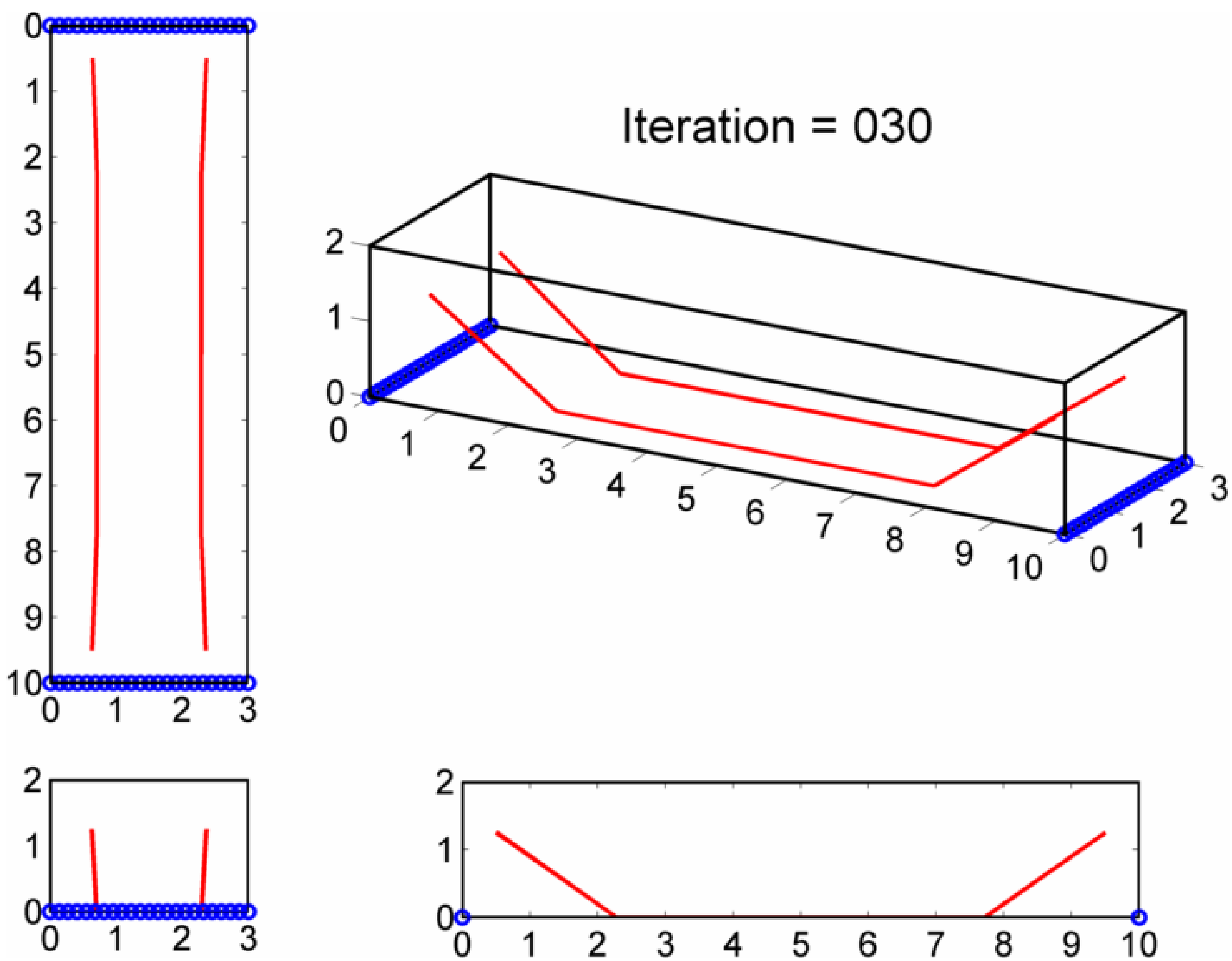
2) TRUSS LAYOUT WITHIN A CONTINUUM

- 3D BEAM WITH REINFORCEMENT

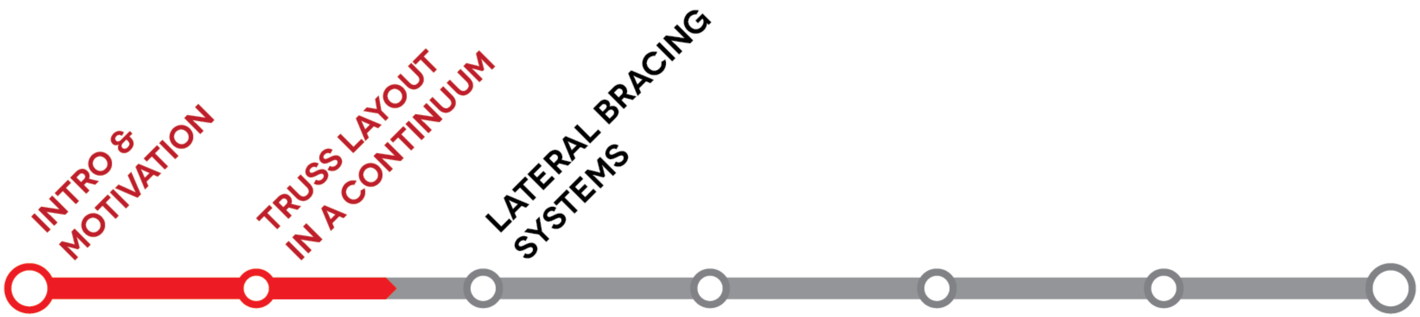


2) TRUSS LAYOUT WITHIN A CONTINUUM

- 3D BEAM WITH REINFORCEMENT



ROADMAP

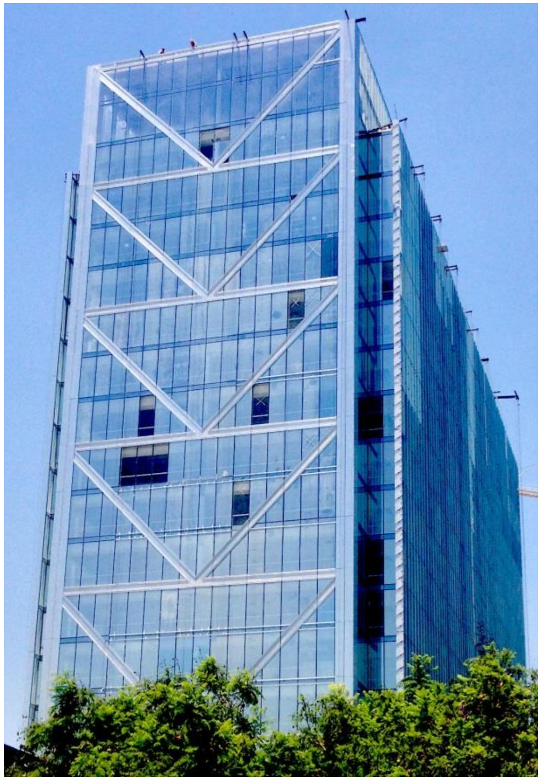

INTRO &
MOTIVATION
TRUSS LAYOUT
IN A CONTINUUM
LATERAL BRACING
SYSTEMS

3) LATERAL BRACING SYSTEMS

- EXAMPLES OF BRACED BUILDINGS

JOHN HANCOCK CENTER (CHICAGO, IL)

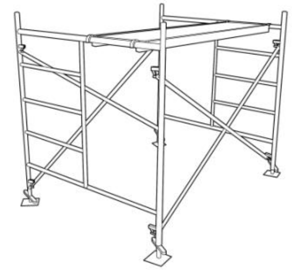
ALCOA BUILDING (SAN FRANCISCO, CA)



BUILDING IN PDTE. RIESCO AVENUE (SANTIAGO, CHILE)

3) LATERAL BRACING SYSTEMS

- OTHER APPLICATIONS



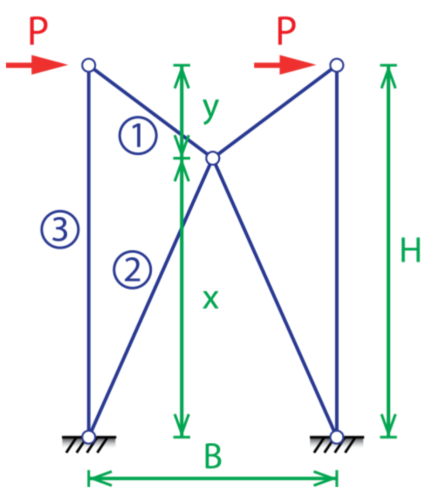
STAGE HIRE

CONSTRUCTION SCAFFOLDING

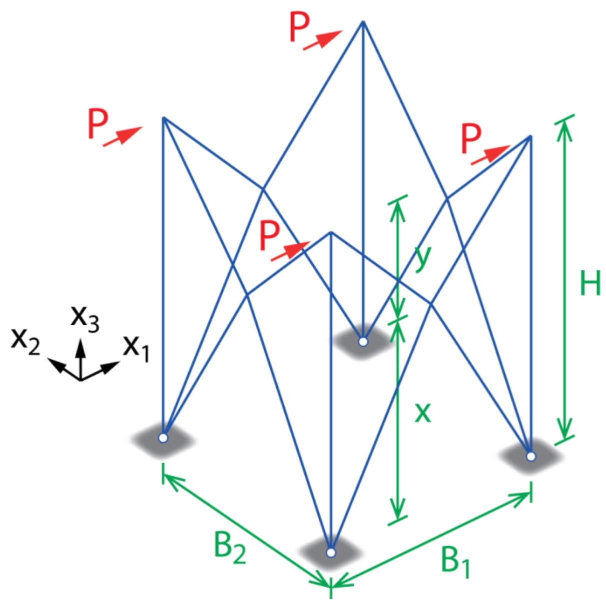
3) LATERAL BRACING SYSTEMS

- BRACING POINT UPPER BOUND

Height x	Weight - Cost		Performance	
	Volume	Load–Path	Compliance	Displacement
2D	$0.75H$	$0.75H$	$0.75H$	$0.75H$
3D	$0.625H$	$0.625H$	$0.6768H$	$0.6768H$

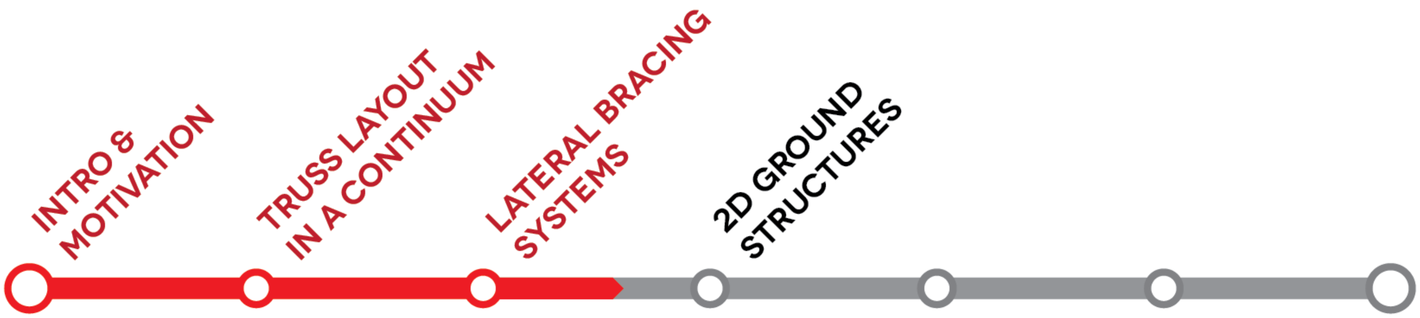


TWO–DIMENSIONAL BRACE



THREE–DIMENSIONAL BRACE

ROADMAP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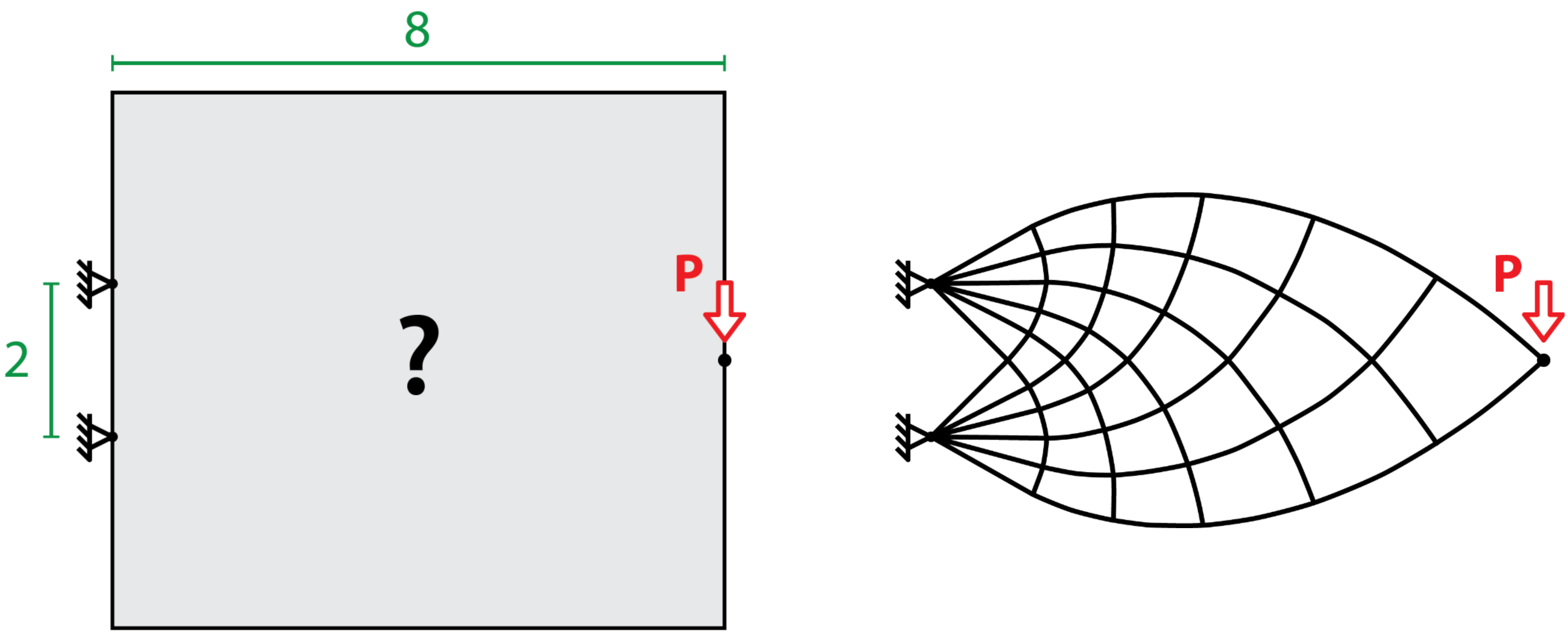
4) GROUND STRUCTURES IN 2D

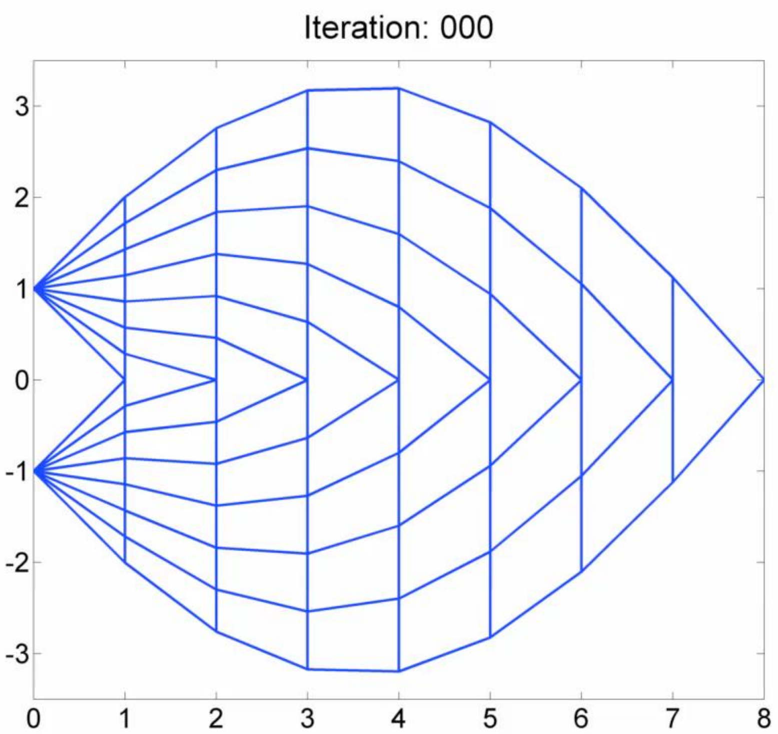
- TRUSS LAYOUT OPTIMIZATION IS HIGHLY NONLINEAR

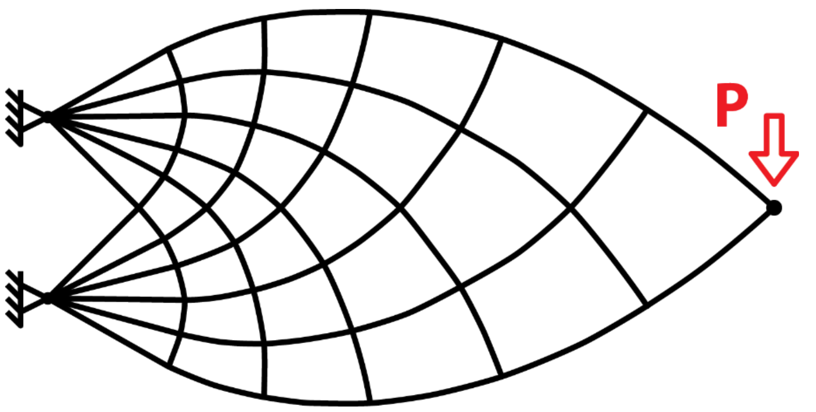


MAZUREK A (2012) – "GEOMETRICAL ASPECTS OF OPTIMUM TRUSS LIKE STRUCTURES FOR THREE-FORCE PROBLEM", STRUCT AND MULTIDISCIPLINARY OPT 45(1), PP 21–32

4) GROUND STRUCTURES IN 2D

- TRUSS LAYOUT OPTIMIZATION IS HIGHLY NONLINEAR





MAZUREK A (2012) – "GEOMETRICAL ASPECTS OF OPTIMUM TRUSS LIKE STRUCTURES FOR THREE-FORCE PROBLEM", STRUCT AND MULTIDISCIPLINARY OPT 45(1), PP 21-32

4) GROUND STRUCTURES IN 2D

- MAIN IDEA:
  CONVERT A GEOMETRY AND SIZE OPTIMIZATION TO A SIZING-ONLY PROBLEM

- PLASTIC FORMULATION:

$$
\begin{aligned}
\min_{\mathbf{a}} \quad & V = \mathbf{l}^T \mathbf{a} \\
\text{s.t.} \quad & \mathbf{B}^T \mathbf{n} = \mathbf{f} \\
& -\sigma_C \leq \sigma_i \leq \sigma_T \qquad \text{if} \qquad a_i > 0 \\
& a_i \geq 0 \quad i = 1, 2 \ldots N_b
\end{aligned}
$$

DORN W S, GOMORY R E, GREENBERG H J (1964) – "AUTOMATIC DESIGN OF OPTIMAL STRUCTURES", JOURNAL DE MECANIQUE 3(1), PP 25–52

4) GROUND STRUCTURES IN 2D

$$
\begin{aligned}
\min_{\mathbf{a}} \quad & V = \mathbf{l}^T \mathbf{a} \\
\text{s.t.} \quad & \mathbf{B}^T \mathbf{n} = \mathbf{f} \\
& -\sigma_C \leq \sigma_i \leq \sigma_T \quad \text{if} \quad a_i > 0 \\
& a_i \geq 0 \quad i = 1, 2 \ldots N_b
\end{aligned}
$$

VANISHING CONSTRAINT

- MULTIPLYING THE INEQUALITY BY CROSS-SECTIONAL AREA

$$
\begin{aligned}
\min_{\mathbf{a}} \quad & V = \mathbf{l}^T \mathbf{a} \\
\text{s.t.} \quad & \mathbf{B}^T \mathbf{n} = \mathbf{f} \\
& -\sigma_C a_i \leq n_i \leq \sigma_T a_i
\end{aligned}
$$

DORN W S, GOMORY R E, GREENBERG H J (1964) – "AUTOMATIC DESIGN OF OPTIMAL STRUCTURES", JOURNAL DE MECANIQUE 3(1), PP 25–52

4) GROUND STRUCTURES IN 2D

$$\min_{\mathbf{a}} \quad V = \mathbf{l}^T \mathbf{a}$$
$$\text{s.t.} \quad \mathbf{B}^T \mathbf{n} = \mathbf{f}$$
$$-\sigma_C a_i \leq n_i \leq \sigma_T a_i$$

- INTRODUCING SLACK VARIABLES

$$\left.\begin{aligned} n_i + 2\frac{\sigma_0}{\sigma_C} s_i^- &= \sigma_T a_i \\ -n_i + 2\frac{\sigma_0}{\sigma_T} s_i^+ &= \sigma_C a_i \\ \sigma_0 = \left(\sigma_T + \sigma_C\right)/2 & \end{aligned}\right\} \quad \begin{aligned} a_i &= \frac{s_i^+}{\sigma_T} + \frac{s_i^-}{\sigma_C} \\ n_i &= s_i^+ - s_i^- \end{aligned}$$

$$\min_{\mathbf{s}^+, \mathbf{s}^-} \quad V = \mathbf{l}^T \left(\frac{\mathbf{s}^+}{\sigma_T} + \frac{\mathbf{s}^-}{\sigma_C} \right)$$
$$\text{s.t.} \quad \mathbf{B}^\mathbf{T} \left(\mathbf{s}^+ - \mathbf{s}^-\right) = \mathbf{f}$$
$$s_i^+, s_i^- \geq 0$$

4) GROUND STRUCTURES IN 2D

- REMARKS
  - DESIGN VARIABLES DOUBLED: S^+ AND S^-
  - NO MORE VANISHING CONSTRAINT
  - DIFFERENT LIMITS IN TENSION AND COMPRESSION
  - LINEAR PROGRAM

    KARMARKAR N (1984) "A NEW POLYNOMIAL-TIME ALGORITHM FOR LINEAR PROGRAMMING." COMBINATORICA, 4(4):373-395.

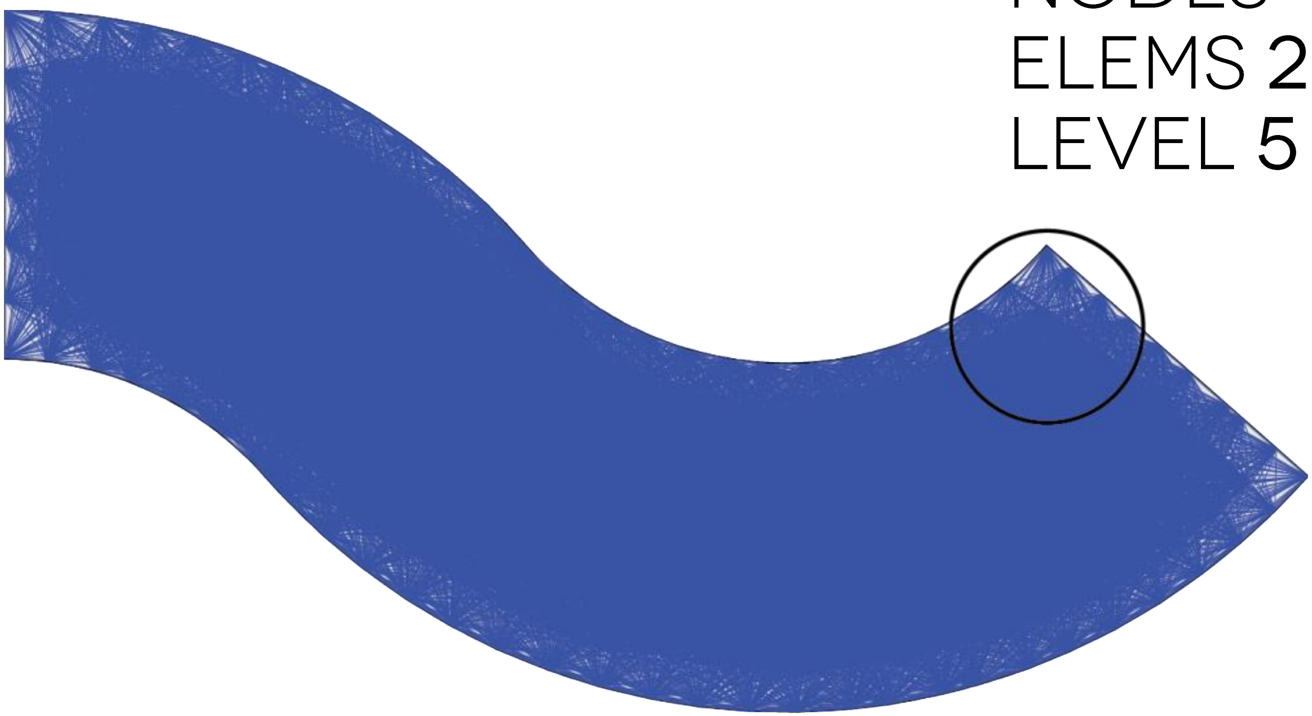
    WRIGHT MH (2004) "THE INTERIOR-POINT REVOLUTION IN OPTIMIZATION: HISTORY, RECENT DEVELOPMENTS, AND LASTING CONSEQUENCES." BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY, 42(1):39-56.

$$
\begin{aligned}
\min_{\mathbf{s}^+,\mathbf{s}^-} \quad & V = \mathbf{1}^T \left(\frac{\mathbf{s}^+}{\sigma_T} + \frac{\mathbf{s}^-}{\sigma_C} \right) \\
\text{s.t.} \quad & \mathbf{B}^{\mathbf{T}} \left(\mathbf{s}^+ - \mathbf{s}^- \right) = \mathbf{f} \\
& s_i^+, s_i^- \geq 0
\end{aligned}
$$

4) GROUND STRUCTURES IN 2D

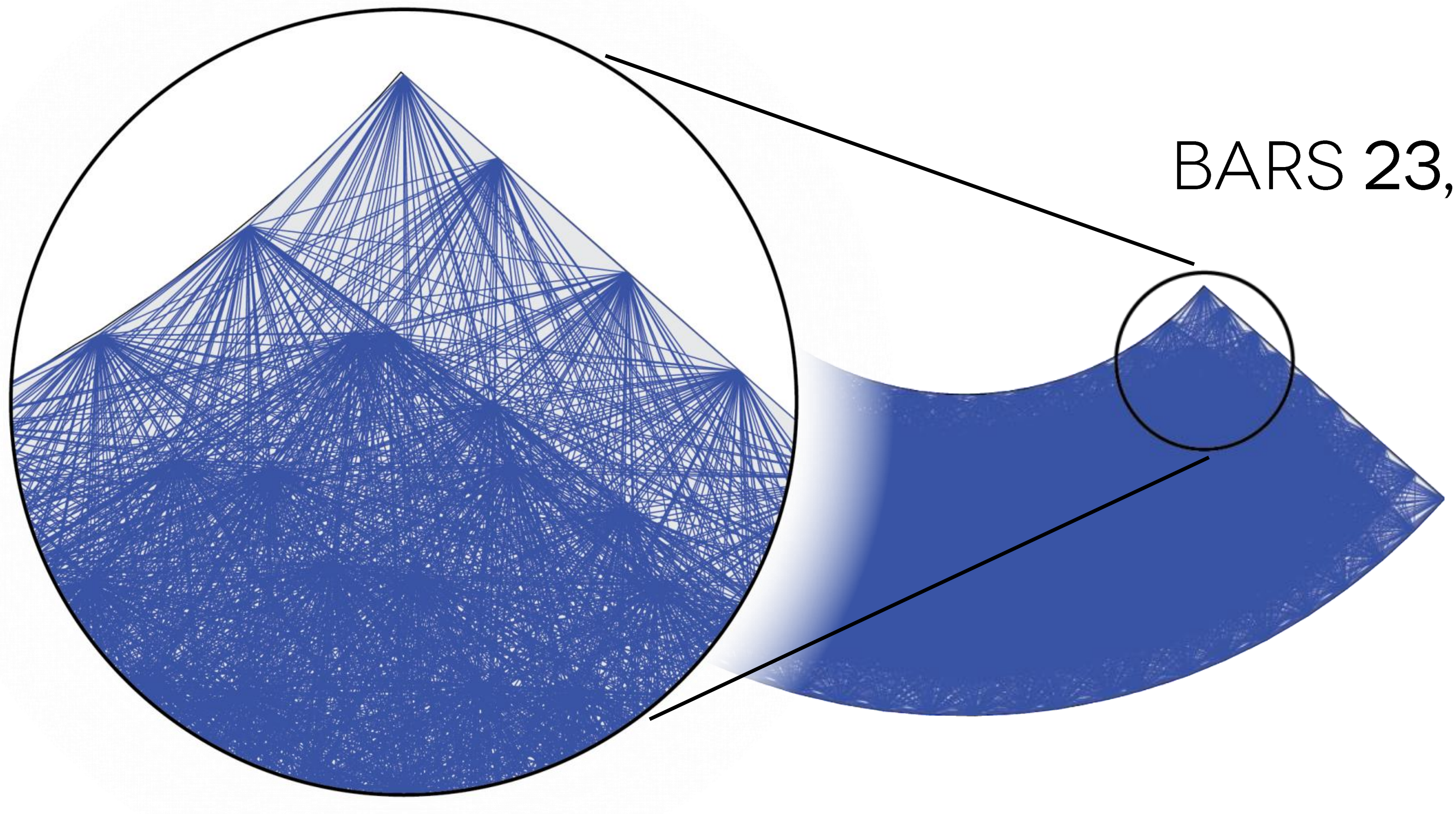
- SIZING OF A HIGHLY **INTERCONNECTED** AND **REDUNDANT** TRUSS

NODES **404**
ELEMS **200**
LEVEL **5**



4) GROUND STRUCTURES IN 2D

- SIZING OF A HIGHLY **INTERCONNECTED** AND **REDUNDANT** TRUSS



4) GROUND STRUCTURES IN 2D

- UNIQUE SOLUTION — NO COLLINEAR BARS

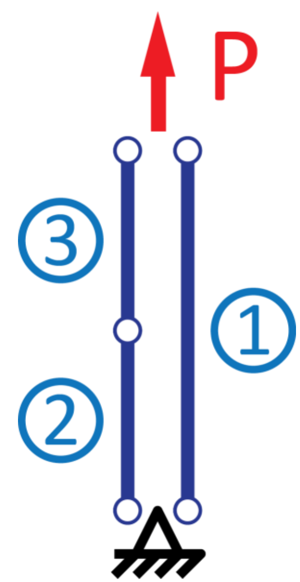
GIVEN $\sigma_T = 1$ AND $P = 1$

$a_1 = 1.0$

$a_1 = 1.0 \quad a_2 = a_3 = 0.0$

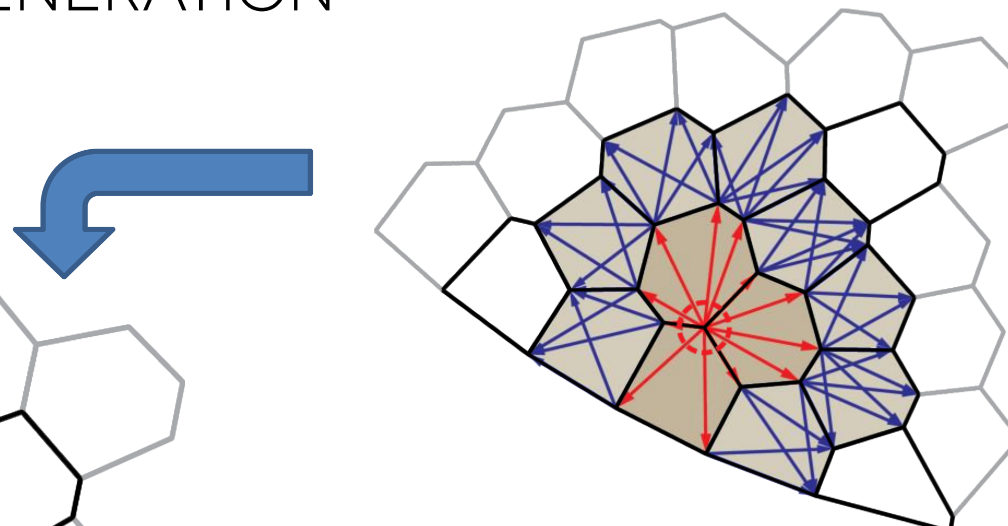
$a_1 = 0.0 \quad a_2 = a_3 = 1.0$

$a_1 = 0.5 \quad a_2 = a_3 = 0.5$



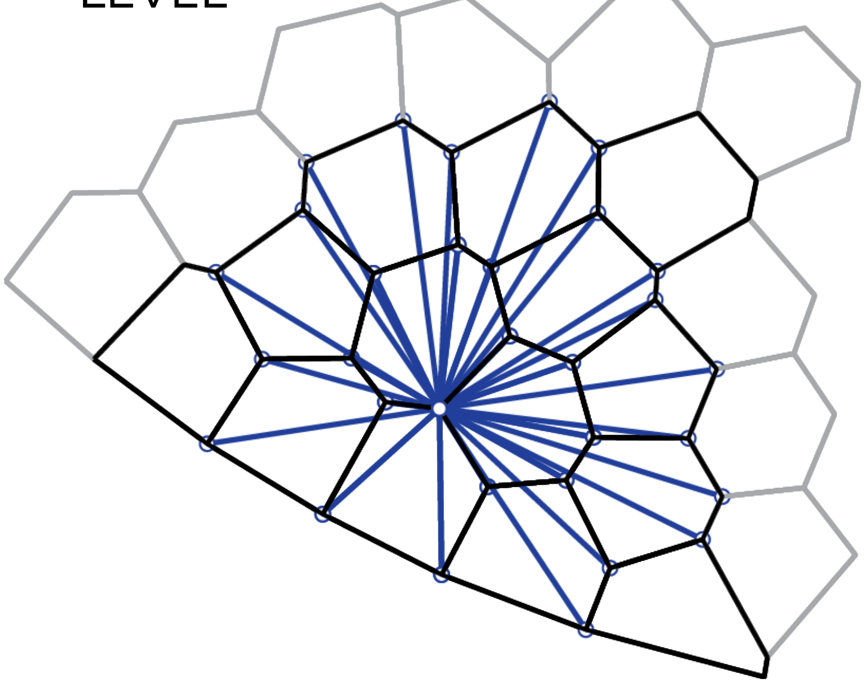
4) GROUND STRUCTURES IN 2D

- HIGHLY INTERCONNECTED TRUSS
  - CONNECTIVITY GENERATION



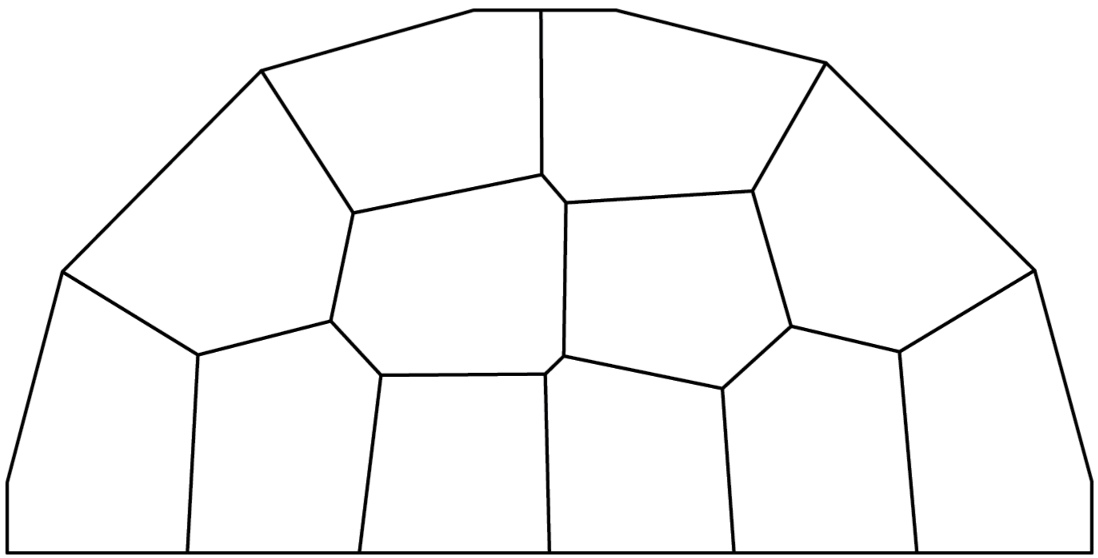
CONNECTION LEVEL: ZERO

TRUSS MEMBERS
AT THIS CONNECTION
LEVEL



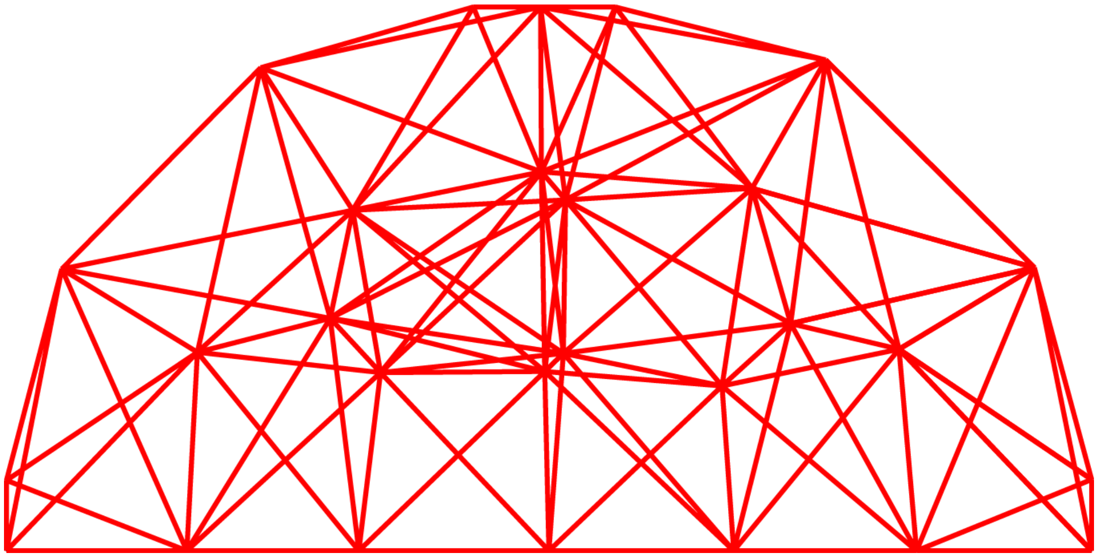
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - BASE MESH



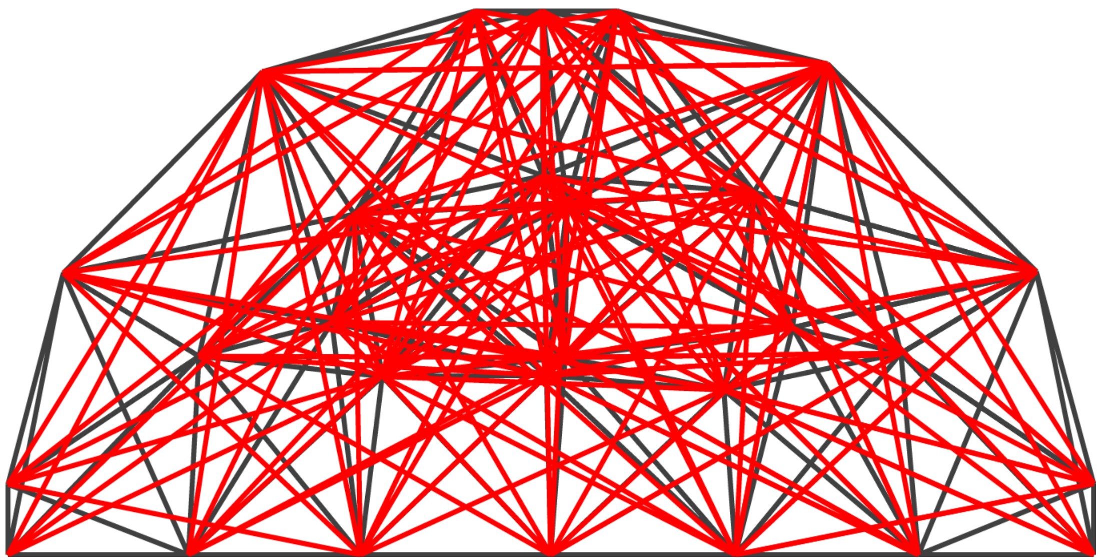
GROUND STRUCTURE METHOD

- EXAMPLE
  - CONNECTIVITY: LEVEL 1



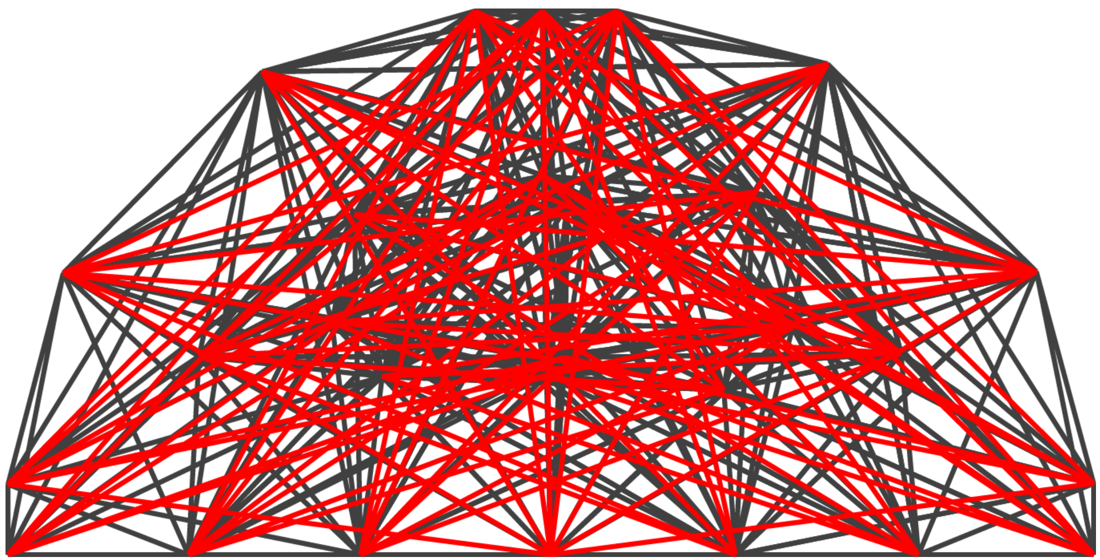
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 2



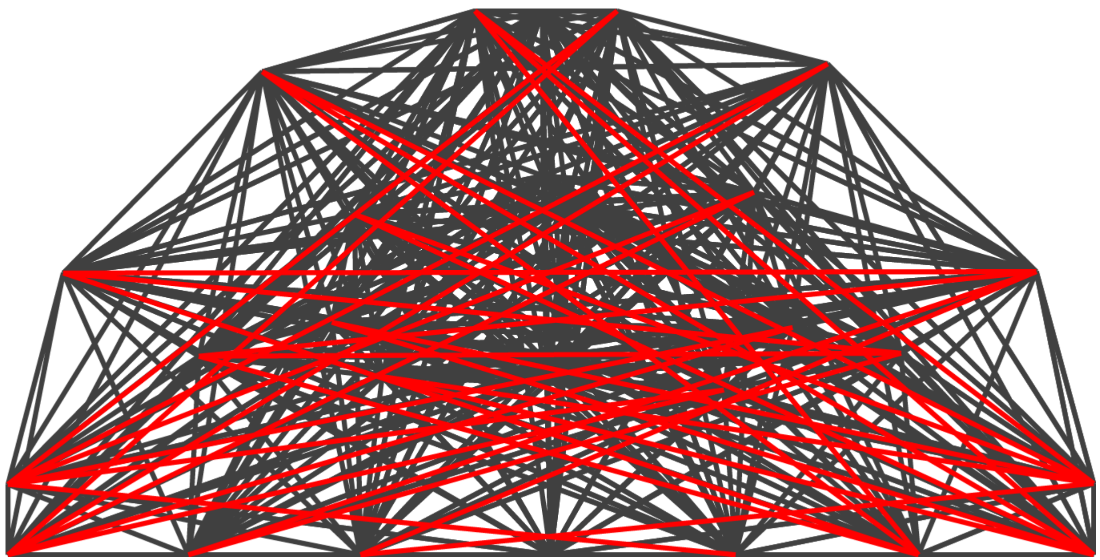
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 3



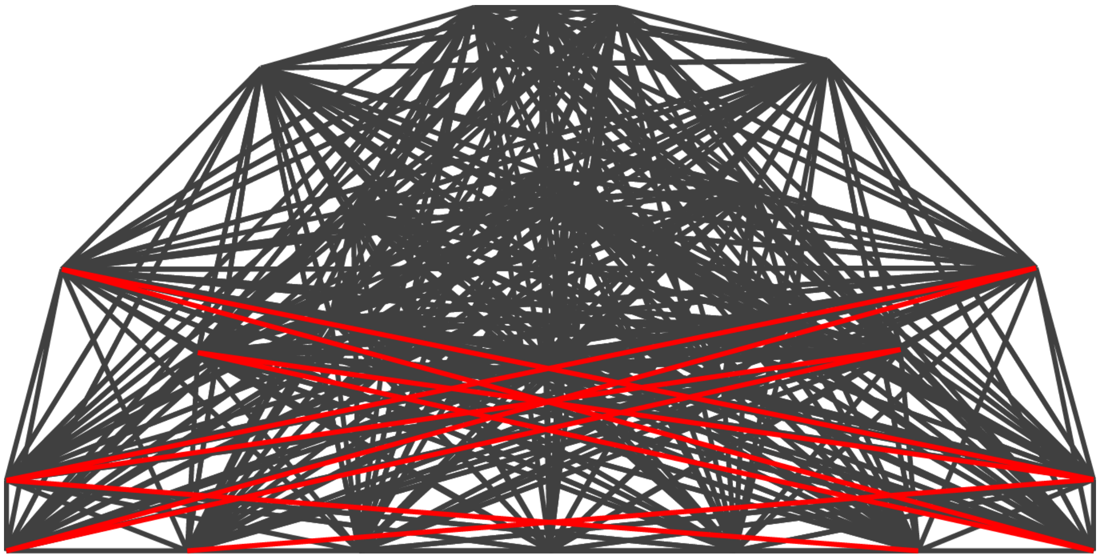
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 4



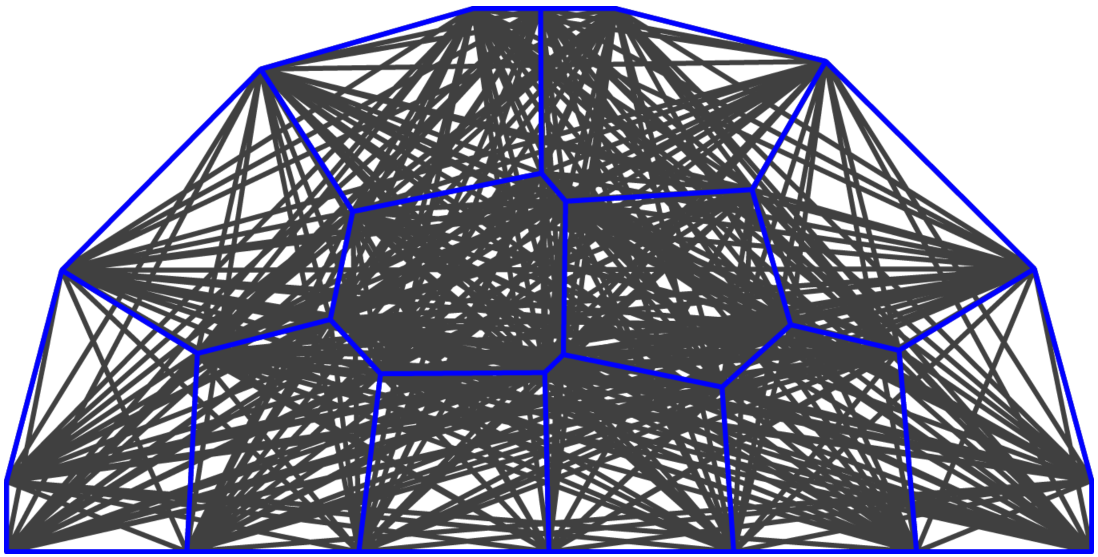
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 5



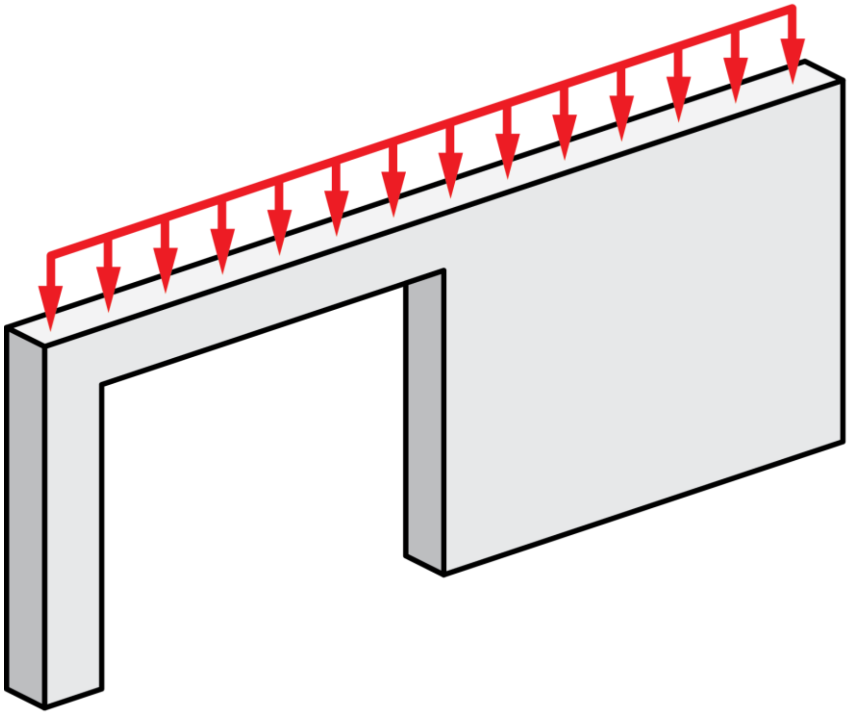
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 5



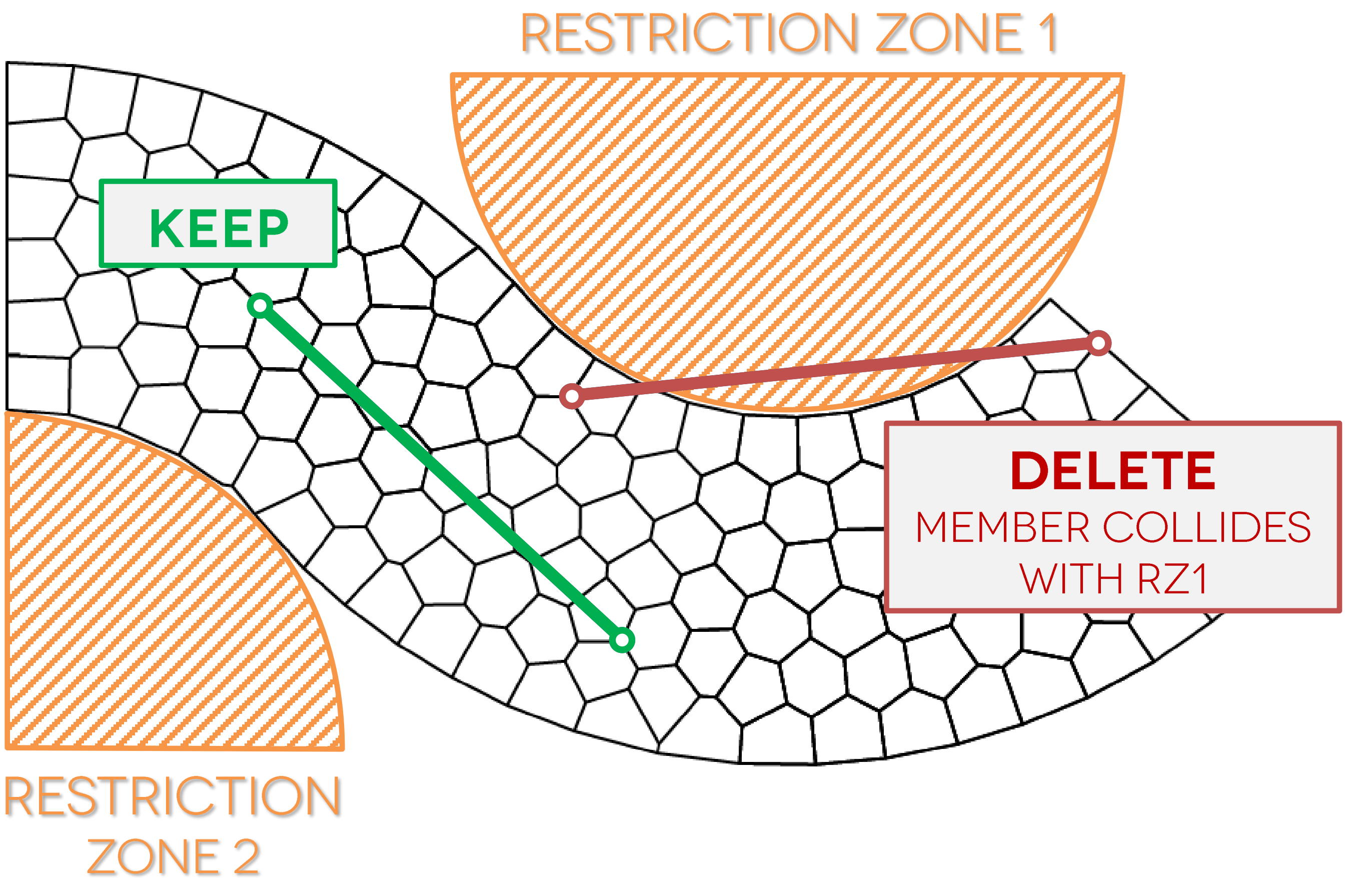
4) GROUND STRUCTURES IN 2D

- THERE CANNOT BE BARS **EVERYWHERE**
  - DEFINE ZONES WHERE NO BARS CAN BE



4) GROUND STRUCTURES IN 2D

- INTERSECTION TESTS FROM VIDEO-GAME AND COMPUTER GRAPHICS INDUSTRY

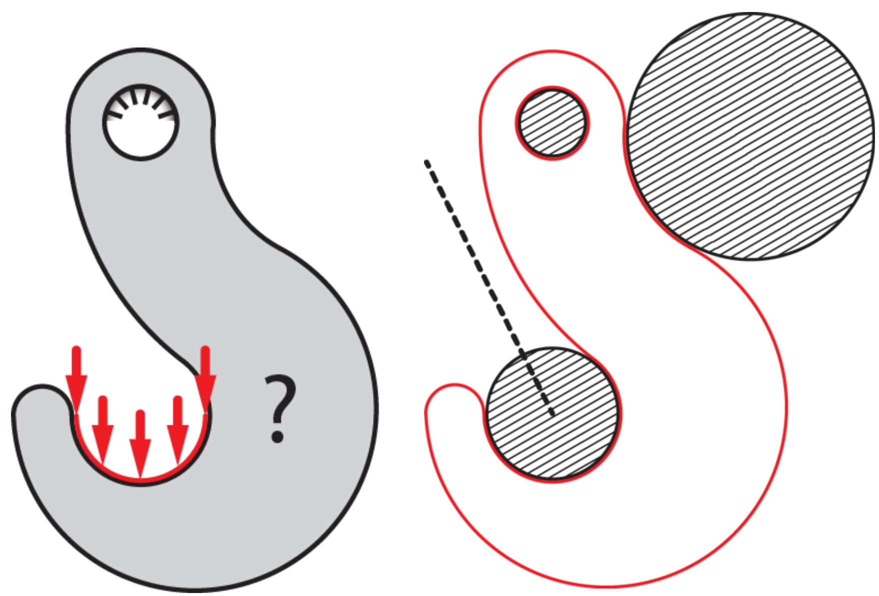


4) GROUND STRUCTURES IN 2D

- RESTRICTION ZONE PRIMITIVES
  - CIRCLE
  - SEGMENT (LINE)
  - RECTANGLE
  - POLYGON

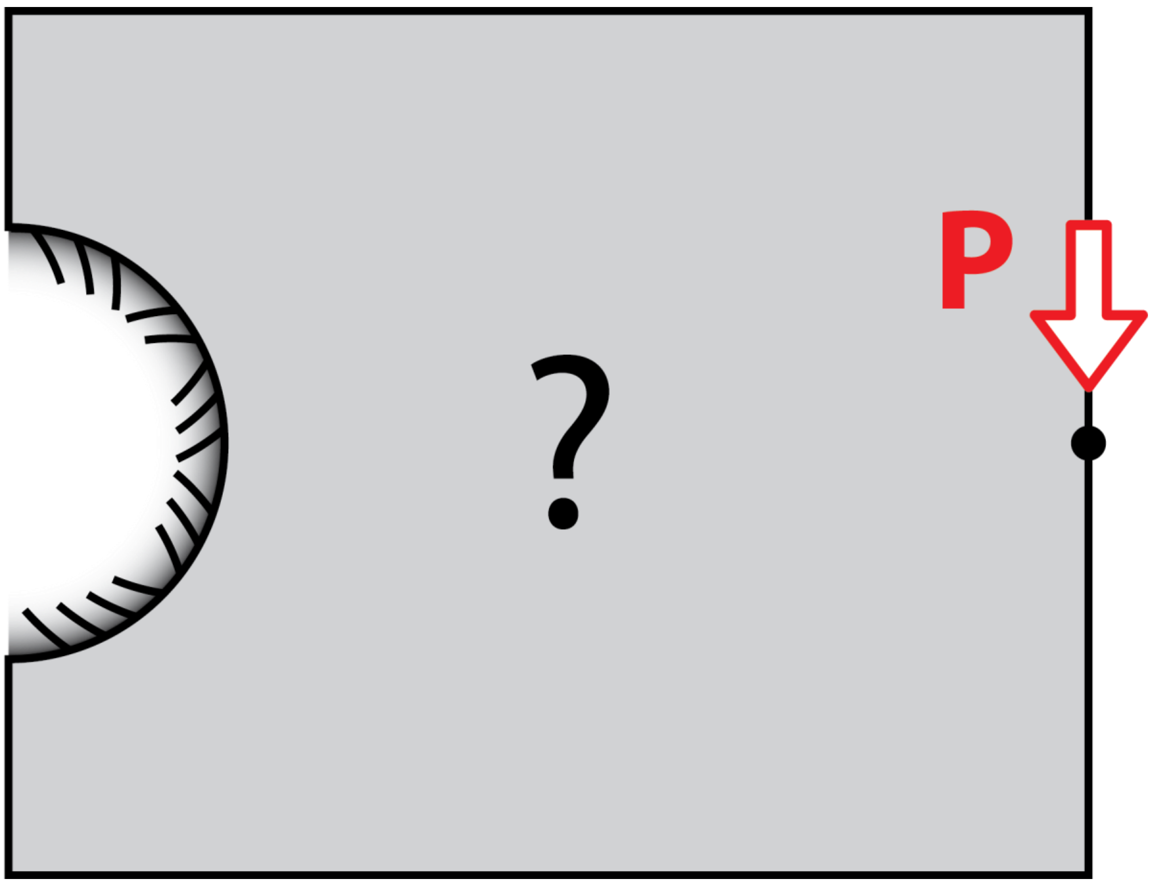
- CAN BE COMBINED...

3 CIRCLES
+
1 SEGMENT



4) GROUND STRUCTURES IN 2D

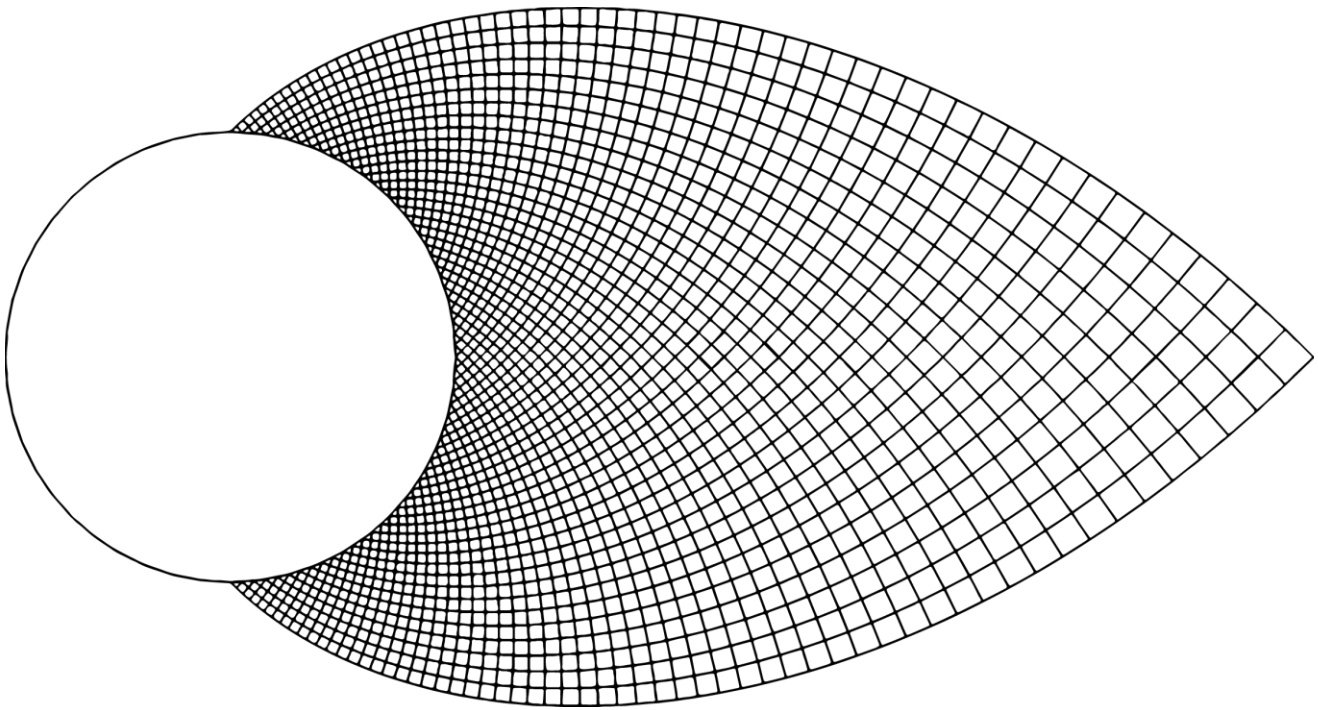
- MICHELL CANTILEVER



MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589-597

4) GROUND STRUCTURES IN 2D

- MICHELL CANTILEVER



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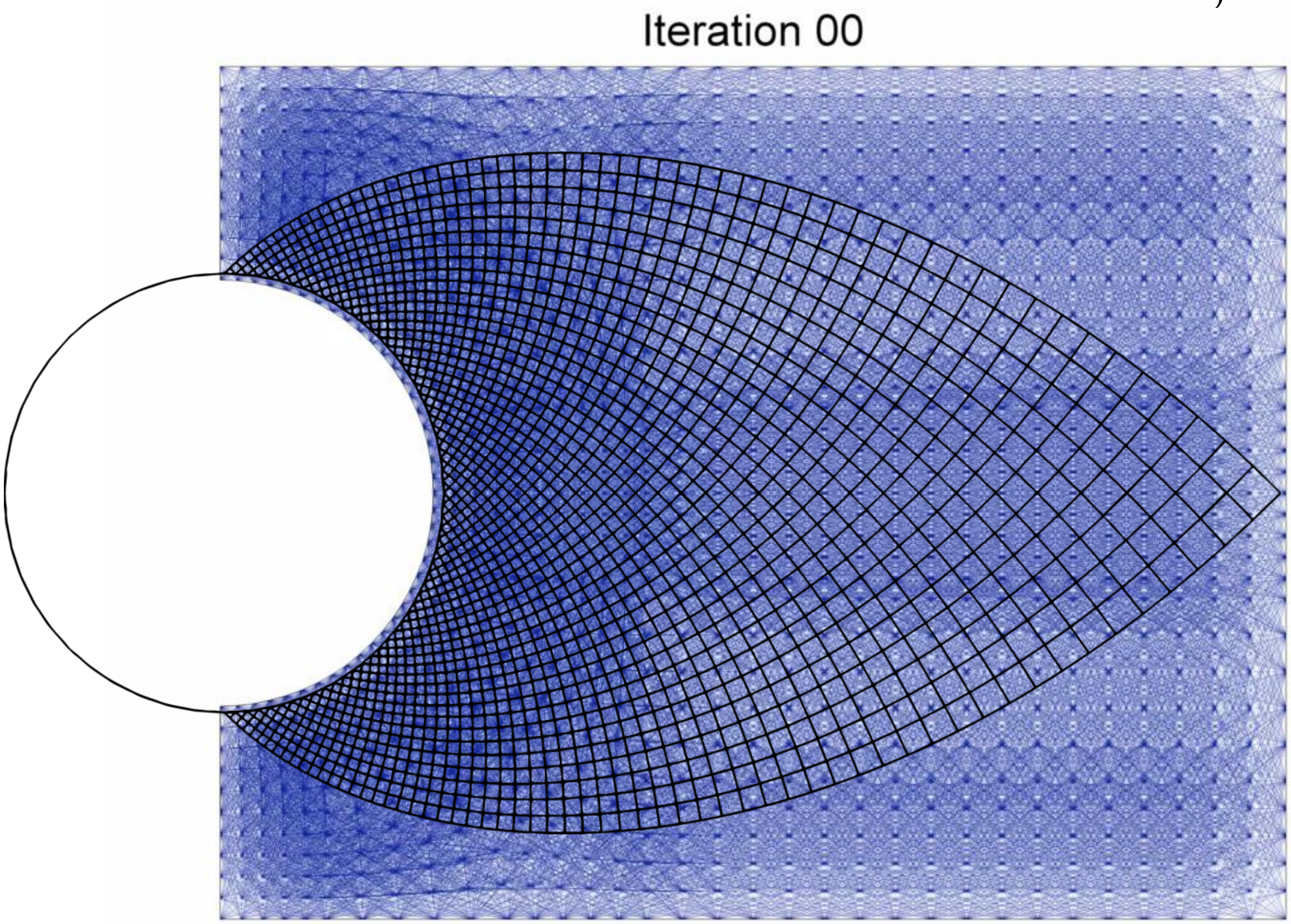
4) GROUND STRUCTURES IN 2D

- MICHELL CANTILEVER 28,256 BARS



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4) GROUND STRUCTURES IN 2D

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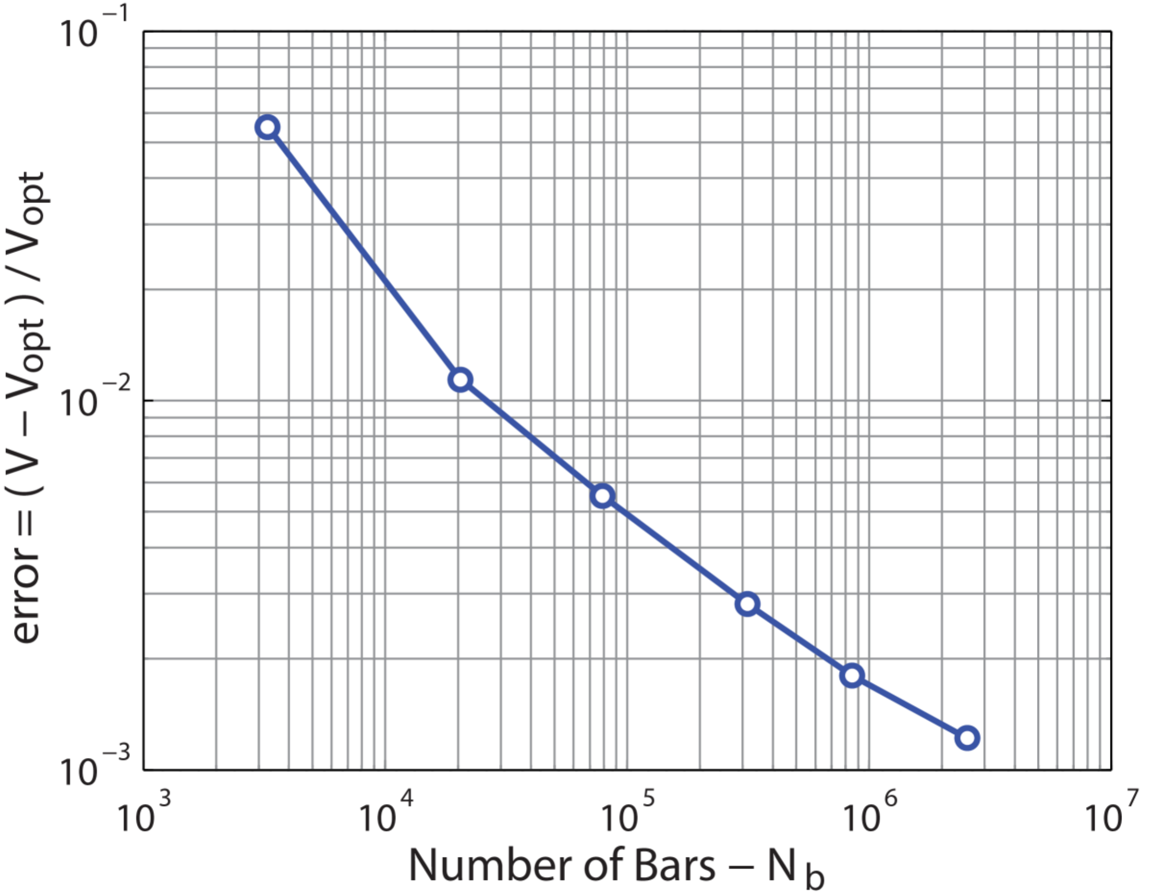
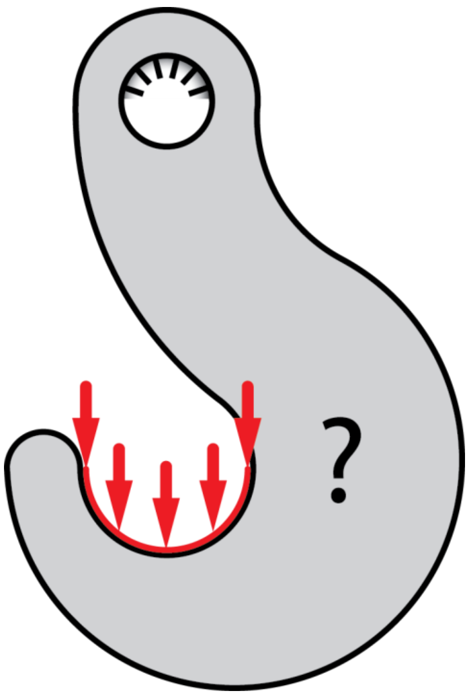
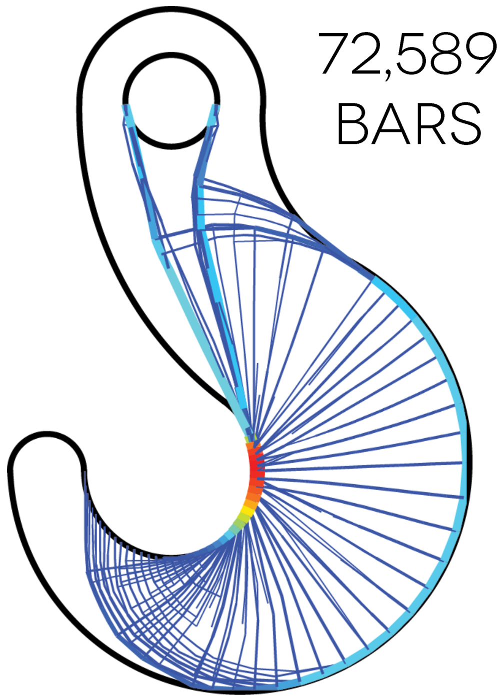


4) GROUND STRUCTURES IN 2D

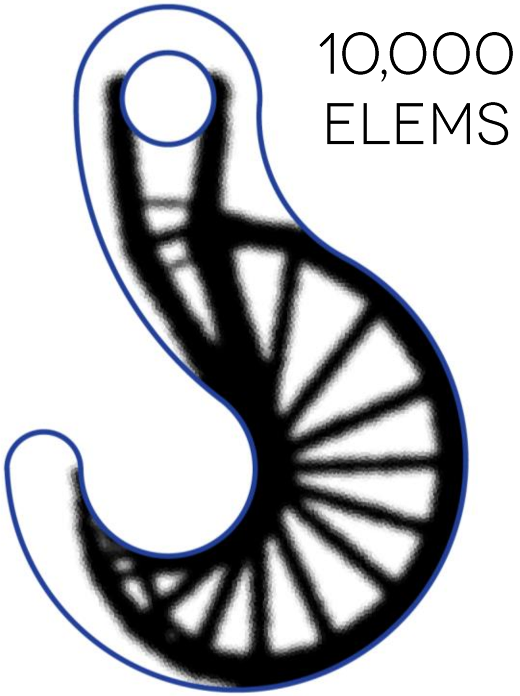
- HOOK PROBLEM



DOMAIN & BCs



GROUND STRUCTURES

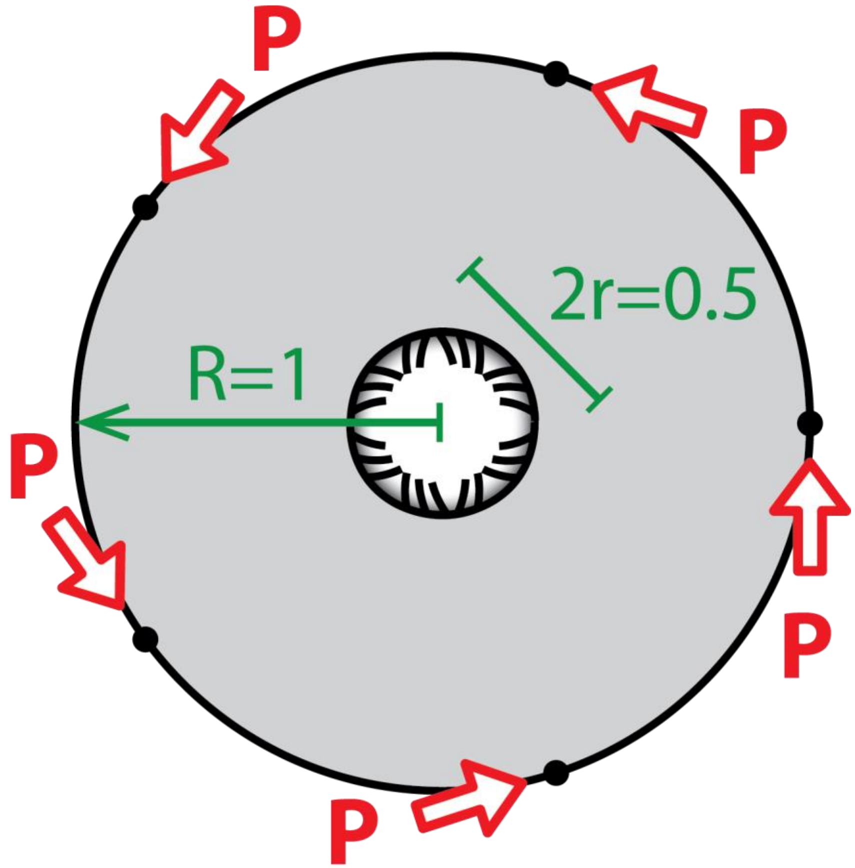


DENSITY–BASED METHOD ◇

◇ TALISCHI C, PAULINO GH, PEREIRA A, MENEZES IFM (2012), "POLYTOP: A MATLAB IMPLEMENTATION OF A GENERAL TOPOLOGY OPTIMIZATION FRAMEWORK USING UNSTRUCTURED POLYGONAL FINITE ELEMENT MESHES", STRUCT. MULTIDISC. OPTIM. 45(3), 329–357

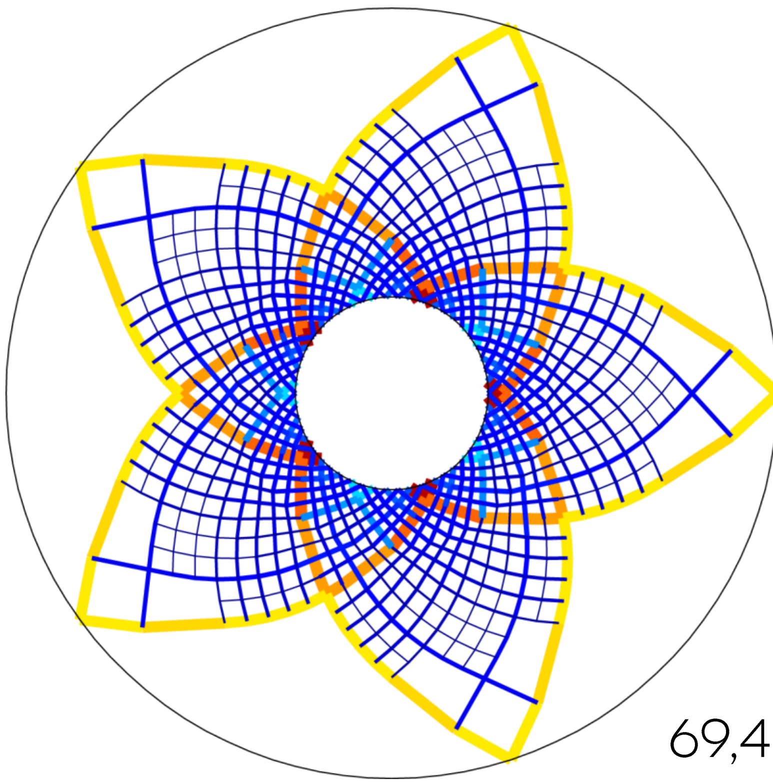
4) GROUND STRUCTURES IN 2D

- FLOWER PROBLEM



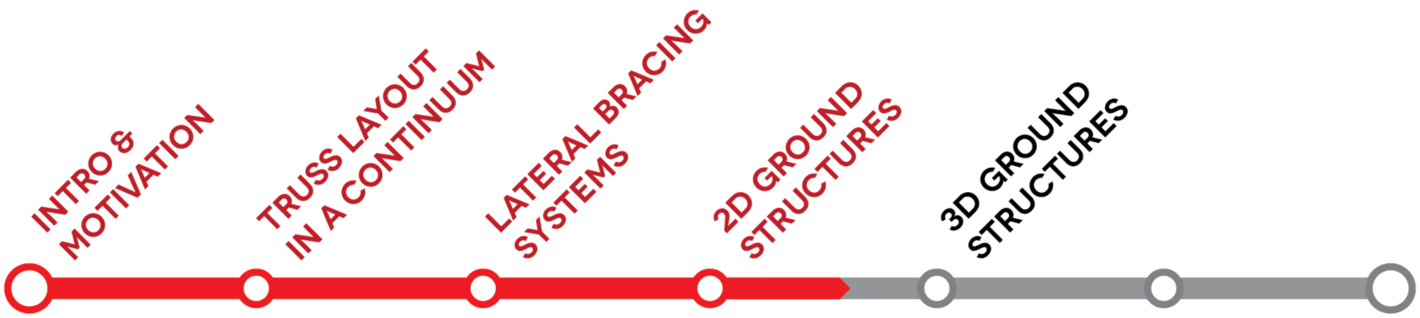
4) GROUND STRUCTURES IN 2D

- FLOWER PROBLEM



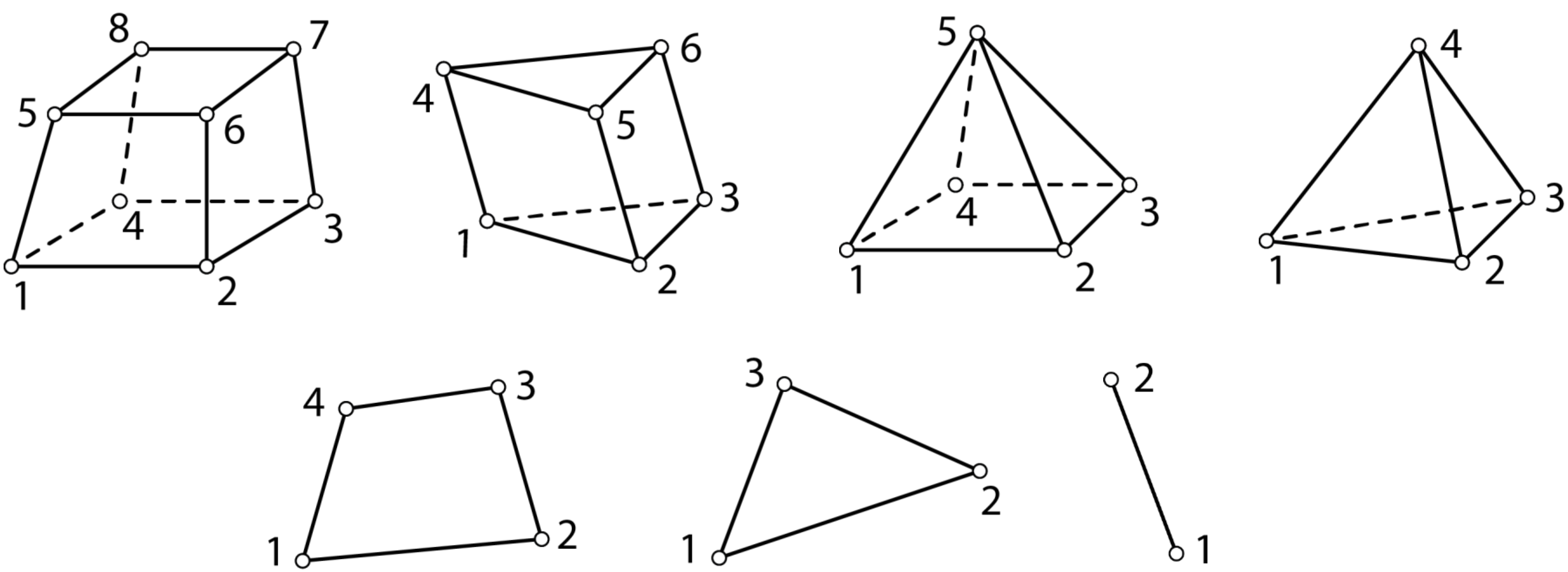
69,400 BARS

ROADMAP


INTRO &
MOTIVATION
TRUSS LAYOUT
IN A CONTINUUM
LATERAL BRACING
SYSTEMS
2D GROUND
STRUCTURES
3D GROUND
STRUCTURES

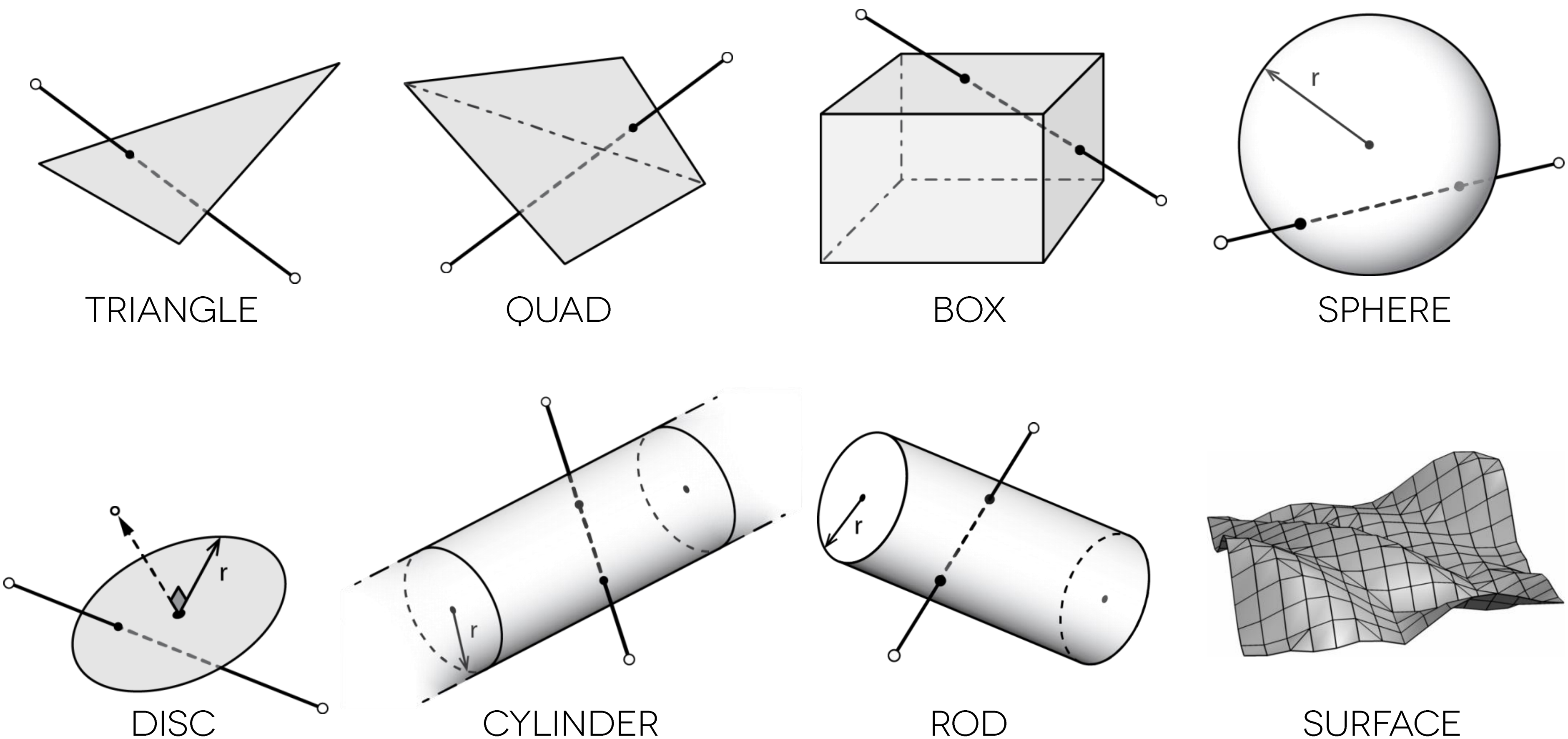
5) GROUND STRUCTURES IN 3D

- BASE-MESH DEFINITION
  - GROUND STRUCTURE ALGORITHM SUPPORTS ANY CONVEX POLYTOPE
  - IMPLEMENTATION IS RESTRICTED TO 7 ELEMENTS: MESH GENERATION AND PLOTTING PURPOSES



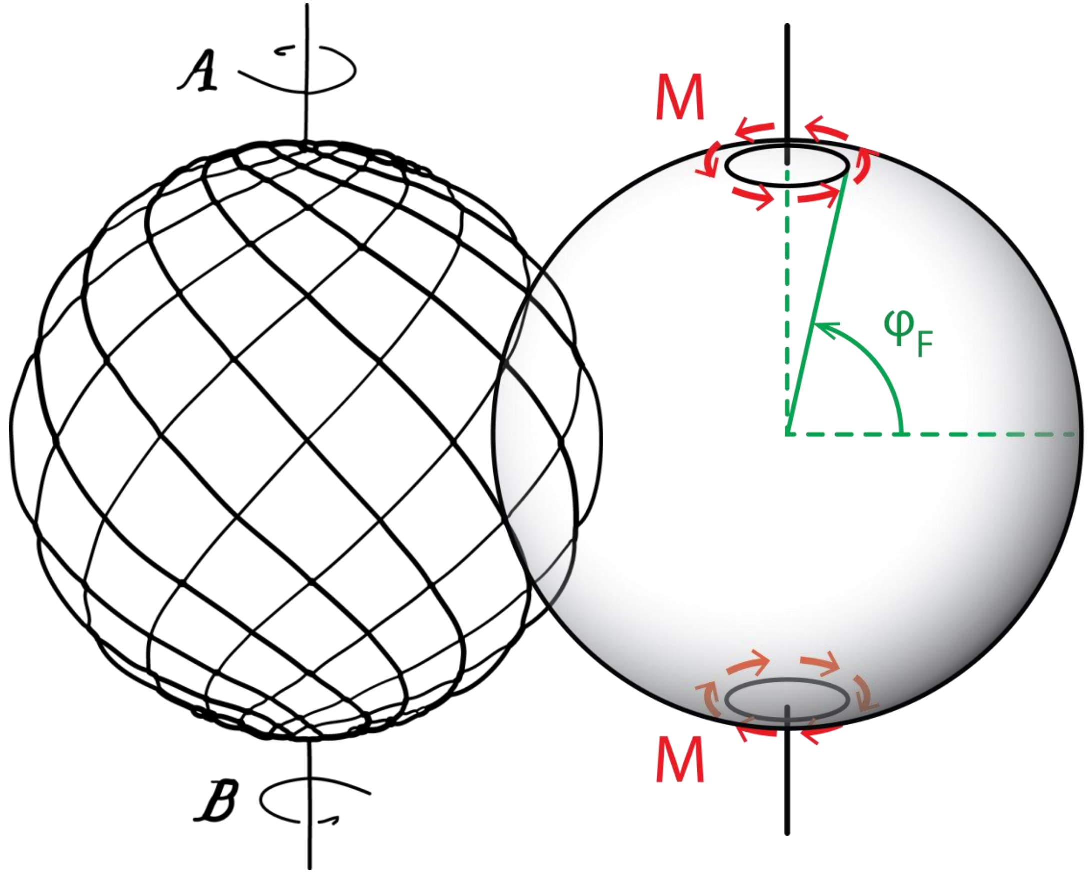
5) GROUND STRUCTURES IN 3D

- RESTRICTION PRIMITIVES:



5) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM

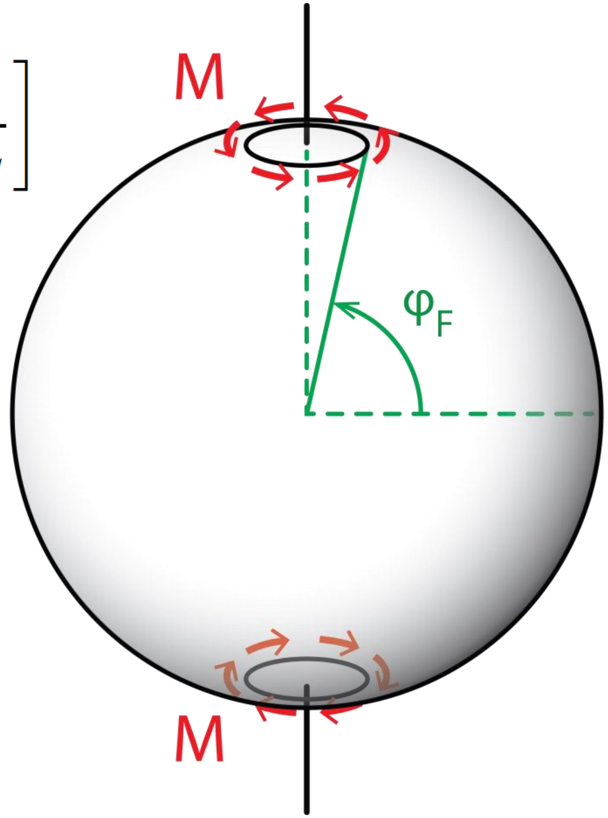


MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589-597

5) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM

$$V_{opt} = 2M \log \left(\tan \left\{ \frac{\pi}{4} + \frac{\phi_F}{2} \right\} \right) \left[\frac{1}{\sigma_T} + \frac{1}{\sigma_C} \right]$$



MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589-597

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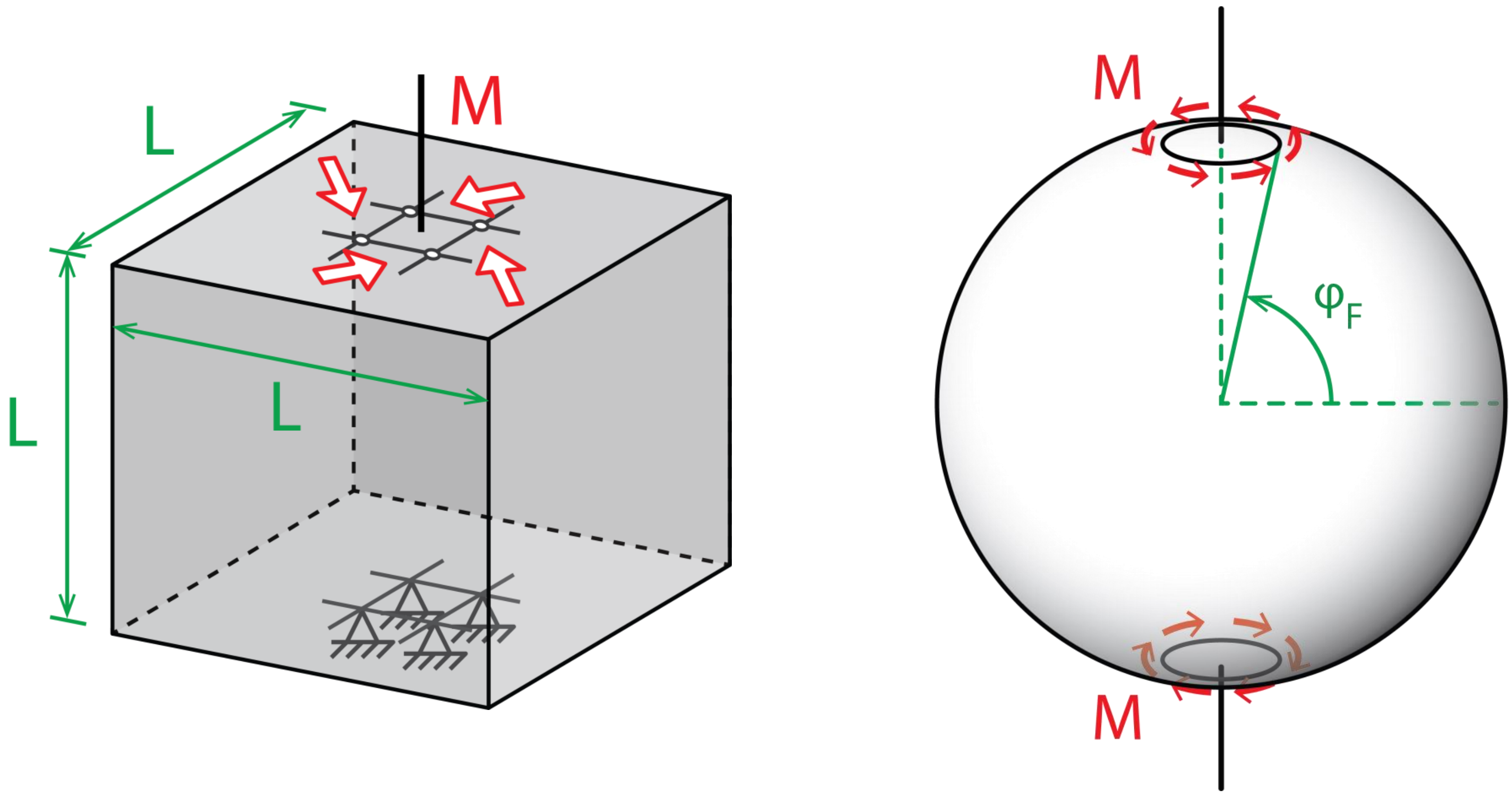
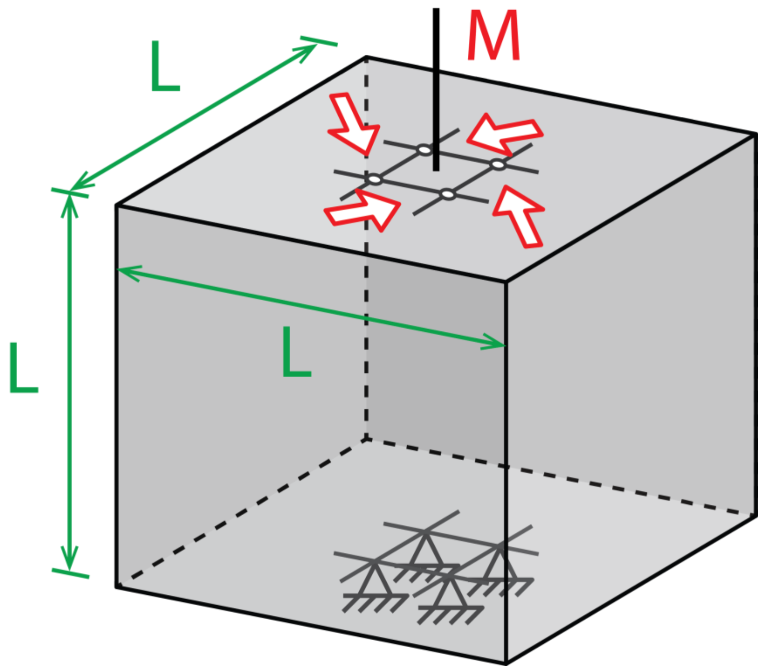
- TORSION BALL PROBLEM

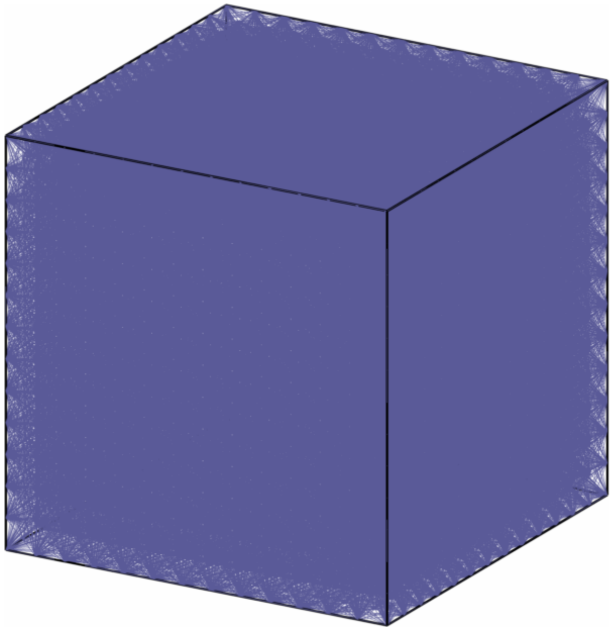


MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589-597

5) GROUND STRUCTURES IN 3D

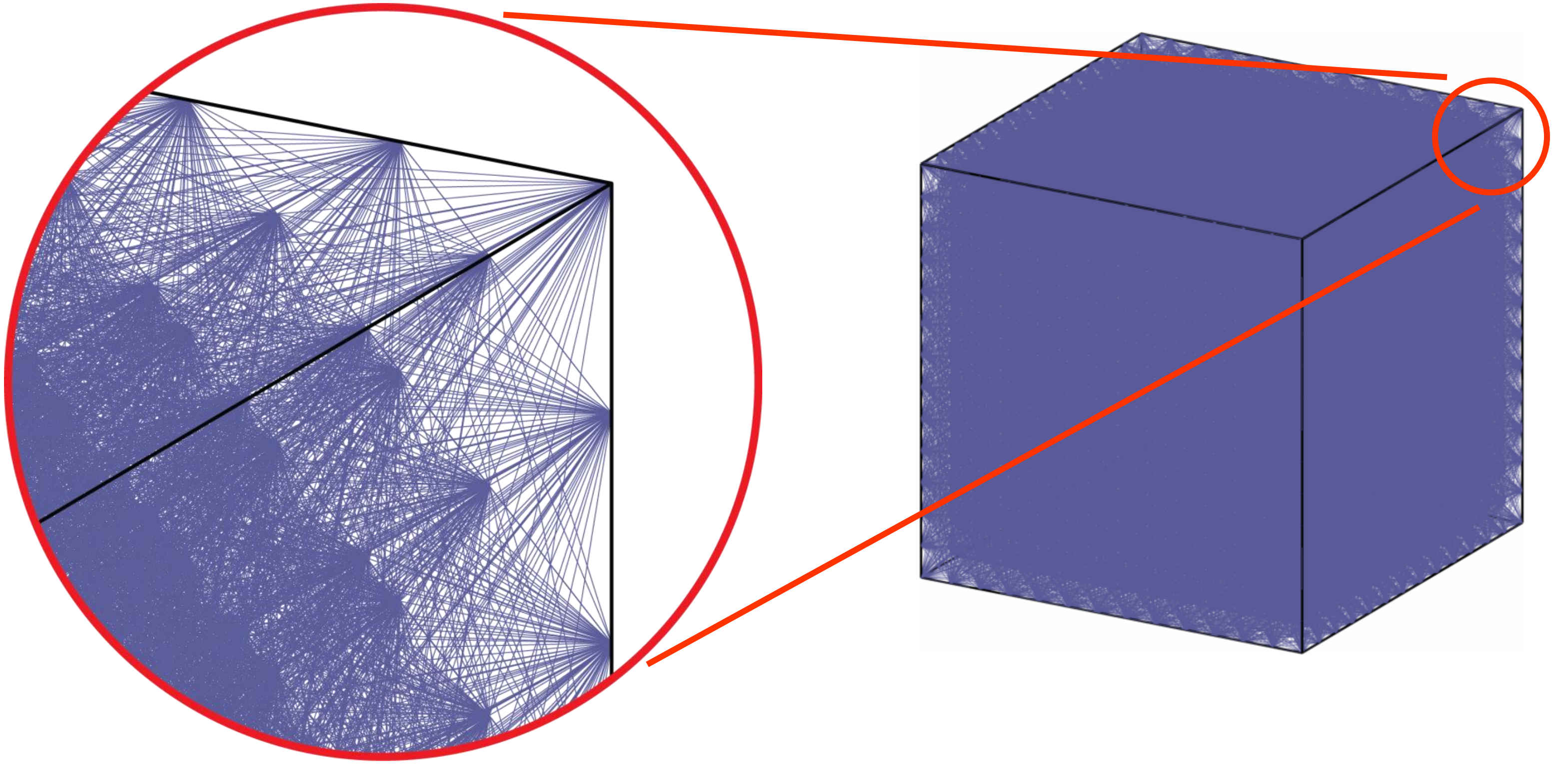
- TORSION BALL PROBLEM





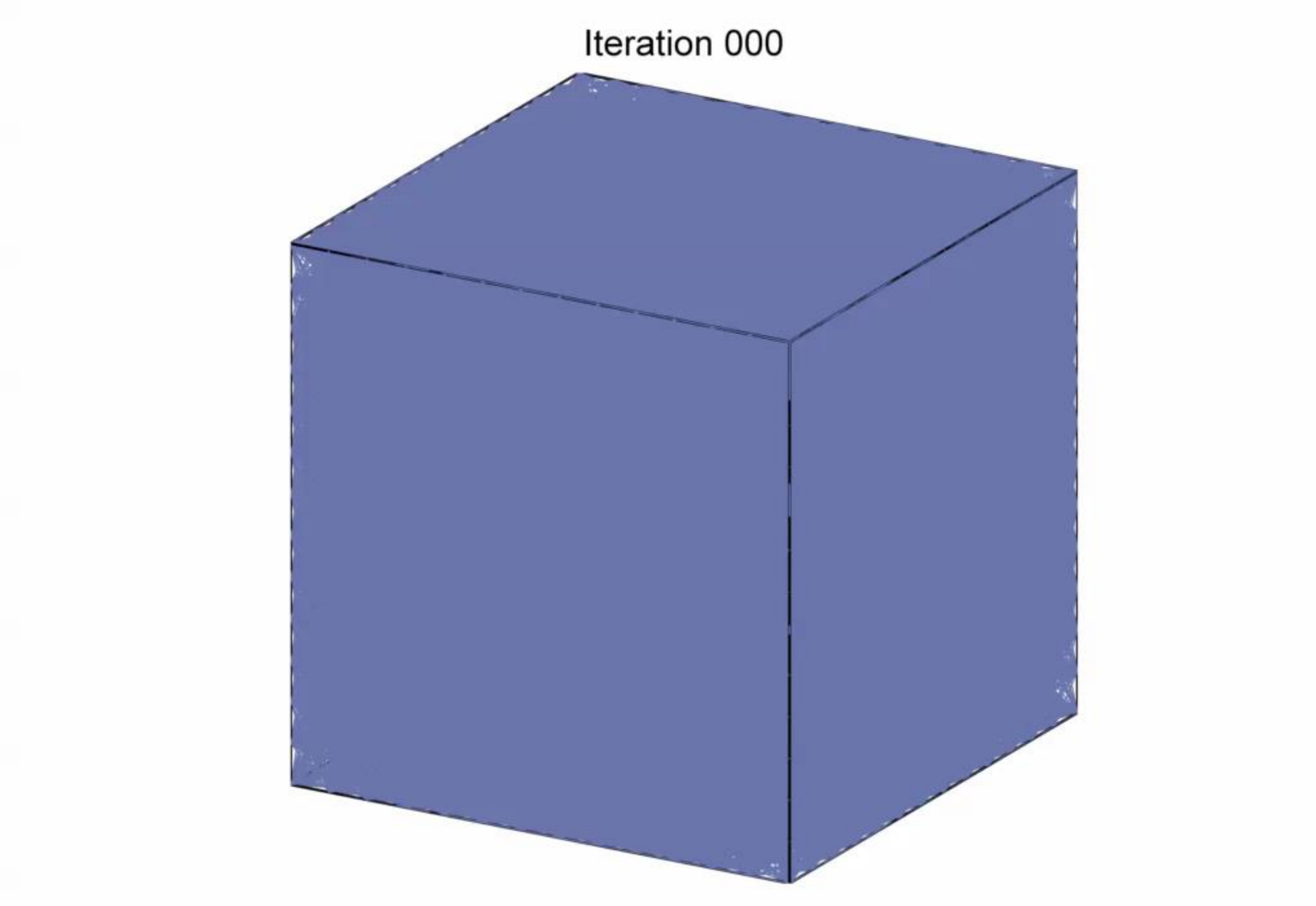
5) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM



5) GROUND STRUCTURES IN 3D

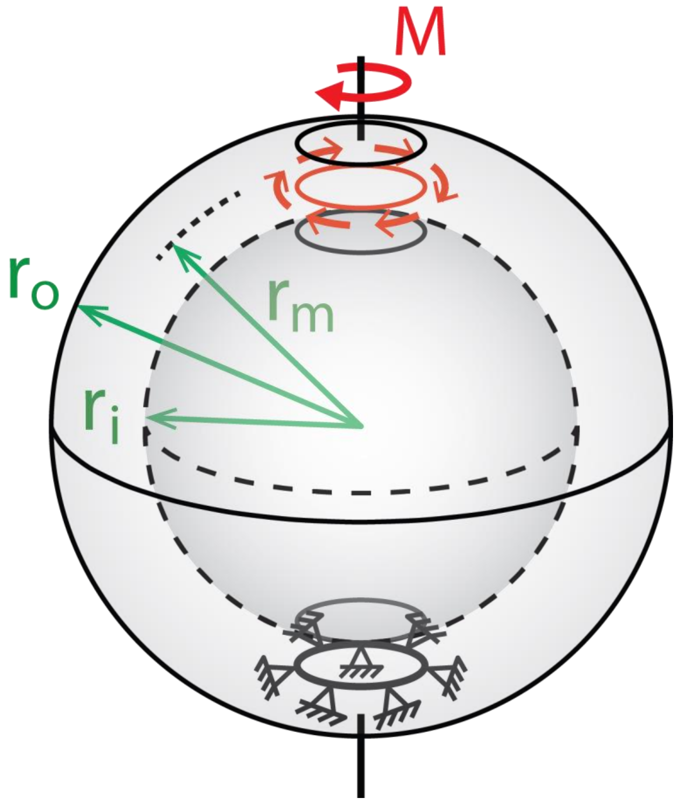
- TORSION BALL PROBLEM

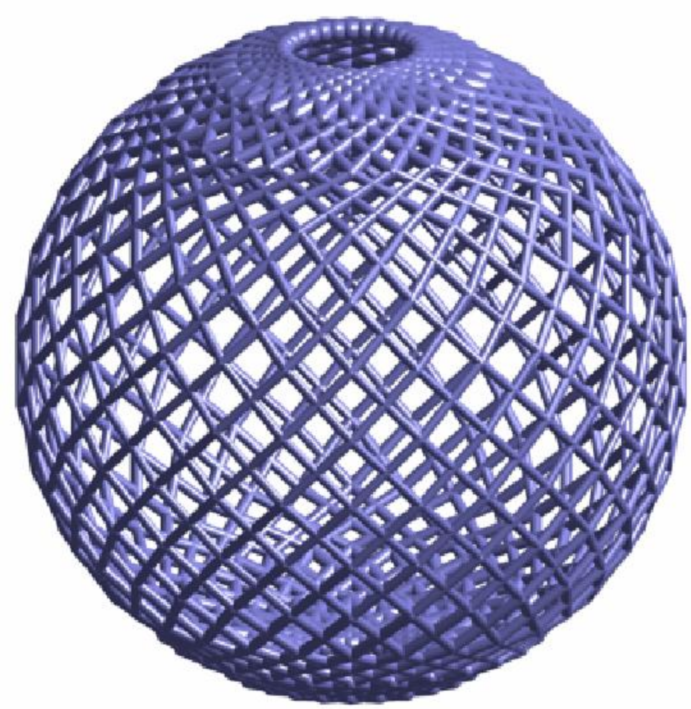


268,636 BARS

5) GROUND STRUCTURES IN 3D

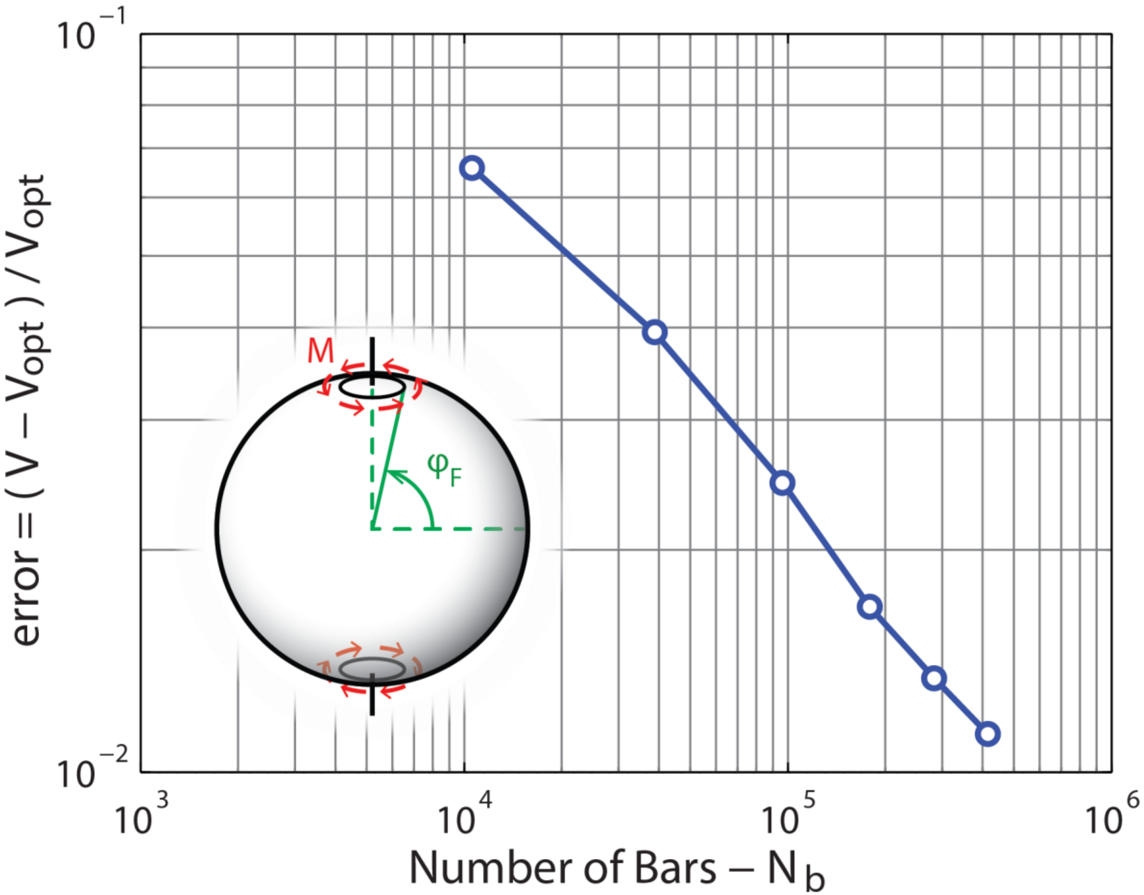
- TORSION BALL
  IMPROVING THE BASE MESH: SPHERICAL COORDINATES





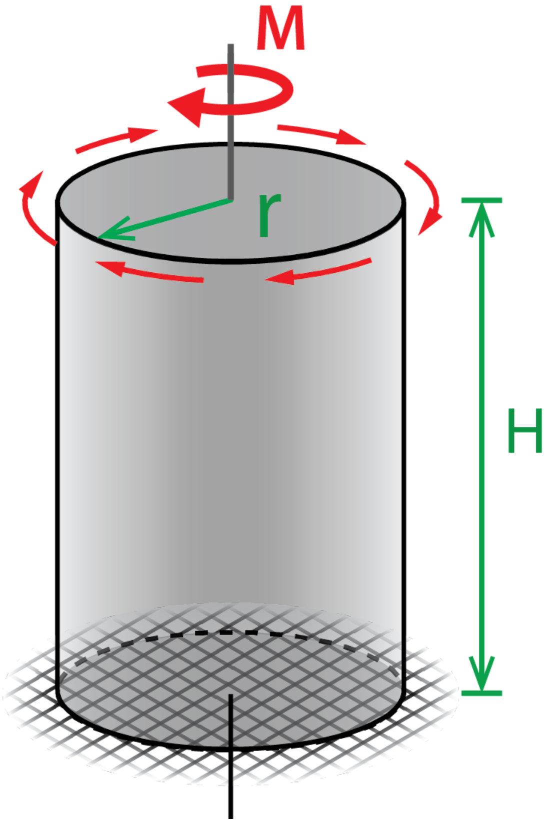
5) GROUND STRUCTURES IN 3D

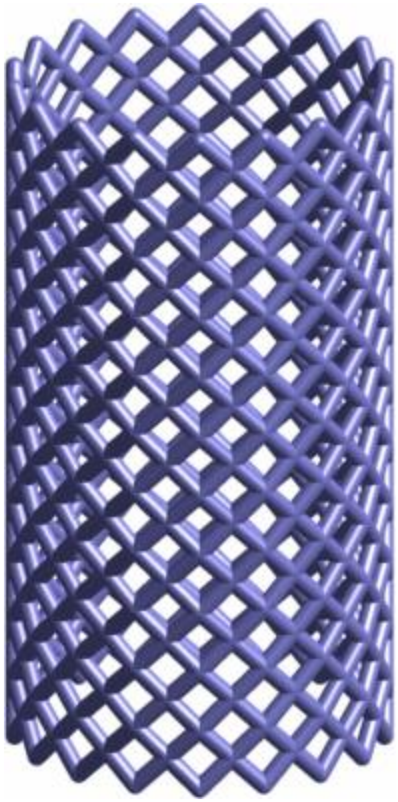
- TORSION BALL
  IMPROVING THE BASE MESH: SPHERICAL COORDINATES



5) GROUND STRUCTURES IN 3D

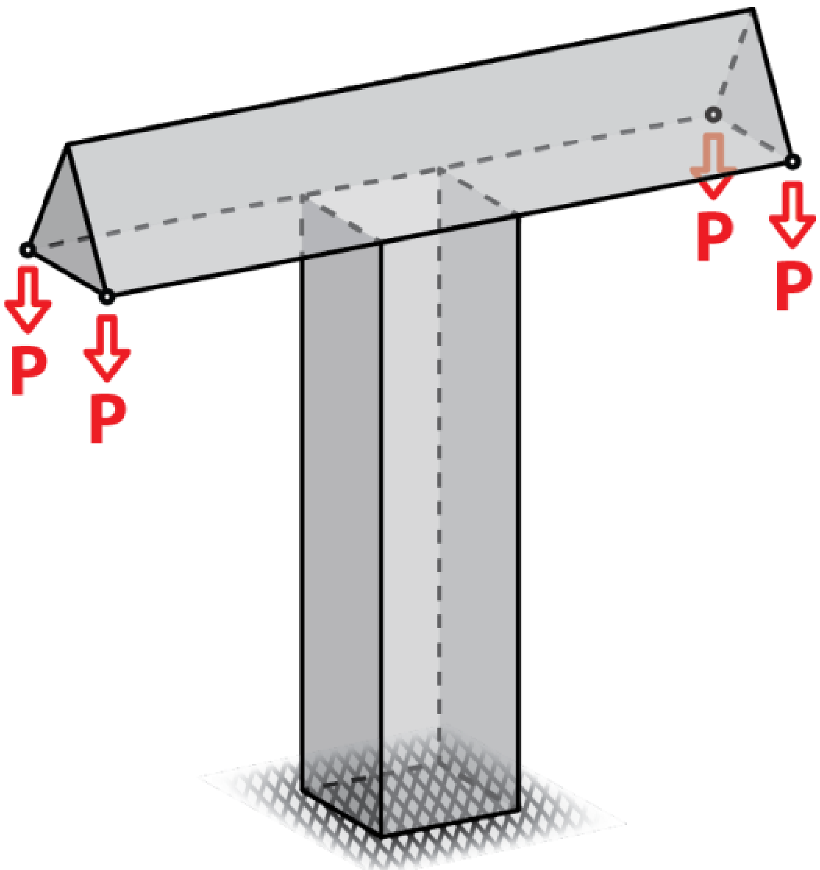
- OTHER KNOWN SOLUTIONS?

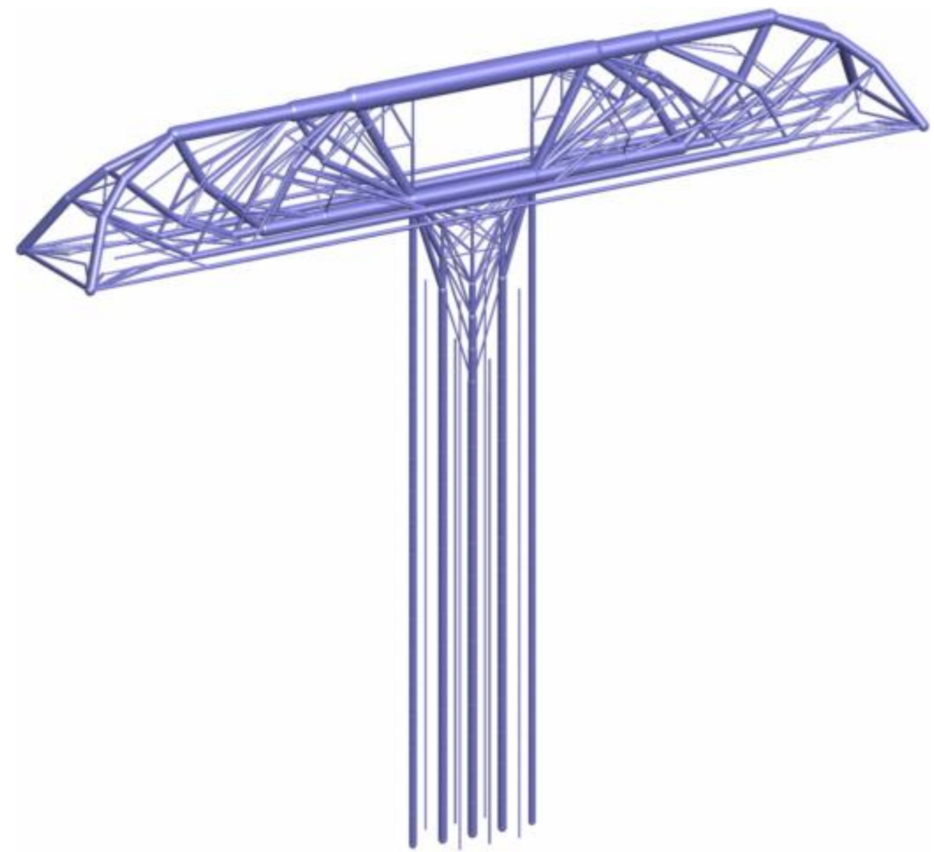




5) GROUND STRUCTURES IN 3D

- MORE APPLIED PROBLEMS?

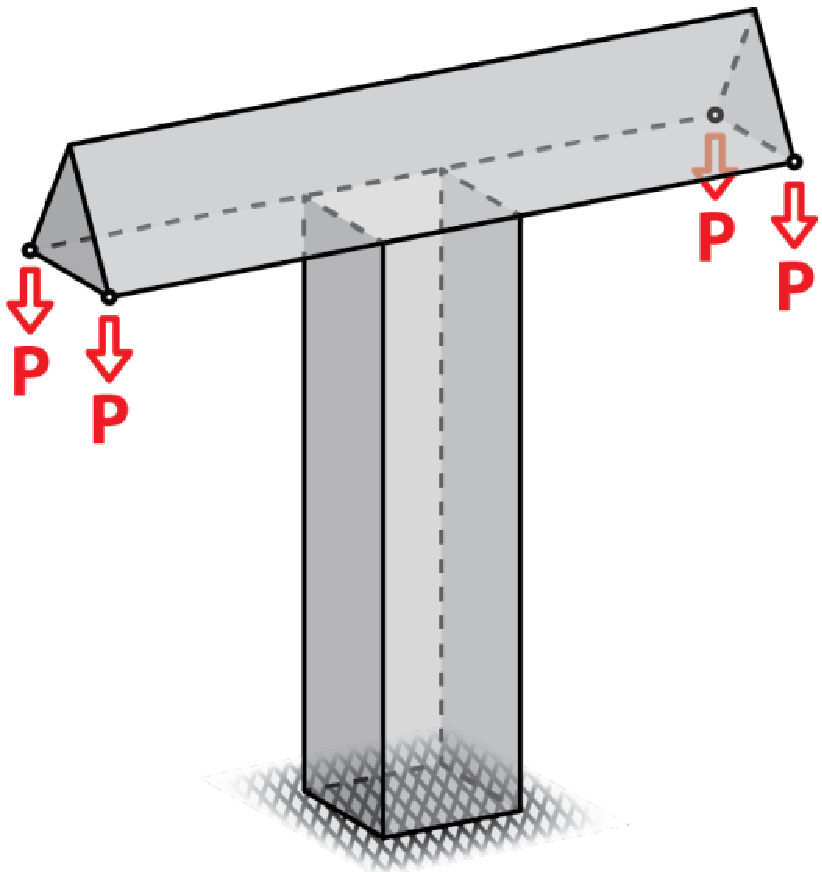


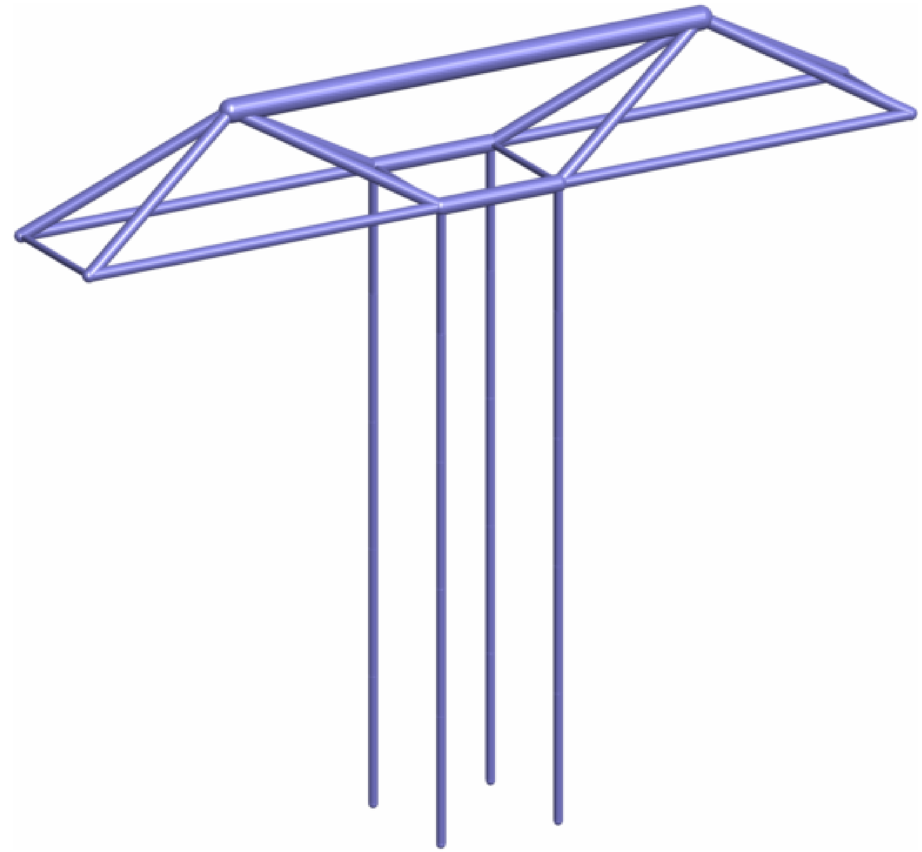


SMITH ODS (1998). "GENERATION OF GROUND STRUCTURES FOR 2D AND 3D DESIGN DOMAINS." ENGINEERING COMPUTATIONS, 15(4):462–500.

5) GROUND STRUCTURES IN 3D

- MORE APPLIED PROBLEMS?

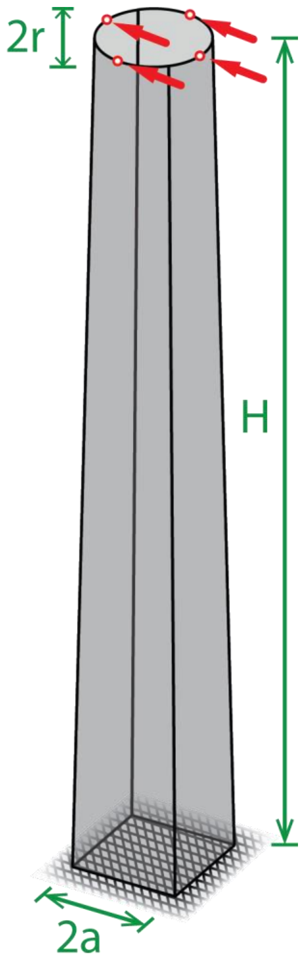


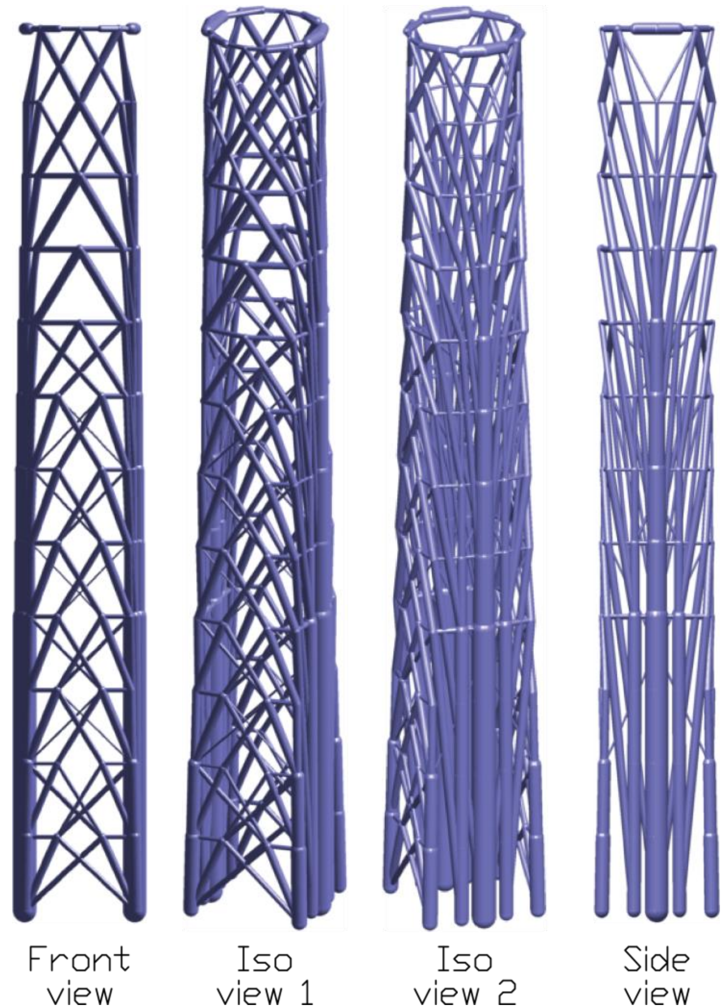


SMITH ODS (1998). "GENERATION OF GROUND STRUCTURES FOR 2D AND 3D DESIGN DOMAINS." ENGINEERING COMPUTATIONS, 15(4):462-500.

5) GROUND STRUCTURES IN 3D

- MORE APPLIED PROBLEMS?

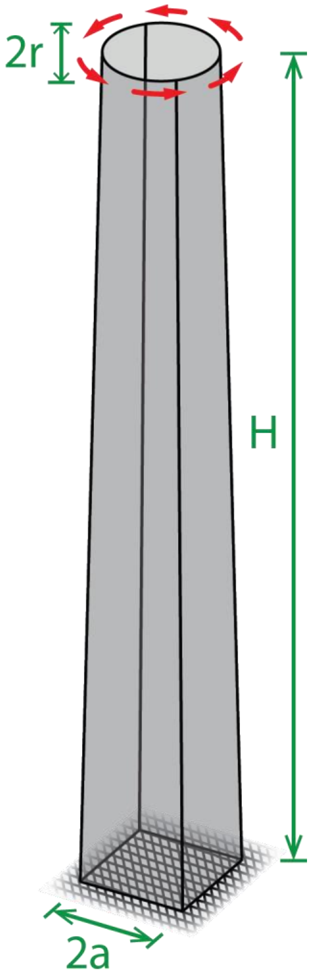


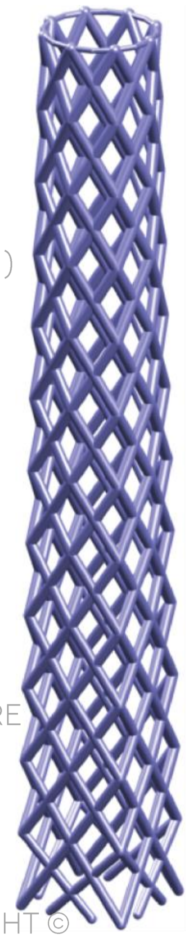


4,100 BARS

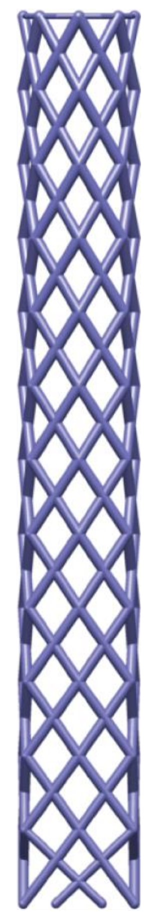
5) GROUND STRUCTURES IN 3D

- MORE APPLIED PROBLEMS?





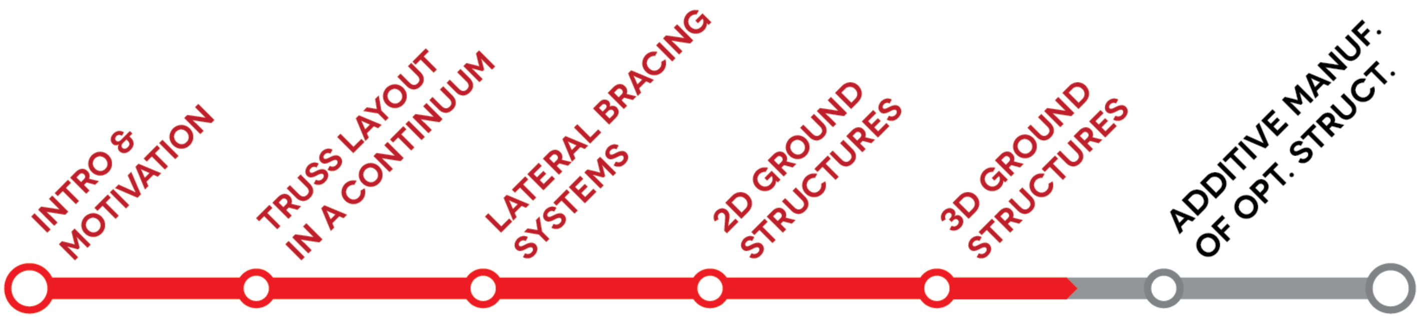
Iso view



Front view

4,100 BARS

ROADMAP



INTRO &
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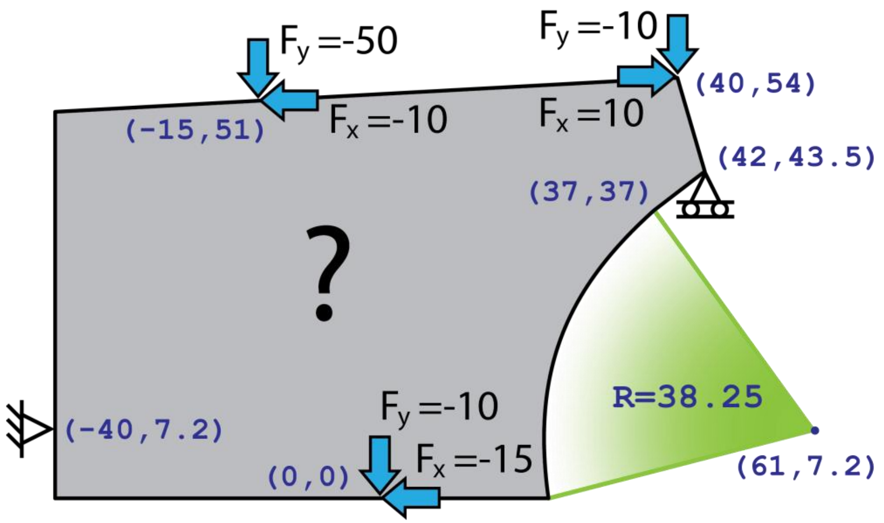
STRUCTURES
3D GROUND
STRUCTURES
ADDITIVE MANUF.
OF OPT. STRUCT.

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- INTRODUCTION TO DENSITY-BASED TOPOLOGY OPTIMIZATION

CANNONDALE CAPO
(URBAN COMMUTER BIKE)

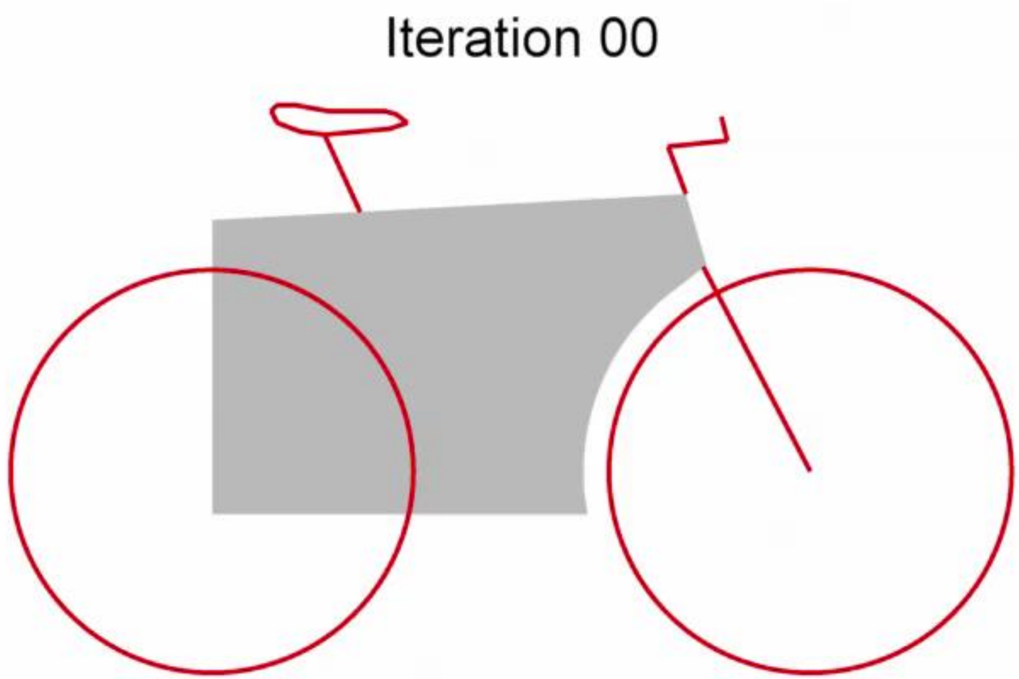


BIKE DOMAIN AND LOADS

ZEGARD T, PAULINO GH (2013) "TOWARD GPU ACCELERATED TOPOLOGY OPTIMIZATION ON UNSTRUCTURED MESHES." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, 48(3):473–485

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- INTRODUCTION TO DENSITY-BASED TOPOLOGY OPTIMIZATION



ZEGARD T, PAULINO GH (2013) "TOWARD GPU ACCELERATED TOPOLOGY OPTIMIZATION ON UNSTRUCTURED MESHES." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, 48(3):473-485

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- DENSITY–BASED (NESTED) FORMULATION:
  - USING A DENSITY FILTER[1]
  - MODIFIED SIMP[2,3,4]

$$
\begin{aligned}
\min_{\boldsymbol{\rho}} \quad & J\left(\boldsymbol{\rho}, \mathbf{u}\left(\boldsymbol{\rho}\right)\right) \\
\text{s.t.} \quad & \overline{\boldsymbol{\rho}} = \mathbf{H}\boldsymbol{\rho} && \text{— FILTERING} \\
& \textstyle\sum_{i}^{N_e} \overline{\rho}_i v_i - (f)\left(V_0\right) \leq 0 && \text{— VOLUME CONSTRAINT} \\
& g_i\left(\boldsymbol{\rho}, \mathbf{u}\left(\boldsymbol{\rho}\right)\right) \leq 0 \qquad i = 1 \ldots N_c \\
& 0 \leq \rho_j \leq 1 \qquad j = 1 \ldots N_e && \text{— 1 = SOLID, 0 = VOID} \\
& E_k\left(\overline{\rho}_k\right) = E_{min} + \overline{\rho}_k^p \left(E_0 - E_{min}\right) \qquad k = 1 \ldots N_e && \text{— MOD–SIMP} \\
\text{with} \quad & \mathbf{K}\left(\overline{\rho}\right)\mathbf{u} = \mathbf{f}
\end{aligned}
$$

1) BOURDIN B (2001) "FILTERS IN TOPOLOGY OPTIMIZATION." INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 50(9):2143–2158

2) BENDSOE MP (1989) "OPTIMAL SHAPE DESIGN AS A MATERIAL DISTRIBUTION PROBLEM." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 1(4):193–202

3) ZHOU M, ROZVANY G (1991) "THE COC ALGORITHM, PART II: TOPOLOGICAL, GEOMETRICAL AND GENERALIZED SHAPE OPTIMIZATION. "COMP METH APPL MECH ENGRG 89:309–336

4) SIGMUND O (2007) "MORPHOLOGY–BASED BLACK AND WHITE FILTERS FOR TOPOLOGY OPTIMIZATION." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, 33(4–5):401–424.

6) ADDITIVE MANUF. OF OPT. STRUCTS.

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  - USING A DENSITY FILTER[1]
  - MODIFIED SIMP[2,3,4]

$$
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\text{s.t.} \quad & \overline{\boldsymbol{\rho}} = \mathbf{H}\boldsymbol{\rho} \\
& \textstyle\sum_i^{N_e} \overline{\rho}_i v_i - (f)\,(V_0) \leq 0 \\
& g_i\left(\boldsymbol{\rho}, \mathbf{u}\left(\boldsymbol{\rho}\right)\right) \leq 0 \qquad i = 1 \ldots N_c \\
& 0 \leq \rho_j \leq 1 \qquad j = 1 \ldots N_e \\
& E_k\left(\overline{\rho}_k\right) = E_{min} + \overline{\rho}_k^{p}\left(E_0 - E_{min}\right) \qquad k = 1 \ldots N_e \\
\text{with} \quad & \mathbf{K}\left(\overline{\rho}\right)\mathbf{u} = \mathbf{f}
\end{aligned}
$$

P=1
VARIABLE THICKNESS SHEET PROBLEM (CONVEX)

1) BOURDIN B (2001) "FILTERS IN TOPOLOGY OPTIMIZATION." INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 50(9):2143-2158

2) BENDSOE MP (1989) "OPTIMAL SHAPE DESIGN AS A MATERIAL DISTRIBUTION PROBLEM." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 1(4):193-202

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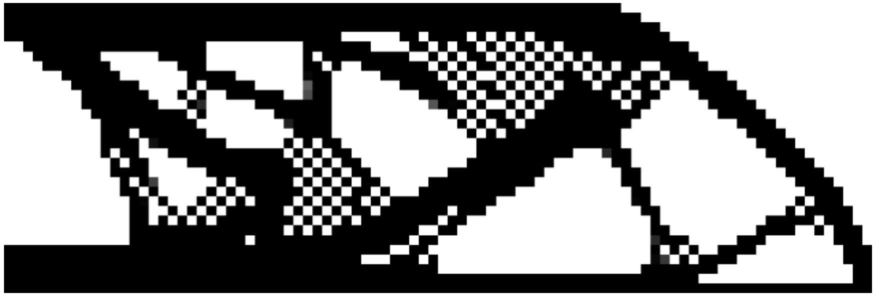
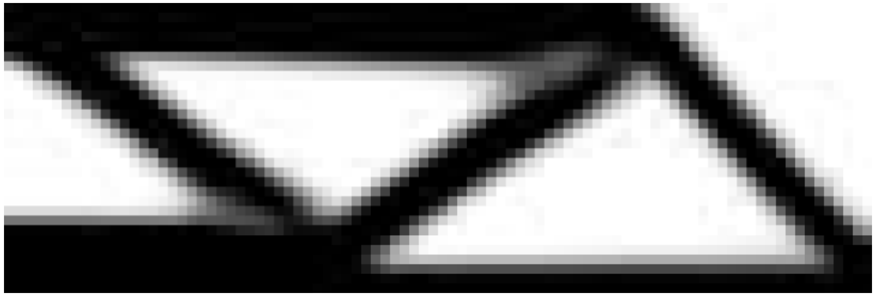
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- FILTERS IN DENSITY-BASED FORMULATION:
  - SENSITIVITY FILTER (1-FIELD)
  - DENSITY FILTER (2-FIELDS)
  - PROJECTION FILTER (3-FIELDS)

USED IN THIS WORK



UNFILTERED
(CHECKERBOARD)

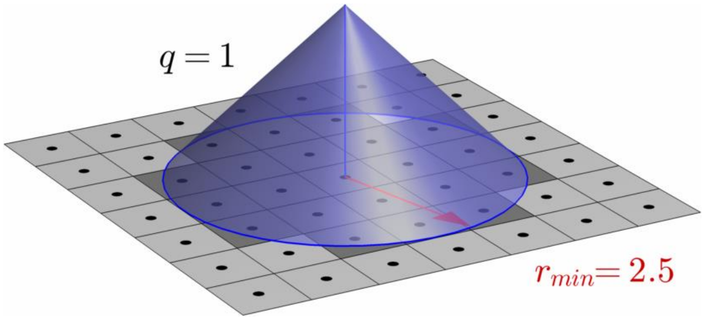


FILTERED

REVIEW ON FILTERING:
SIGMUND O, MAUTE K (2013) "TOPOLOGY OPTIMIZATION APPROACHES." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 48(6):1031-1055

6) ADDITIVE MANUF. OF OPT. STRUCTS.

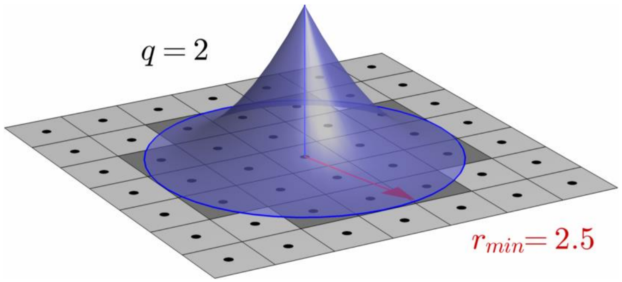
- CONVOLUTION (BLURRING) OF THE DENSITY FIELD

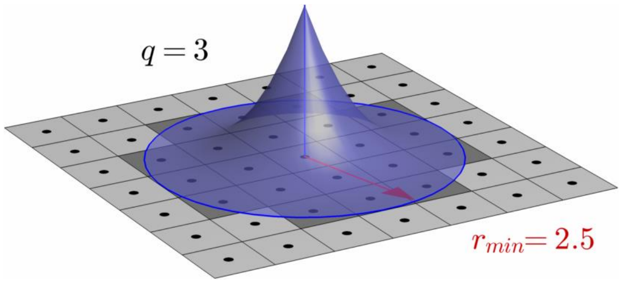


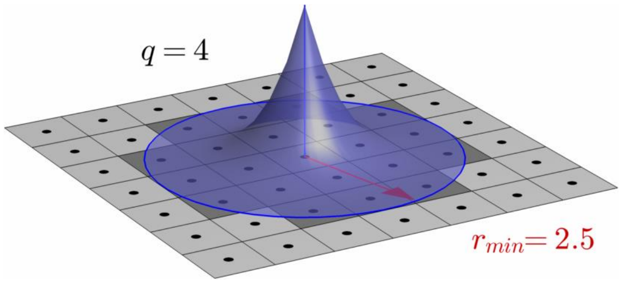
$$\overline{\boldsymbol{\rho}} = \mathbf{H}\boldsymbol{\rho}$$

$$\text{with} \quad \mathbf{H}_{ij} = \frac{h(i,j)\, v_j}{\sum_k^{N_e} h(i,k)\, v_k}$$

$$h(i,j) = \begin{cases} [r_{min} - \text{dist}(i,j)]^q & \text{for} \quad r_{min} - \text{dist}(i,j) > 0 \\ 0 & \text{otherwise} \end{cases}$$

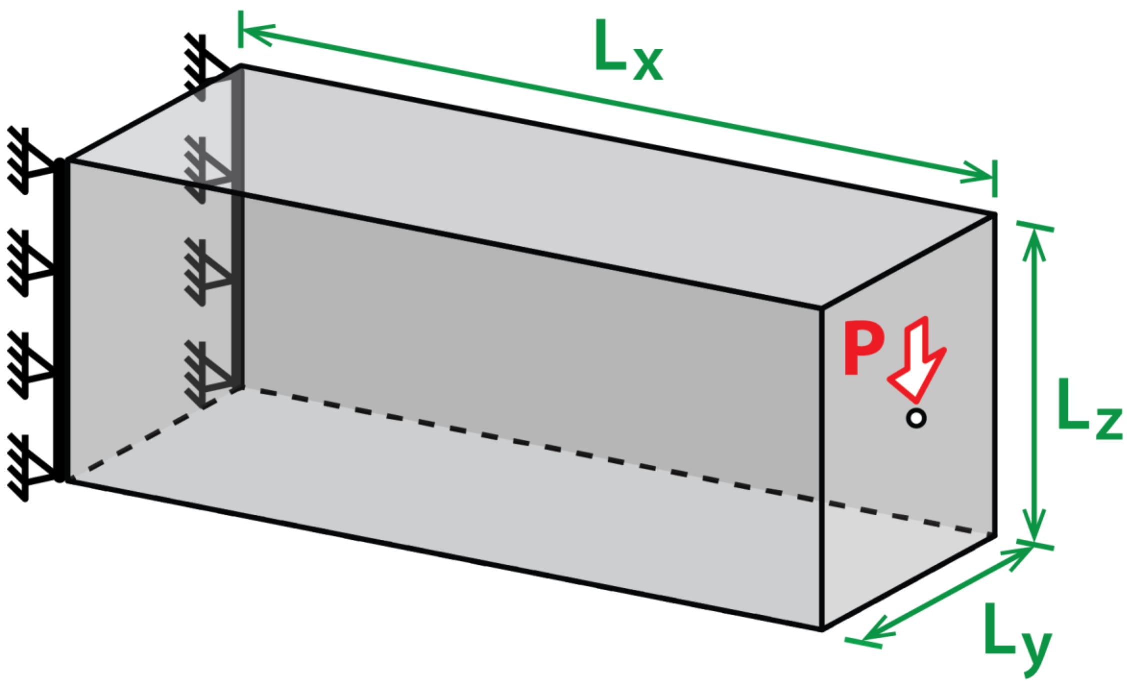






6) ADDITIVE MANUF. OF OPT. STRUCTS.

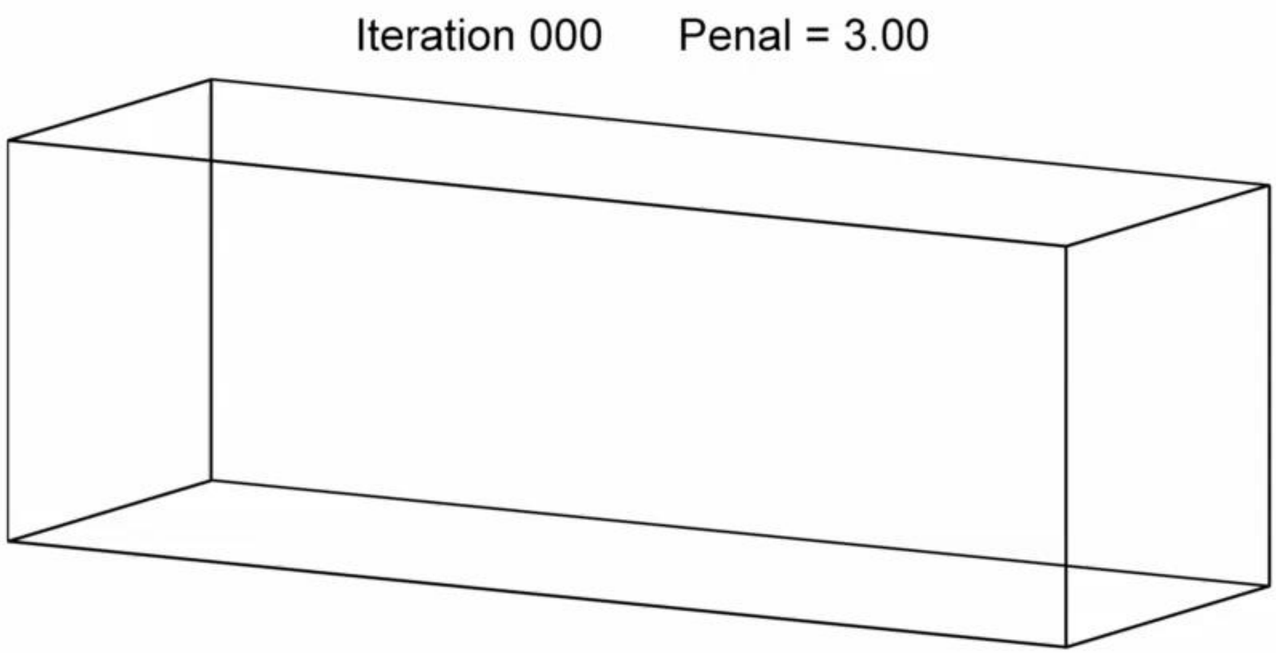
- EDGE-SUPPORTED CANTILEVER BEAM
  LX=3, LY=LZ=1, Q=1, R=5 AND **VOLFRAC=10%**



559,872 DVS FOR ½
(1,119,744 TOTAL)

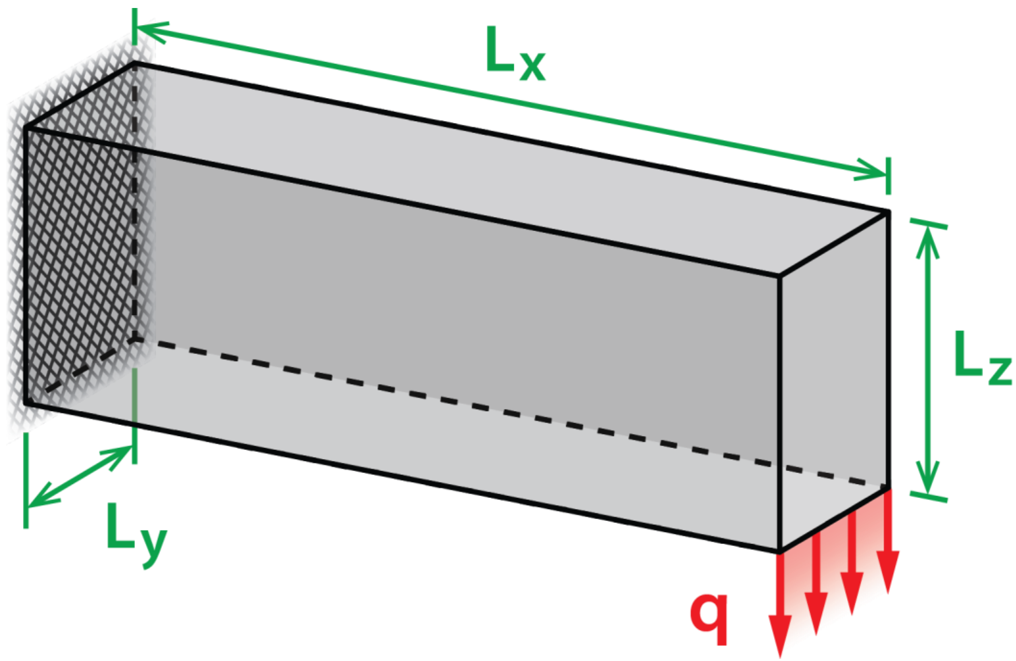
6) Additive Manuf. of Opt. Structs.

- Edge-supported cantilever beam
  Lx=3, Ly=Lz=1, Q=1, R=5 and volfrac=10%



6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-LOADED CANTILEVER BEAM
  LX=3, LY=LZ=1
  VOLFRAC=10%, R=6, Q=1 AND P=3



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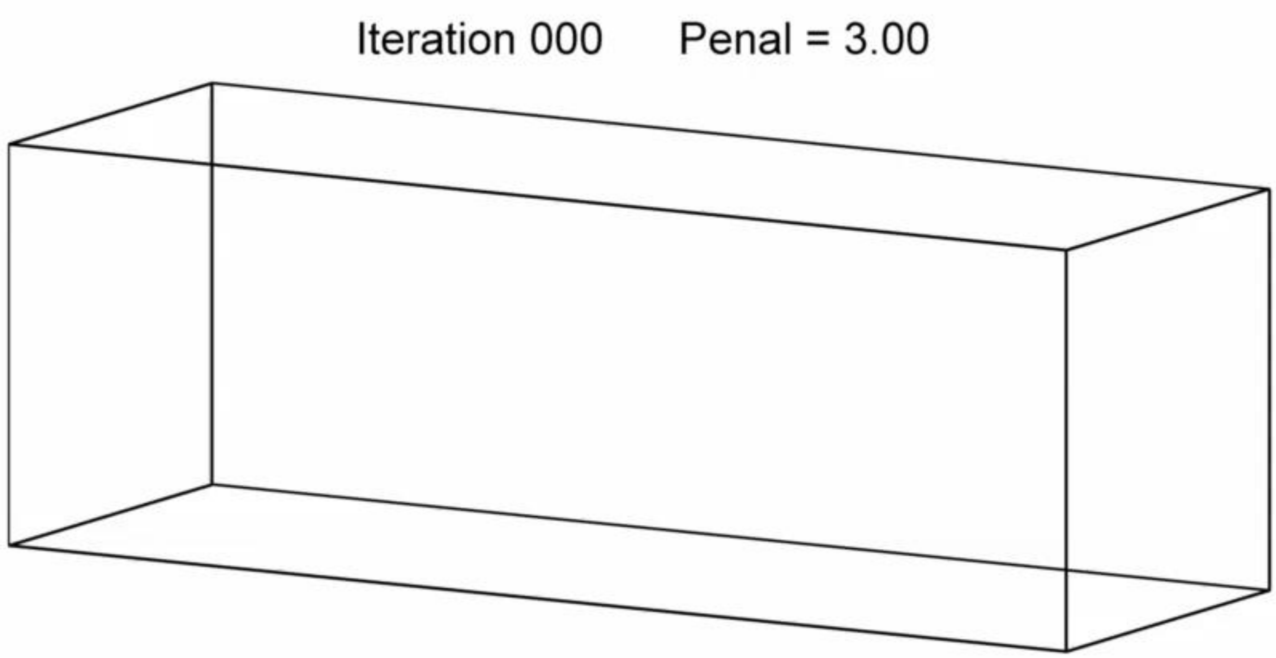
  Lx=3, Ly=Lz=1, Volfrac=10%, R=6, Q=1 and P=3



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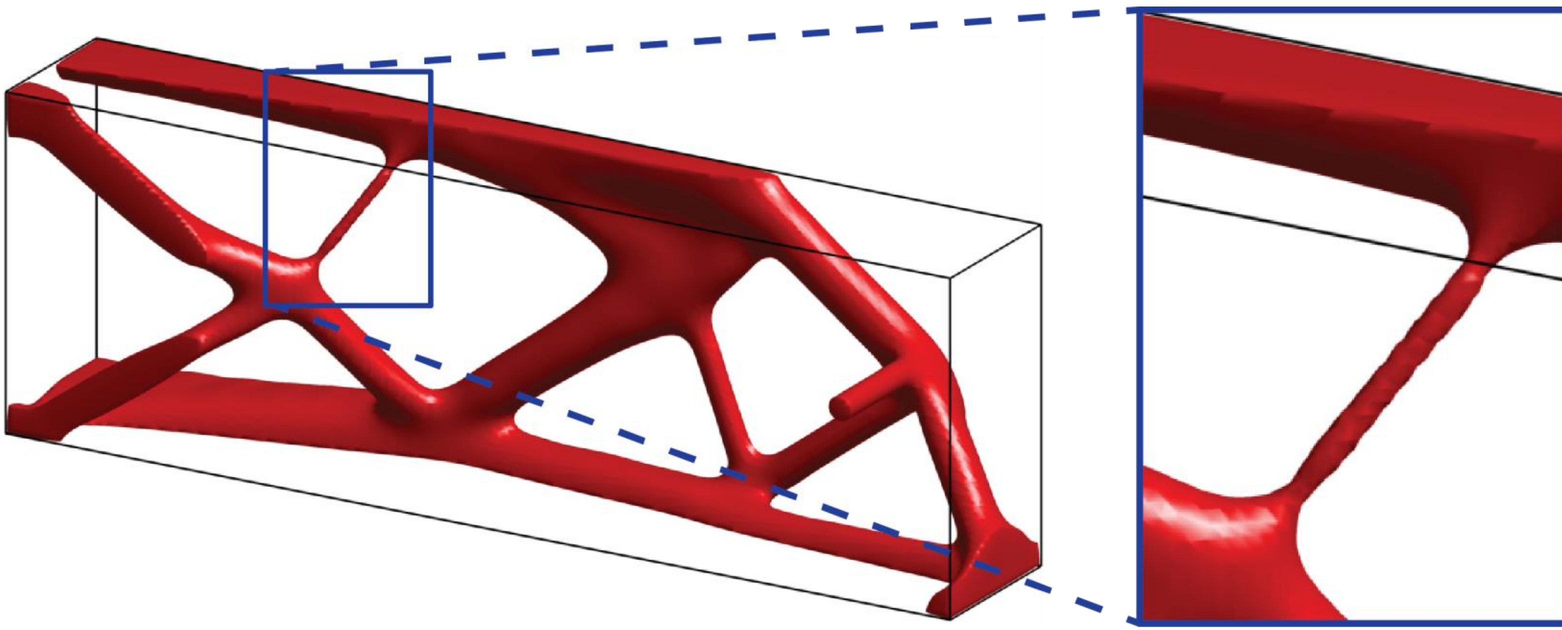
  Lx=3, Ly=Lz=1
  Volfrac=10%, R=6, Q=1 and P=3



559,872 DVs for ½
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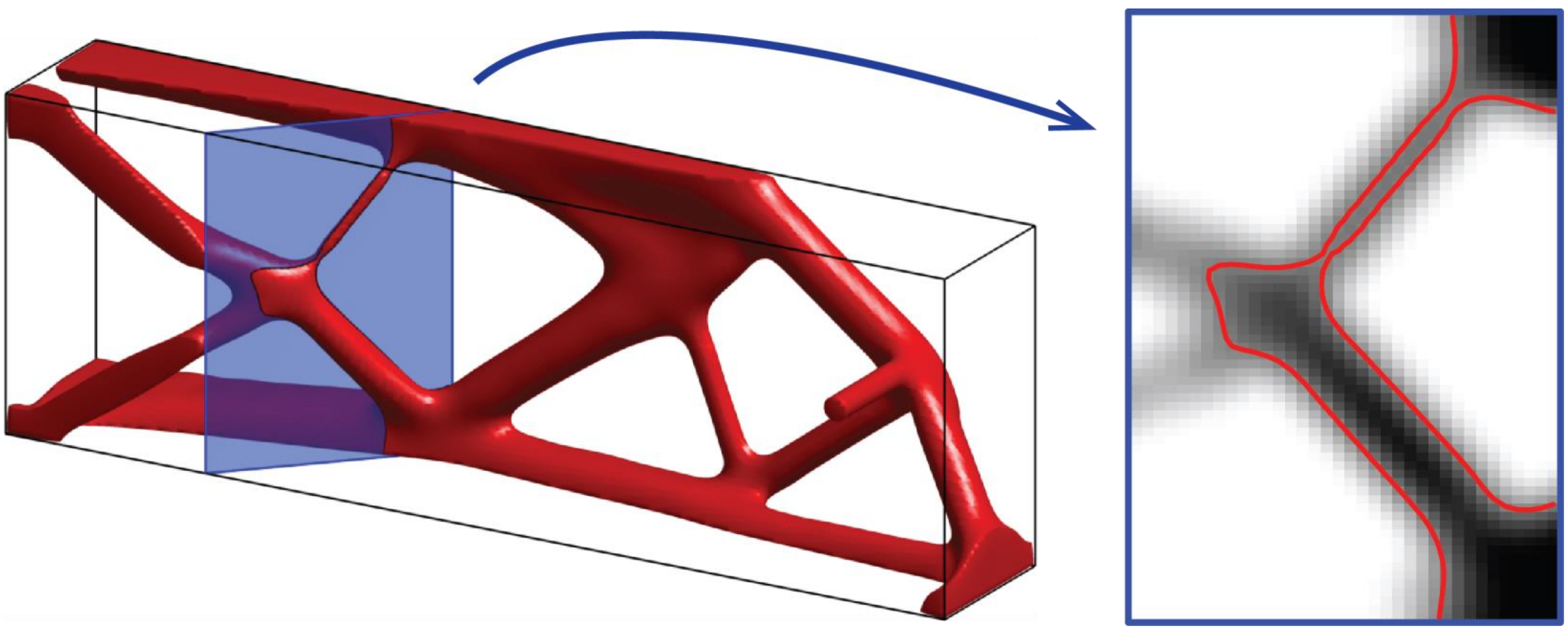
6) Additive Manuf. of Opt. Structs.

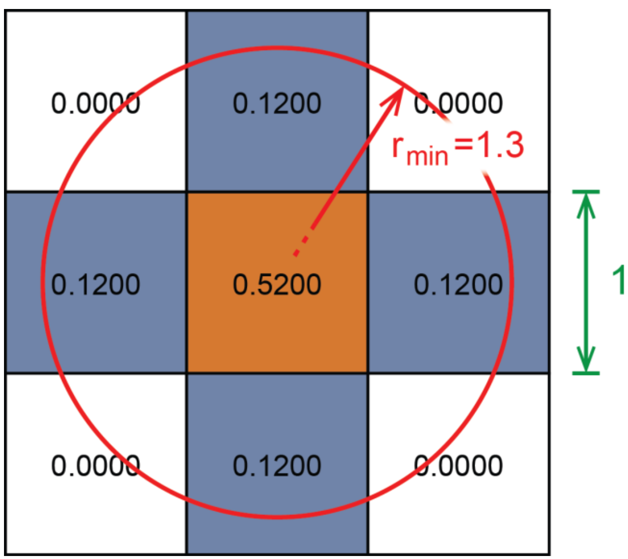
- Edge-loaded cantilever
  Lx=3, Ly=Lz=1
  Volfrac=10%, R=6, Q=1 and P=3



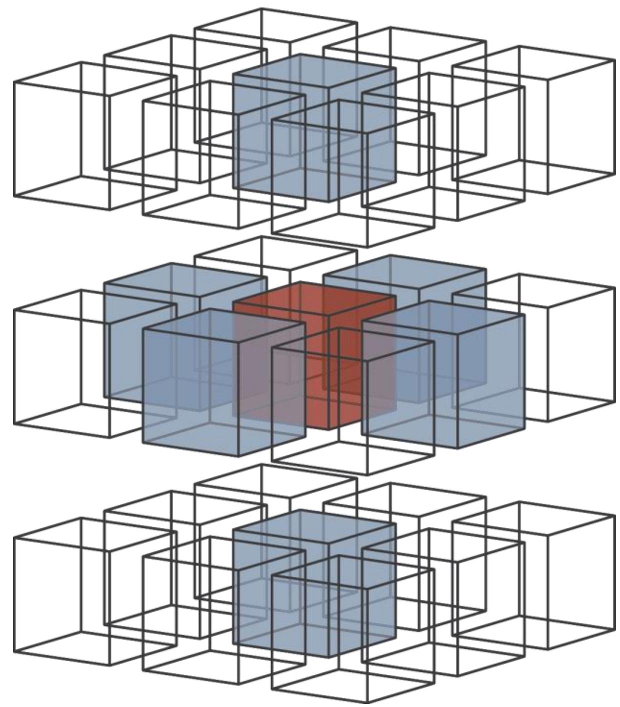
559,872 DVs for ½
(1,119,744 total)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- FILTER'S WEIGHTS FOR A REGULAR MESH
  R_{min}=1.3, Q=1 AND ELEM SIZE IS L=1



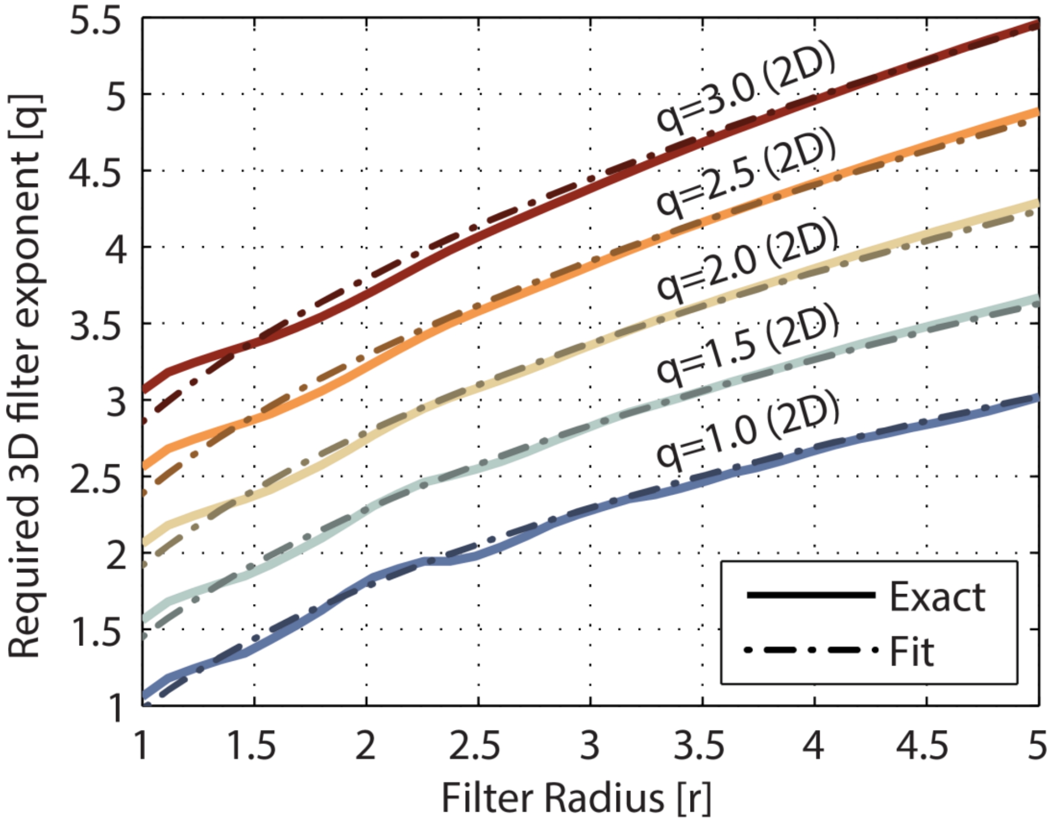
TWO-DIMENSIONS



THREE-DIMENSIONS
(H_{ii} = 0.4194)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

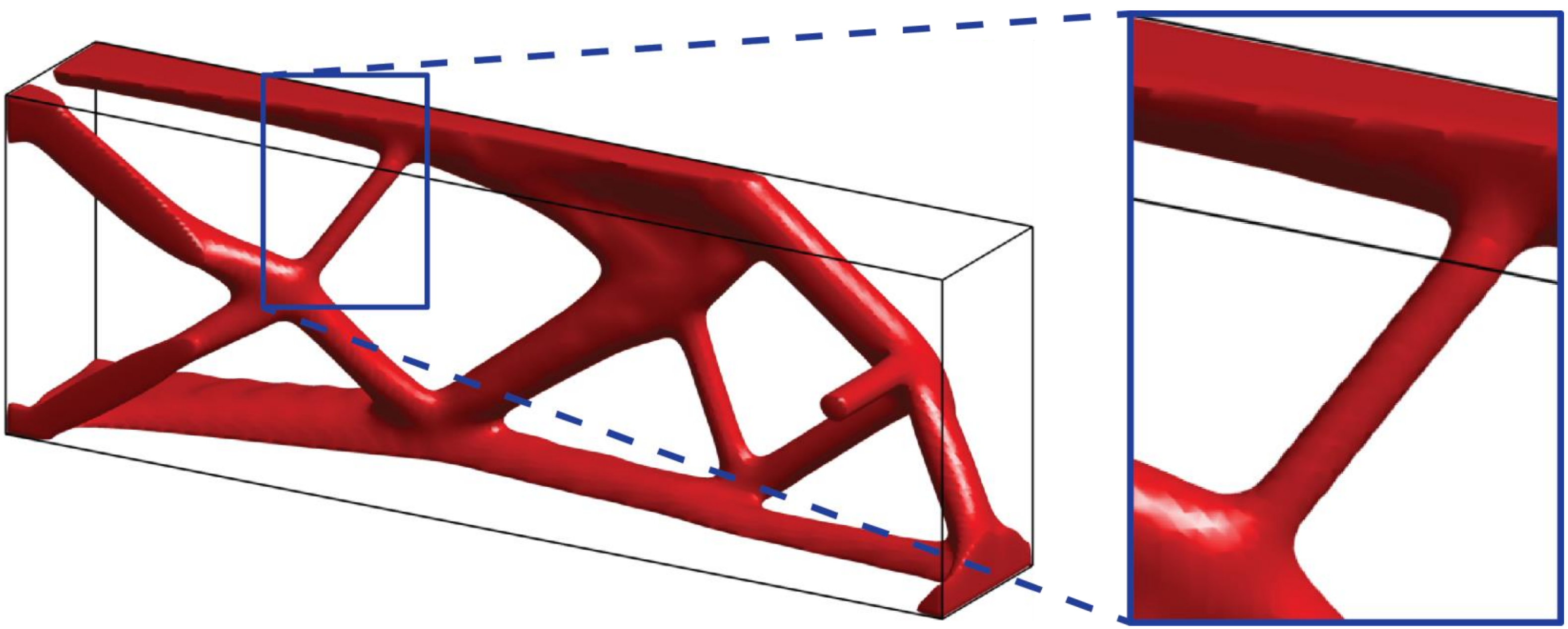
IDEA: WHAT EXPONENT Q MAKES $H_{ii}^{(2D)} = H_{ii}^{(3D)}$?



$$q^{(3D)} = \log(r_{min}) + \frac{17}{20}q^{(2D)} + \frac{4}{57}q^{(2D)}r_{min} + \frac{4}{87}r_{min}$$

6) Additive Manuf. of Opt. Structs.

- Edge-loaded cantilever
  $L_x=3$, $L_y=L_z=1$
  Volfrac=10%, R=6, Q=3 and P=3



559,872 DVs for ½
(1,119,744 total)

6) Additive Manuf. of Opt. Structs.

- Edge-loaded cantilever
  Lx=3, Ly=Lz=1
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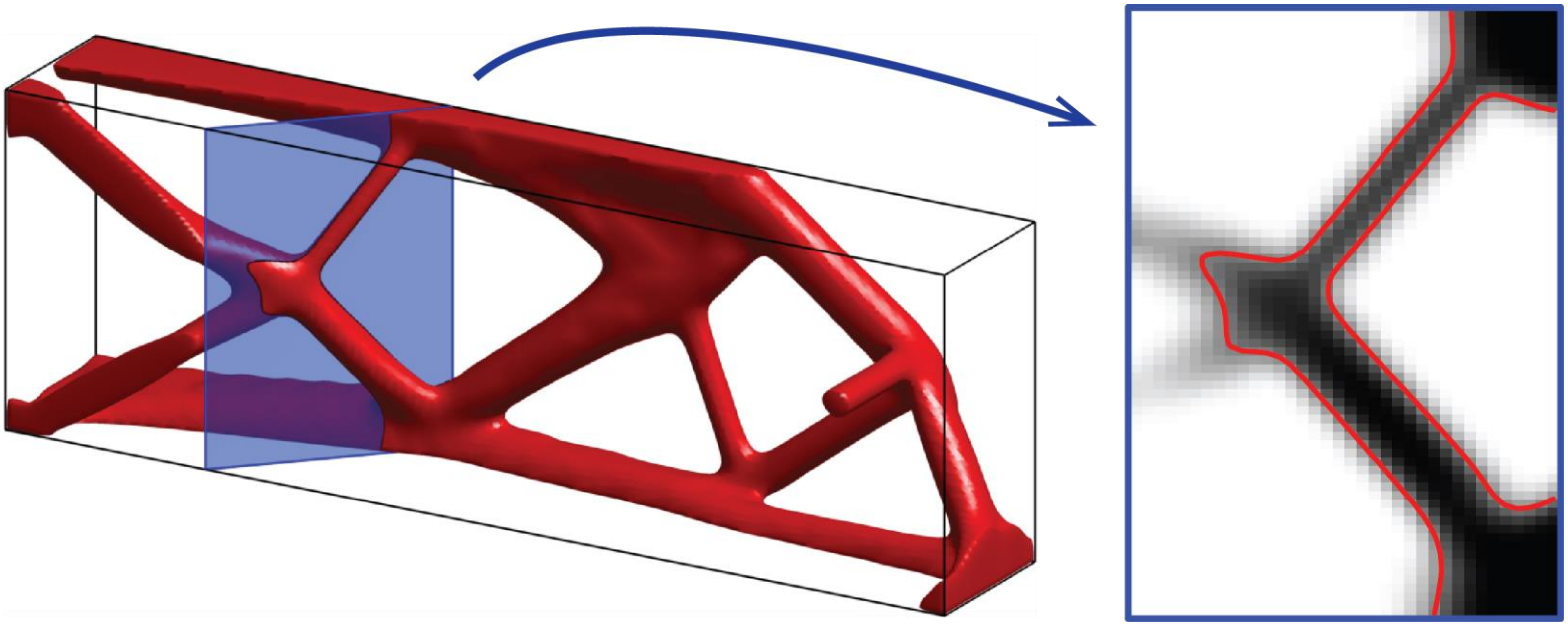
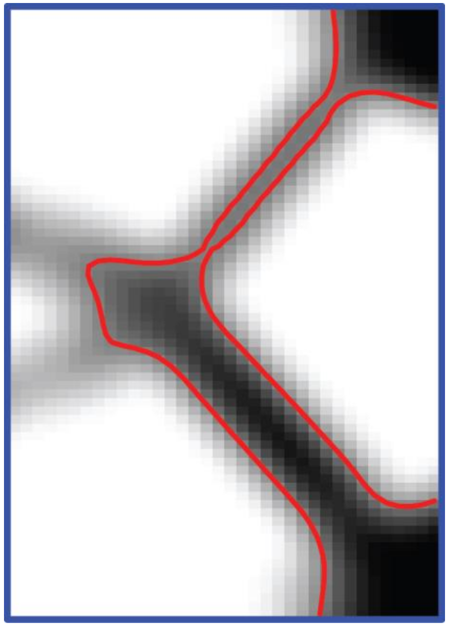


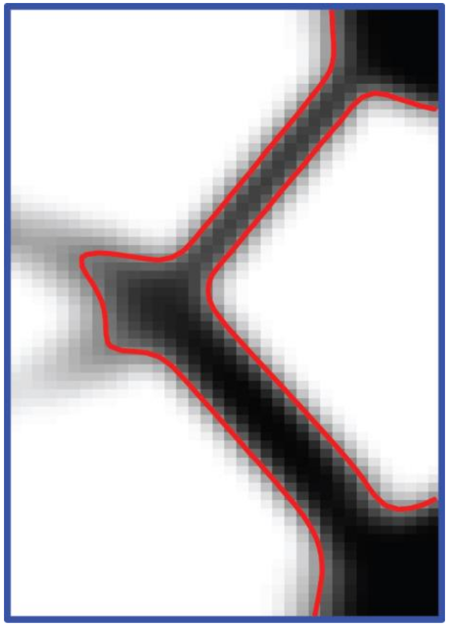
559,872 DVs for ½
(1,119,744 total)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-LOADED CANTILEVER
  DENSITY FILTER: R=6



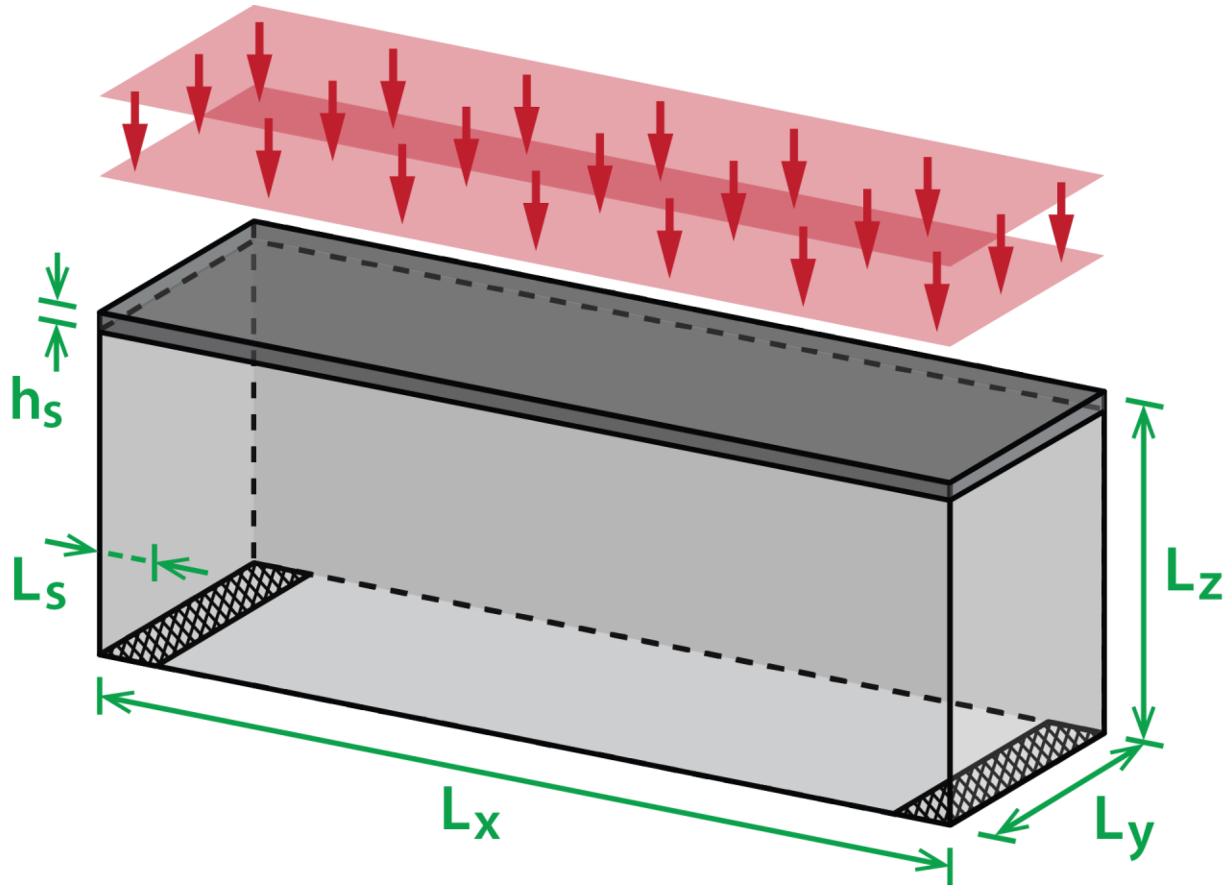
LINEAR DENSITY FILTER



CUBIC DENSITY FILTER

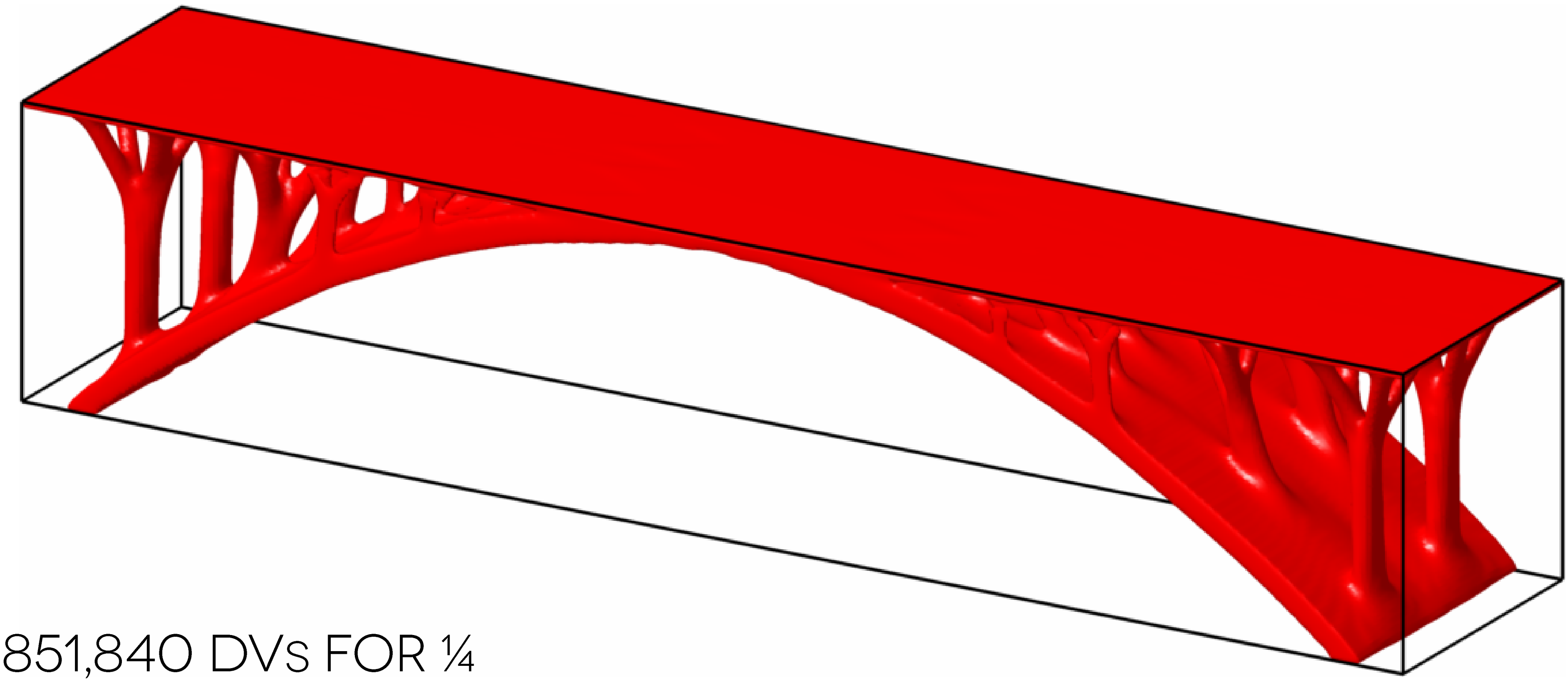
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- BRIDGE PROBLEM
  LX=XXX, LY=LZ=YYY, Q=ZZZ, R=5 AND VOLFRAC=10%



6) Additive manuf. of opt. structs.

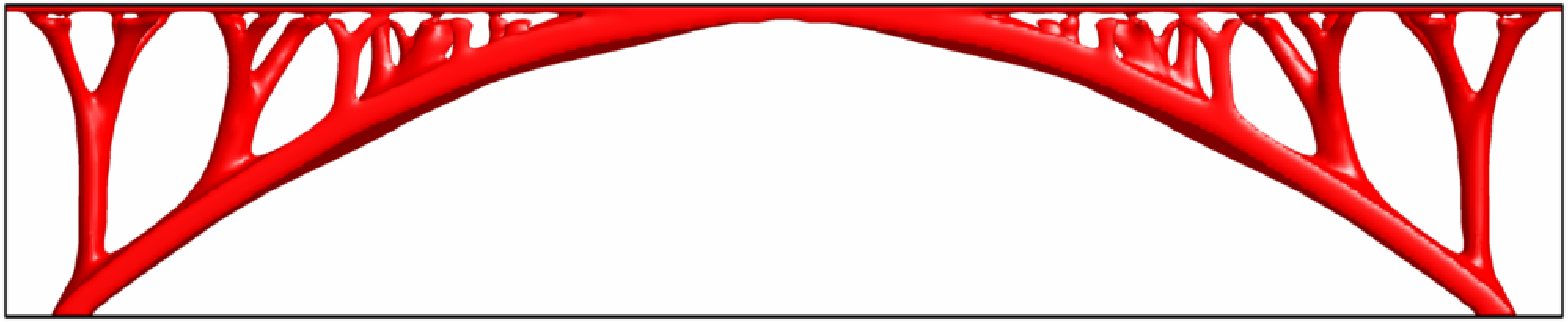
- Bridge problem
  $L_x=25$, $L_y=L_z=5$
  Volfrac=10%, R=5, Q=3 and P=3



851,840 DVs for ¼
(3,407,360 total)

6) Additive Manuf. of Opt. Structs.

- Bridge problem
  Lx=25, Ly=Lz=5
  Volfrac=10%, R=5, Q=3 and P=3



851,840 DVs for ¼
(3,407,360 total)

6) Additive Manuf. of Opt. Structs.

- Bridge problem
  $L_x=25$, $L_y=L_z=5$
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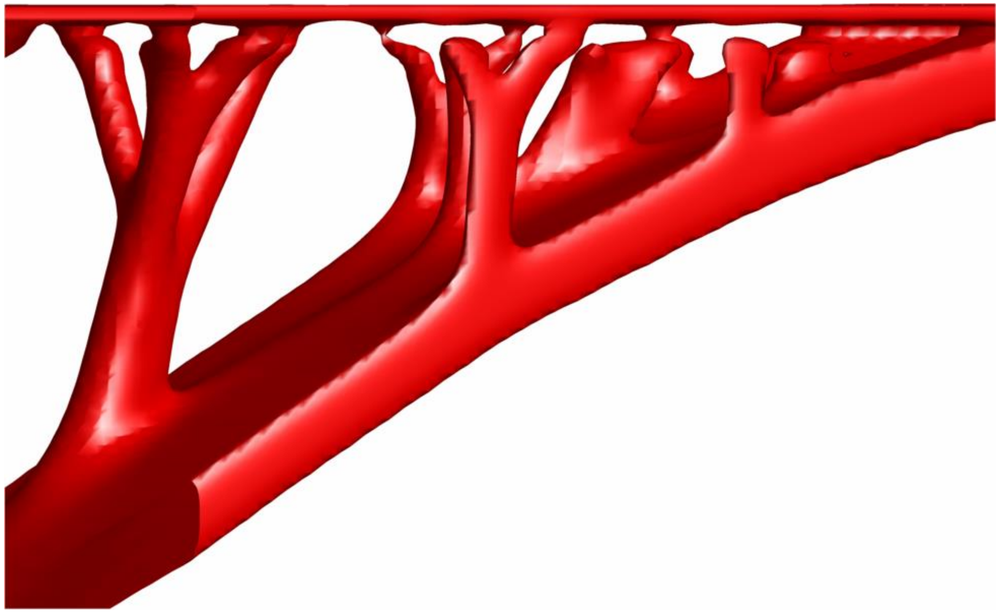
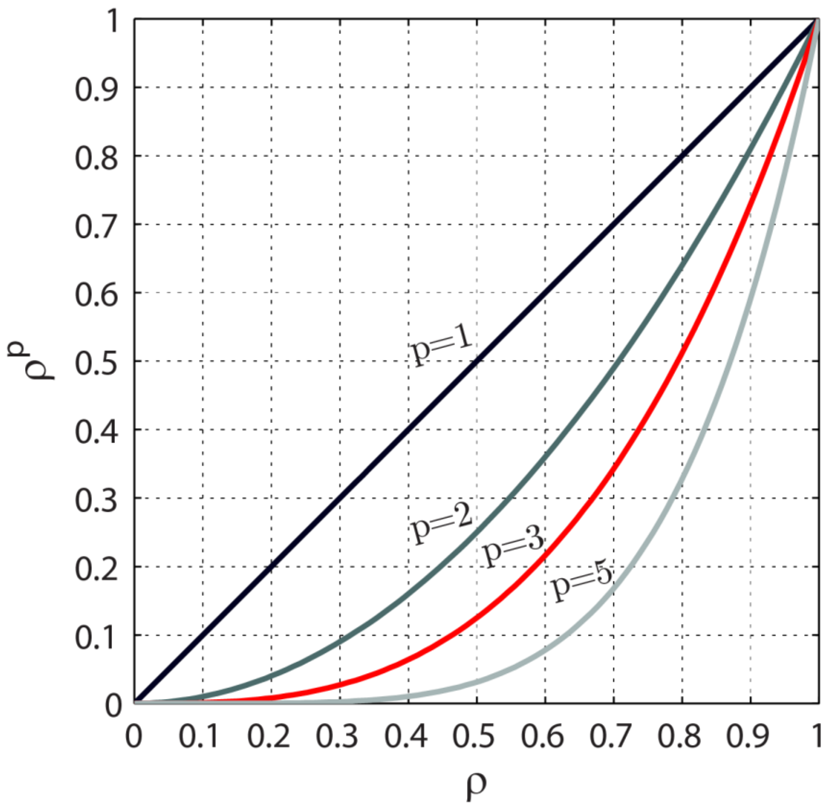


851,840 DVs for ¼
(3,407,360 total)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SIMP'S POWER-LAW:



SIMP:

$$E_i\left(\rho_i\right) = \rho_i^p E_0$$

$$0 \lesssim \rho_{min} \leq \rho_j \leq 1$$

MODIFIED SIMP:

$$E_k\left(\rho_k\right) = E_{min} + \rho_k^p \left(E_0 - E_{min}\right)$$

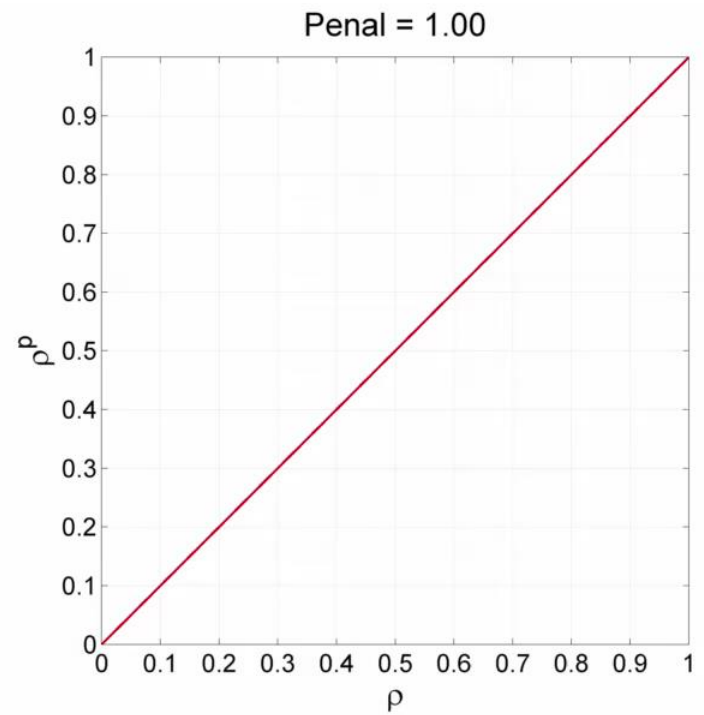
$$0 \leq \rho_j \leq 1$$

2) BENDSOE MP (1989) "OPTIMAL SHAPE DESIGN AS A MATERIAL DISTRIBUTION PROBLEM." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 1(4):193–202

3) ZHOU M, ROZVANY G (1991) "THE COC ALGORITHM, PART II: TOPOLOGICAL, GEOMETRICAL AND GENERALIZED SHAPE OPTIMIZATION. "COMP METH APPL MECH ENGRG 89:309–336

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- CONTINUATION OF "P" PARAMETER



ALLAIRE G, FRANCFORT G (1993) "A NUMERICAL ALGORITHM FOR TOPOLOGY AND SHAPE OPTIMIZATION." IN TOPOLOGY DESIGN OF STRUCTURES, SPRINGER

ALLAIRE G, KOHN R (1993) "TOPOLOGY OPTIMIZATION AND OPTIMAL SHAPE DESIGN USING HOMOGENIZATION." IN TOPOLOGY DESIGN OF STRUCTURES, SPRINGER

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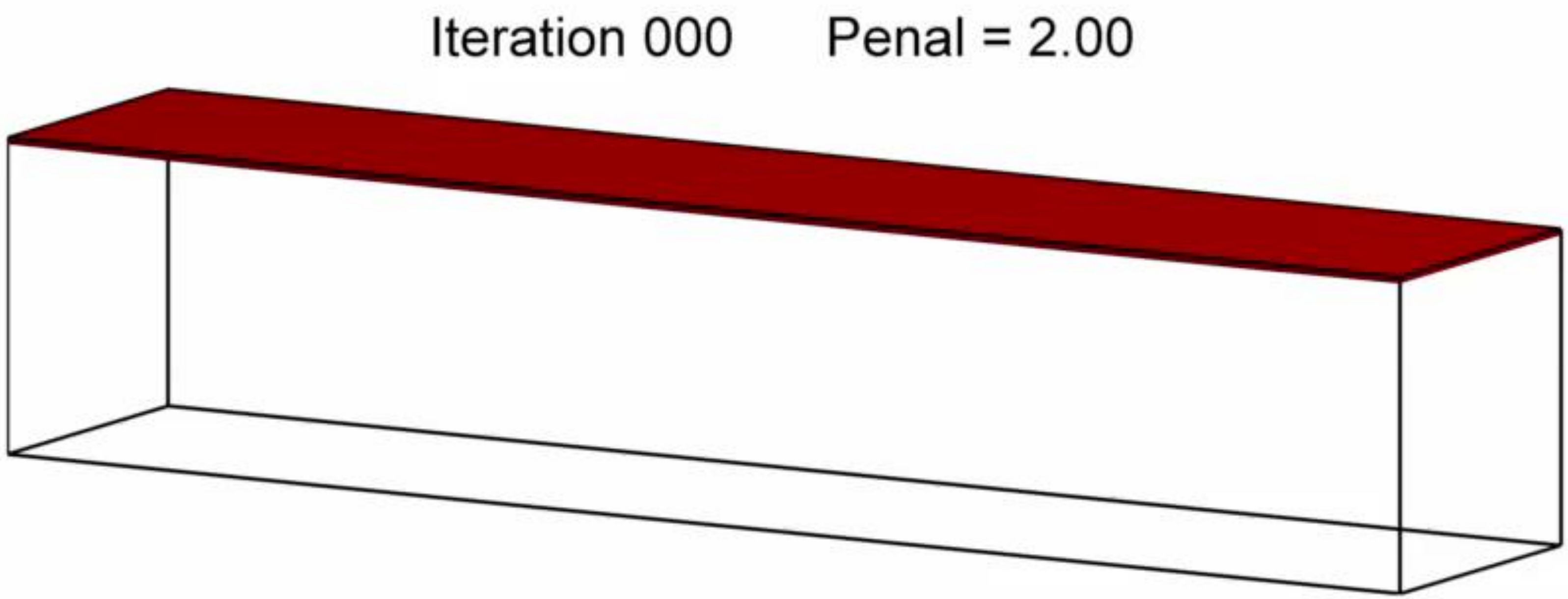
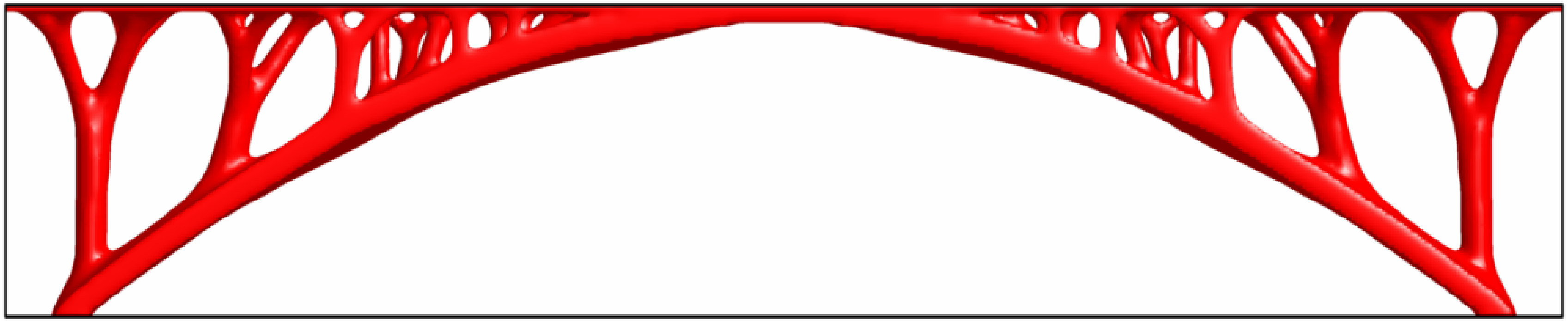
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- BRIDGE PROBLEM
  LX=25, LY=LZ=5, VOLFRAC=10%, R=5, Q=3 AND P=[CONT]



6) Additive Manuf. of Opt. Structs.

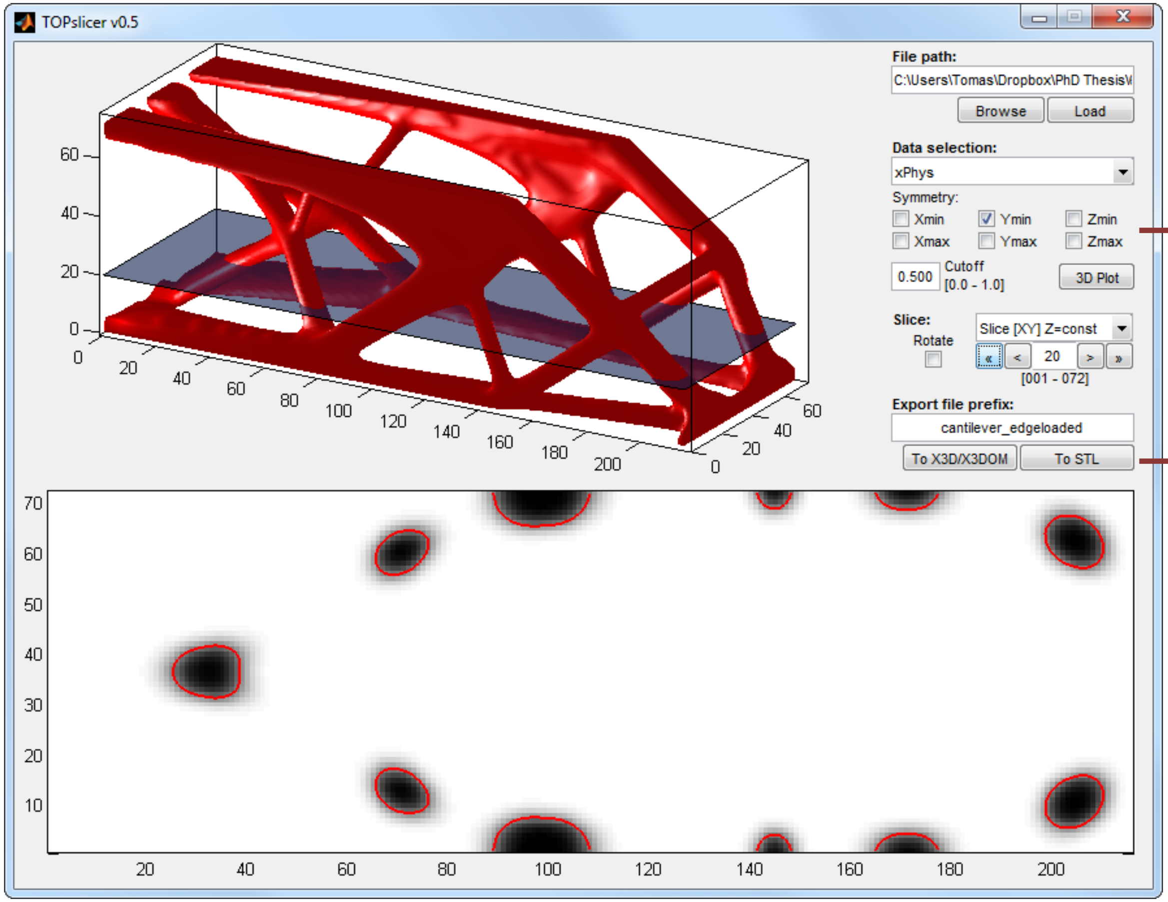
- Bridge Problem
  Lx=25, Ly=Lz=5, Volfrac=10%, R=5, Q=3 and P=[cont]



851,840 DVs for ¼
(3,407,360 total)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- TOPSLICER: AN INSPECTOR/EXPORTER OF 3D DENSITY-BASED TOPOLOGIES

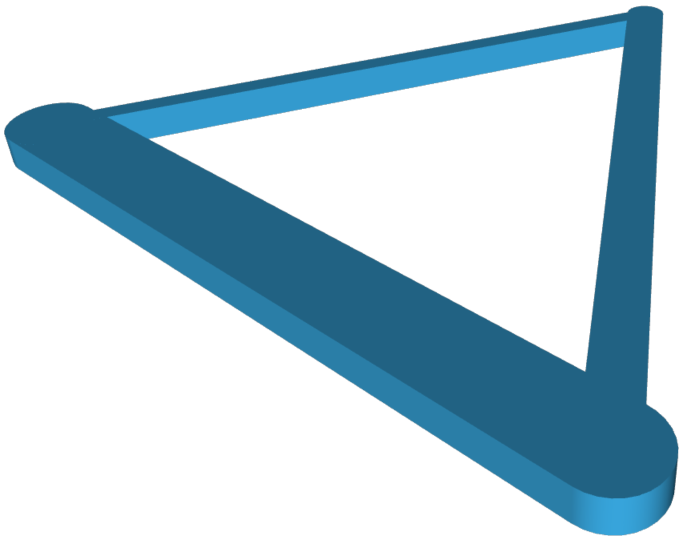


RESTORE SYMMETRY

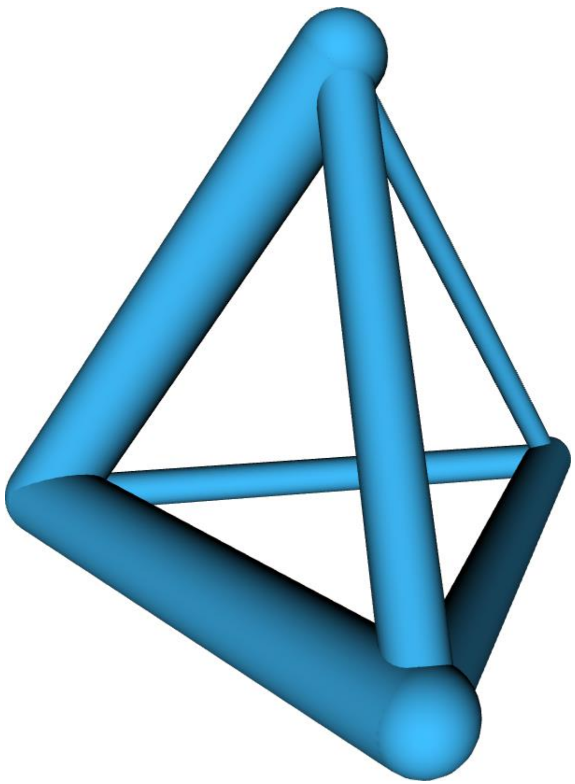
EXPORT TO X3D OR STL

6) Additive Manuf. of Opt. Structs.

- Manufacturing of ground structures



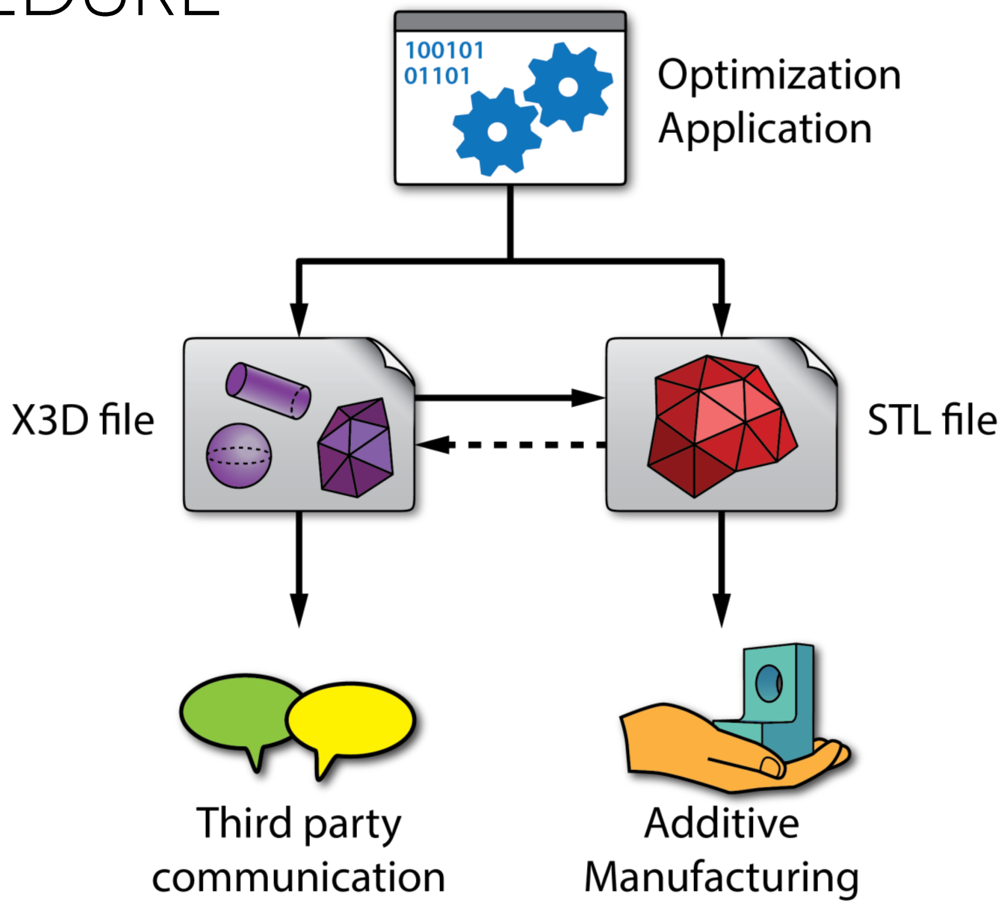
2D ground structures



3D ground structures

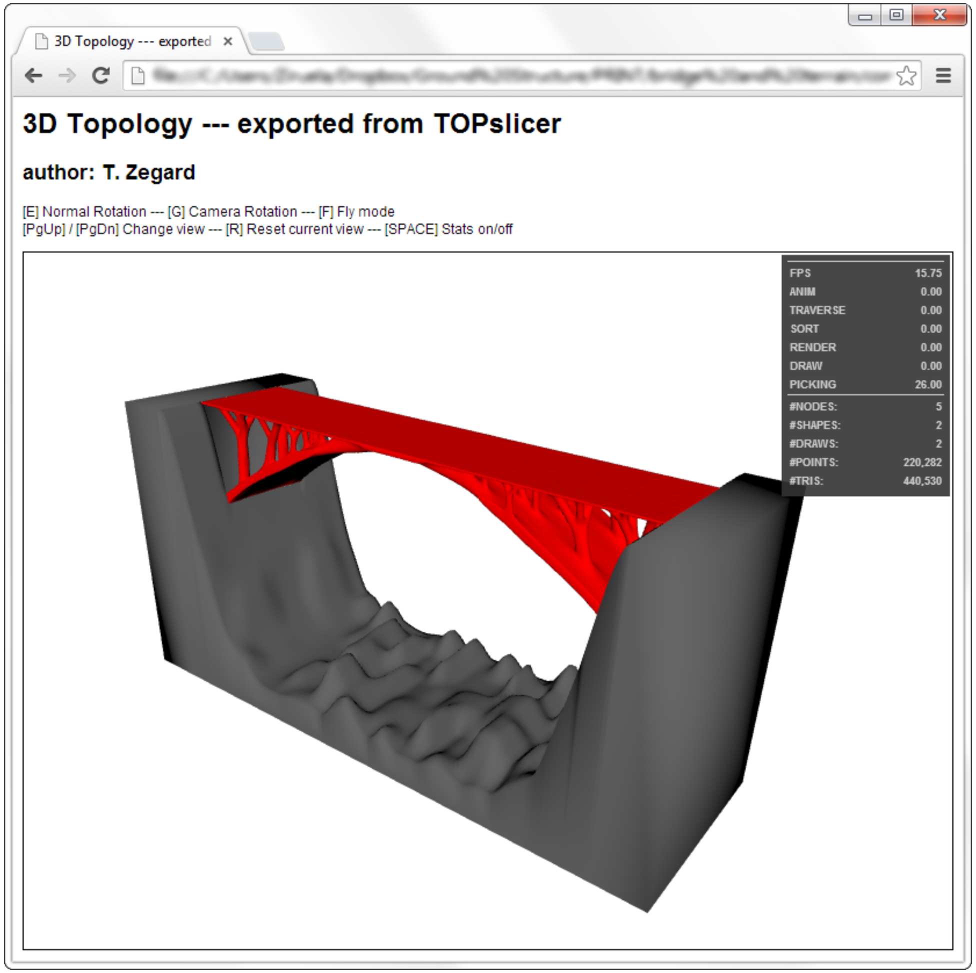
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- PROCEDURE



6) Additive Manuf. of Opt. Structs.

- X3D: Royalty-free format for representing 3D computer graphics. Managed by the Web3D Consortium.



6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL:

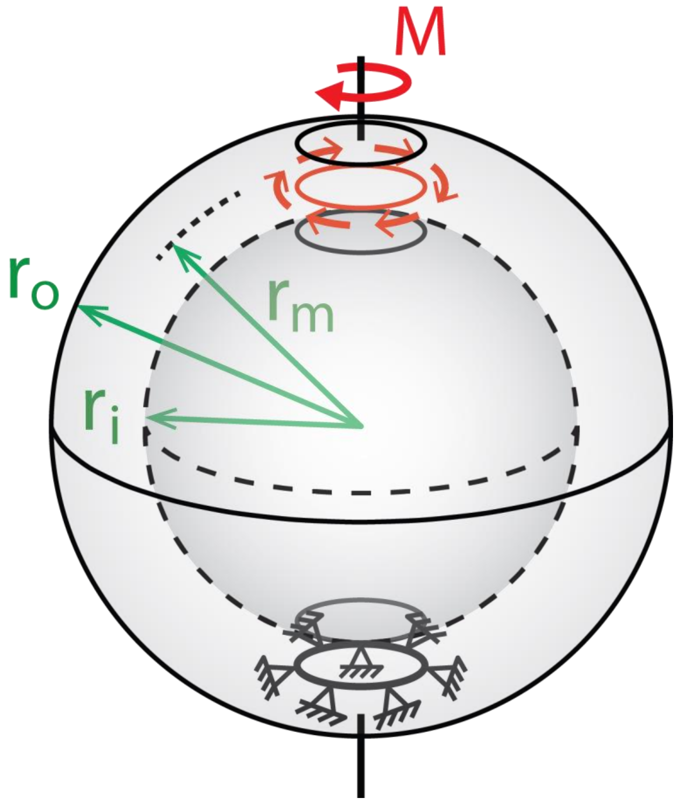
- COLOR CODE

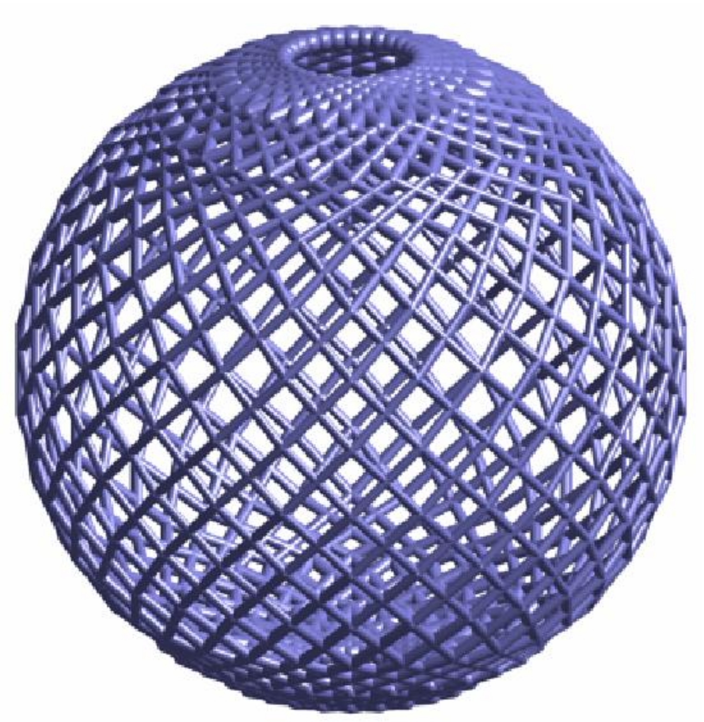
  WHITE — 3D GROUND STRUCTURES
  BLUE — 2D GROUND STRUCTURES
  RED — 3D DENSITY METHOD
  BLACK — APPLICATION-ORIENTED

- MANUFACTURED USING:
  - FDM: FUSED DEPOSITION MODELING
  - SLS: SELECTIVE LASER SINTERING

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: TORSION BALL



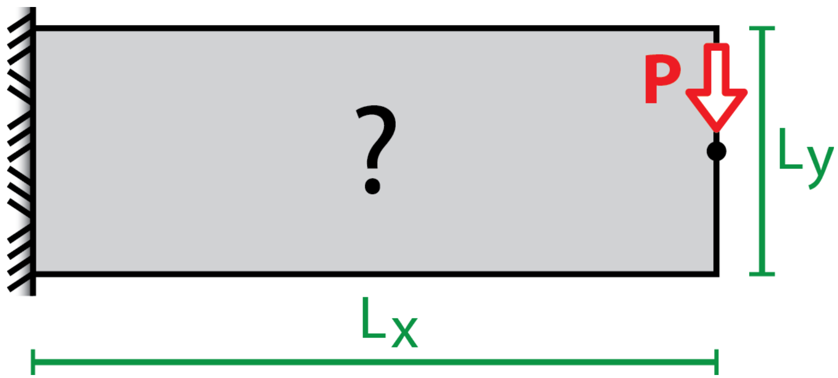


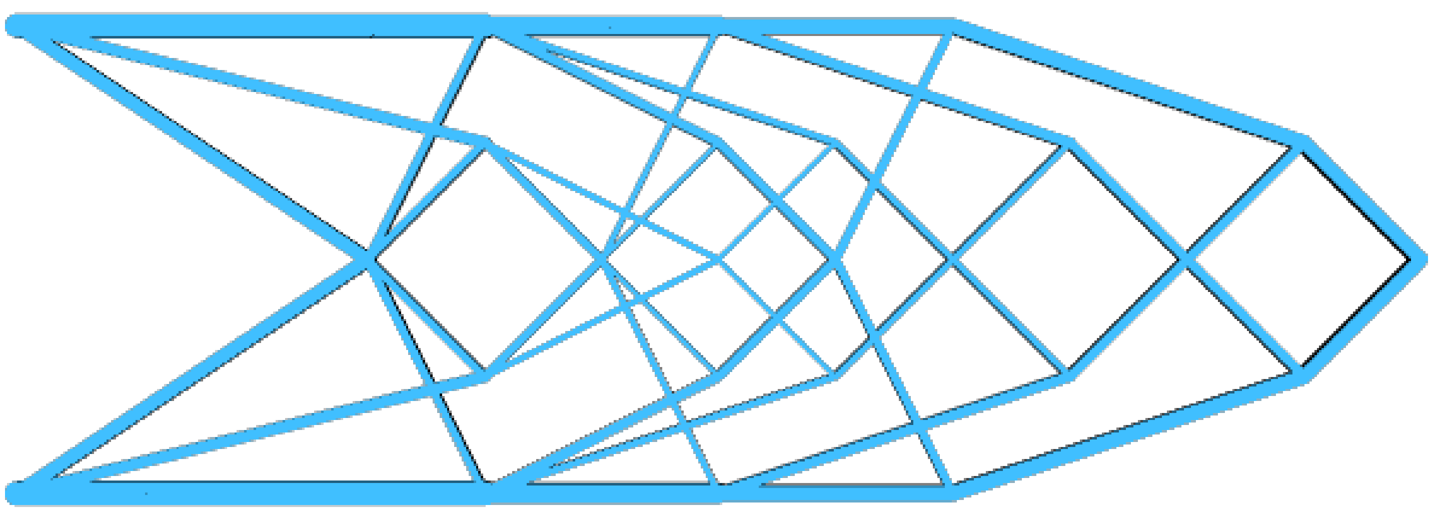
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: TORSION BALL

6) ADDITIVE MANUF. OF OPT. STRUCTS.

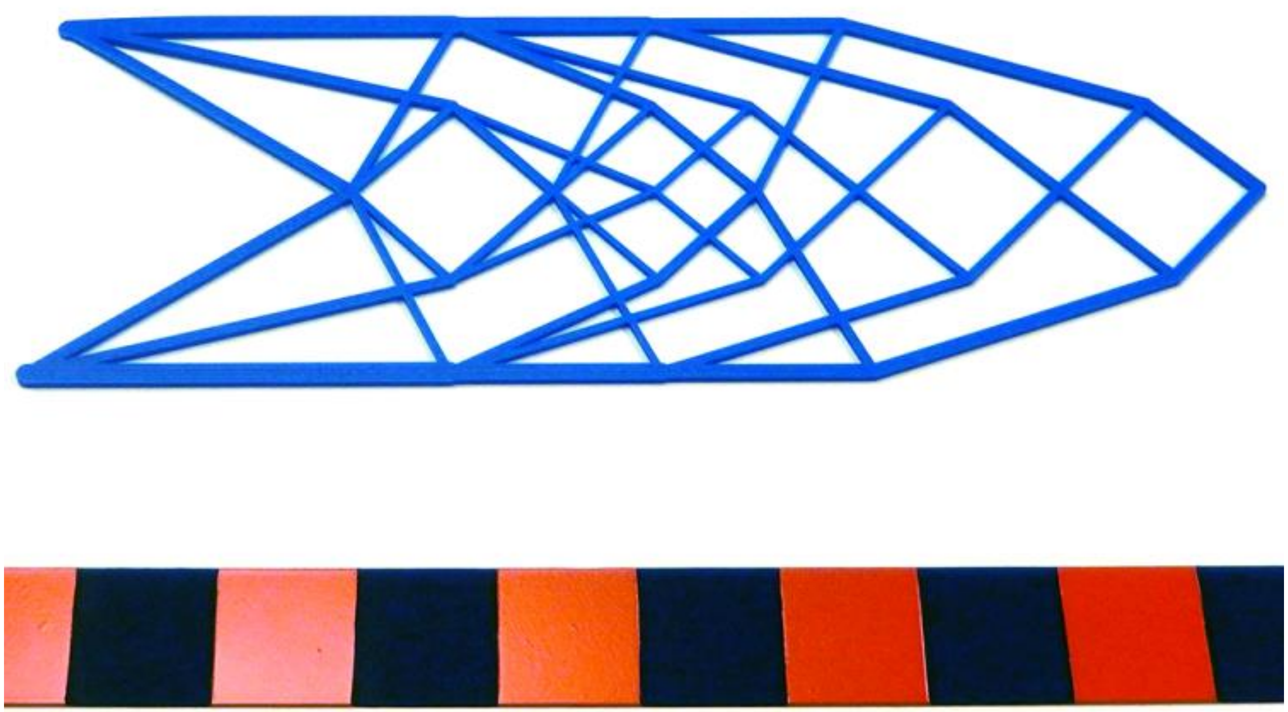
- SHOW AND TELL: CANTILEVER





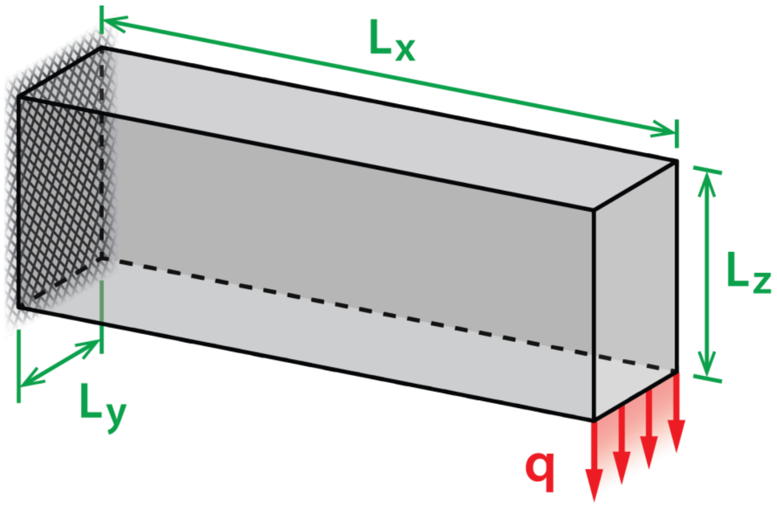
6) Additive Manuf. of Opt. Structs.

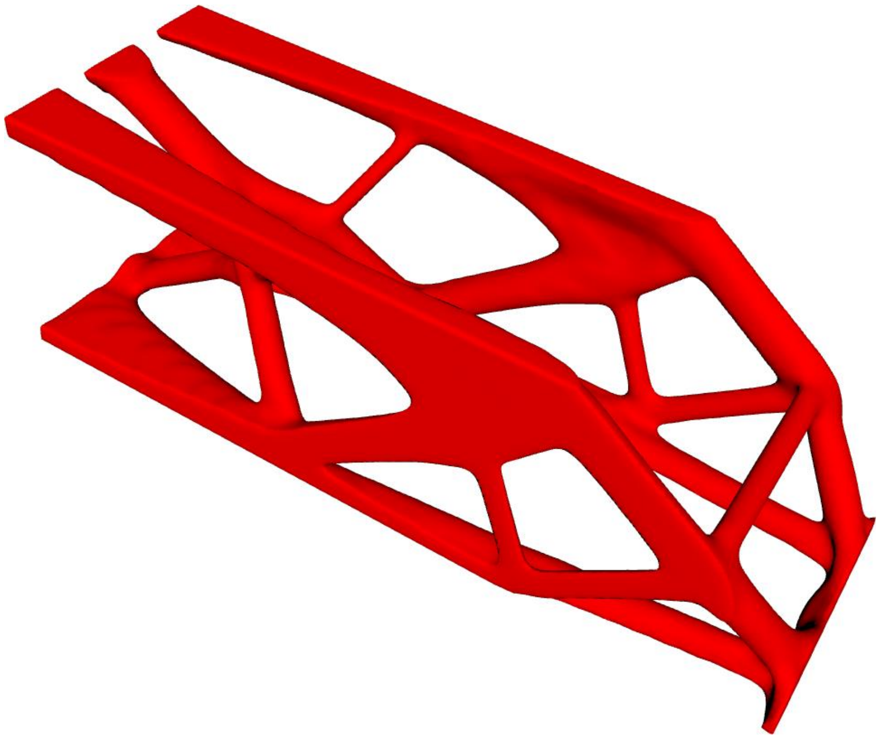
- Show and tell: Cantilever



6) ADDITIVE MANUF. OF OPT. STRUCTS.

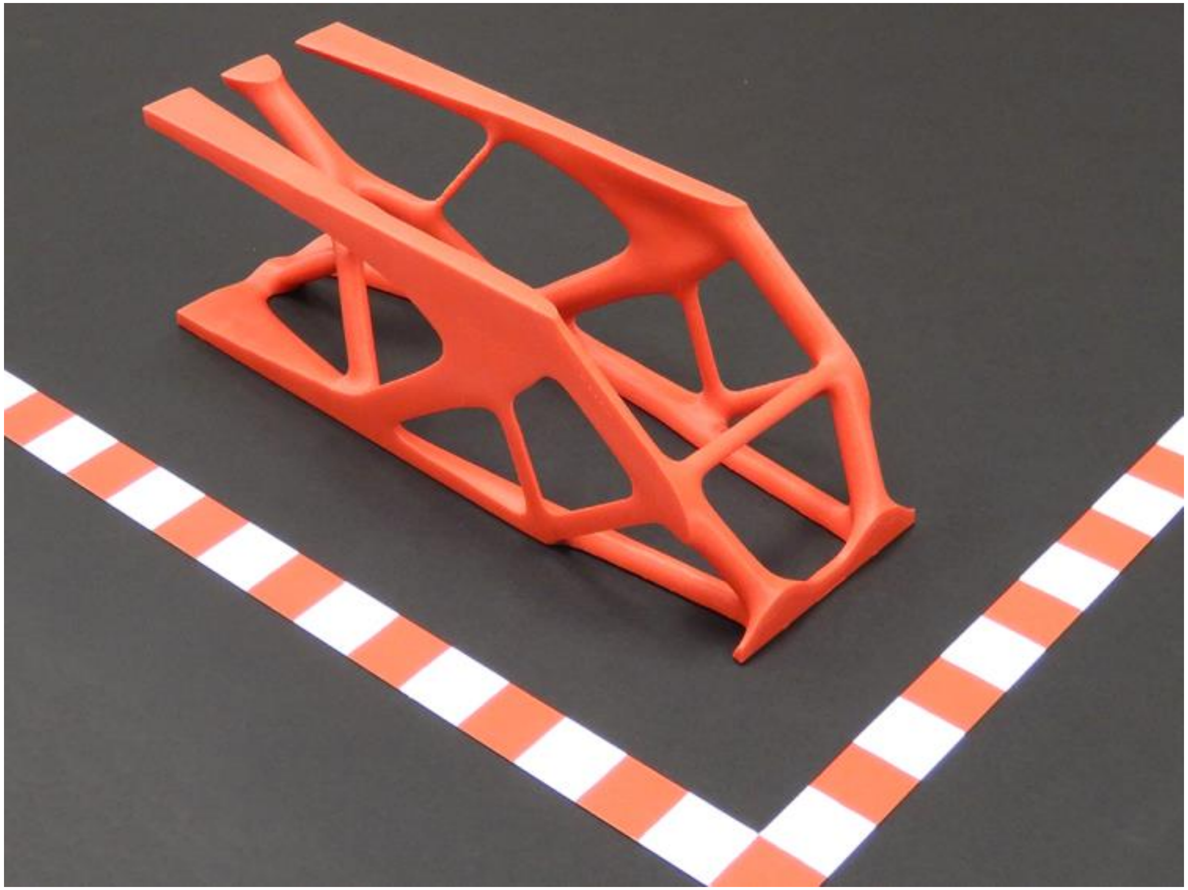
- SHOW AND TELL:
  EDGE-LOADED 3D CANTILEVER (NO FIX)





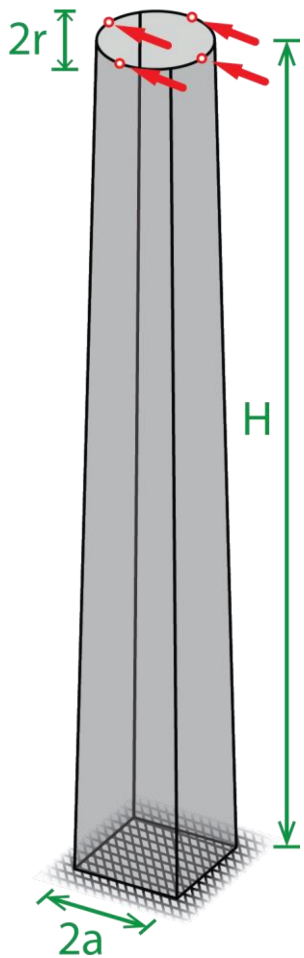
6) ADDITIVE MANUF. OF OPT. STRUCTS.

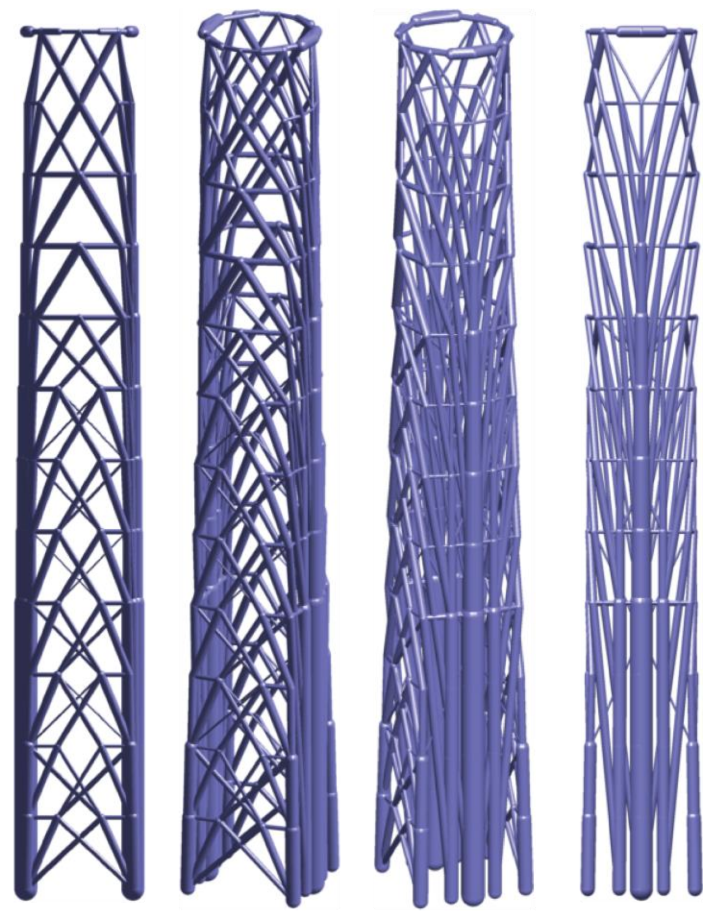
- SHOW AND TELL:
  EDGE-LOADED 3D CANTILEVER (NO FIX)



6) ADDITIVE MANUF. OF OPT. STRUCTS.

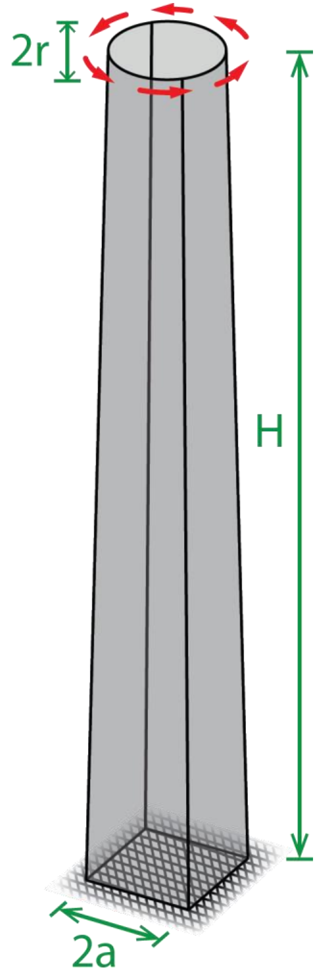
- APPLICATION-ORIENTED: LOTTE TOWER

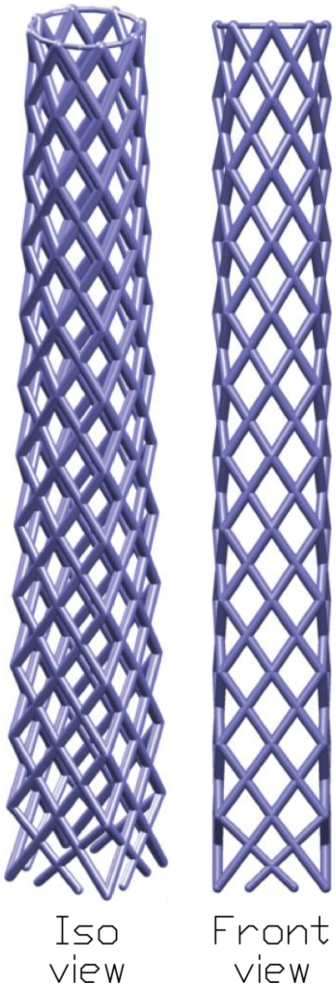




6) Additive Manuf. of Opt. Structs.

- Application-Oriented: Lotte Tower



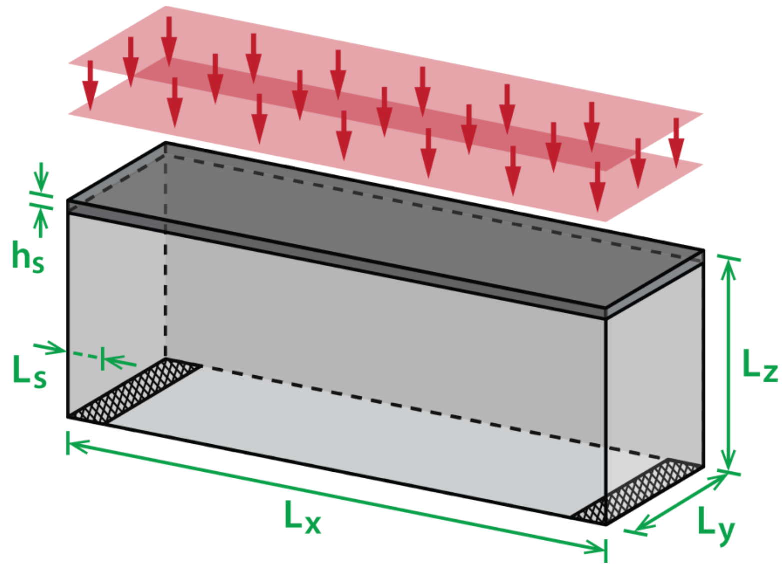


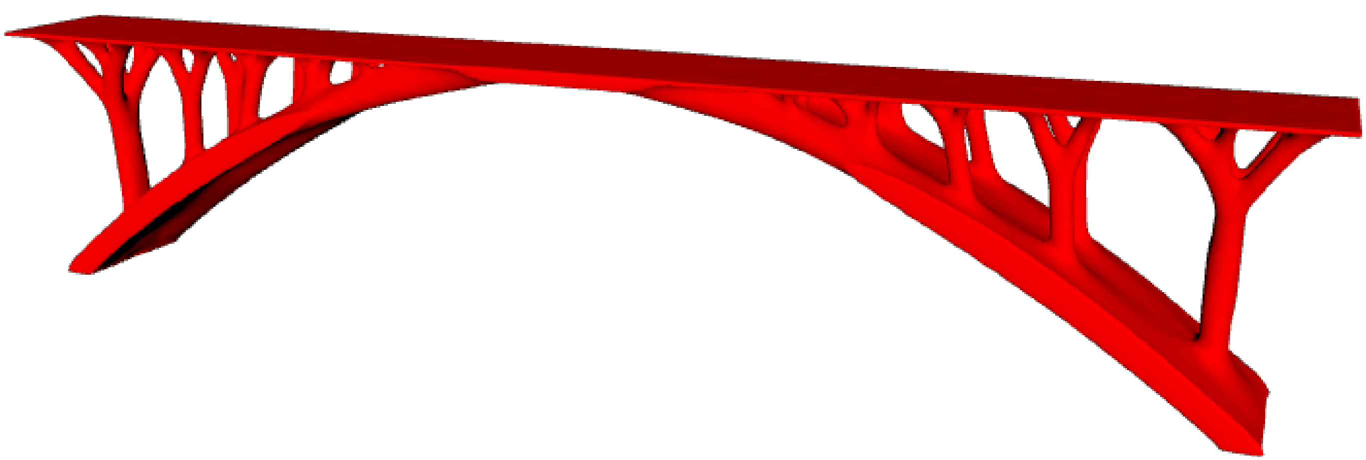
6) Additive manuf. of opt. structs.

- Application-oriented: Lotte Tower

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: BRIDGE



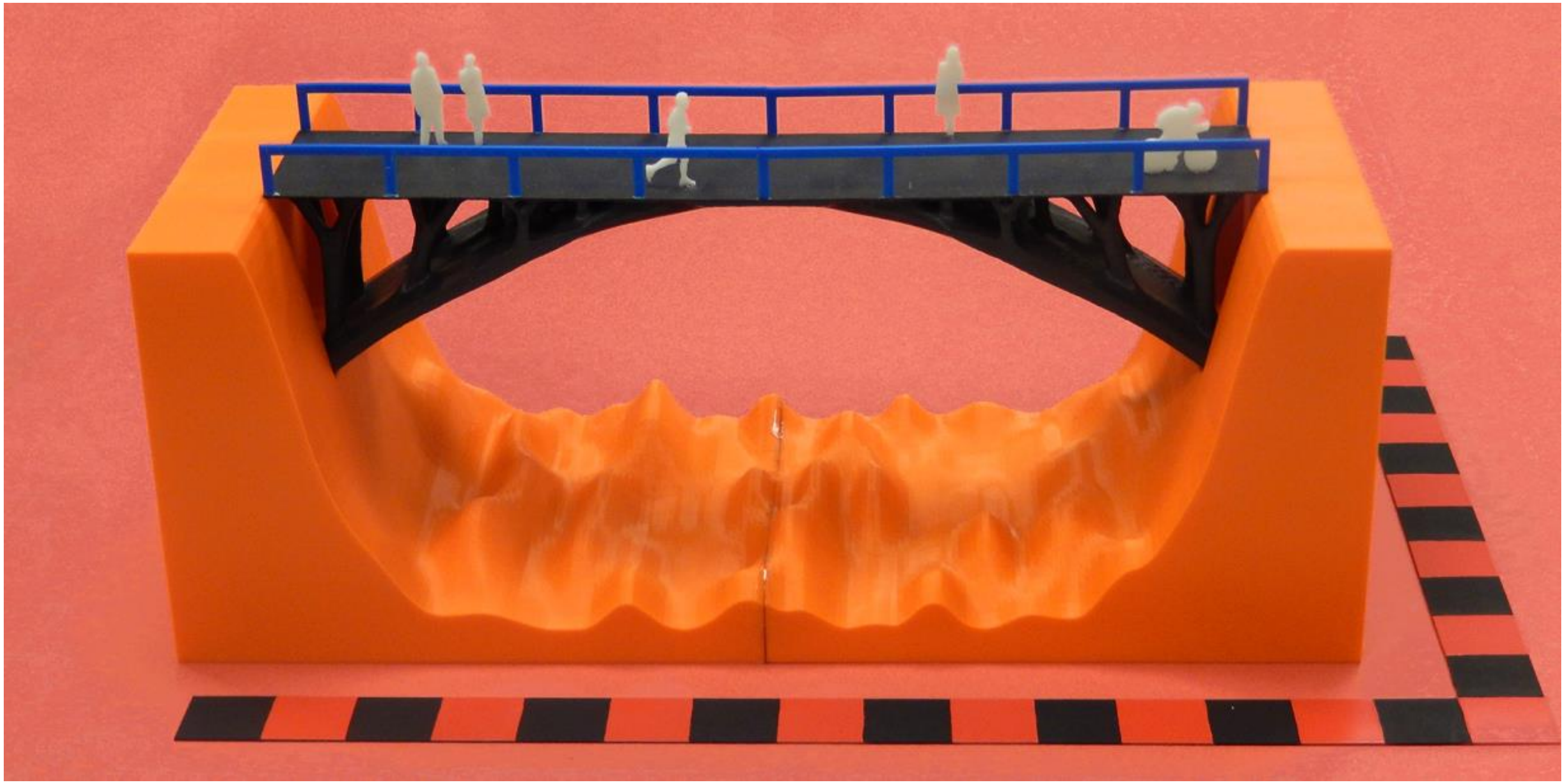


6) Additive Manuf. of Opt. Structs.

- Application-oriented: Bridge

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: BRIDGE

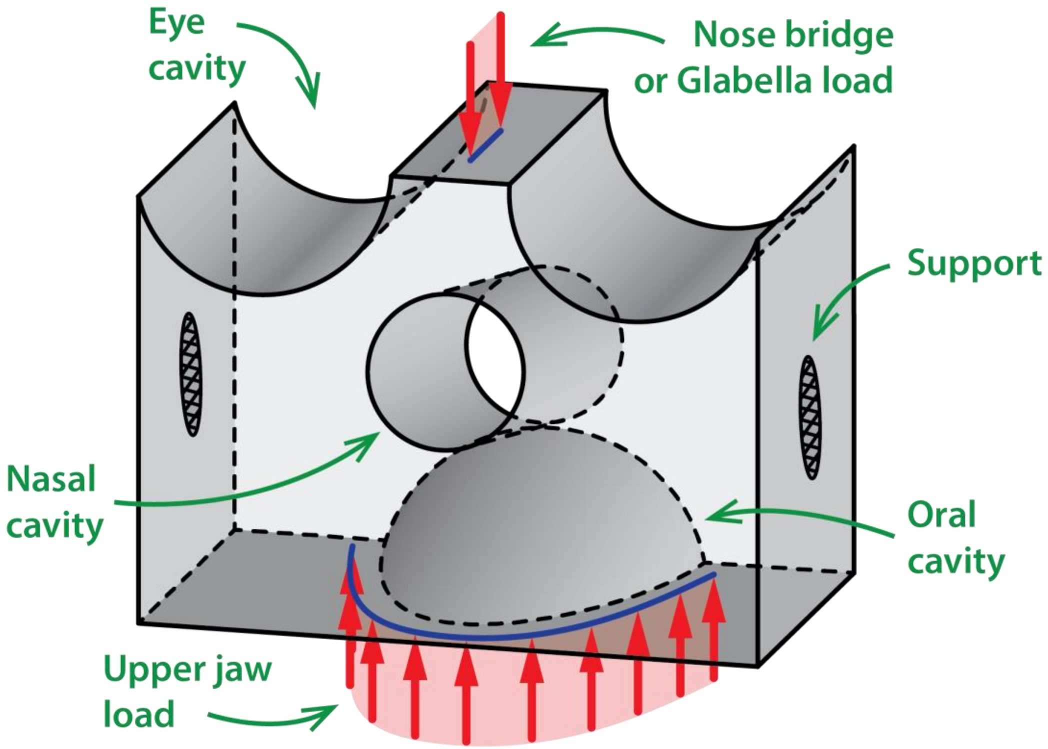


6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: BRIDGE
  ACHIEVING LARGE SCALES

6) ADDITIVE MANUF. OF OPT. STRUCTS.

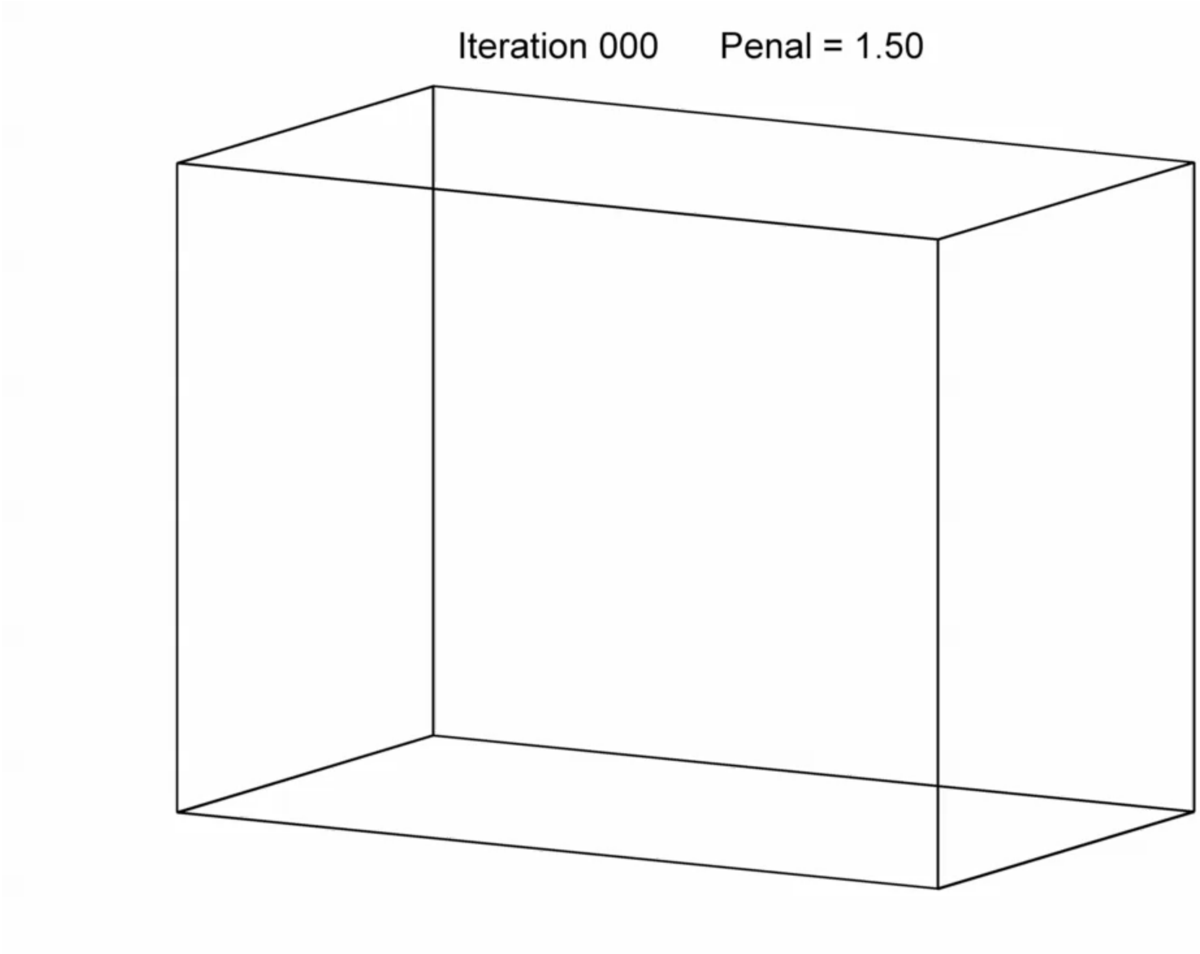
- APPLICATION-ORIENTED:
  CRANIOFACIAL RECONSTRUCTION



SUTRADHAR A, PAULINO GH, MILLER MJ, NGUYEN TH (2010) "TOPOLOGICAL OPTIMIZATION FOR DESIGNING PATIENT-SPECIFIC LARGE CRANIOFACIAL SEGMENTAL BONE REPLACEMENTS." PNAS, 107(30):13222-13227

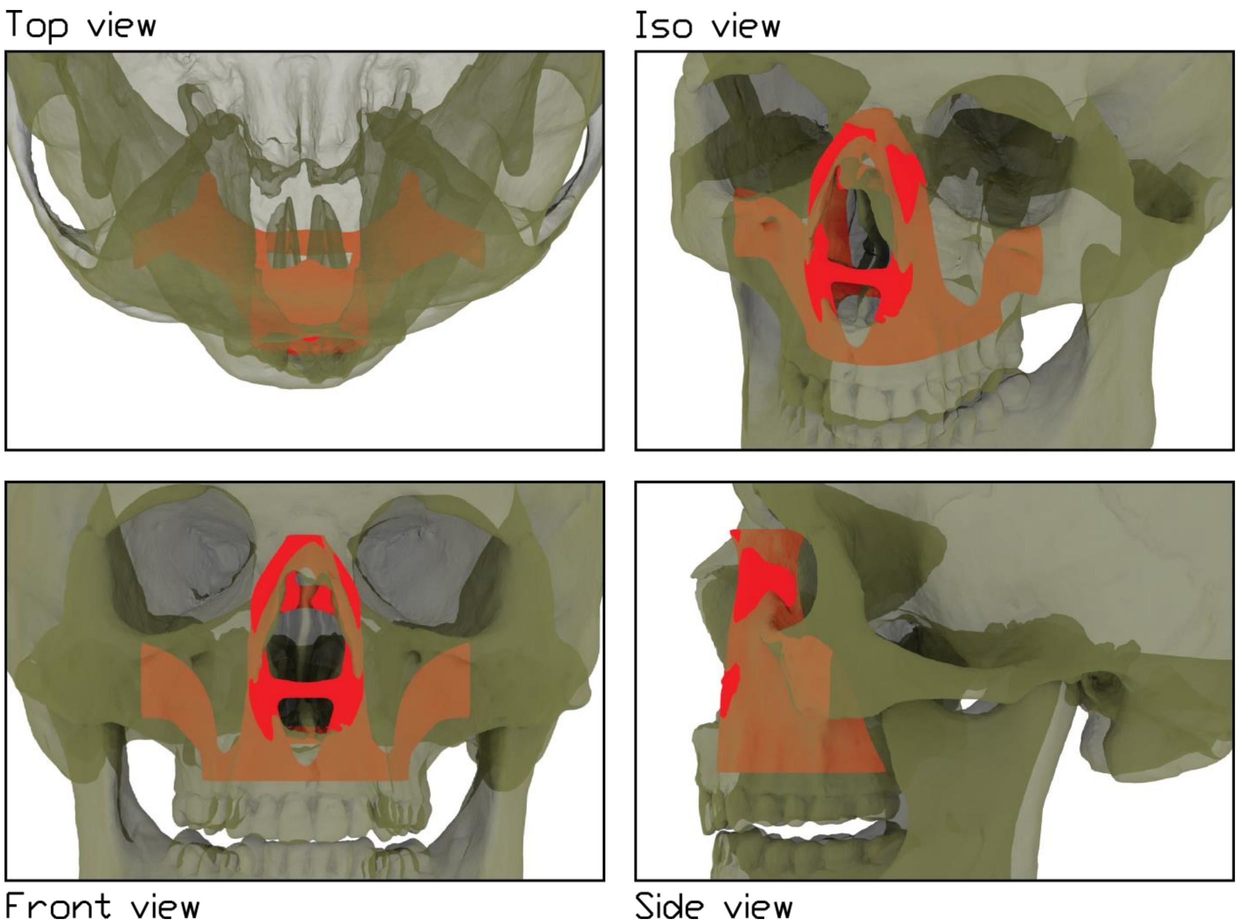
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED:
  CRANIOFACIAL RECONSTRUCTION



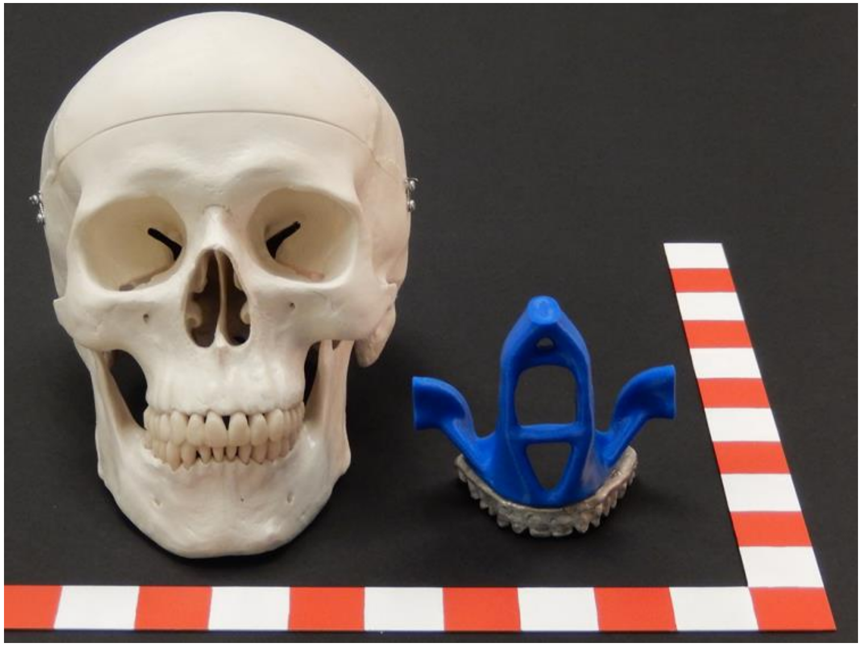
6) ADDITIVE MANUF. OF OPT. STRUCTS.

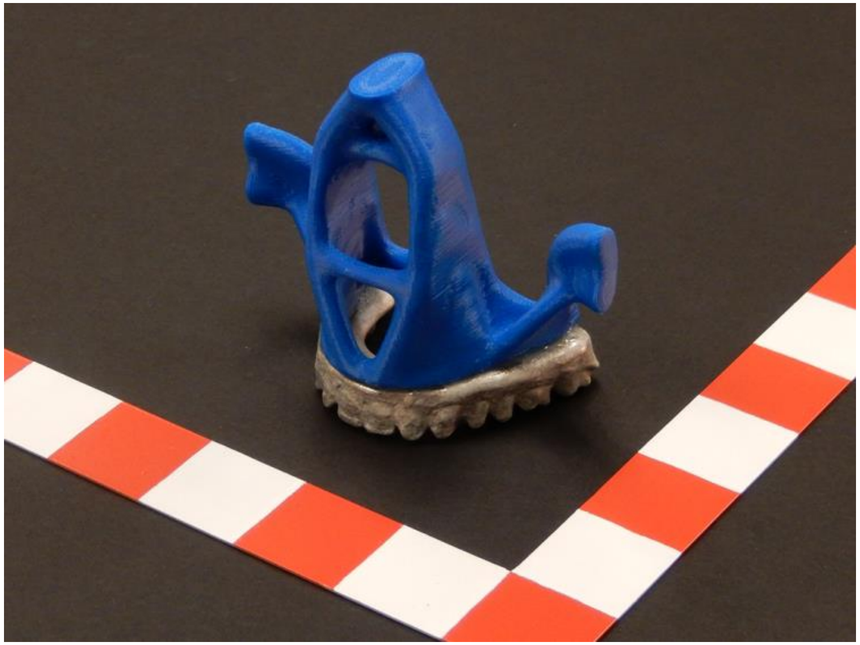
- APPLICATION-ORIENTED:
  CRANIOFACIAL RECONSTRUCTION



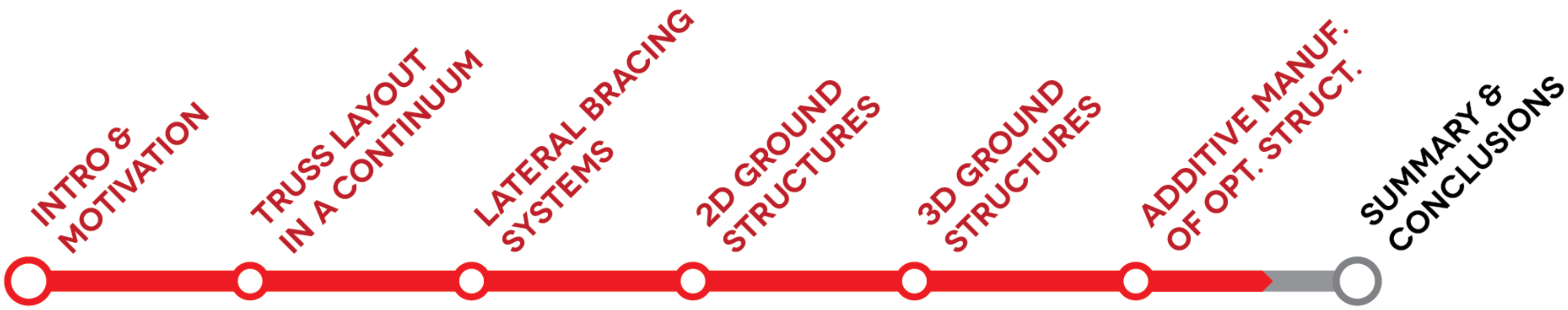
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED:
  CRANIOFACIAL RECONSTRUCTION





ROADMAP

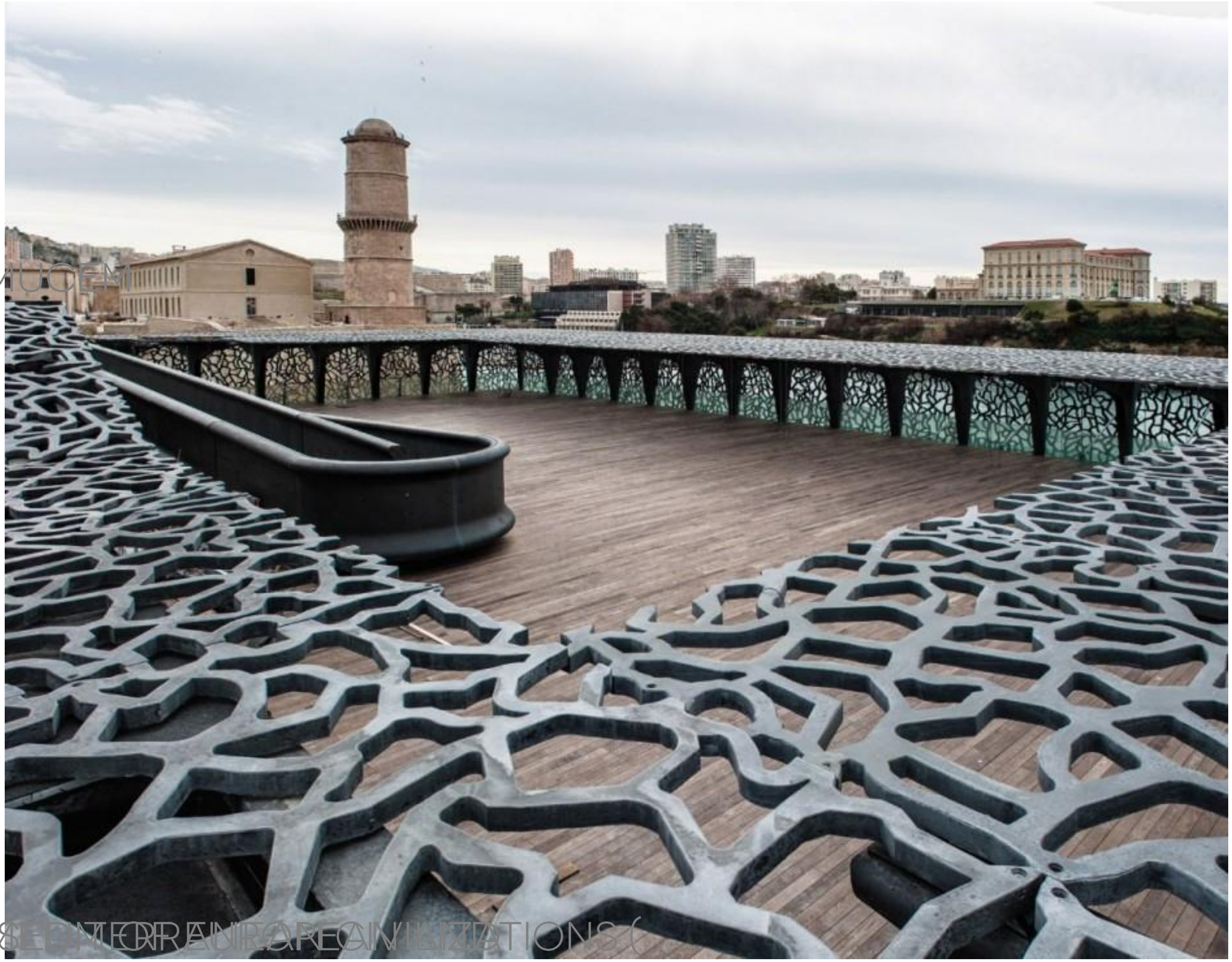


INTRO & MOTIVATION
TRUSS LAYOUT IN A CONTINUUM
LATERAL BRACING SYSTEMS
2D GROUND STRUCTURES
3D GROUND STRUCTURES
ADDITIVE MANUF. OF OPT. STRUCT.
SUMMARY & CONCLUSIONS

7) Summary and Conclusions

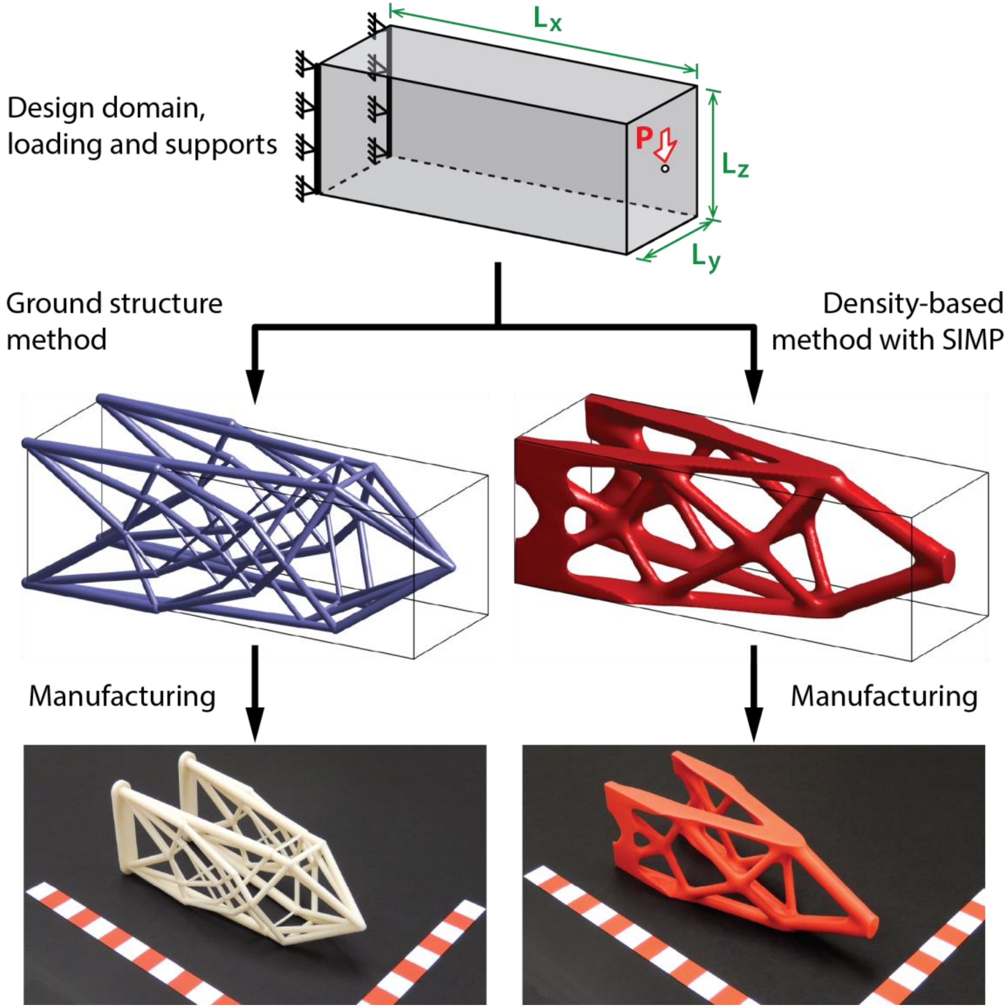
- Optimization:
  1. Essential for sustainability
  2. Can be incorporated into design today
  3. Give a design, and I will try to make it better
  4. Different methods for different problems
  5. Yes, we can manufacture this
  6. Design guided by functionality and not just beauty

7) SUMMARY AND CONCLUSIONS



7) SUMMARY AND CONCLUSIONS

- INTEGRATED DESIGN PROCESS:
  START TO FINISH



AKNOWLEDGEMENTS

- FULBRIGHT–CONICYT SCHOLARSHIP
- SKIMORE, OWINGS & MERRILL LLP
- CEE @ ILLINOIS
- ADVISER: GLAUCIO H. PAULINO
- RESEARCH GROUP
- PHD COMMITTEE
- MY FAMILY AND FRIENDS

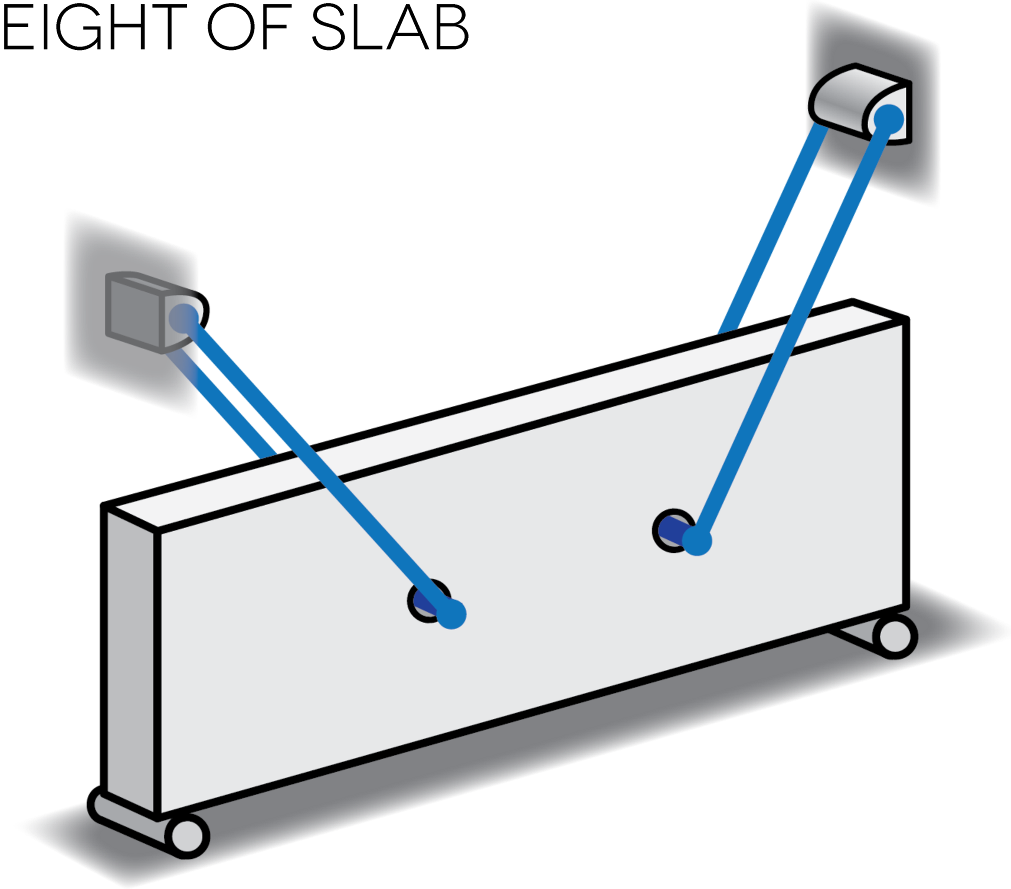
THE END

?

2) TRUSS LAYOUT WITHIN A CONTINUUM

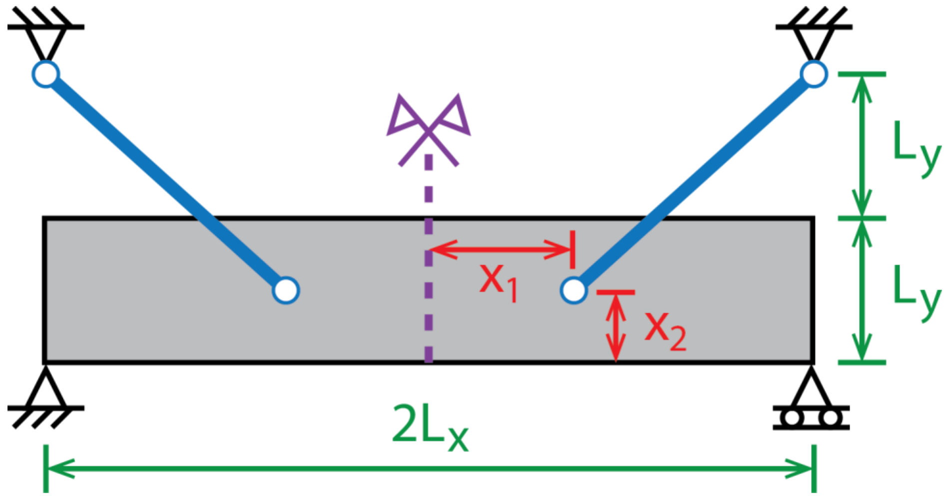
- SLAB WITH SUPPORTING CABLES

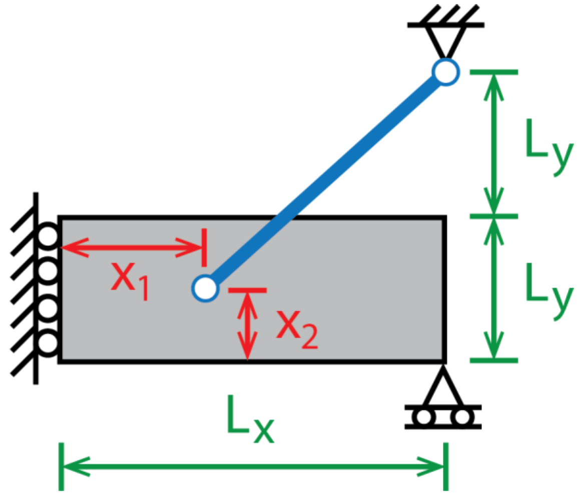
SELF WEIGHT OF SLAB



2) TRUSS LAYOUT WITHIN A CONTINUUM

- SLAB WITH SUPPORTING CABLES





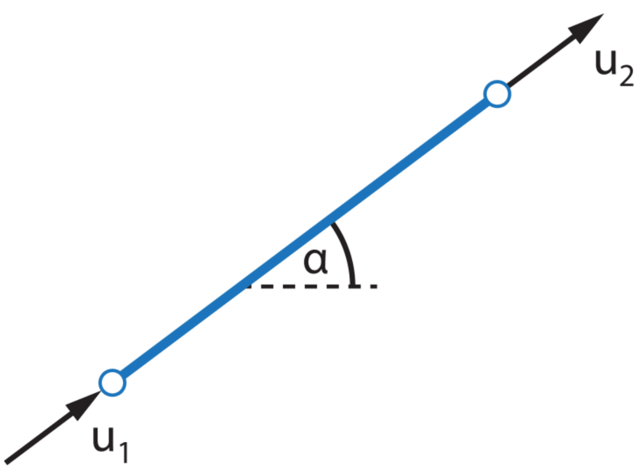
2) TRUSS LAYOUT WITHIN A CONTINUUM

- TRUSS ELEMENT

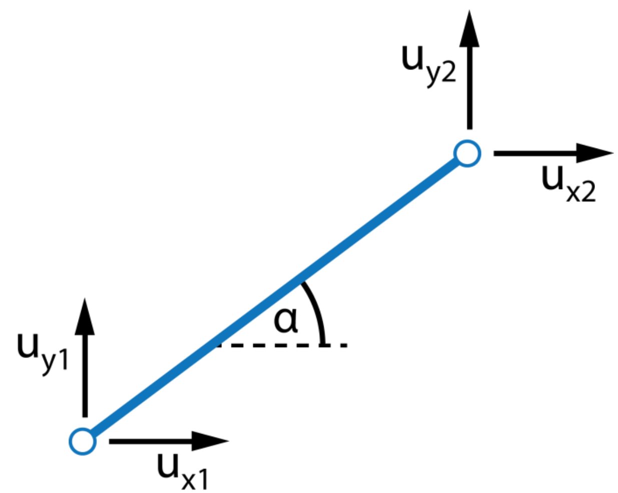
$$\mathbf{K}^{\star} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\mathbf{K}_e = \mathbf{T}_e^T \mathbf{K}_e^{\star} \mathbf{T}_e \qquad \mathbf{T} = \begin{bmatrix} \mathbf{d} & \mathbf{0} \\ \mathbf{0} & \mathbf{d} \end{bmatrix}$$

$$\mathbf{d} = \frac{1}{L}\left[x_2 - x_1, y_2 - y_1, z_2 - z_1\right]$$



LOCAL COORDINATES



GLOBAL COORDINATES

2) TRUSS LAYOUT WITHIN A CONTINUUM

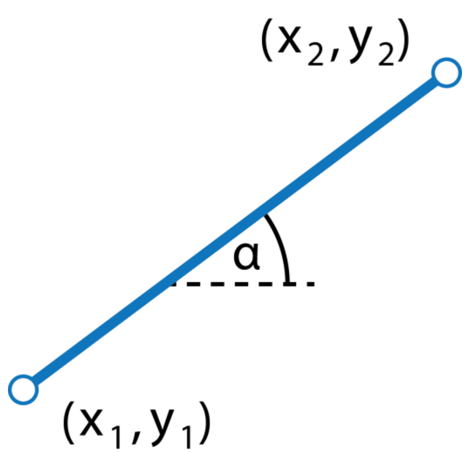
- SENSITIVITY W.R.T COORD 'N' OF NODE 'J'

$$\frac{\partial \mathbf{K}_e}{\partial n_j} = \frac{\partial \mathbf{T}_e^T}{\partial n_j}\mathbf{K}_e^\star \mathbf{T}_e + \mathbf{T}_e^T \frac{\partial \mathbf{K}_e^\star}{\partial L}\frac{\partial L}{\partial n_j}\mathbf{T}_e + \mathbf{T}_e^T \mathbf{K}_e^\star \frac{\partial \mathbf{T}_e}{\partial n_j}$$

$$\mathbf{J}_{(1)}(\mathbf{d}) = \frac{1}{L}\left(\mathbf{d}^T\mathbf{d} - \mathbf{I}\right) \qquad \mathbf{J}_{(2)}(\mathbf{d}) = -\mathbf{J}_{(1)}(\mathbf{d})$$

$$\frac{\partial L}{\partial n_1} = -\mathbf{d}_n \qquad \frac{\partial L}{\partial n_2} = \mathbf{d}_n$$

$$\frac{\partial \mathbf{K}^\star}{\partial L} = -\frac{AE}{L^2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$



HANSEN S R, VANDERPLAATS G N (1988) – "AN APPROXIMATION METHOD FOR CONFIGURATION OPTIMIZATION OF TRUSSES", AAIA JOURNAL 28(1), PP 161–168

2) TRUSS LAYOUT WITHIN A CONTINUUM

- DISPLACEMENTS "U" ANYWHERE IN Ω USING FEM SHAPE FUNCTIONS

$$\mathbf{u} = \mathbf{N}\mathbf{u}_c$$

- CONFORMING COUPLING
  - TRUSS MEMBER'S $\mathbf{K}_e$ IS COUPLED BY AN EQUIVALENT $\mathbf{K}_e^+$ MATRIX

$$\mathbf{u}^T\mathbf{K}_e\mathbf{u} = \mathbf{u}_c^T\mathbf{K}_e^+\mathbf{u}_c$$

$$(\mathbf{N}\mathbf{u}_c)^T\,\mathbf{K}_e\,(\mathbf{N}\mathbf{u}_c) = \mathbf{u}_c^T\mathbf{K}_e^+\mathbf{u}_c$$

$$\mathbf{u}_c^T\,\left(\mathbf{N}^T\mathbf{K}_e\mathbf{N}\right)\,\mathbf{u}_c = \mathbf{u}_c^T\mathbf{K}_e^+\mathbf{u}_c$$

$$\boxed{\mathbf{N}^T\mathbf{K}_e\mathbf{N} = \mathbf{K}_e^+}$$

2) TRUSS LAYOUT WITHIN A CONTINUUM

- PROOF-OF-CONCEPT:
  COMPLIANCE FORMULATION

$$
\begin{aligned}
\min_{\mathbf{A},\mathbf{x}} \quad & C = \mathbf{u}^T \mathbf{K} \mathbf{u} = \mathbf{u}^T \mathbf{f} \\
\text{s.t.} \quad & \textstyle\sum_i A_i L_i \leq \bar{V} \\
\text{with} \quad & \mathbf{K}\mathbf{u} = \mathbf{f}
\end{aligned}
$$

← REQUIRED IF MEMBERS ARE ALSO SIZED

VOLUME CONSTRAINT IS NOT ALWAYS NECESSARY IN A GEOMETRY-ONLY OPTIMIZATION PROBLEM

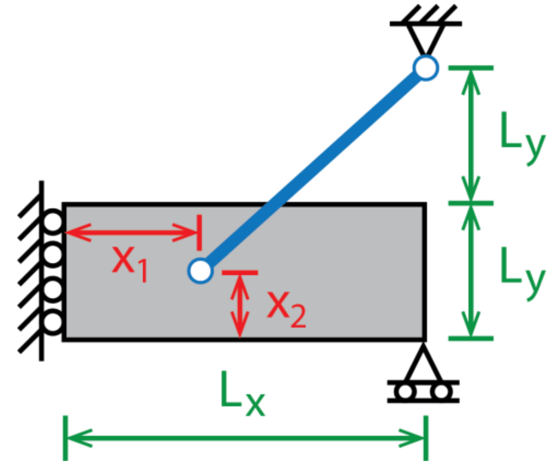
2) TRUSS LAYOUT WITHIN A CONTINUUM

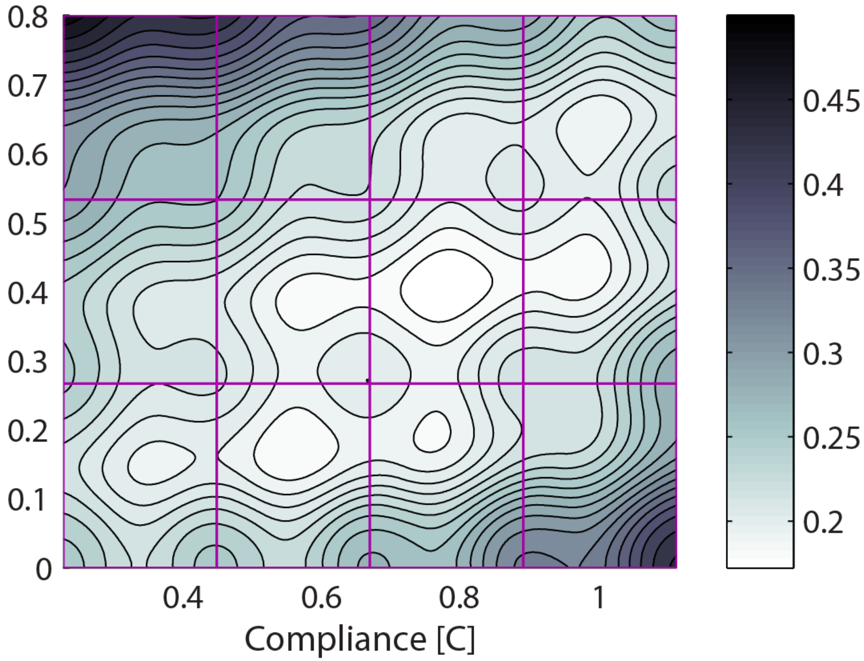
- 3X9 MESH (Q4 ELEMS)

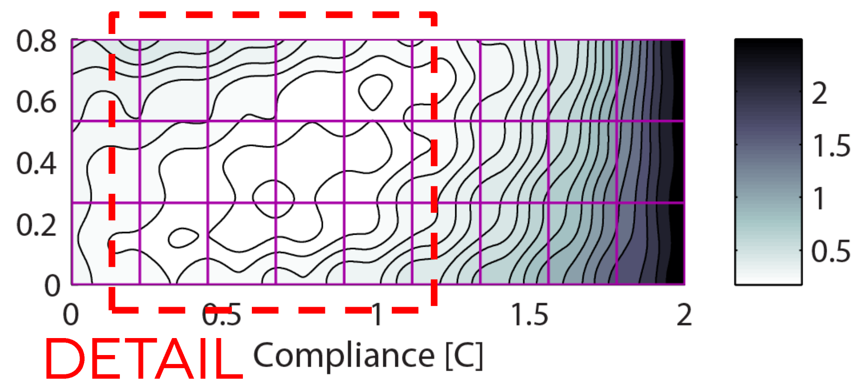
  SLAB: LX=2 LY=0.8 E=100 ν =0.3

  CABLE: AE=300

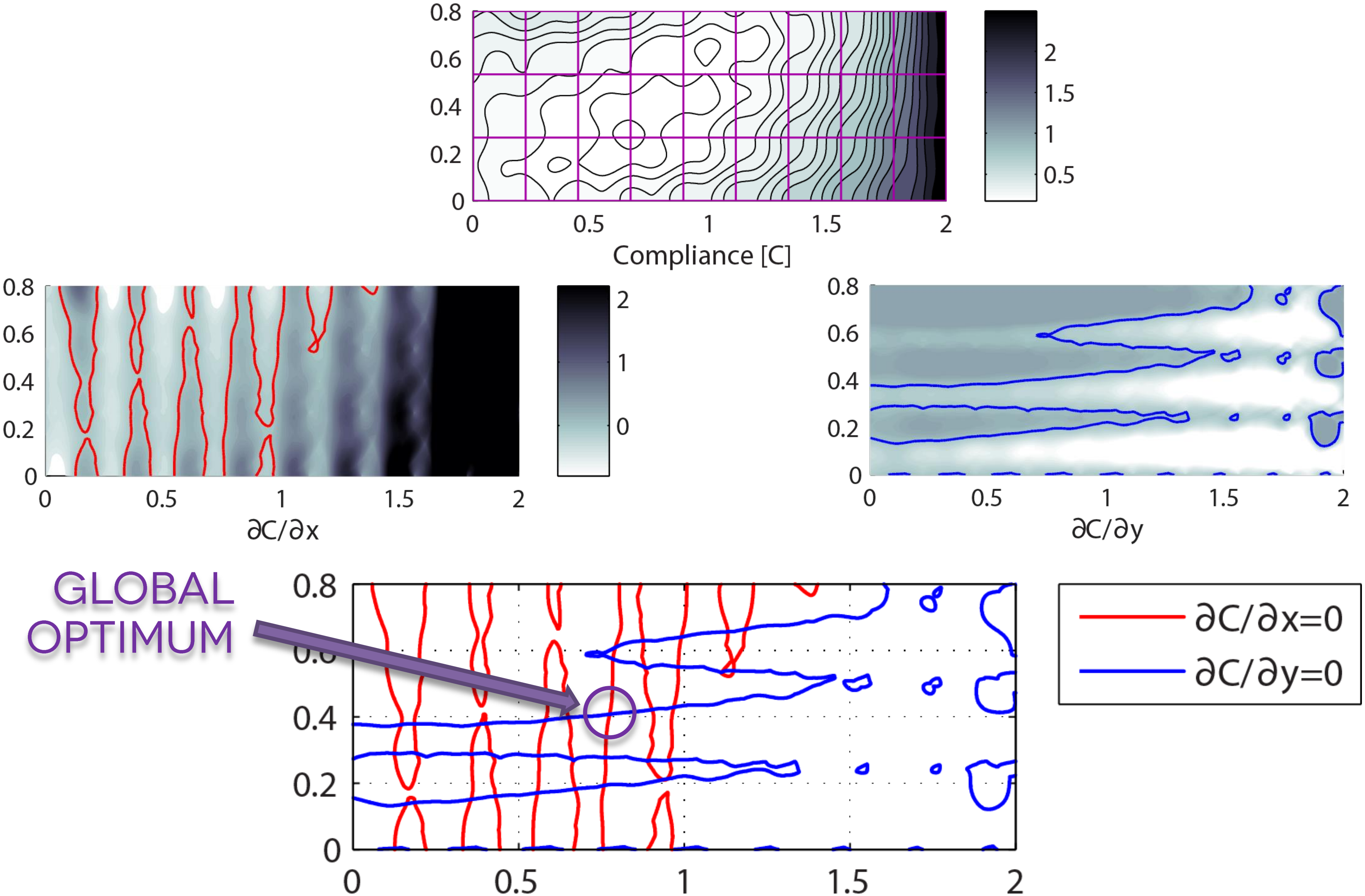
  LOAD: B=[0 −2]



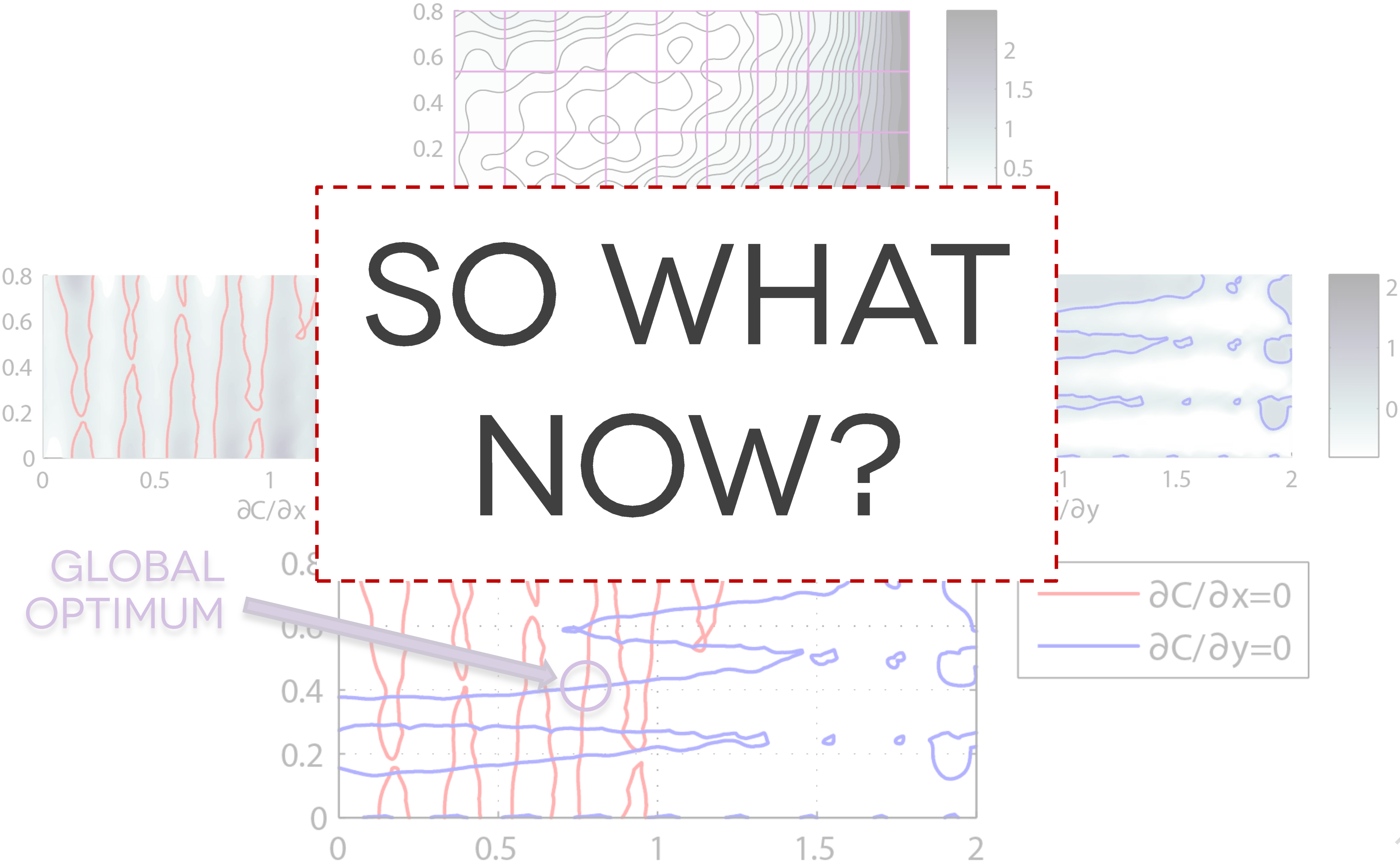




2) TRUSS LAYOUT WITHIN A CONTINUUM

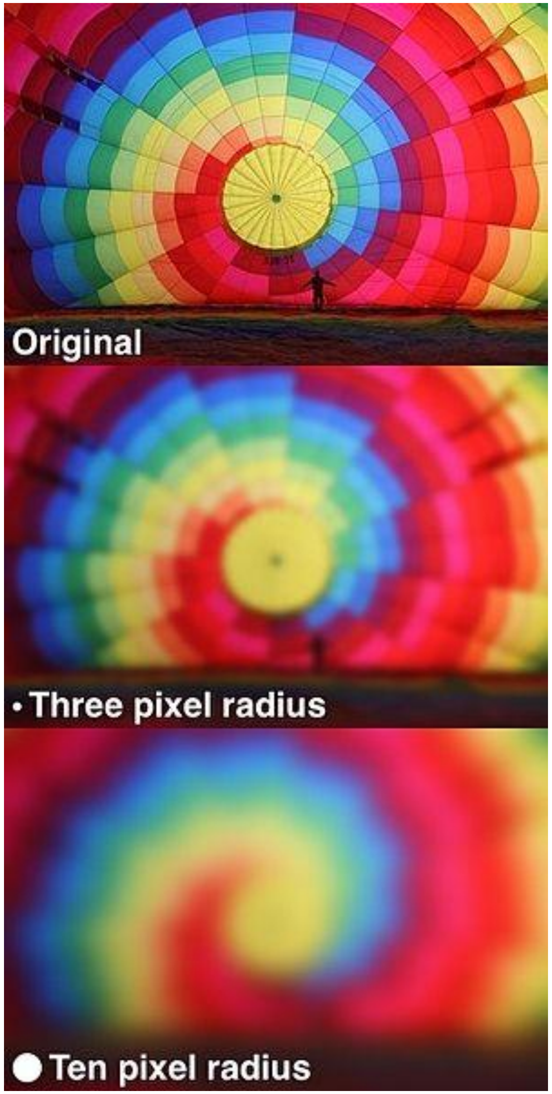


2) TRUSS LAYOUT WITHIN A CONTINUUM



2) TRUSS LAYOUT WITHIN A CONTINUUM

- GAUSSIAN BLUR:
  - CONVOLUTION WITH A GAUSSIAN FUNCTION



IDEA:

- LETS "BLUR" THE FIRST DERIVATIVE!

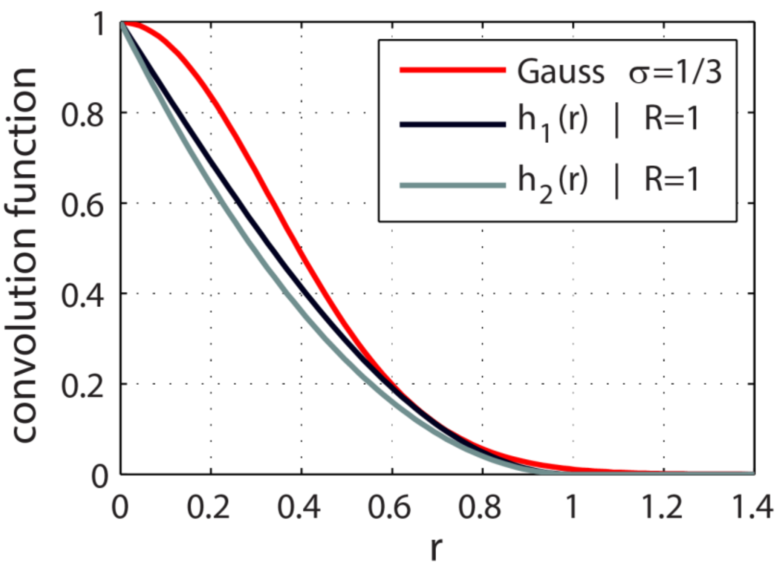
HTTP://EN

2) TRUSS LAYOUT WITHIN A CONTINUUM

- ARBITRARILY SMOOTH CONVOLUTION?

$$h(0) = 1$$
$$h(r \geq R) = 0$$
$$\left.\frac{dh}{dr}\right|_{r=R} = 0$$

REQUIREMENTS



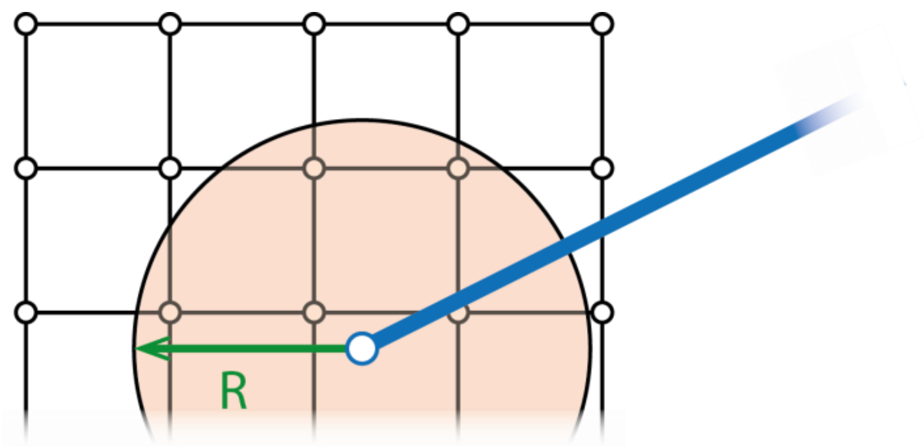
$$h_1(r) = \begin{cases} 1 - \sin\left(\frac{r\pi}{2R}\right) & r \leq R \\ 0 & r > R \end{cases}$$

$$h_2(r) = \begin{cases} \left(\frac{r}{R}\right)^2 - 2\left(\frac{r}{R}\right) + 1 & r \leq R \\ 0 & r > R \end{cases}$$

2) TRUSS LAYOUT WITHIN A CONTINUUM

- CONVOLUTION-BASED SHAPE FUNCTIONS
  - PARTITION OF UNITY REQUIREMENT

$$\tilde{N}_a = \frac{h\left(r_a\right)}{\sum_k h\left(r_k\right)}$$

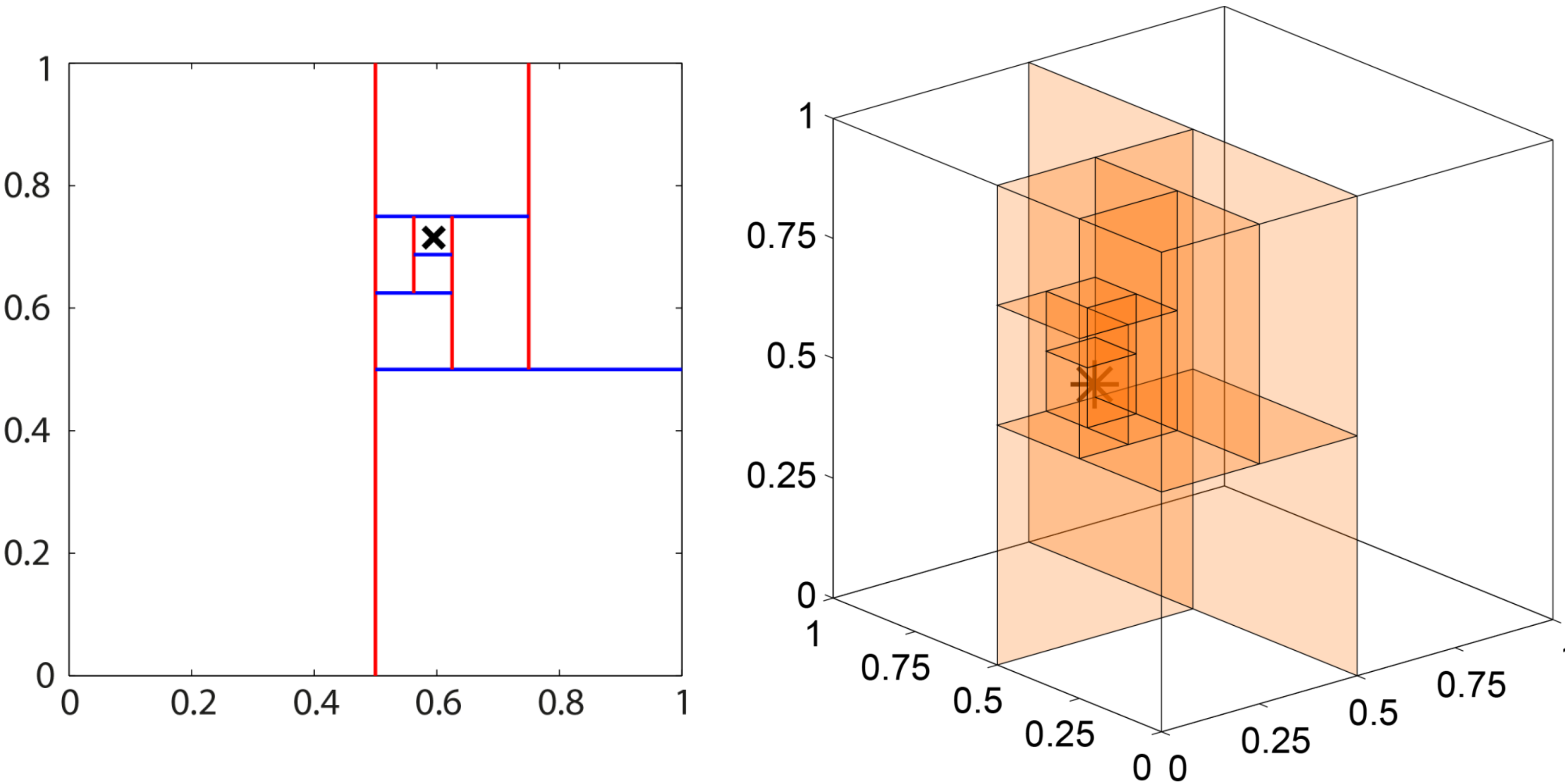


$$\mathbf{K}_e^+ = \tilde{\mathbf{N}}^T \mathbf{K}_e \tilde{\mathbf{N}}$$

$$\frac{\partial \mathbf{K}_e^+}{\partial n_j} = \frac{\partial \tilde{\mathbf{N}}^T}{\partial n_j} \mathbf{K}_e \tilde{\mathbf{N}} + \tilde{\mathbf{N}}^T \frac{\partial \mathbf{K}_e}{\partial n_j} \tilde{\mathbf{N}} + \tilde{\mathbf{N}}^T \mathbf{K}_e \frac{\partial \tilde{\mathbf{N}}}{\partial n_j}$$

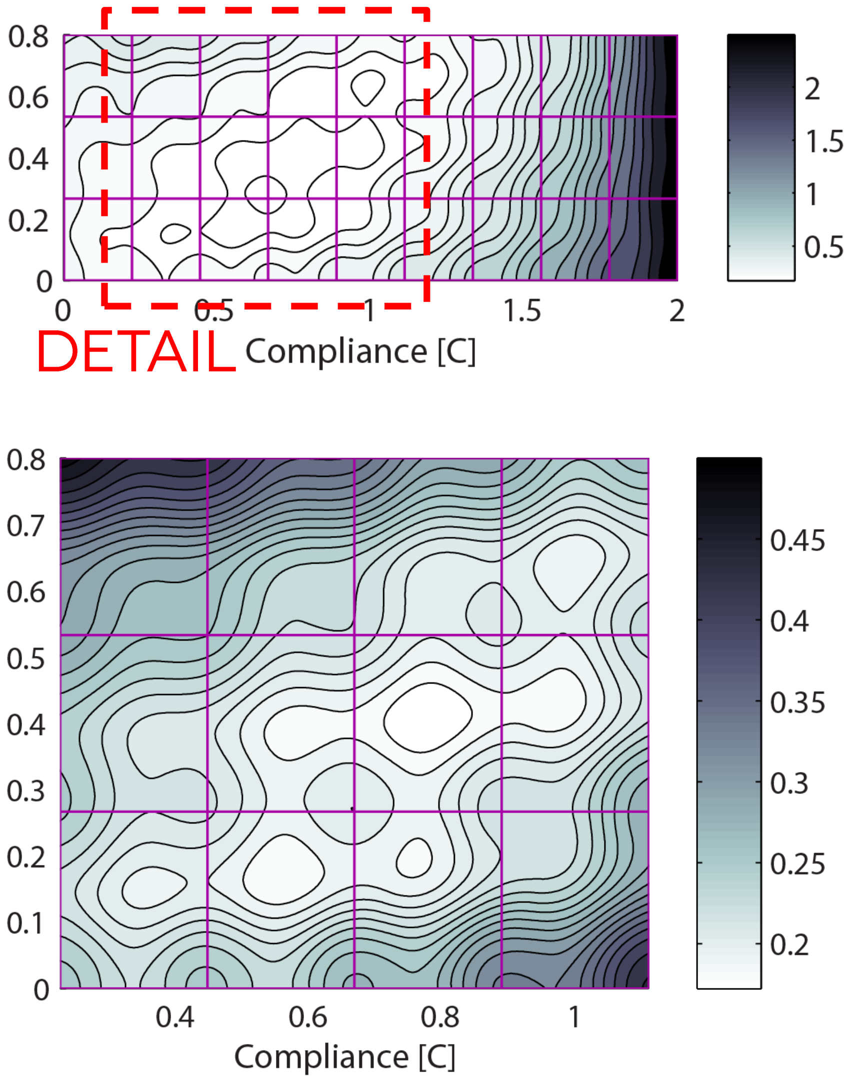
2) TRUSS LAYOUT WITHIN A CONTINUUM

- CONVOLUTION FUNCTION LOCALITY
  - SEARCH WITH BINARY, QUAD & OCT TREES

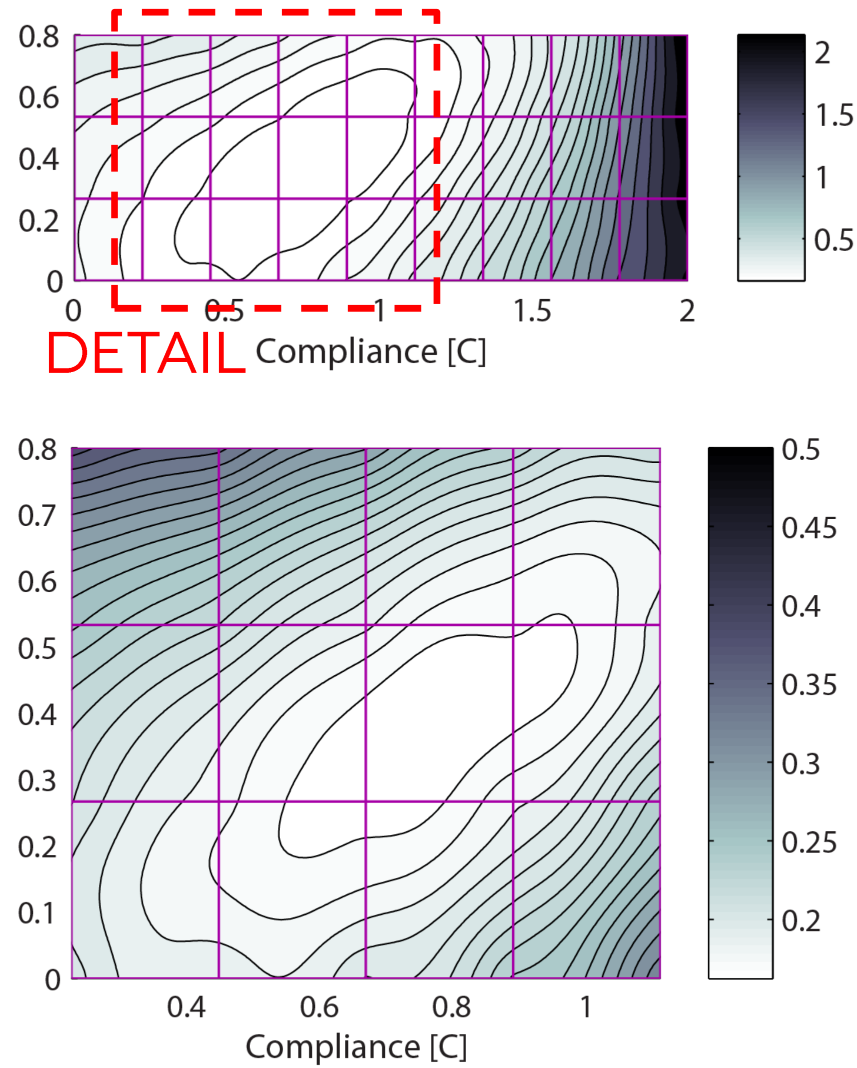


2) TRUSS LAYOUT WITHIN A CONTINUUM

FEM SHAPE FUNCTIONS

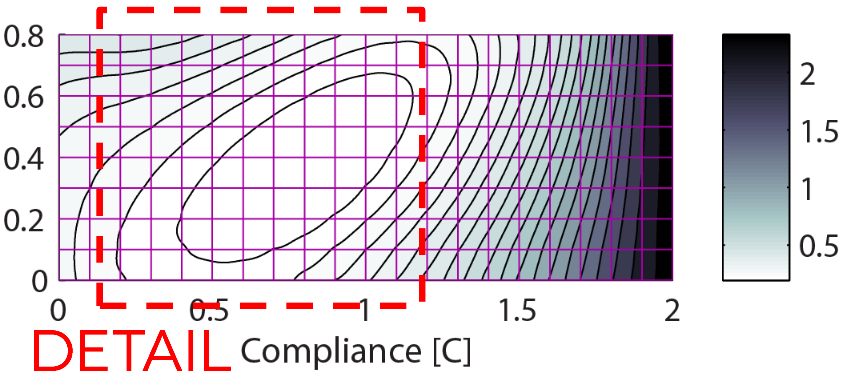


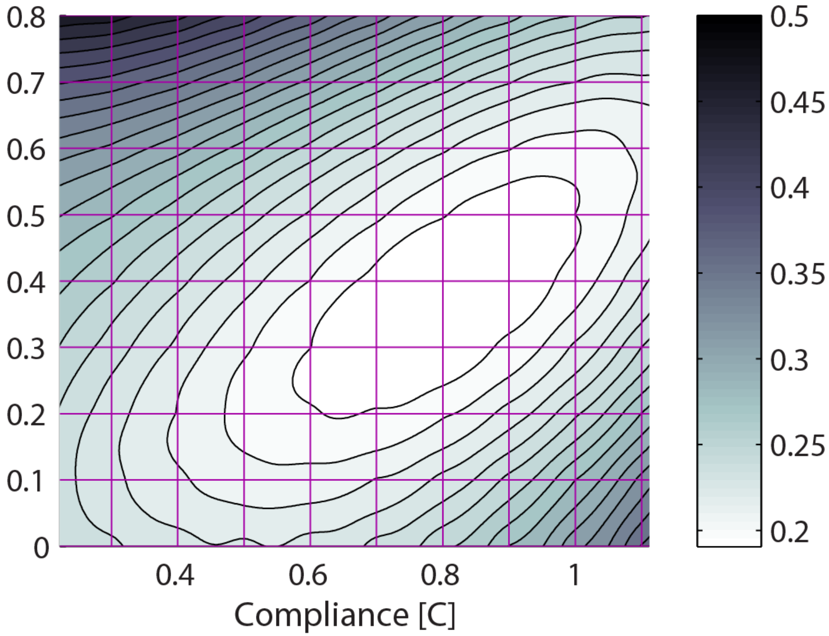
CONVOLUTION (R=0.5)



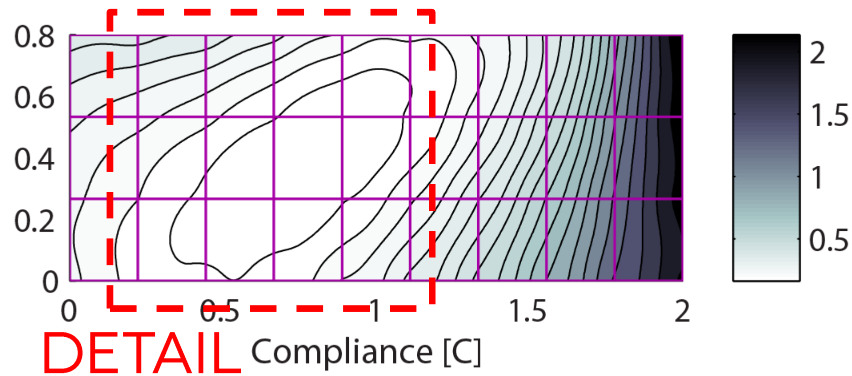
2) TRUSS LAYOUT WITHIN A CONTINUUM

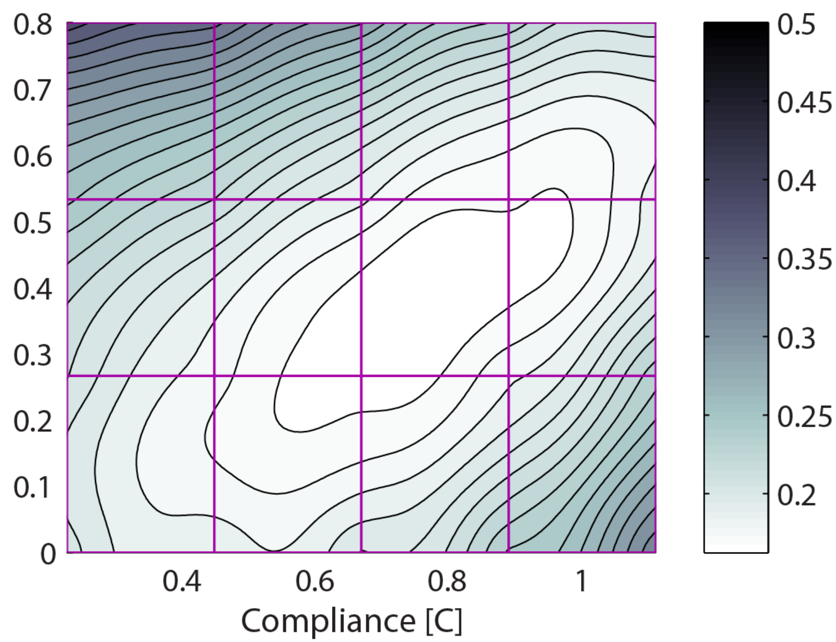
CONVOLUTION (R=0.3)





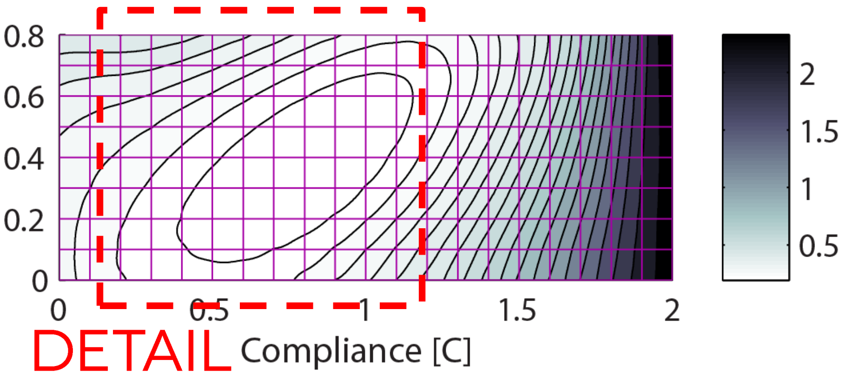
CONVOLUTION (R=0.5)

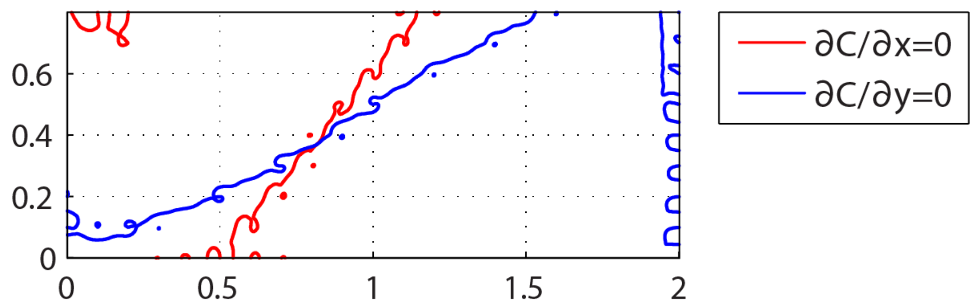




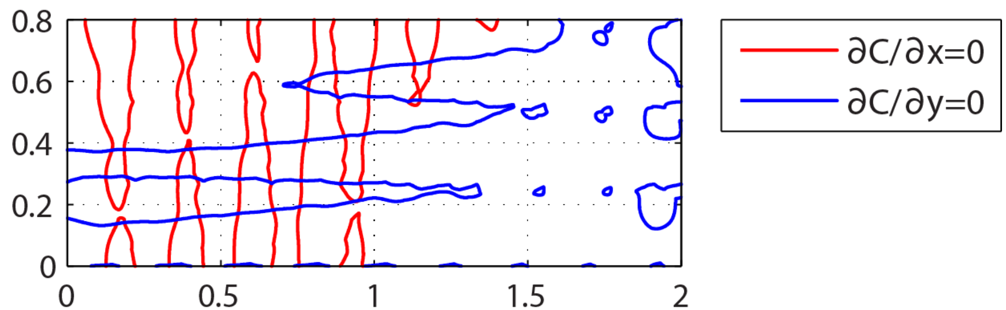
2) TRUSS LAYOUT WITHIN A CONTINUUM

CONVOLUTION (R=0.3)





PREVIOUS CASE



2) TRUSS LAYOUT WITHIN A CONTINUUM

- SLAB WITH SUPPORTING CABLES (8X21 Q9)

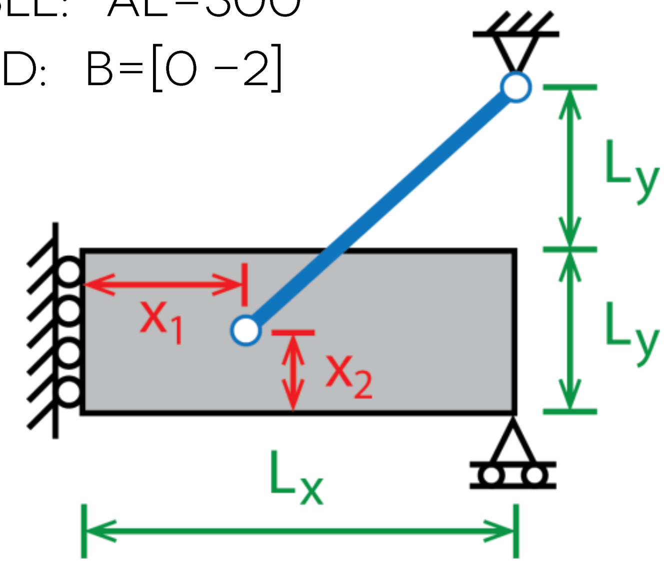
VIDEO

3X9 MESH (Q4 ELEMS)

SLAB: LX=2 LY=0.8 E=100 ν =0.3

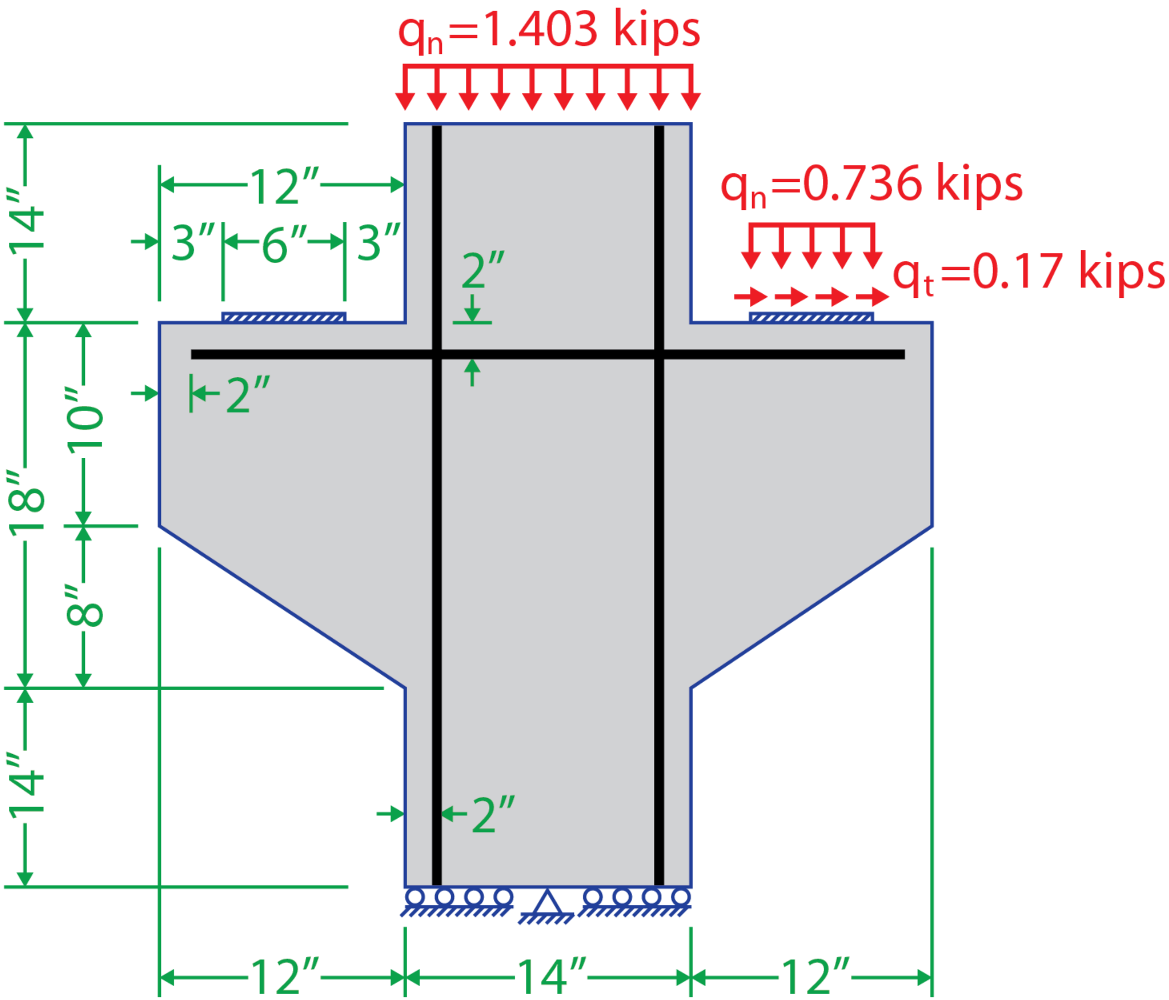
CABLE: AE=300

LOAD: B=[0 −2]



2) TRUSS LAYOUT WITHIN A CONTINUUM

- DOUBLE CORBEL (UNSTRUCTURED MESH)



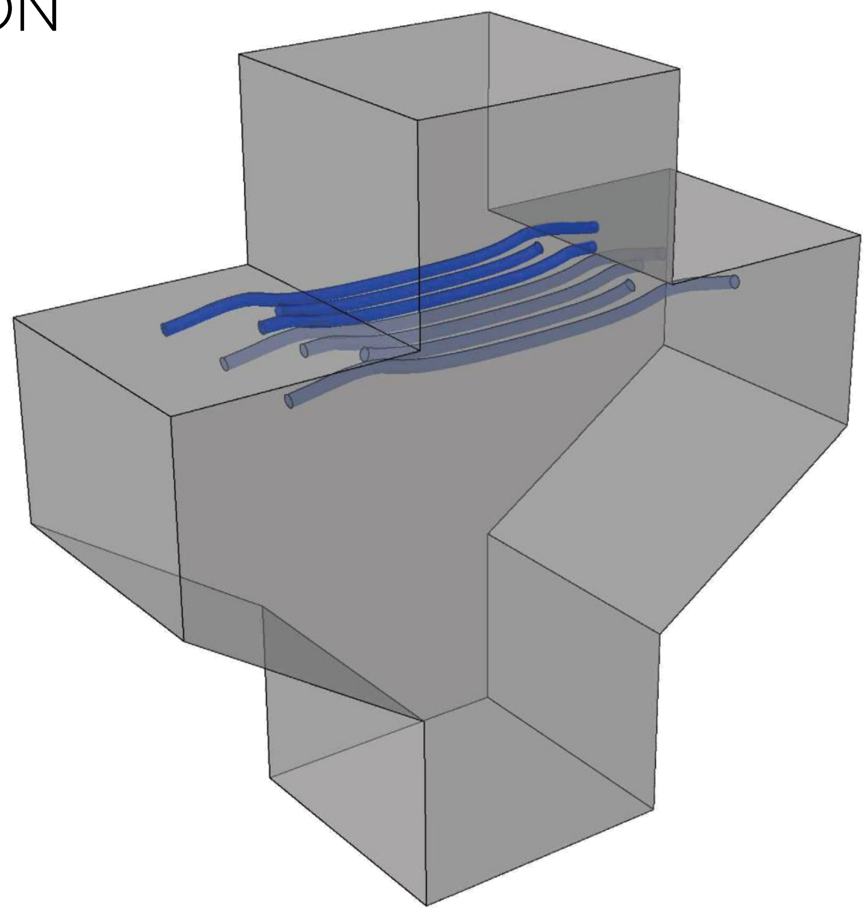
2) Truss layout within a continuum

- Double corbel (unstructured mesh)

VIDEO

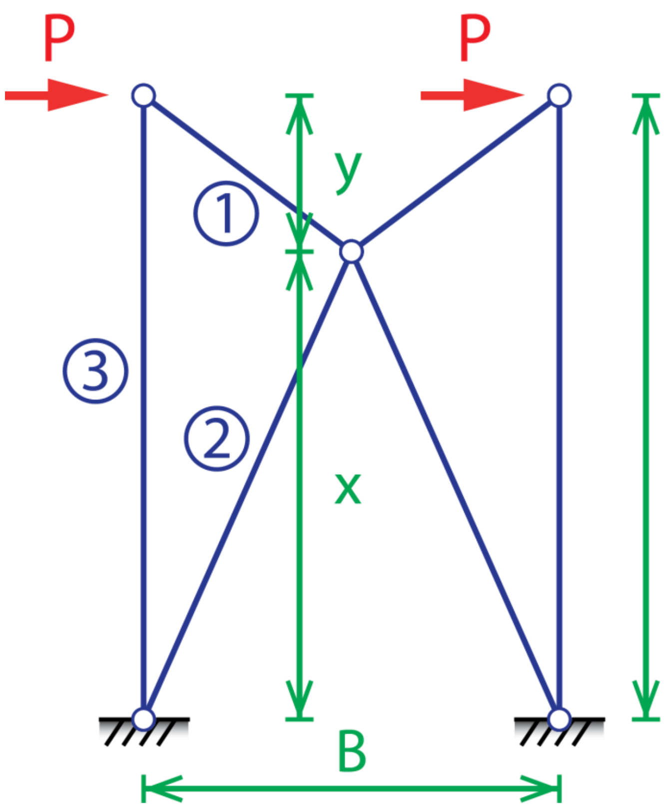
2) Truss layout within a continuum

- Double corbel (unstructured mesh)
  - Interpretation

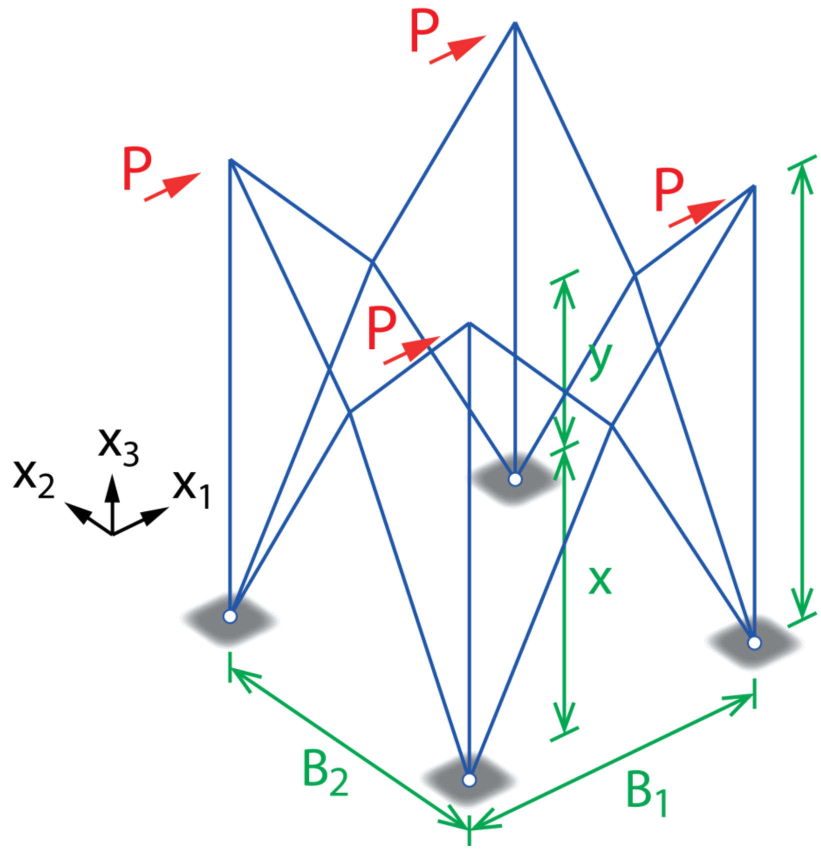


3) LATERAL BRACING SYSTEMS

- UNIT BRACES IN 2 AND 3-DIMENSIONS



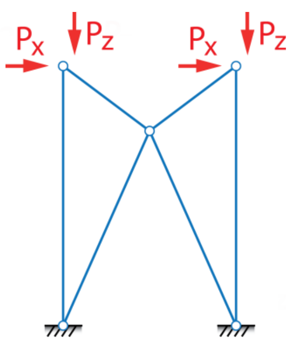
TWO-DIMENSIONAL BRACE



THREE-DIMENSIONAL BRACE

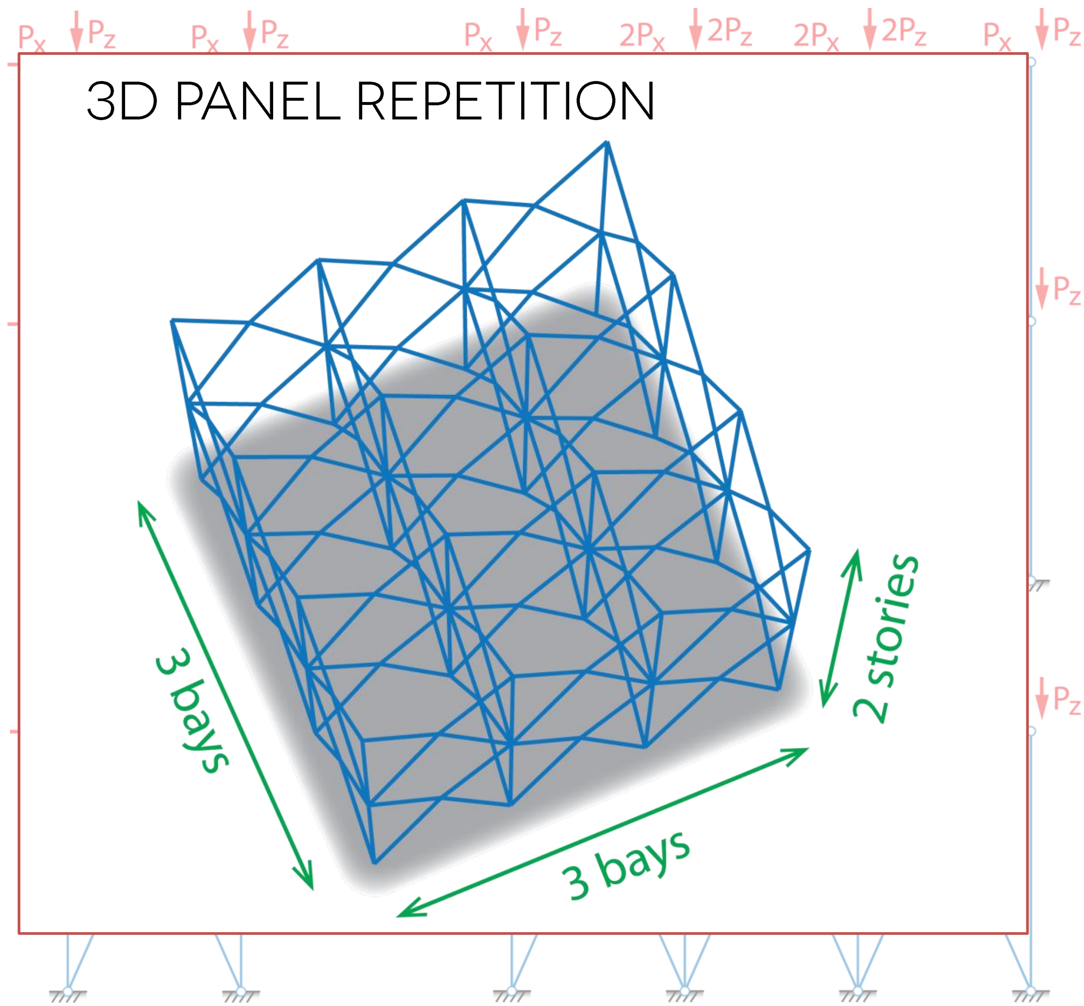
3) LATERAL BRACING SYSTEMS

- MULTIPLE STORIES – MULTIPLE BAYS



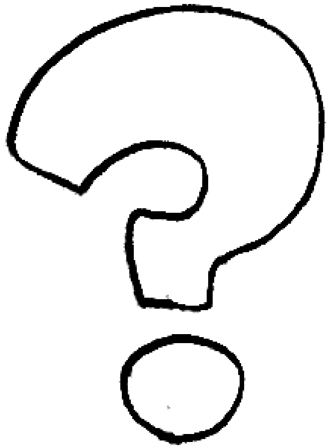
3) LATERAL BRACING SYSTEMS

- MULTIPLE STORIES – MULTIPLE BAYS



3) LATERAL BRACING SYSTEMS

- WHAT IS OPTIMAL?
  - LEAST **WEIGHT**
  - MINIMUM **COMPLIANCE**
  - SMALLEST **DISPLACEMENT**
  - OTHER…



- ASSUMPTIONS
  - ZERO CONNECTION COST
  - STATIC, LINEAR & ELASTIC
  - TRUSS MEMBERS

3) LATERAL BRACING SYSTEMS

- FORMULATIONS (1/2)
  - MINIMUM VOLUME

$$\begin{aligned} \min_{\mathbf{A},\mathbf{x}} \quad & V = \mathbf{A}^T\mathbf{L} \\ \text{s.t.} \quad & \sigma_c \leq \sigma_i \leq \sigma_t \qquad \forall i = 1 \ldots n_e \\ \text{with} \quad & \mathbf{Ku} = \mathbf{f} \end{aligned}$$

  - MINIMUM LOAD-PATH

$$\begin{aligned} \min_{\mathbf{A},\mathbf{x}} \quad & Z = \textstyle\sum_i |N_i|\, L_i \\ \text{s.t.} \quad & \textstyle\sum_i A_i L_i \leq \bar{V} \\ \text{with} \quad & \mathbf{Ku} = \mathbf{f} \end{aligned}$$

3) LATERAL BRACING SYSTEMS

- FORMULATIONS (2/2)
  - MINIMUM COMPLIANCE

$$\begin{aligned} \min_{\mathbf{A},\mathbf{x}} \quad & C = \mathbf{u}^T \mathbf{K} \mathbf{u} = \mathbf{u}^T \mathbf{f} \\ \text{s.t.} \quad & \textstyle\sum_i A_i L_i \leq \bar{V} \\ \text{with} \quad & \mathbf{K}\mathbf{u} = \mathbf{f} \end{aligned}$$

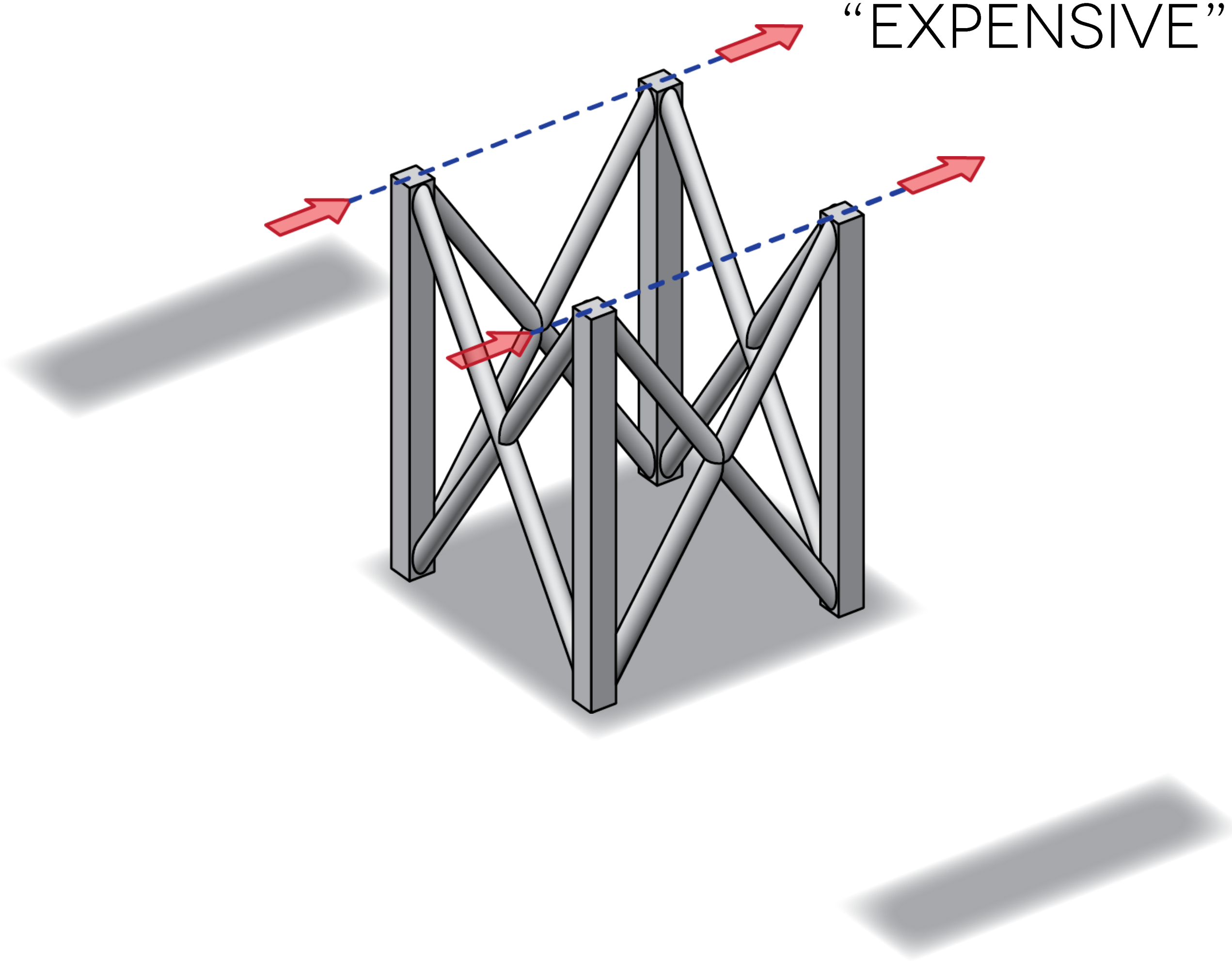
  - MINIMUM DISPLACEMENT

$$\begin{aligned} \min_{\mathbf{A},\mathbf{x}} \quad & \Delta = u_j \\ \text{s.t.} \quad & \textstyle\sum_i A_i L_i \leq \bar{V} \\ \text{with} \quad & \mathbf{K}\mathbf{u} = \mathbf{f} \end{aligned}$$

3) LATERAL BRACING SYSTEMS

3D SYMMETRY:

BRACES ARE TWICE AS "EXPENSIVE" AS IN 2D



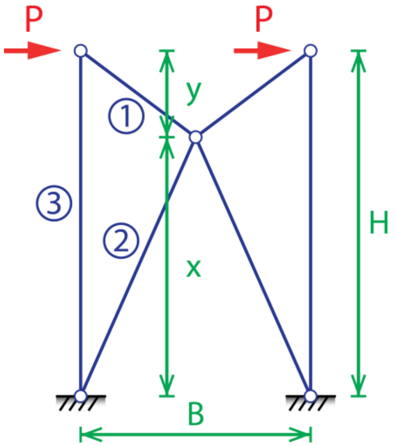
3) LATERAL BRACING SYSTEMS

- MIN VOLUME **ANALYTICAL** SOLUTION
  - 2D BRACE: α=1
  - 3D BRACE: α=2

$$\mathcal{L} = \alpha A_1 L_1 + \alpha A_2 L_2 + A_3 H + \lambda_{11}\left(-A_1\bar{\sigma} - N_1\right) + \lambda_{12}\left(-A_1\bar{\sigma} + N_1\right) + \ldots$$

$$\lambda_{21}\left(-A_2\bar{\sigma} - N_2\right) + \lambda_{22}\left(-A_2\bar{\sigma} + N_2\right) + \ldots$$

$$\lambda_{31}\left(-A_3\bar{\sigma} - N_3\right) + \lambda_{32}\left(-A_3\bar{\sigma} + N_3\right)$$



$$\lambda_{11} = \alpha L_1/\bar{\sigma} \qquad \lambda_{12} = 0 \qquad x = \frac{2\alpha + 1}{4\alpha} H$$

$$\lambda_{21} = 0 \qquad \lambda_{22} = \alpha L_2/\bar{\sigma}$$

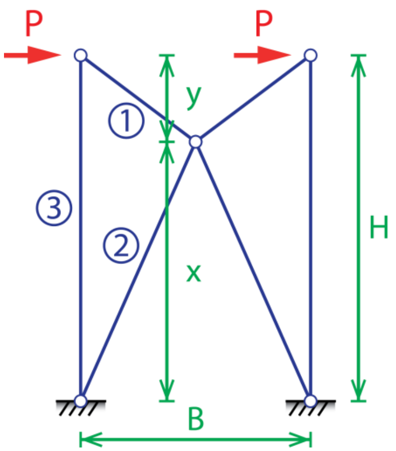
$$\lambda_{31} = 0 \qquad \lambda_{32} = H/\bar{\sigma} \qquad y = \frac{2\alpha - 1}{4\alpha} H$$

3) LATERAL BRACING SYSTEMS

- MIN COMPLIANCE **ANALYTICAL** SOLUTION
  - 2D BRACE: $\alpha=1$
  - 3D BRACE: $\alpha=2$

$$C = \frac{4P^2}{EB^2}\left[\frac{L_1^3}{A_1} + \frac{L_2^3}{A_2} + \frac{L_3}{A_3}y^2\right]$$

$$\mathcal{L} = \frac{4P^2}{EB^2}\left[\frac{L_1^3}{A_1} + \frac{L_2^3}{A_2} + \frac{L_3}{A_3}y^2\right] + \lambda\left(\alpha A_1 L_1 + \alpha A_2 L_2 + A_3 H - \bar{V}\right)$$



$$\lambda = \frac{4P^2y^2}{EB^2A_3^2}$$

$$x = \frac{2\sqrt{\alpha}+1}{4\sqrt{\alpha}}H$$

$$y = \frac{2\sqrt{\alpha}-1}{4\sqrt{\alpha}}H$$

3) LATERAL BRACING SYSTEMS

- ANALYTICAL SOLUTION FOR A SINGLE BAY
  - 2D BRACE: $\alpha=1$
  - 3D BRACE: $\alpha=2$

$$x = \frac{2\alpha + 1}{4\alpha} H$$

WEIGHT &
LOAD–PATH

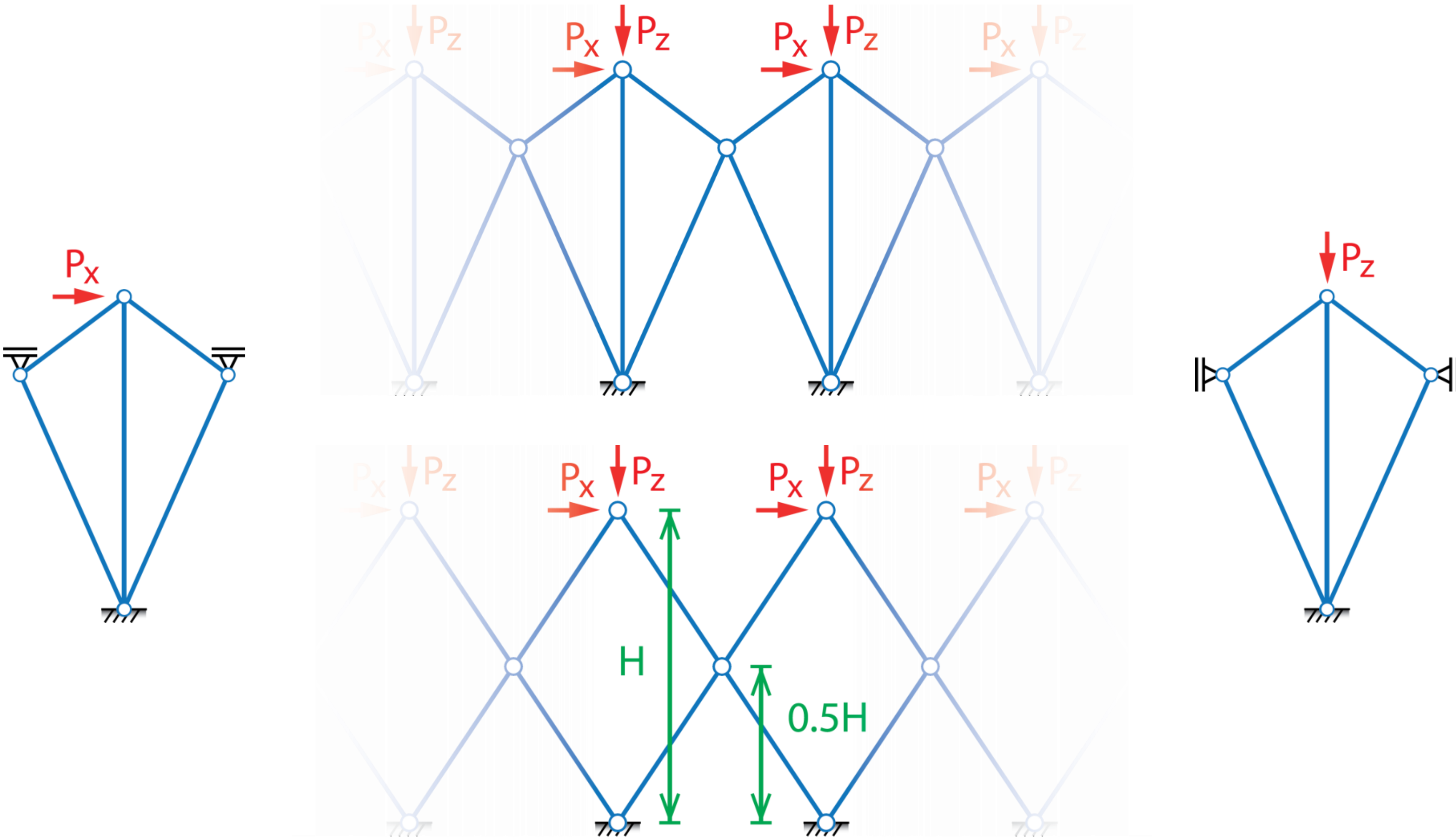
$$x = \frac{2\sqrt{\alpha} + 1}{4\sqrt{\alpha}} H$$

COMPLIANCE &
DISPLACEMENT

Height x	Weight - Cost		Performance	
	Volume	Load-Path	Compliance	Displacement
2D	$0.75H$	$0.75H$	$0.75H$	$0.75H$
3D	$0.625H$	$0.625H$	$0.6768H$	$0.6768H$

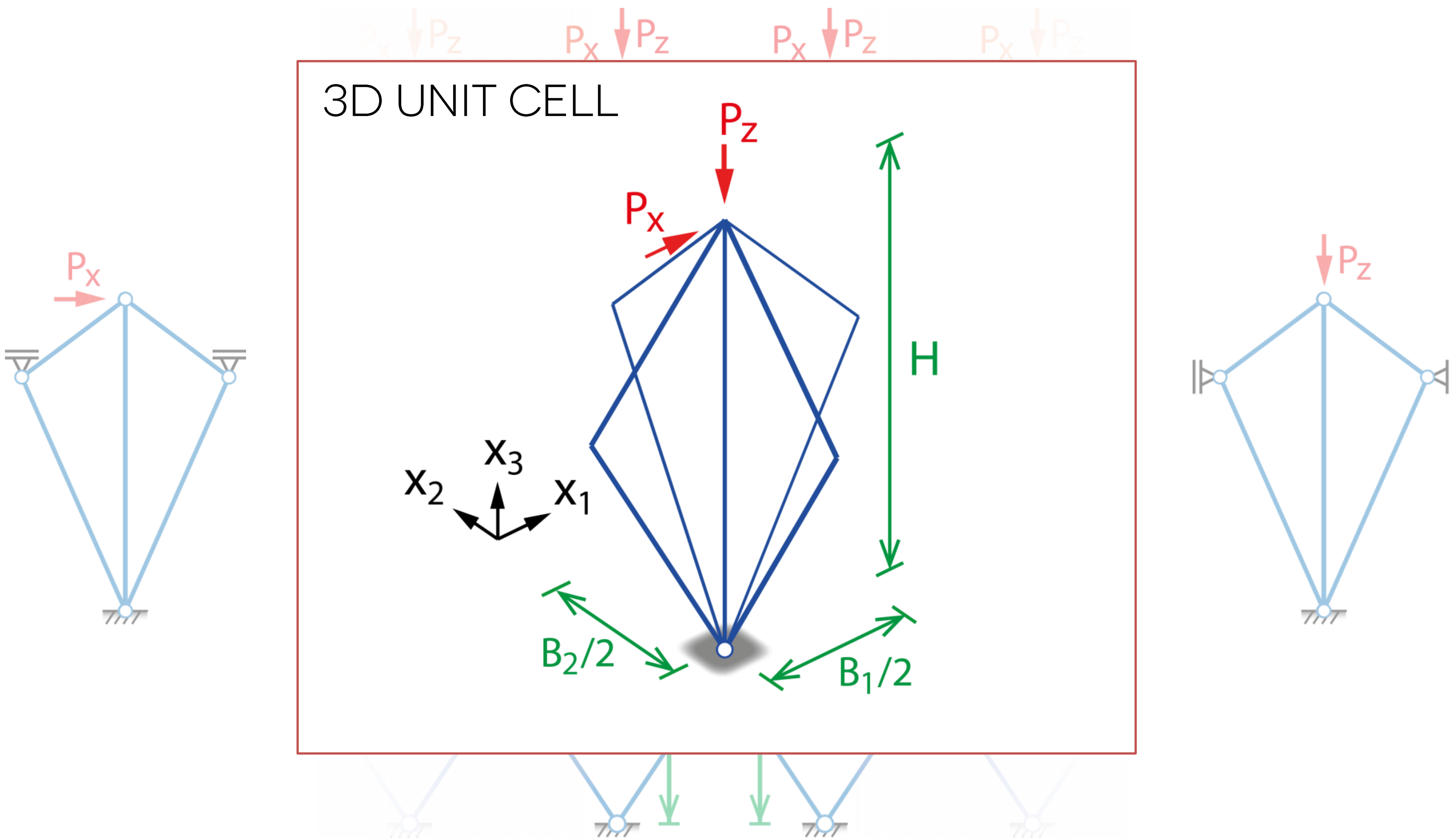
3) LATERAL BRACING SYSTEMS

- LIMIT CASE OF ∞ BAYS



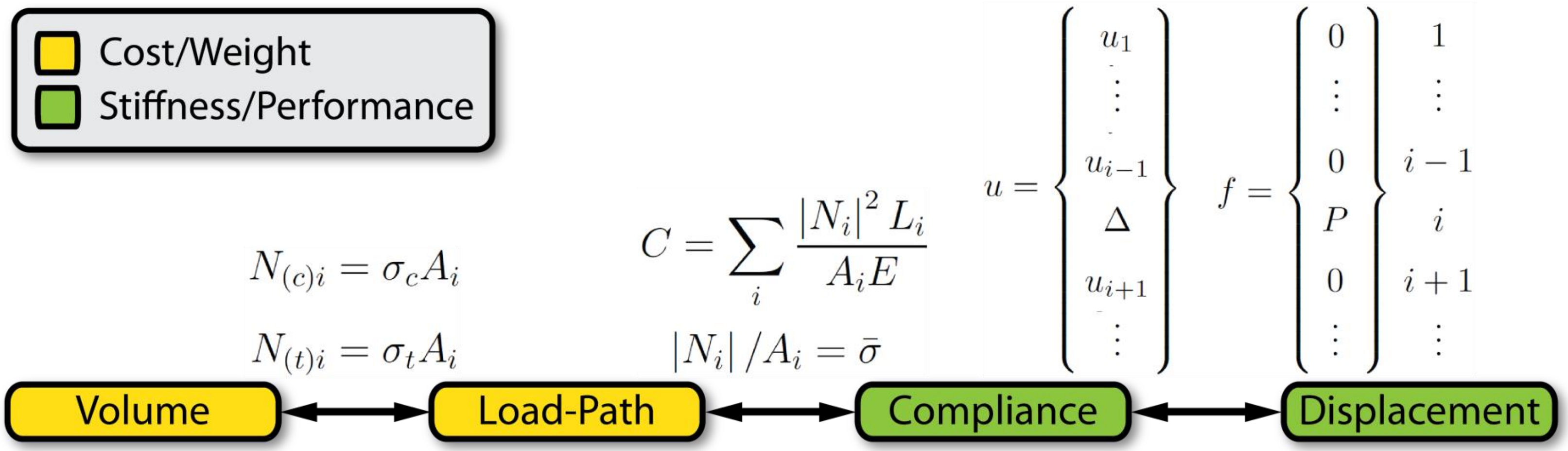
3) LATERAL BRACING SYSTEMS

- LIMIT CASE OF ∞ BAYS



3) LATERAL BRACING SYSTEMS

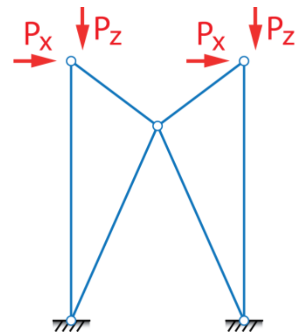
- 3) LATERAL BRACING SYSTEMS

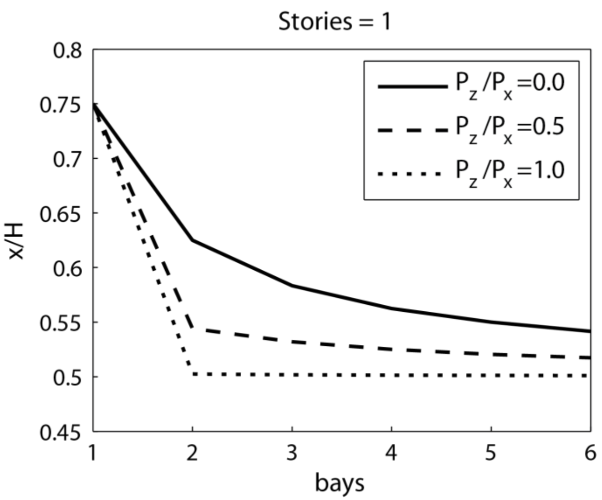


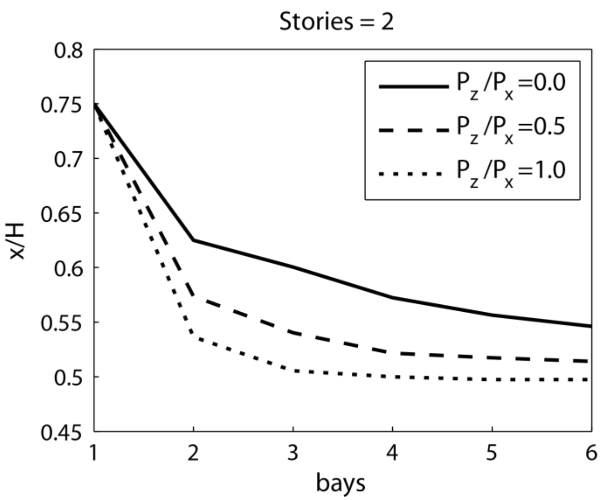
Height x	Weight - Cost		Performance	
	Volume	Load-Path	Compliance	Displacement
2D	$0.75H$	$0.75H$	$0.75H$	$0.75H$
3D	$0.625H$	$0.625H$	$0.6768H$	$0.6768H$

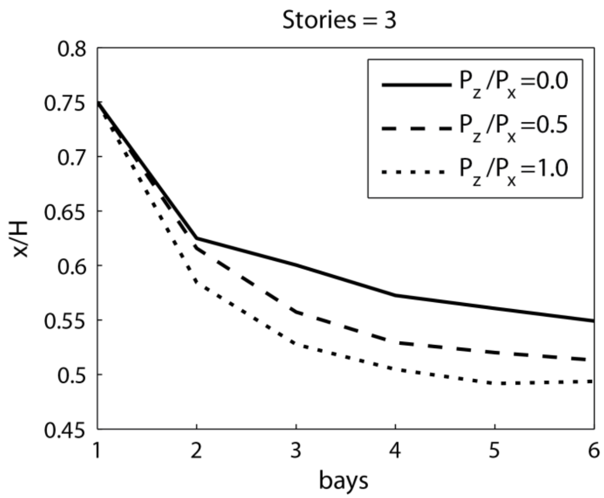
3) LATERAL BRACING SYSTEMS

- OPTIMAL BRACING POINT FOR TWO-DIMENSIONAL BRACES



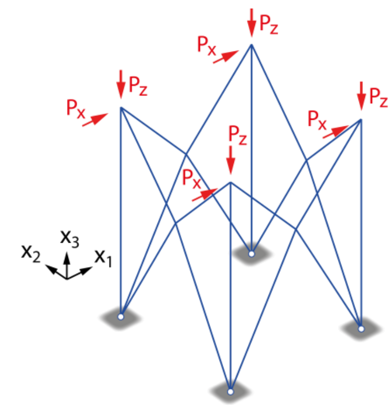


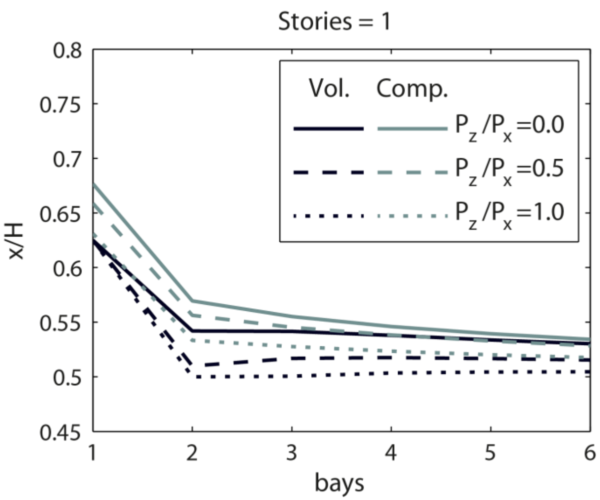


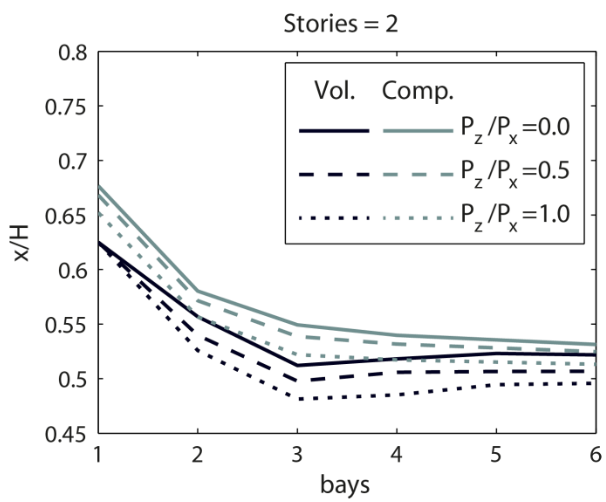


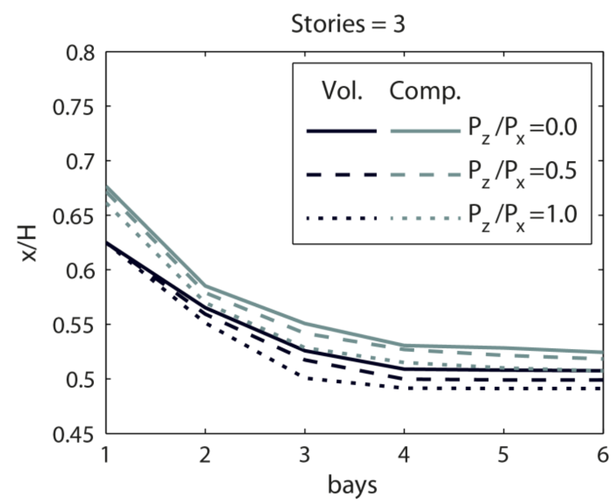
3) Lateral Bracing Systems

- Optimal bracing point for three-dimensional braces



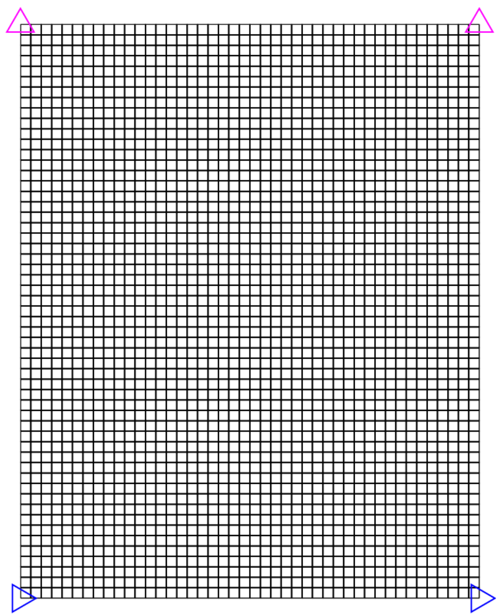




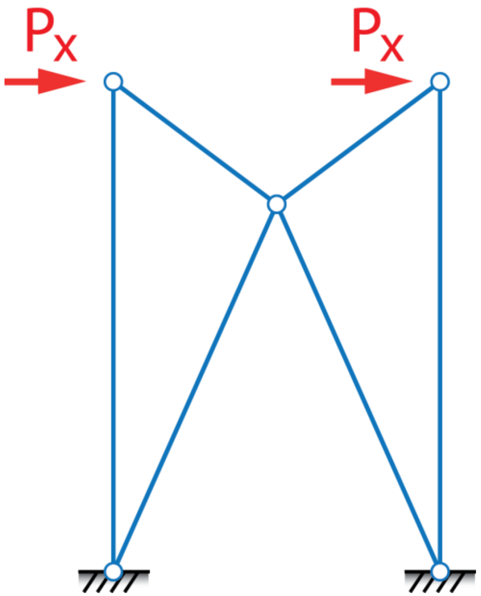


3) LATERAL BRACING SYSTEMS

- GROUND STRUCTURE METHOD
  - WEIGHT MINIMIZATION WITH SYMMETRY



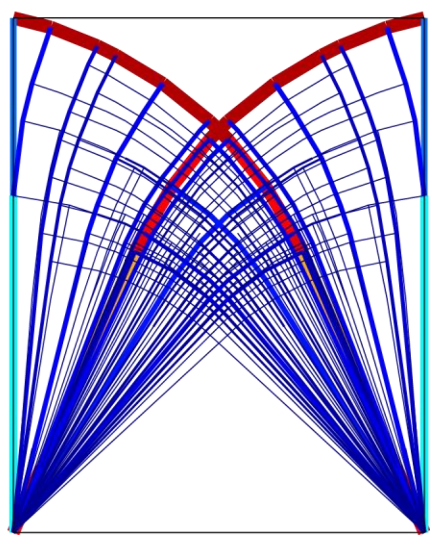
BASE MESH



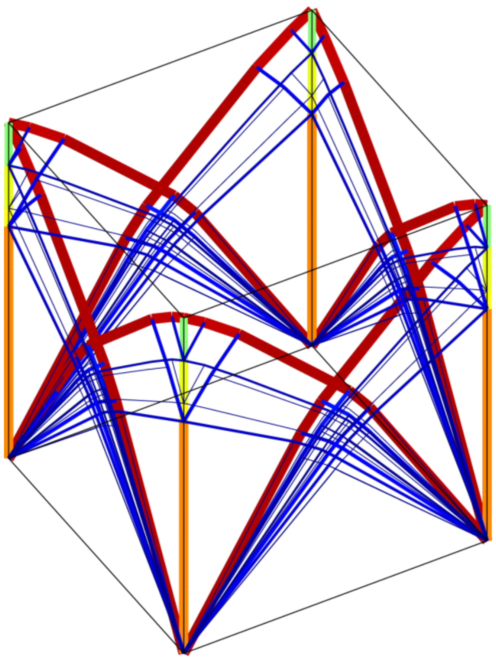
DORN W S, GOMORY R E, GREENBERG H J (1964) – "AUTOMATIC DESIGN OF OPTIMAL STRUCTURES", JOURNAL DE MECANIQUE 3(1), PP 25–52

3) LATERAL BRACING SYSTEMS

- GROUND STRUCTURE METHOD
  - WEIGHT MINIMIZATION WITH SYMMETRY



2D RESULT

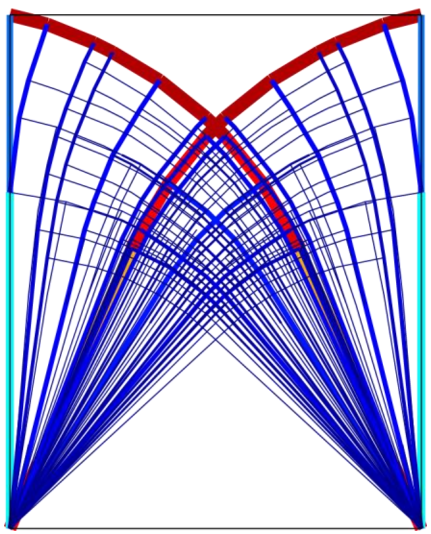


3D RESULT

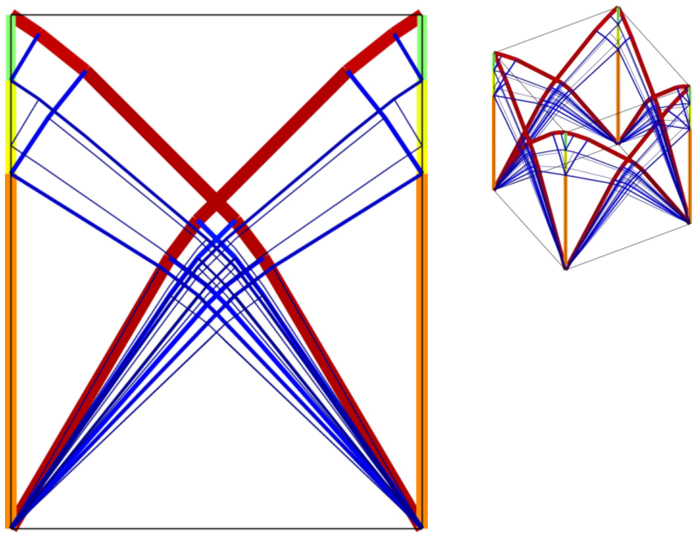
DORN W S, GOMORY R E, GREENBERG H J (1964) – "AUTOMATIC DESIGN OF OPTIMAL STRUCTURES", JOURNAL DE MECANIQUE 3(1), PP 25–52

3) Lateral Bracing Systems

- Ground Structure Method
  - Weight minimization with symmetry



2D Result

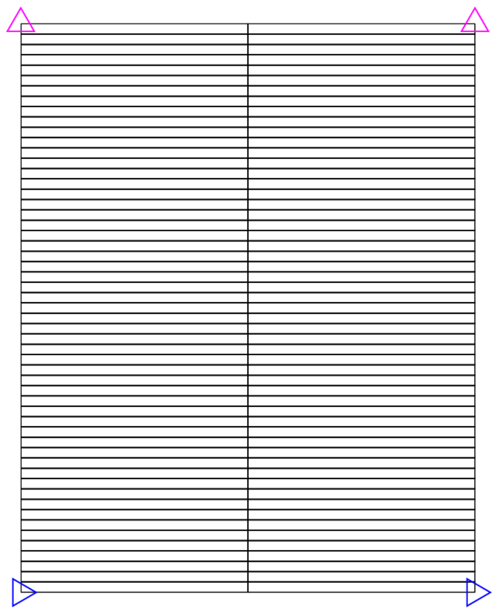


3D Result

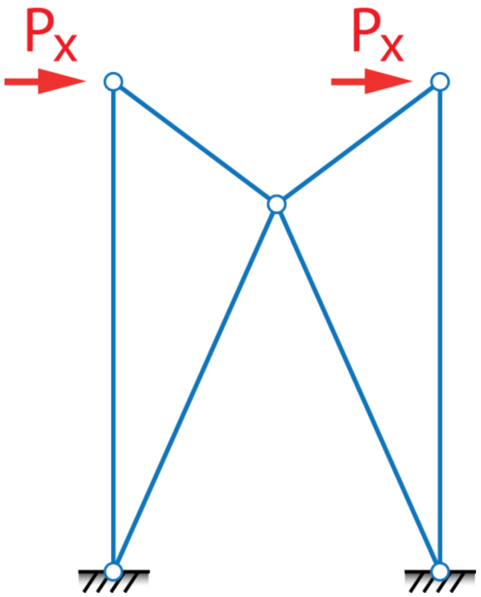
Dorn W S, Gomory R E, Greenberg H J (1964) – "Automatic Design of Optimal Structures", Journal de Mecanique 3(1), pp 25–52

3) LATERAL BRACING SYSTEMS

- GROUND STRUCTURE METHOD
  - WEIGHT MINIMIZATION WITH SYMMETRY



BASE MESH



DORN W S, GOMORY R E, GREENBERG H J (1964) – "AUTOMATIC DESIGN OF OPTIMAL STRUCTURES", JOURNAL DE MECANIQUE 3(1), PP 25–52

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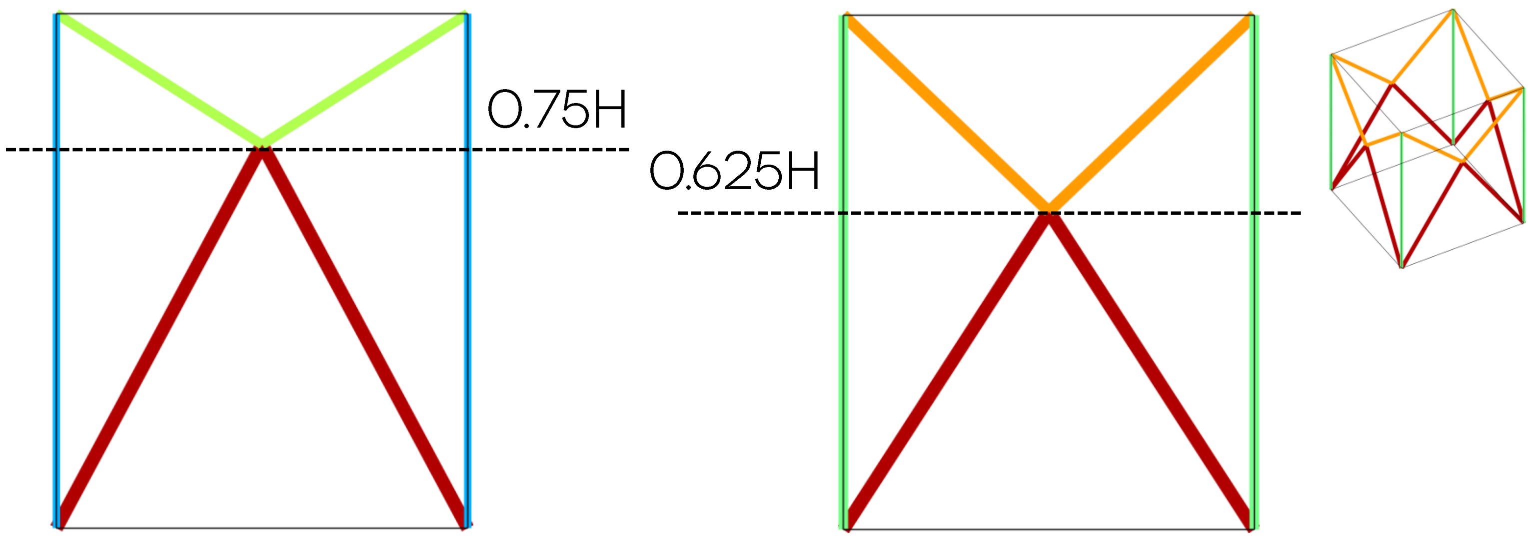
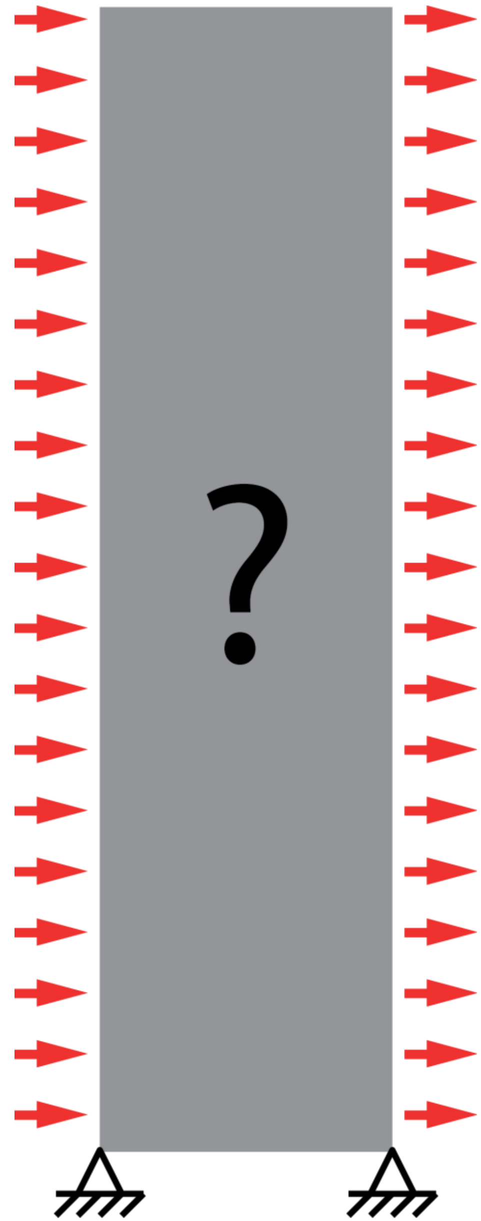


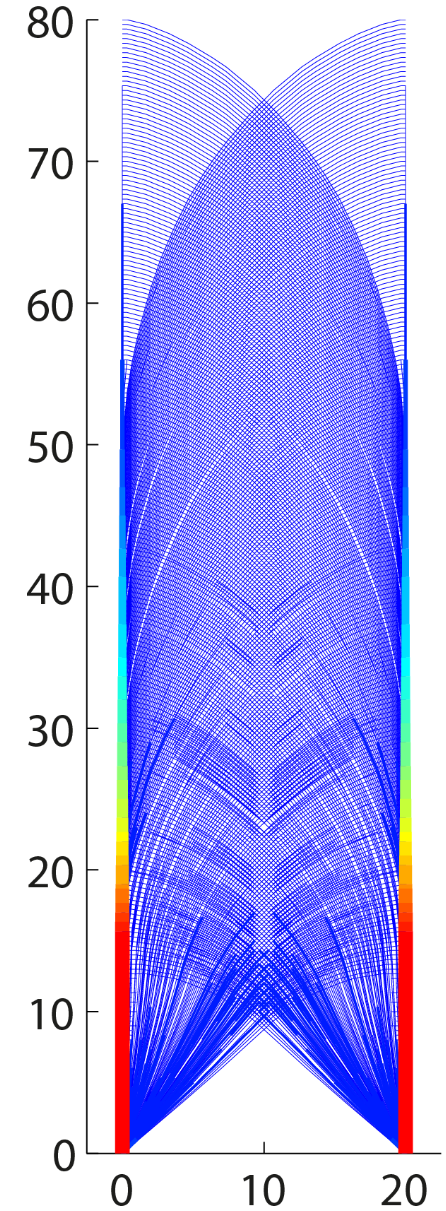
2D RESULT 3D RESULT

DORN W S, GOMORY R E, GREENBERG H J (1964) – "AUTOMATIC DESIGN OF OPTIMAL STRUCTURES", JOURNAL DE MECANIQUE 3(1), PP 25–52

4) GROUND STRUCTURES IN 2D

- BRACED TOWER

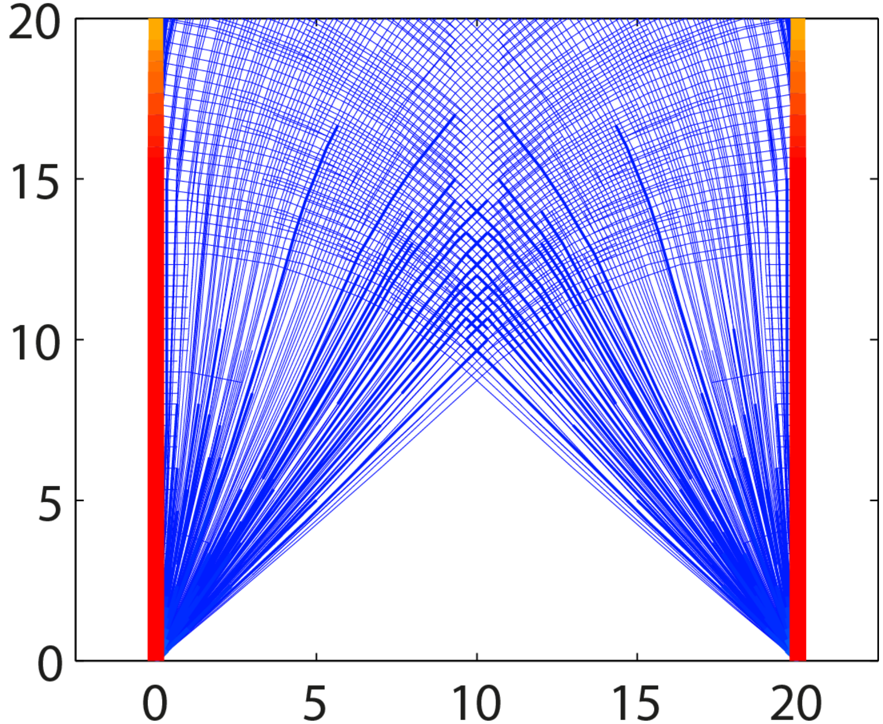


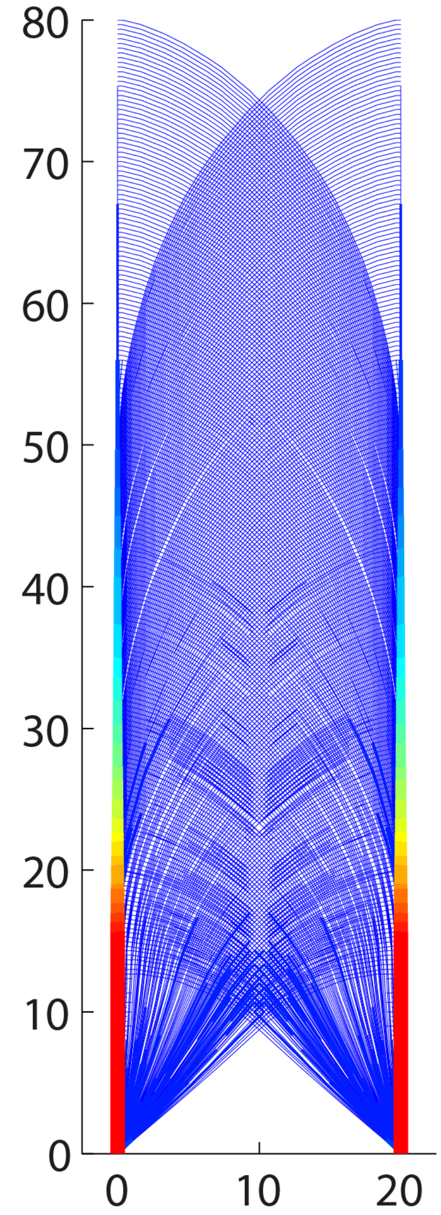


4) GROUND STRUCTURES IN 2D

- BRACED TOWER

11.5 MILLION BARS





5) GROUND STRUCTURES IN 3D

- DIRECT EXTENSION OF THE 2D METHOD
  - FORMULATION REMAINS UNMODIFIED

$$\min_{\mathbf{s}^+,\mathbf{s}^-} \quad V^\star = \frac{V}{\sigma_T} = \underbrace{\{\mathbf{l}^T \quad \kappa\,\mathbf{l}^T\}}_{1\times 2N_b} \underbrace{\begin{Bmatrix}\mathbf{s}^+ \\ \mathbf{s}^-\end{Bmatrix}}_{2N_b\times 1}$$

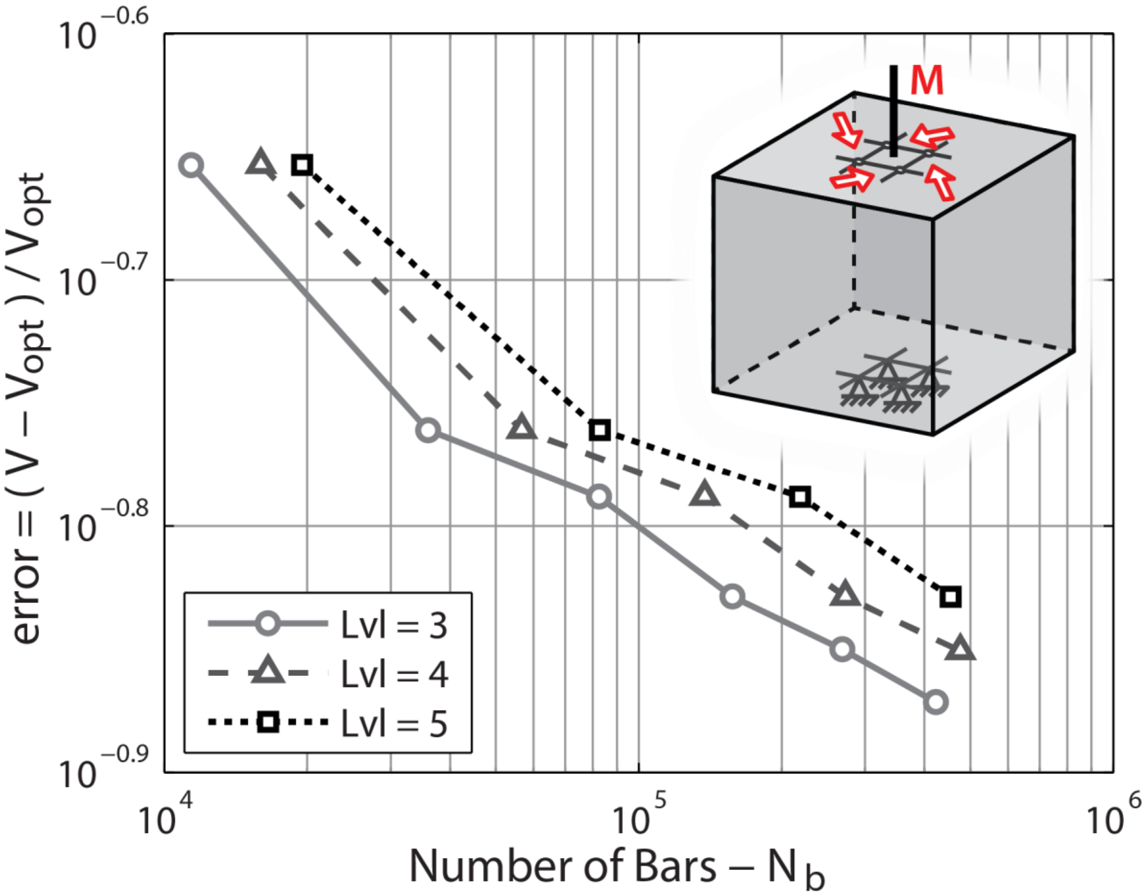
$$\text{s.t.} \quad \underbrace{[\mathbf{B}^T \quad -\mathbf{B}^T]}_{N_{dof}\times 2N_b} \underbrace{\begin{Bmatrix}\mathbf{s}^+ \\ \mathbf{s}^-\end{Bmatrix}}_{2N_b\times 1} = \underbrace{\mathbf{f}}_{N_{dof}\times 1}$$

$$s_i^+, s_i^- \geq 0$$

$$\kappa = \frac{\sigma_T}{\sigma_C}$$

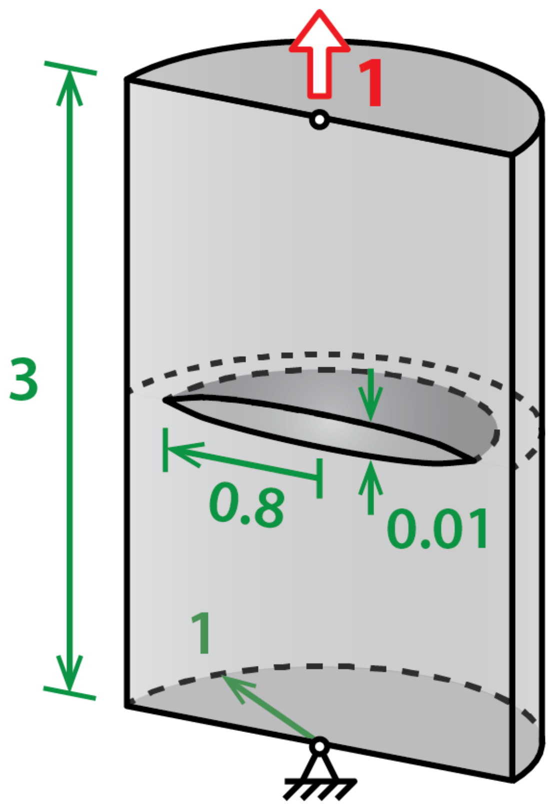
5) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM
  - ERROR DECREASES FROM ~23% TO ~12%

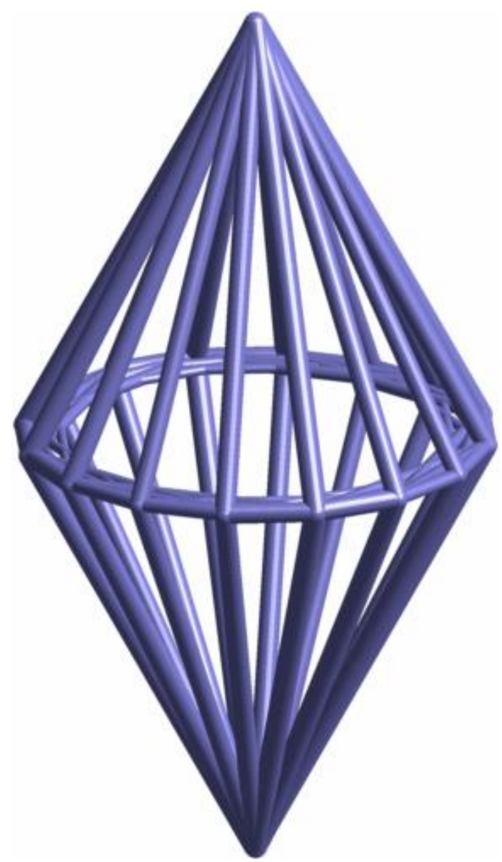


6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: DIAMOND

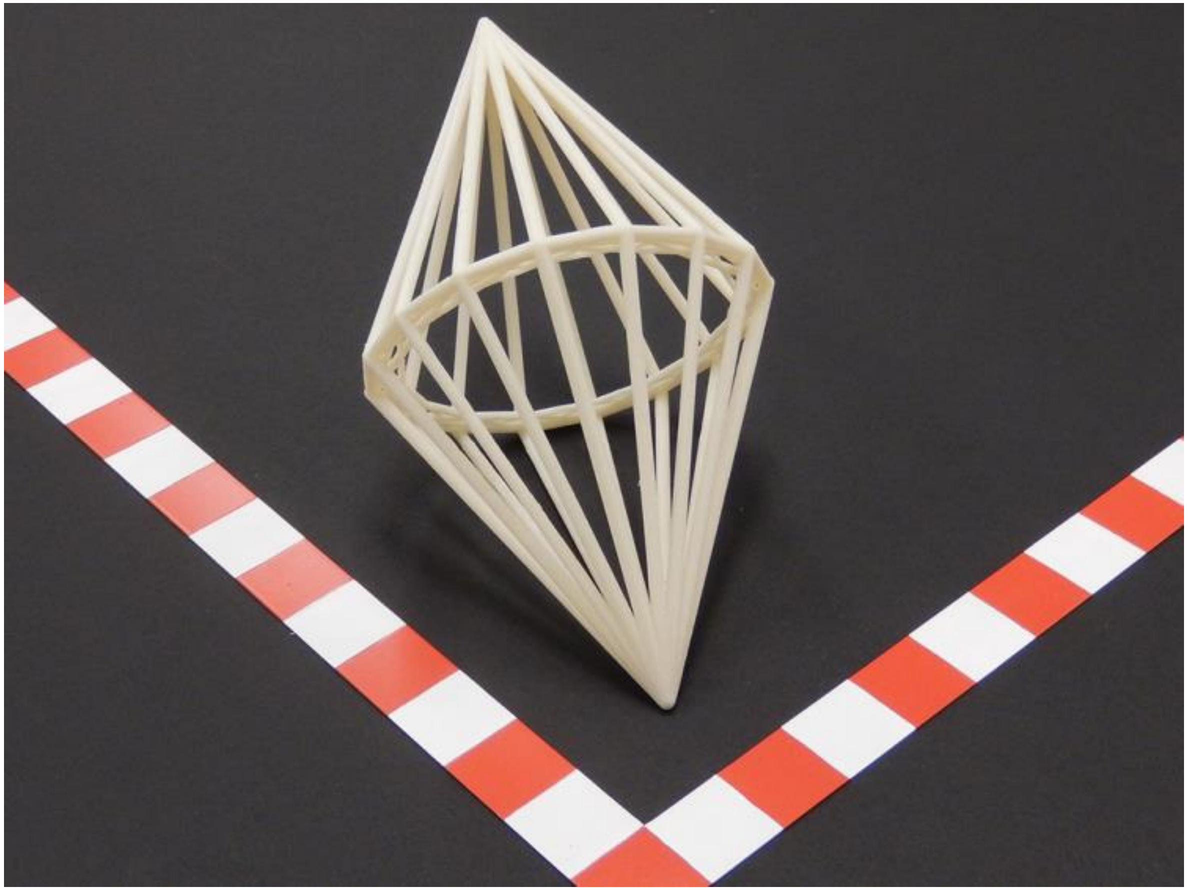


NOTE: HALF-DOMAIN



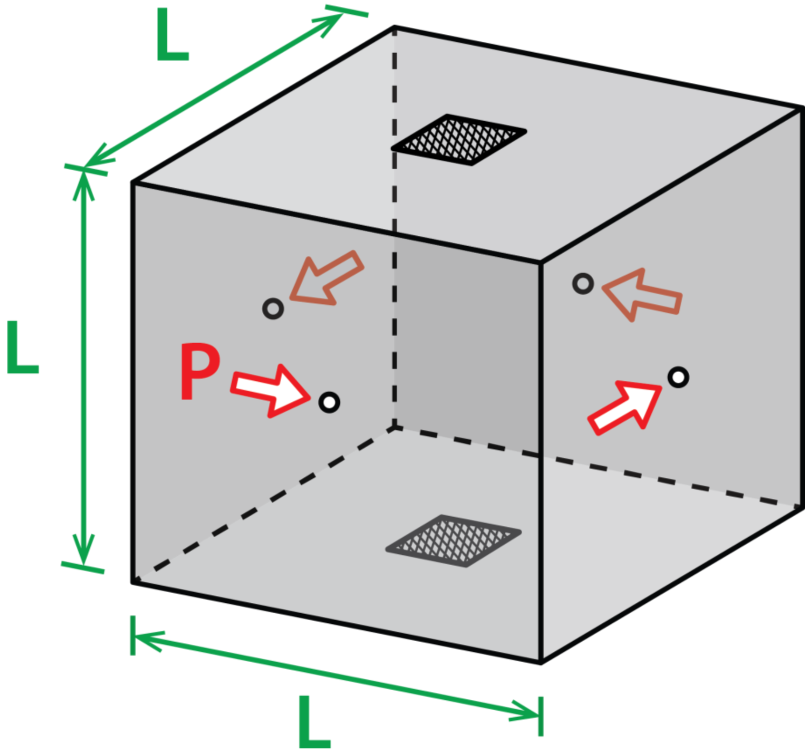
6) ADDITIVE MANUF. OF OPT. STRUCTS.

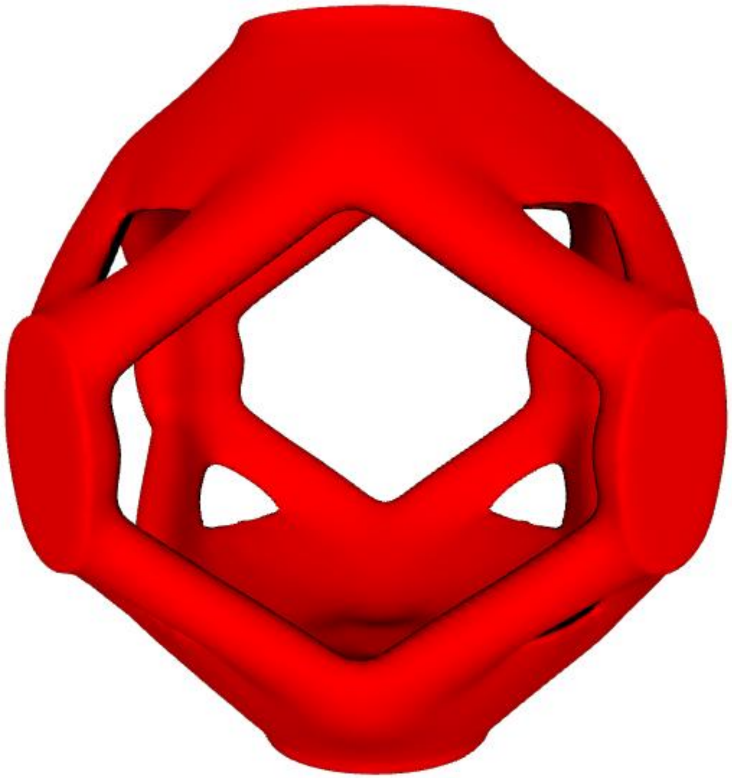
- SHOW AND TELL: DIAMOND



6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: SHEAR BOX

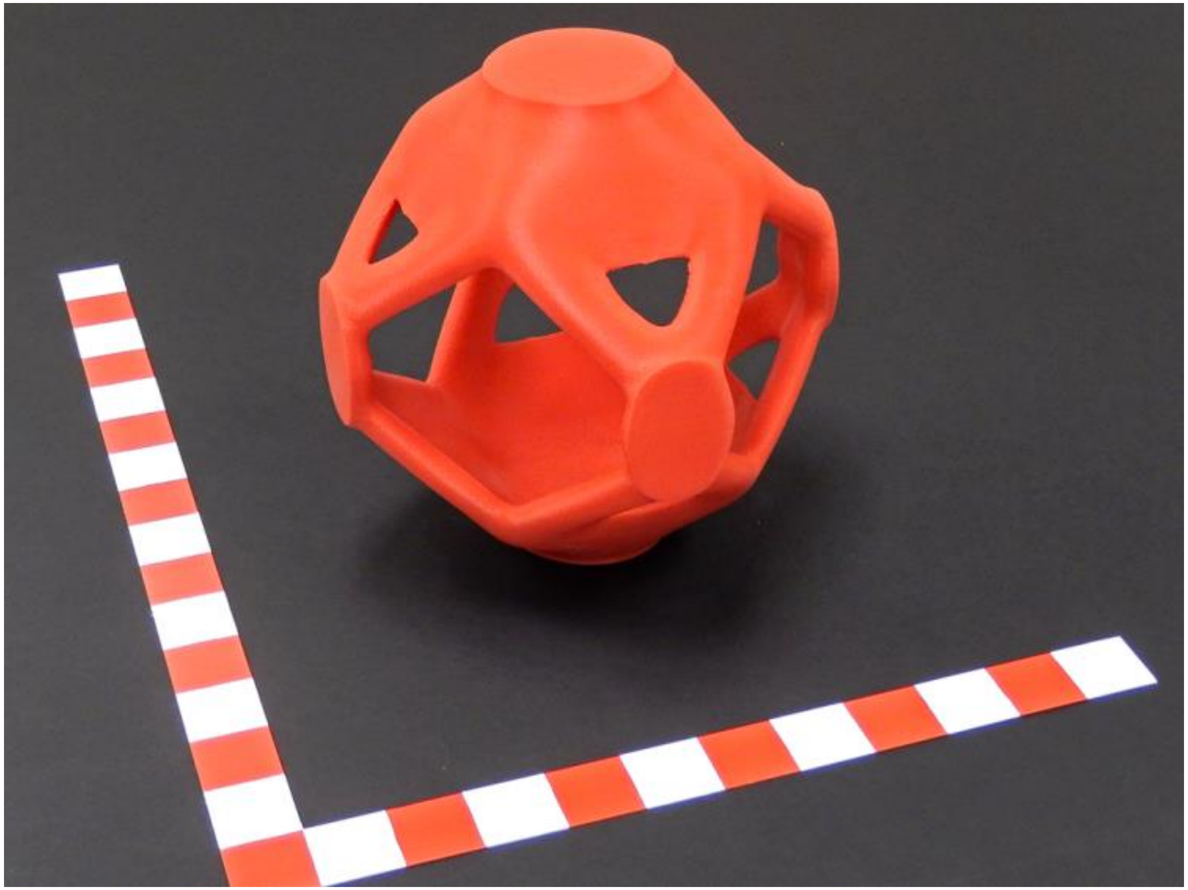




NGUYEN TH, PAULINO GH, SONG J, LE CH (2009) "A COMPUTATIONAL PARADIGM FOR MULTIRESOLUTION TOPOLOGY OPTIMIZATION (MTOP)." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, 41(4):525–39.

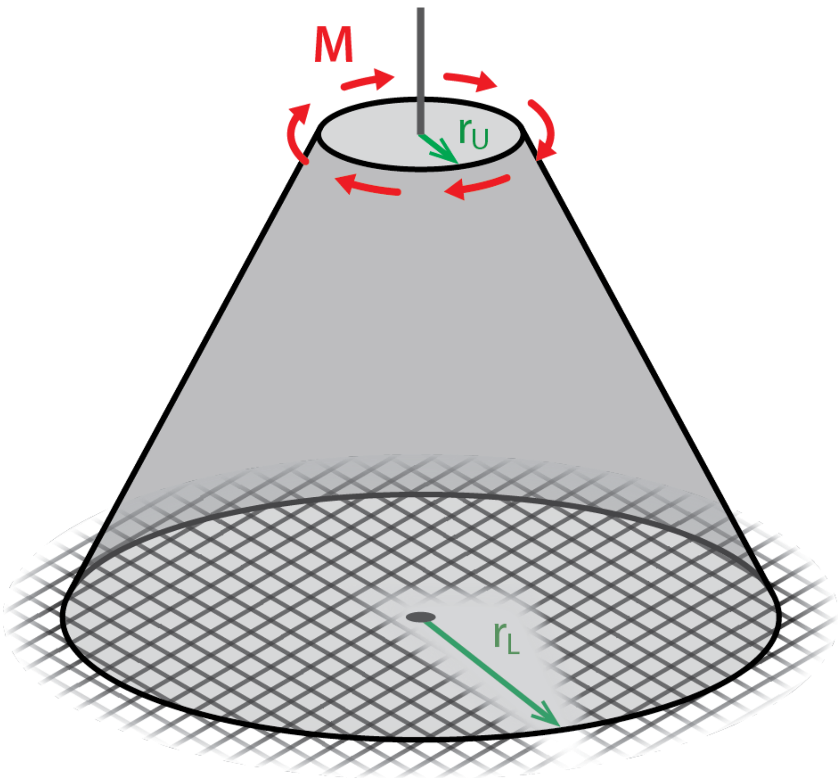
6) Additive Manuf. of Opt. Structs.

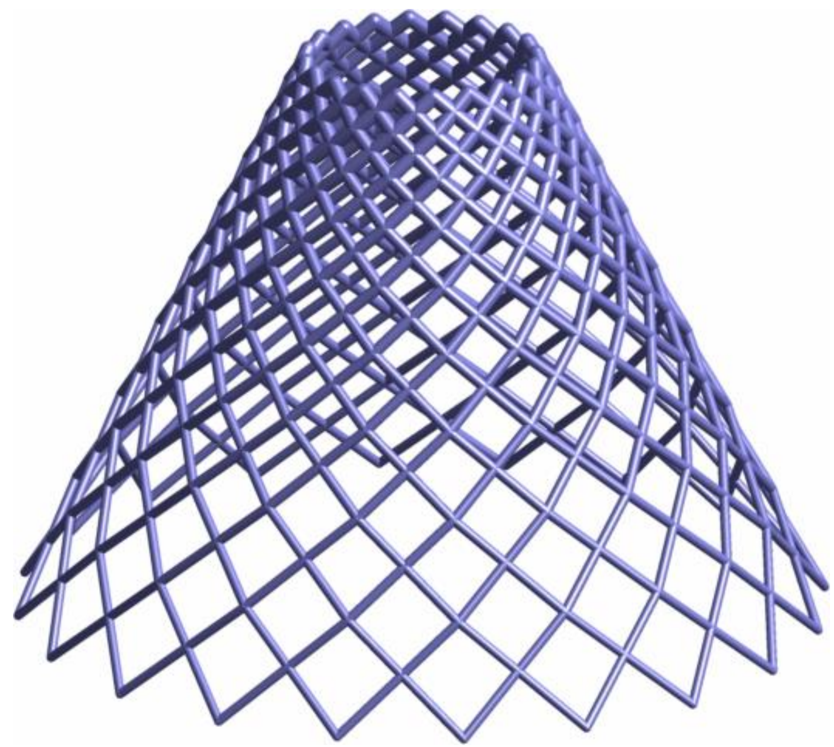
- Show and tell: Shear box



6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: TORSION CONE





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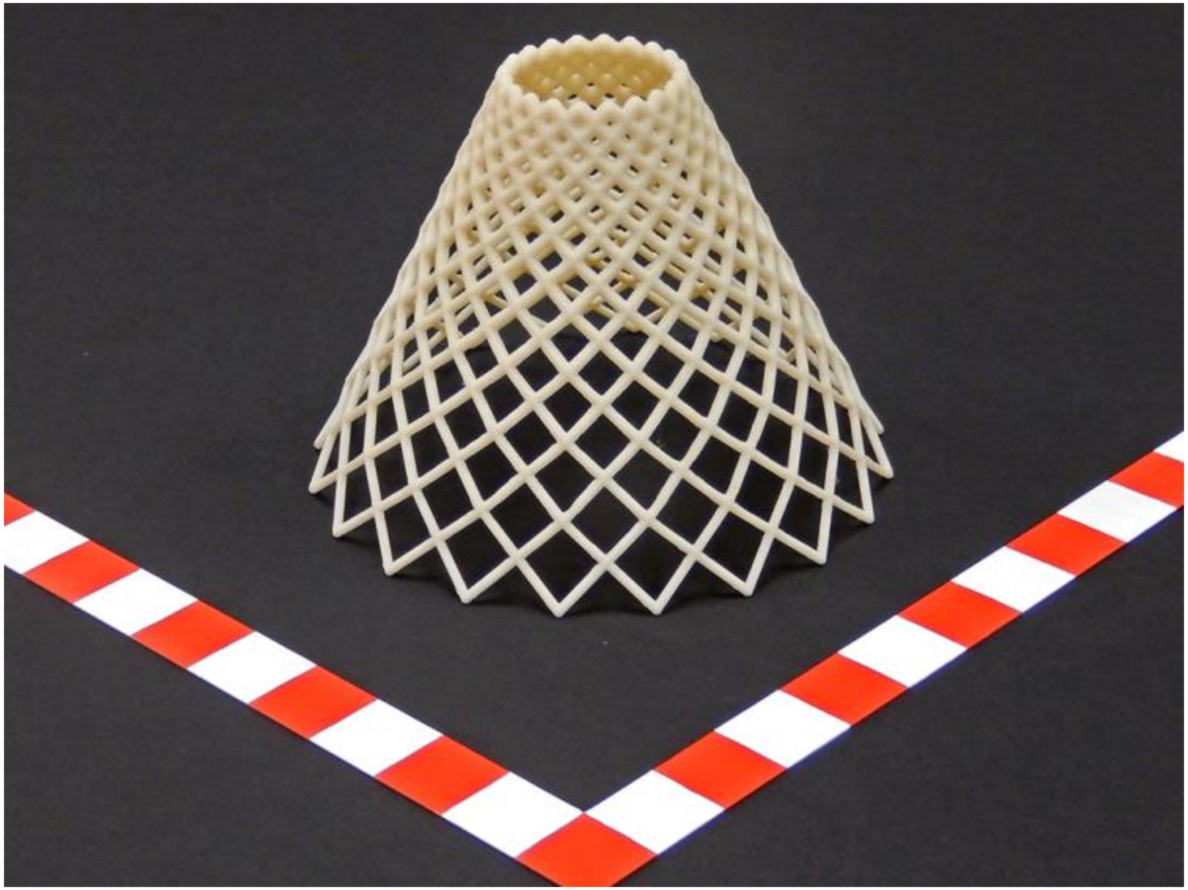
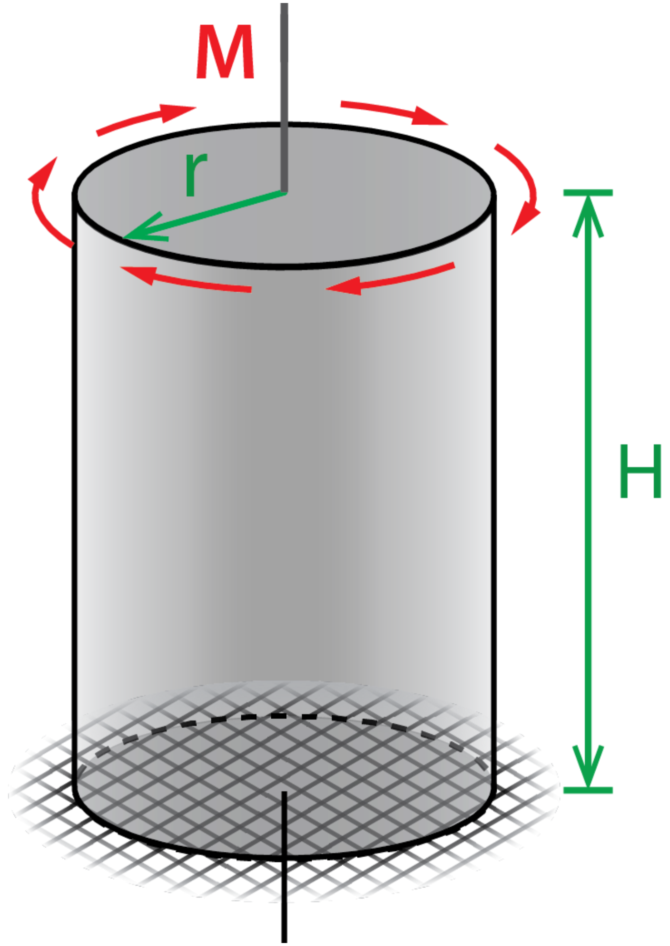
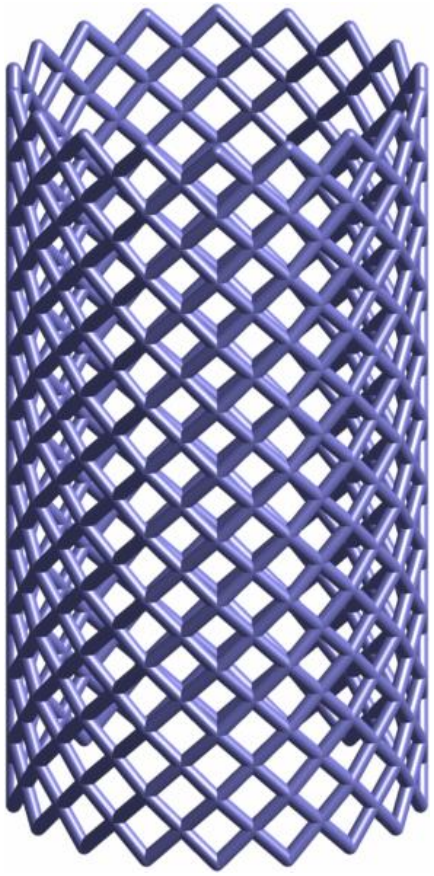


6) Additive Manuf. of Opt. Structs.

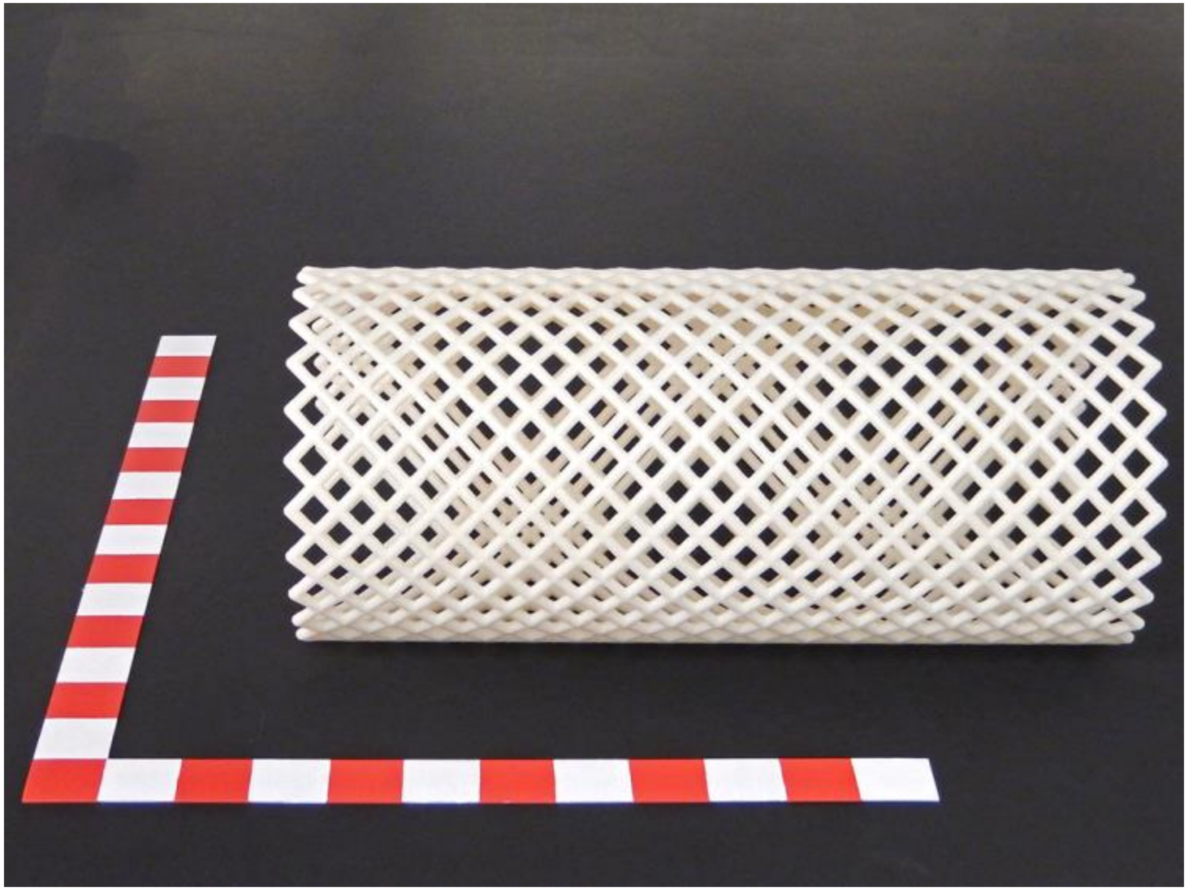
- Show and tell: Torsion cylinder





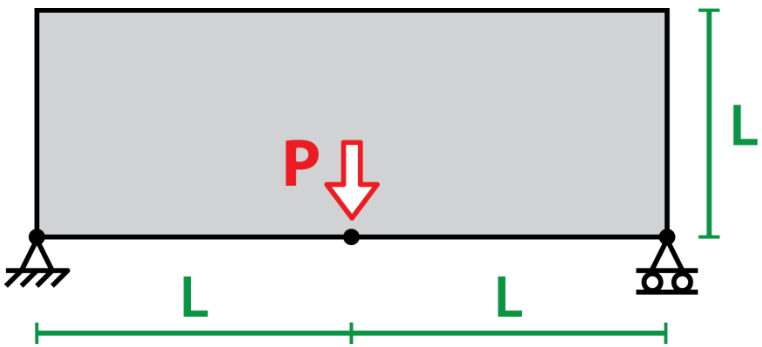
6) ADDITIVE MANUF. OF OPT. STRUCTS.

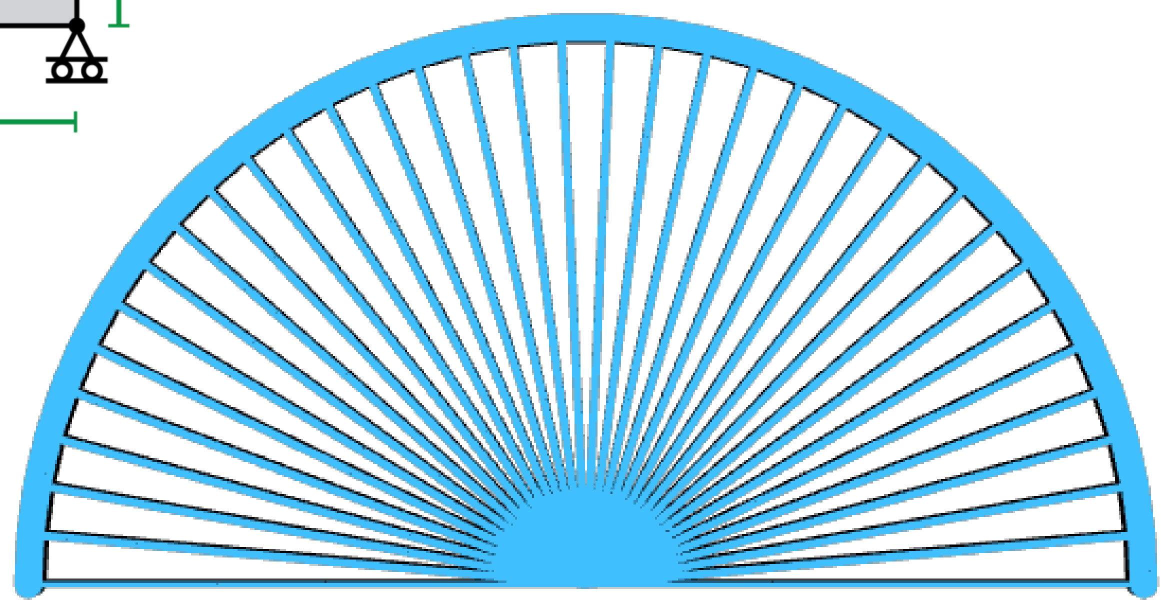
- SHOW AND TELL: TORSION CYLINDER



6) Additive Manuf. of Opt. Structs.

- Show and tell: PINWHEEL





6) Additive Manuf. of Opt. Structs.

- Show and tell: Pinwheel