PhD Defense

Reliability-based topology optimization frameworks for the design of structures subjected to random excitations

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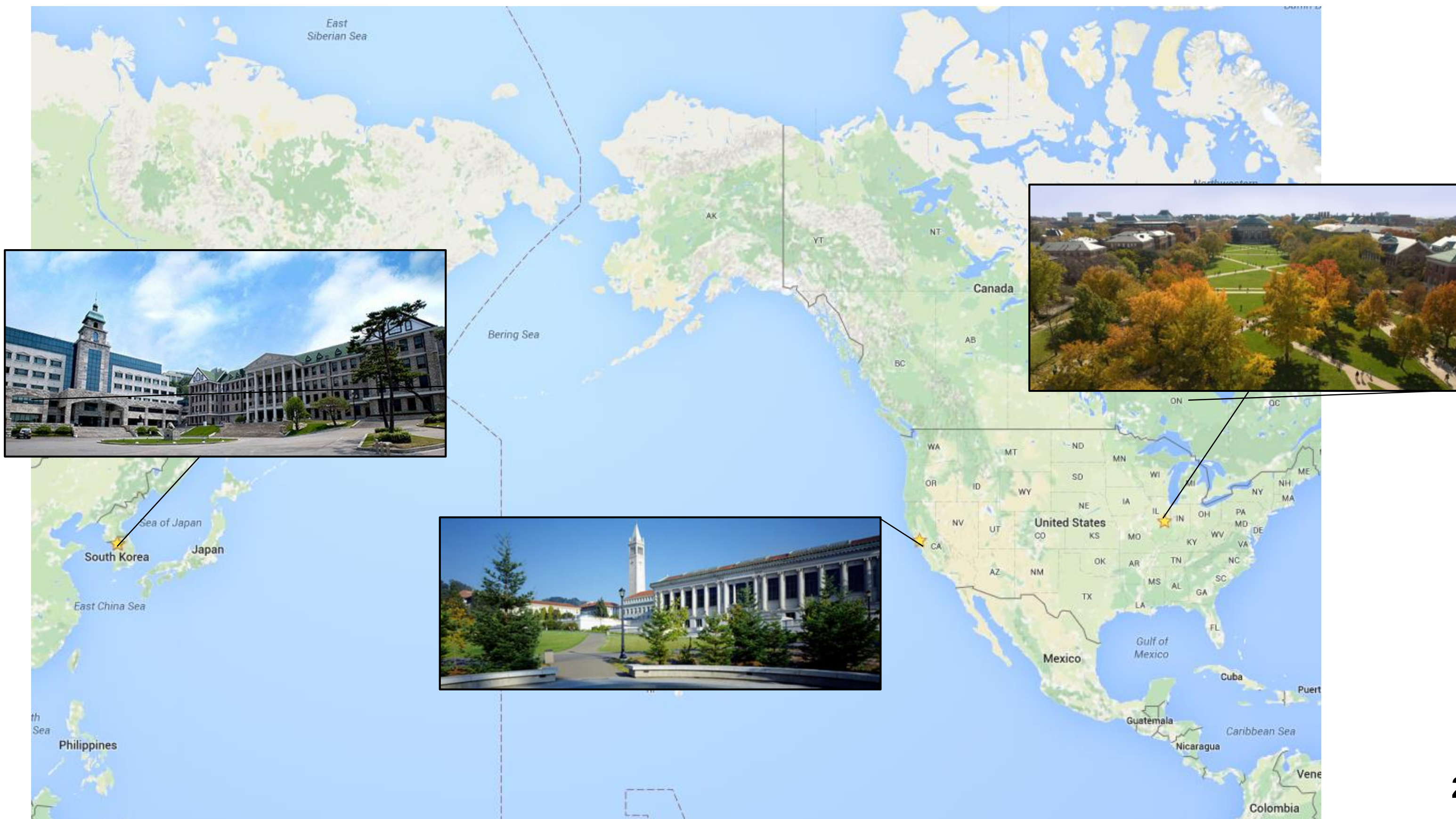
Educational background

Education

B.S (2006, Architectural Engineering), *Hanyang University*, Seoul, Korea

M.S (2007, Structural Engineering), *University of California, Berkeley*, CA

Ph.D (2010-present, Structural Engineering), *University of Illinois, Urbana-Champaign*, IL. **Advisors**: Glaucio H. Paulino, Junho Song

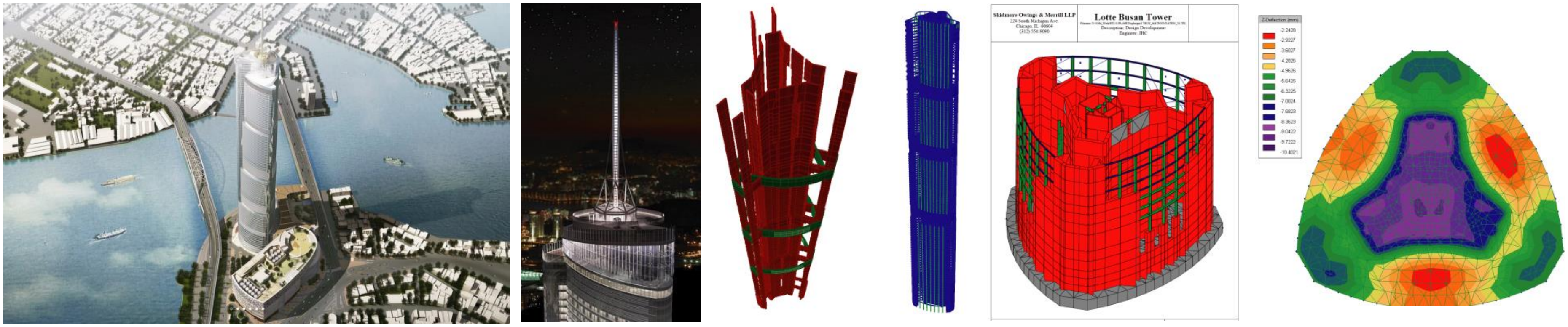


Professional background

Professional experience

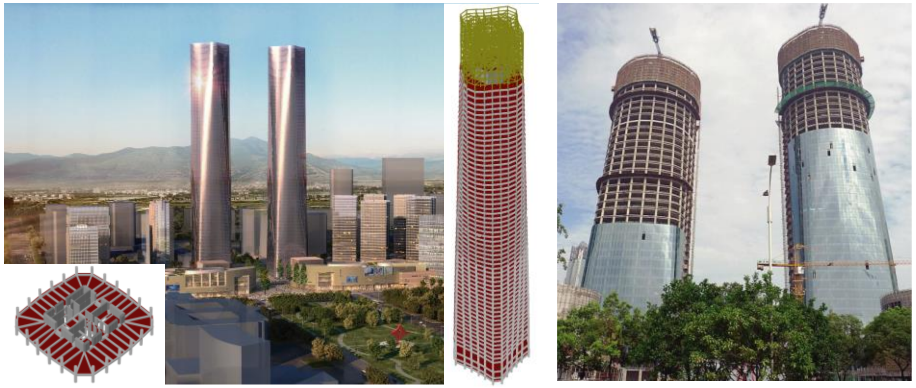
Skidmore, Owings, & Merrill LLP, Chicago, IL, USA

Structural Engineer (September 2007 - June 2010)



Busan lotter tower (Courtesy of Skidmore, Owings & Merrill, LLP)

China merchant (Courtesy of Skidmore, Owings & Merrill, LLP)



Nanchang Greenland (Courtesy of Skidmore, Owings & Merrill, LLP)

Contributions

Discussed in this presentation:

Chun, J., Song, J., Paulino, G.H. (2015). Parameter sensitivity of system reliability using sequential compounding method. *Structural safety*, 55: 26−36. (**Chapter 4**)

Chun, J., Song, J., Paulino, G.H. System reliability-based design/topology optimization of structures constrained by first passage probability. To be submitted for journal publication. (**Chapter 5**)

Chun, J., Paulino, G.H, Song, J. Reliability-based topology optimization of truss structures using a discrete filtering technique. To be submitted for journal publication. (**Chapter 6**)

Not covered in this presentation:

Filipov*, E.T., **Chun*, J.**, Paulino, G.H., Song, J. (2016). Polygonal multiresolution topology optimization (PolyMTOP) for structural dynamics. *Structural and Multidisciplinary Optimization*, 53(4): 673-694. (**Chapter 2**)

Chun, J., Song, J., Paulino, G.H. (2016). Structural topology optimization under constraints on instantaneous failure probability. Structural and Multidisciplinary Optimization. 53(4): 773-799. (**Chapter 3**)

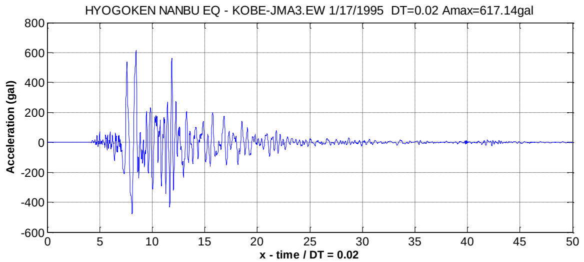
Structural engineering under natural hazards and risks

One of the most **fundamental requirements** on building structures is to **withstand** various **uncertain loads.**

San Francisco Earthquake, 1906

Kobe Earthquake, 1995

The structural design needs to ensure **safe** and **reliable operations** over a prolonged period of time despite random excitations caused by hazardous events.



Random Excitations

Random processes
Non-deterministic excitations
Many possibilities of the process

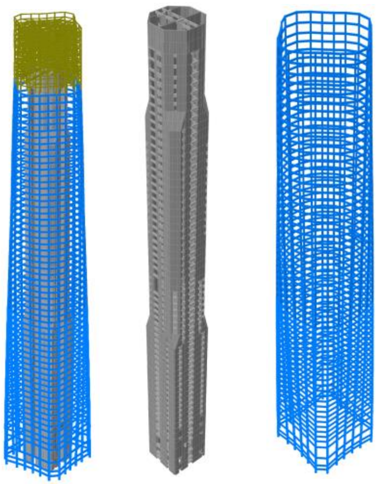
1. http://www.documentingreality.com, 2. Photograph: Kimimasa Mayama/Reuters

Research aim: identification of the optimal structure and system under dynamic and stochastic excitations

Structural Design

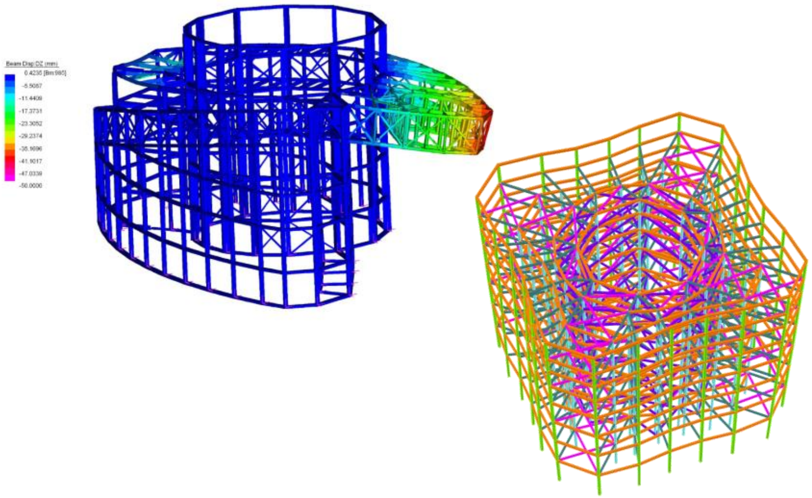


Courtesy of Skidmore, Owing and Merrill, LLP

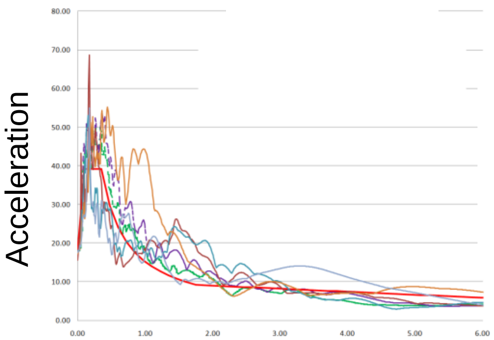


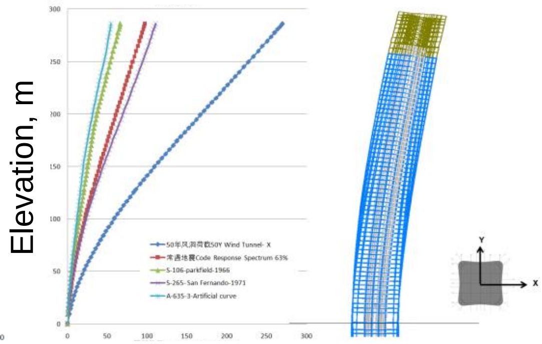
Structural system

Structural elements optimization



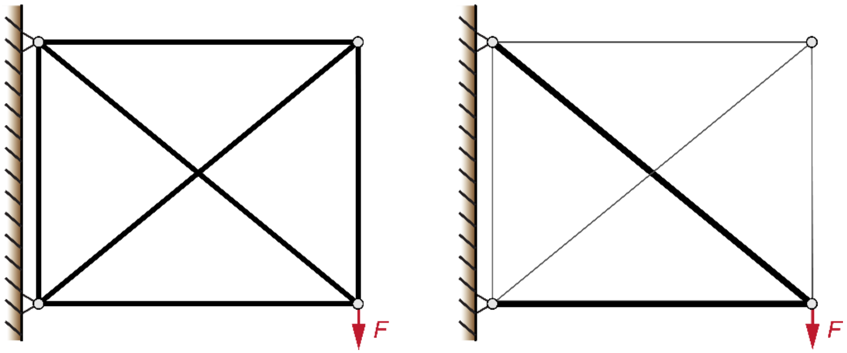
Structural performance optimization



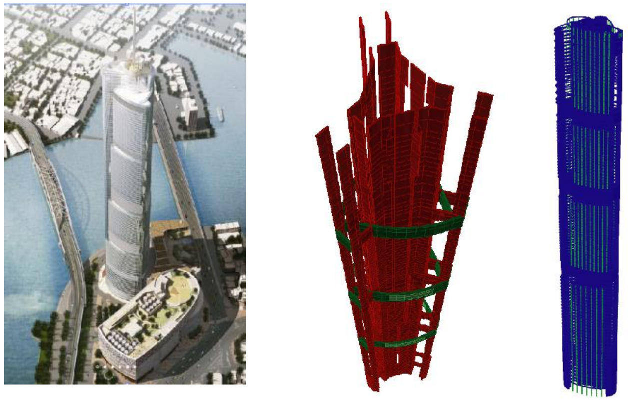


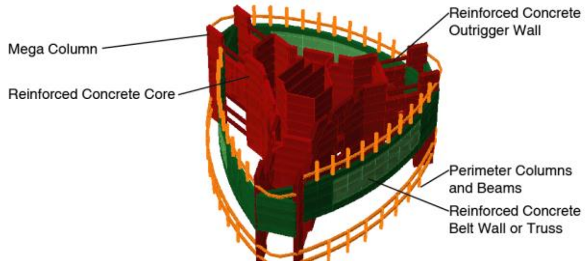
Categorization of structural optimization

Size Optimization



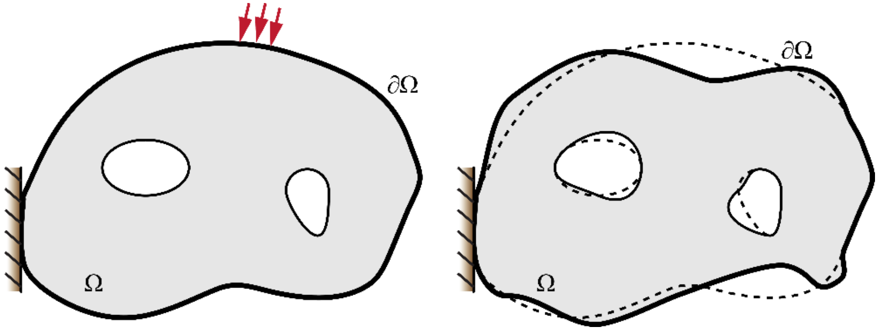
Design variable, *d*: **area**



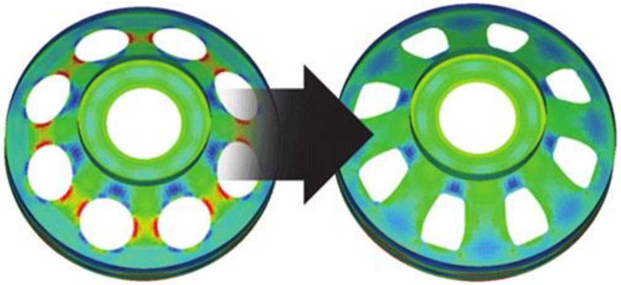


Courtesy of Skidmore, Owing and Merrill, LLP

Shape Optimization



Design variable, *d*: **boundary** of shape



http://www.altairhyperworks.com/

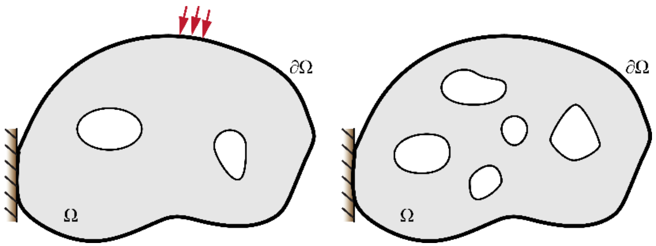
Initial design Optimized design

Stress contour

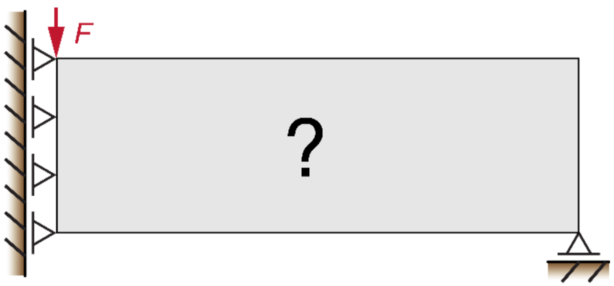
Categorization of structural optimization

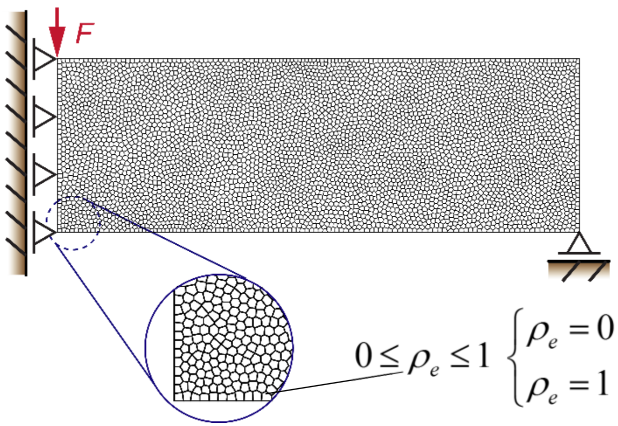
Topology Optimization: Continuum

Design variable, d: **density**



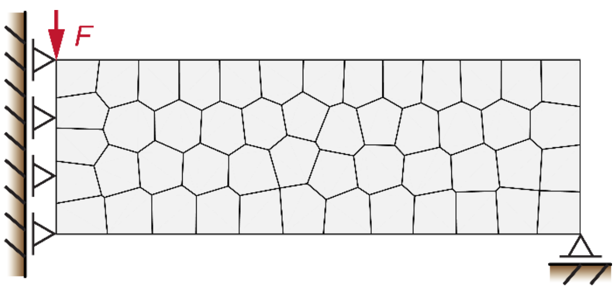
$$\begin{array}{ll} \max\limits_{\mathbf{d}} & \text{stiffness} \\ s.t & \text{volume fraction} \end{array}$$

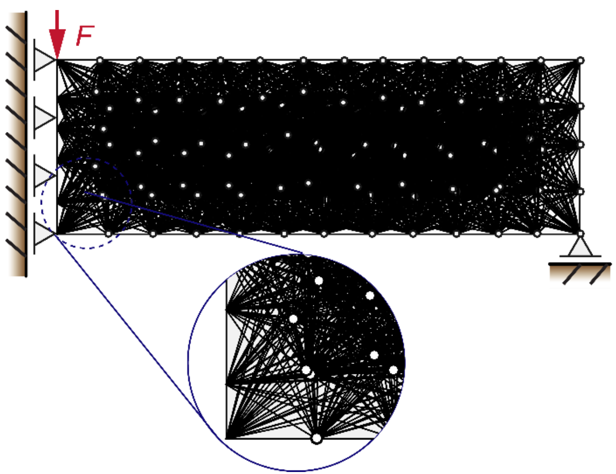


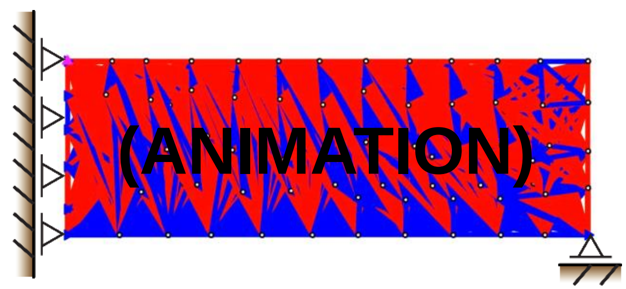


$$0 \le \rho_e \le 1 \quad \begin{cases} \rho_e = 0 & \text{void} \\ \rho_e = 1 & \text{solid} \end{cases} \qquad \mathbf{K}^e(\rho_e) = \rho_e^{\,p} \int_{\Omega_e} \mathbf{B}^{\mathrm{T}} \mathbf{D}_0 \mathbf{B} \, \Omega_e$$

Topology Optimization: Ground structure



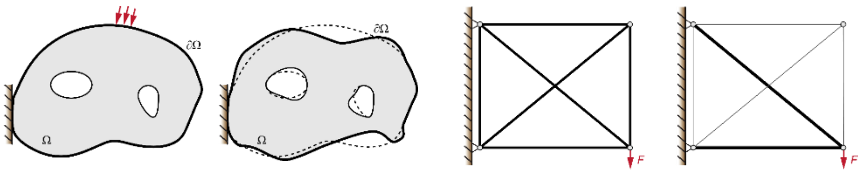




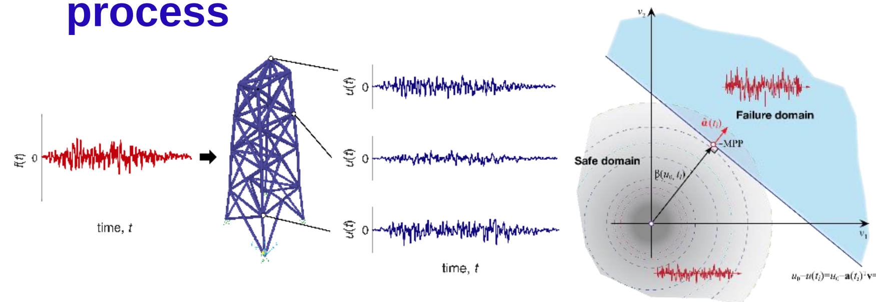
Design variable, d: **area**

Presentation outline

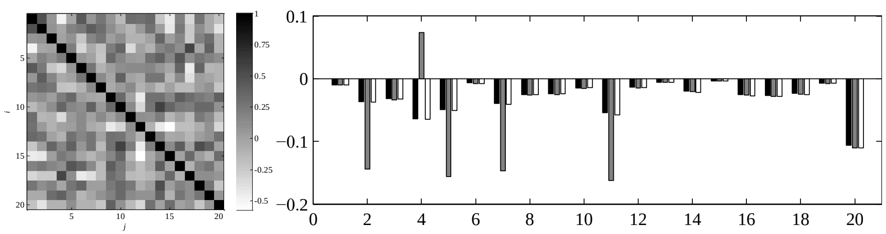
1. Introduction



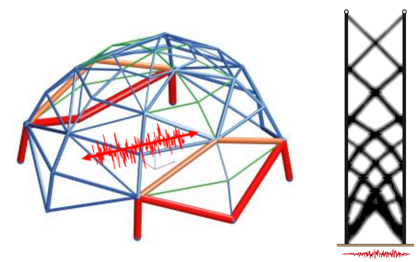
2. Discrete representation of stochastic process



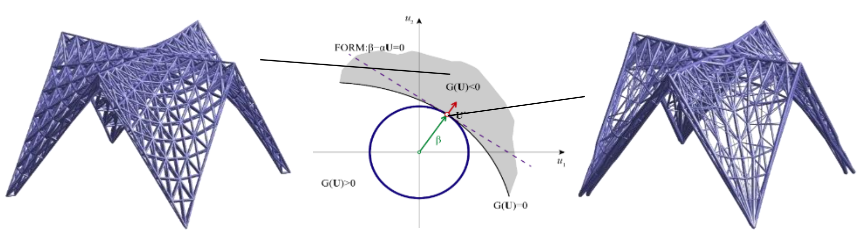
3. Parameter sensitivity of system reliability



4. Structural design and topology optimization under stochastic excitations



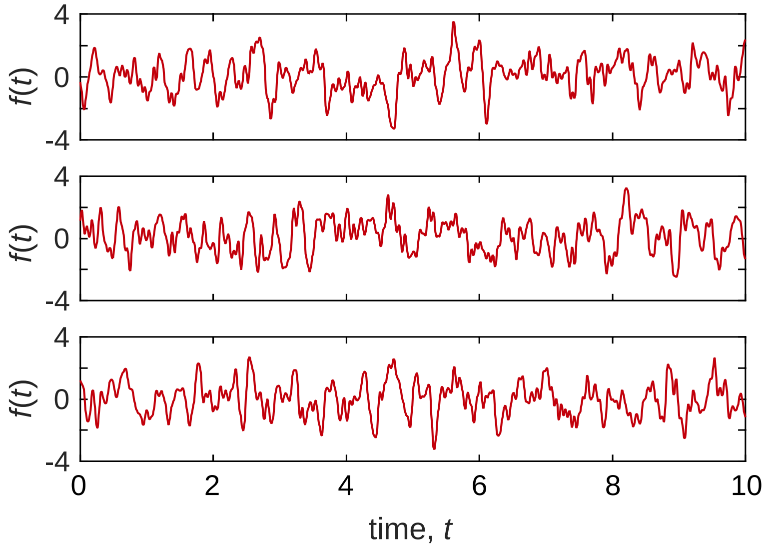
5. Reliability-based topology optimization by ground structure method



Reasonable representation of the random process is needed to obtain a meaningful solution

Stochastic excitation is often described by a random process

Random process can be understood as a collection of random variables defined along the time axis



☐ Discrete Representation of Stochastic Excitation

The **stochastic excitation** is represented by a linear combination of **basis functions,** s(t), with standard normal **independent random variables, v:**

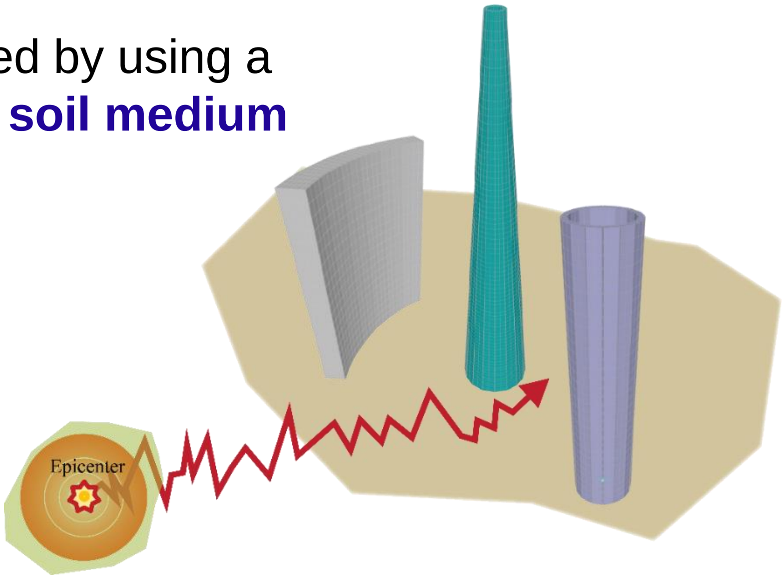
$$f(t) = \mu(t) + \sum_{i=1}^{n} v_i s_i(t) = \mu(t) + \mathbf{s}(t)^{\mathrm{T}} \mathbf{v}$$

Der Kiureghian, A. (2000). The geometry of random vibrations and solutions by FORM and SORM. *Probabilistic Engineering Mechanics*, 15(1),: 81-90.

Stochastic ground motion is modeled as the response of a linear filter to a random pulse train

Modeling Ground Excitations - Filtered Gaussian Process

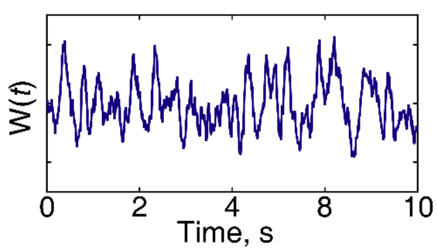
Stochastic ground excitations can be modeled by using a **filter** representing the **characteristic of the soil medium** and Gaussian process.



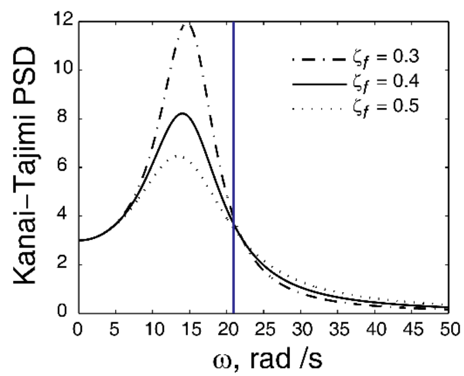
$$f(t) = \int_0^t W(\tau) h_f(t-\tau) d\tau$$

$$\cong \sum_{i=1}^{n} W_i \cdot h_f(t-t_i)\Delta t$$

$$= \sum_{i=1}^{n} \sqrt{2\pi\Phi_0 / \Delta t} \cdot v_i \cdot h_f(t-t_i)\Delta t \;=\; \mathbf{s}(t)^{\mathrm{T}} \mathbf{v}$$

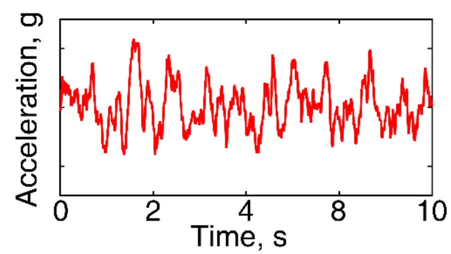


Gaussian process



Soil Medium (Filter)

Filter parameters



Ground acceleration

(Filtered Gaussian Process)

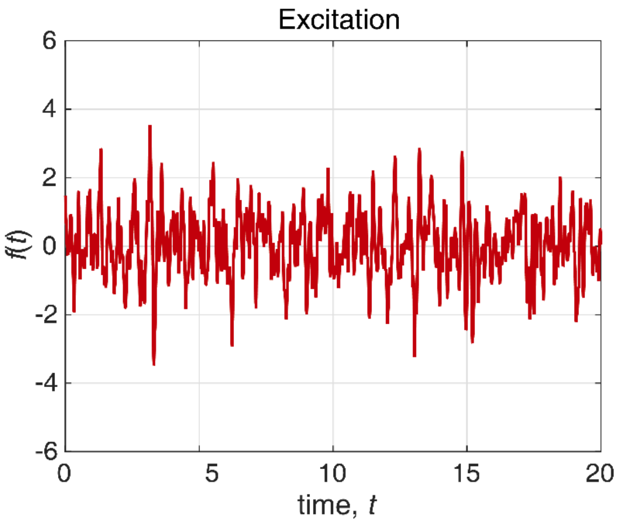
Der Kiureghian, A. (2000). The geometry of random vibrations and solutions by FORM and SORM. *Probabilistic Engineering Mechanics*, 15(1): 81-90.

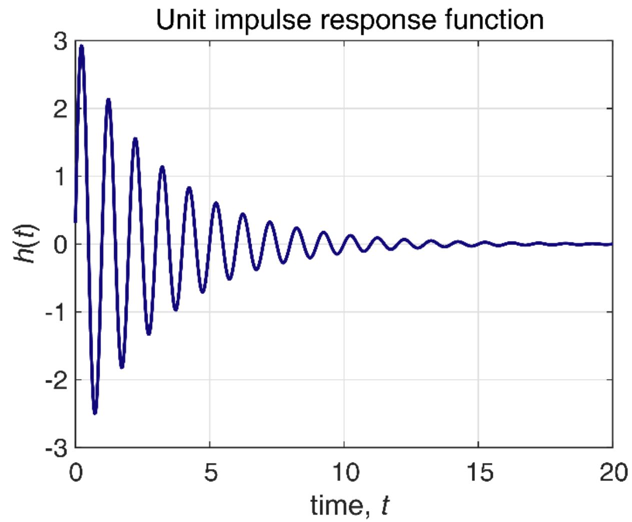
Stochastic response of a structure is described by a finite number of random variables

Dynamic Response

Convolution integral for responses of linear systems subjected to the stationary process can be developed with the impulse response function.

$$u(t) = \int_0^t f(\tau) h_s(t-\tau) d\tau$$





$$u(t) = \int_0^t \sum_{i=1}^n v_i s_i(\tau) h_s(t-\tau) d\tau = \sum_{i=1}^n v_i a_i(t) = \mathbf{a}(t)^{\mathrm{T}} \mathbf{v} \qquad a_i(t) = \int_0^t s_i(\tau) h_s(t-\tau) d\tau, \quad i = 1, \ldots, n$$

Deterministic, time-dependent
- filter + structure

Random, time-independent

Failure event is defined in terms of dynamic response in discretization representation form

Instantaneous Failure Probability

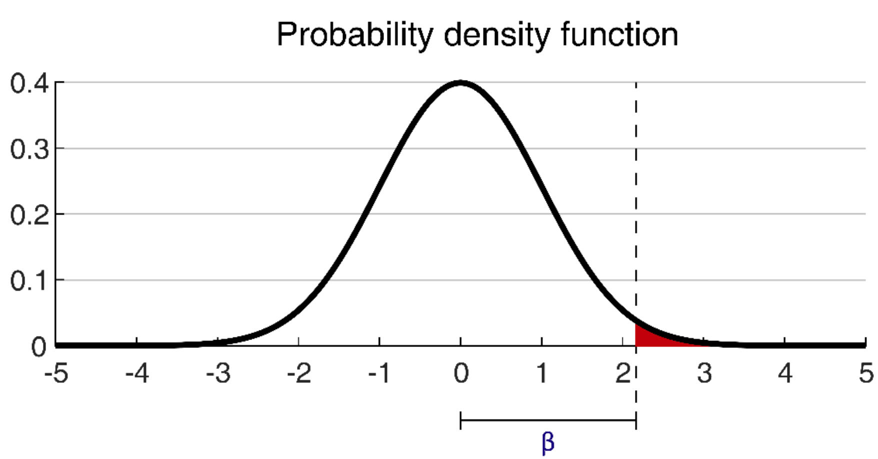
Failure event of a linear system at a certain time t_i is defined as

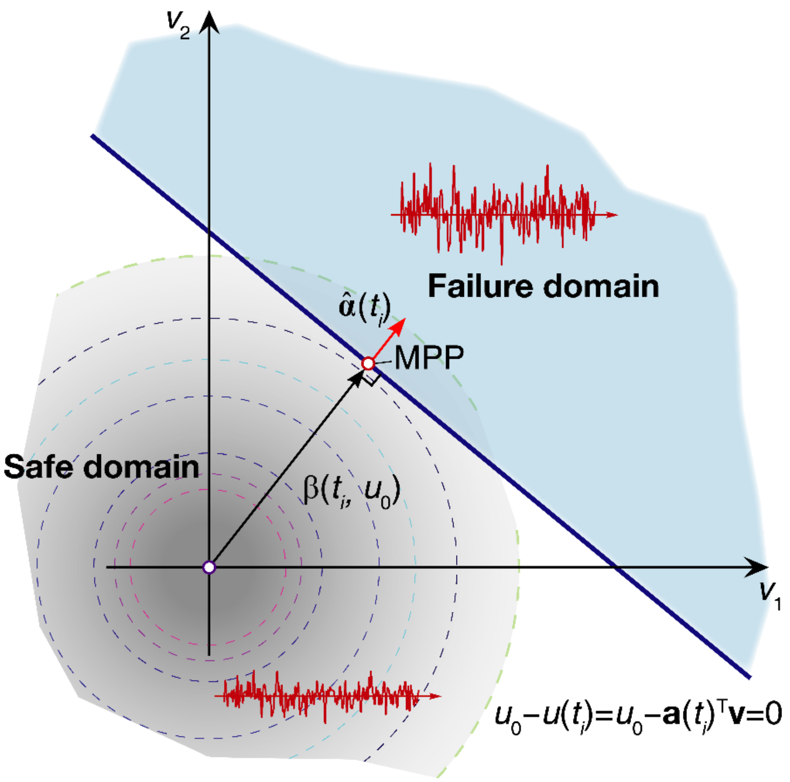
$$E_f : g(t_i, u_0) \le 0 \equiv E_f : u(t_i) \ge u_0 \equiv E_f : \mathbf{a}(t_i)^{\mathrm{T}} \mathbf{v} \ge u_0$$

Failure Probability P_f is computed as

$$\beta(t_i, u_0) = \frac{u_0}{\left\| \mathbf{a}(t_i) \right\|}$$

$$\begin{aligned} P_f\left(E_f : g(t_i, u_0) \le 0\right) &= \int_{-\infty}^{-\beta} \frac{1}{\sqrt{2\pi}} \exp(-0.5\, x^2)\, dx \\ &= \Phi\left[-\beta(t_i, u_0)\right] \end{aligned}$$



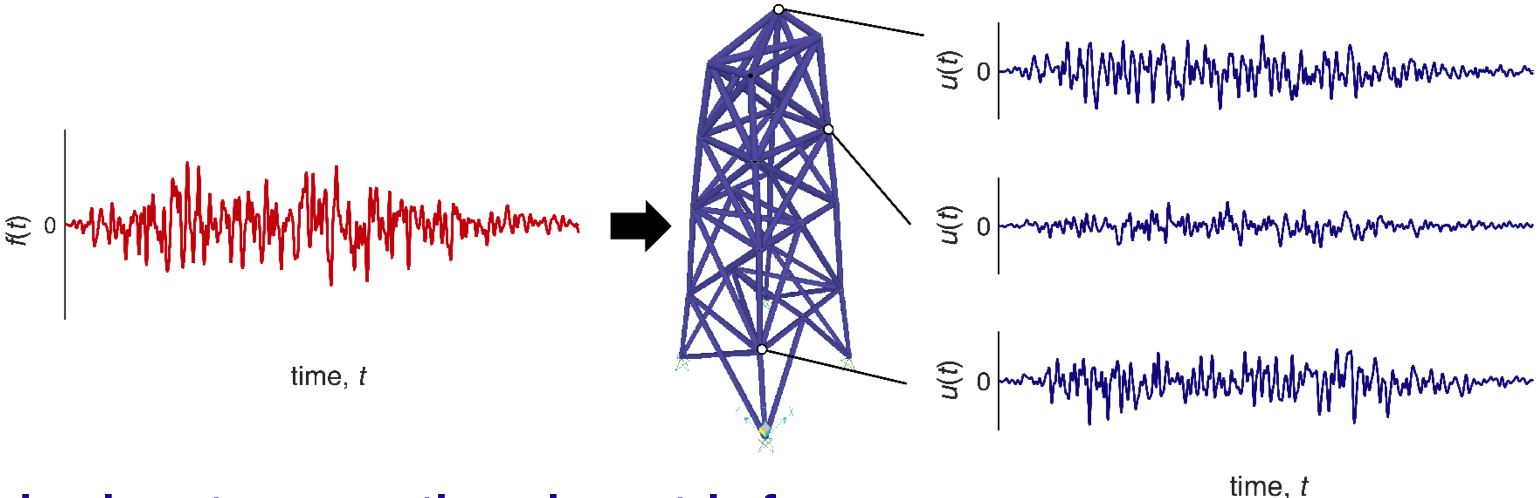


A new procedure to facilitate identifying a(*t*) without derivation of the impulse response function

Numerical Integration (i.e. Newmark method)

$$\dot{\mathbf{u}}(t_{j+1}) = \dot{\mathbf{u}}(t_j) + [(1-\gamma)\Delta t]\ddot{\mathbf{u}}(t_j) + (\gamma\Delta t)\ddot{\mathbf{u}}(t_{j+1})$$

$$\mathbf{u}(t_{j+1}) = \mathbf{u}(t_j) + \Delta t\dot{\mathbf{u}}(t_j) + [(0.5-\eta)\Delta t^2]\ddot{\mathbf{u}}(t_j) + [\eta\Delta t^2]\ddot{\mathbf{u}}(t_{j+1})$$



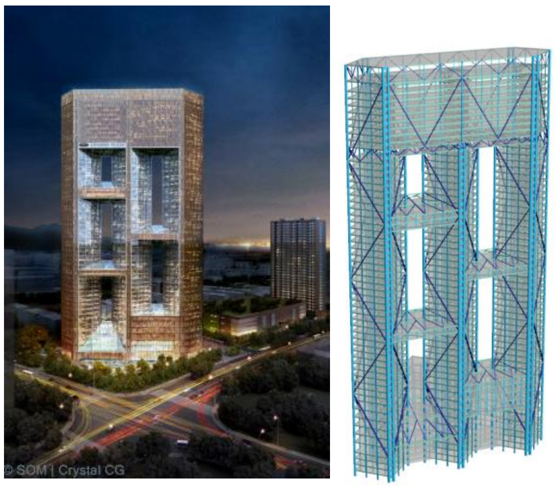
Derived system equations in matrix form

$$\begin{pmatrix} u(t_1) \\ u(t_2) \\ \vdots \\ u(t_{n-1}) \\ u(t_n) \end{pmatrix} = \begin{pmatrix} u(\Delta t) \\ u(2\Delta t) \\ \vdots \\ u(t_0 - \Delta t) \\ u(t_0) \end{pmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & v_1 \\ 0 & 0 & \cdots & v_1 & v_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & v_1 & \cdots & v_{n-2} & v_{n-1} \\ v_1 & v_2 & \cdots & v_{n-1} & v_n \end{bmatrix} \begin{pmatrix} a_1(t_0) \\ a_2(t_0) \\ \vdots \\ a_{n-1}(t_0) \\ a_n(t_0) \end{pmatrix} \Rightarrow \mathbf{a}(t_0)$$

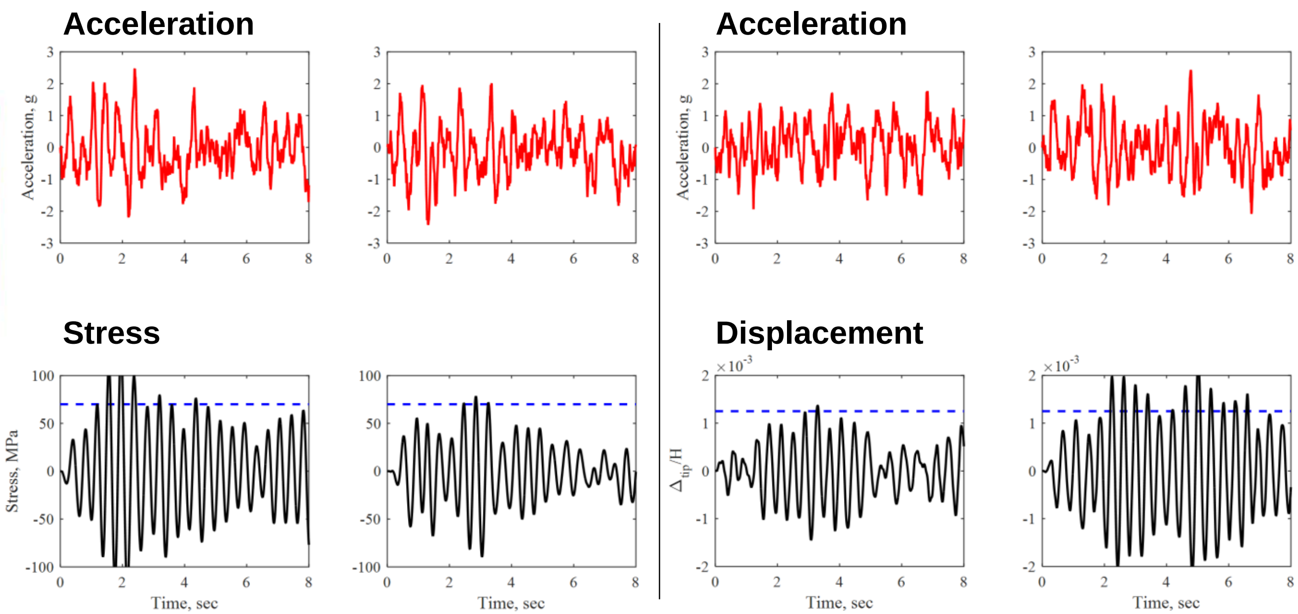
Chun, J., Song, J., Paulino, G.H. (2016). Structural topology optimization under constraints on instantaneous failure probability. Structural and Multidisciplinary Optimization. 53(4): 773-799.

Probability of the occurrence of at least one failure event over a time interval needs to be evaluated

Randomness in Dynamic Responses



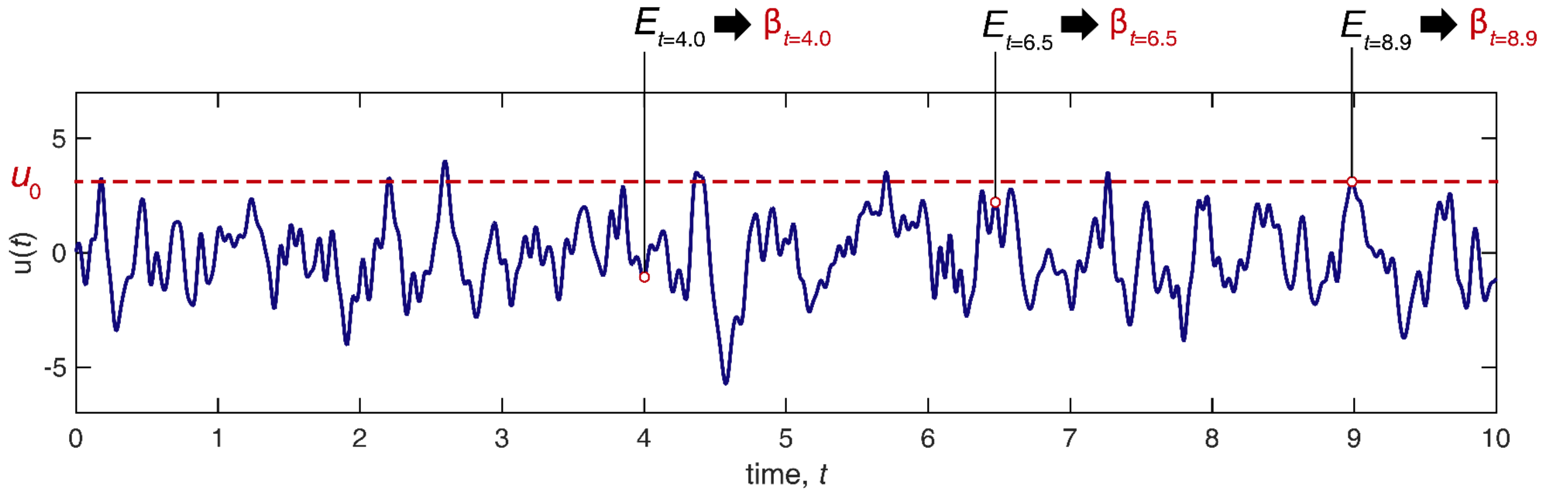
Ssiger International Plaza
Courtesy of Skidmore, Owing and Merrill, LLP



In the reliability analysis of **dynamic system** subjected to **stochastic excitations**, a significant problem is determining the **first passage probability** that **any one of output states** of interest **exceeds** a certain threshold value within a given time duration *T*.

First passage probability is formulated as a series system problem

☐ First Passage Probability P_{fp}



$$P_{fp}(E_{sys}) = P(u_0 < \max_{0<t<t_n} |u(t)|) = P\left(\bigcup_{i=1}^{n} |u(t_i)| > u_0\right) = P\left(\bigcup_{i=1}^{n} E_i\right)$$

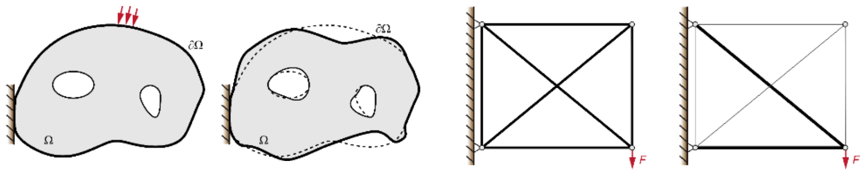
$$P_{fp} = \int_{\Omega} \frac{1}{(2\pi)^{n/2}\sqrt{\det \mathbf{R}}} \exp\left(-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\mathbf{R}\mathbf{z}\right) d\mathbf{z} \qquad \Omega = \left\{(z_1, z_2, \ldots, z_n) \,|\, \left[(z_1 \le -\beta_1) \cup \cdots (z_n \le -\beta_n)\right]\right\}$$

Main issue: 1) **Evaluation** of first passage probability in an efficient way
2) **Sensitivity analysis** for the gradient-based optimization algorithms

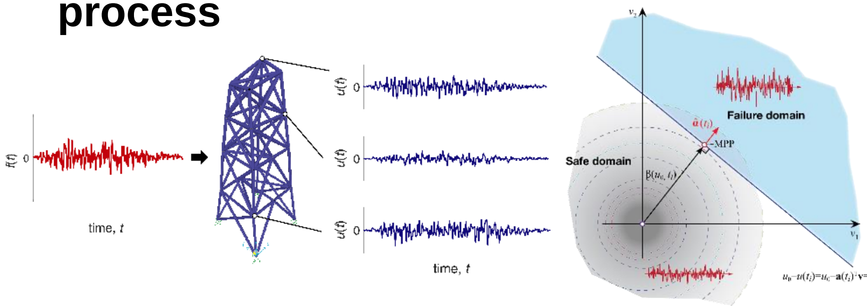
Song, J., and A. Der Kiureghian (2006). Joint first-passage probability and reliability of systems under stochastic excitation. *J. Engineering Mechanics*, ASCE, 132(1):65-77.
Fujimura, K. and A. Der Kiureghian (2007). Tail-Equivalent Linearization Method for Nonlinear Random Vibration. *Probabilistic Engineering Mechanics*, 22: 63-76

Presentation outline

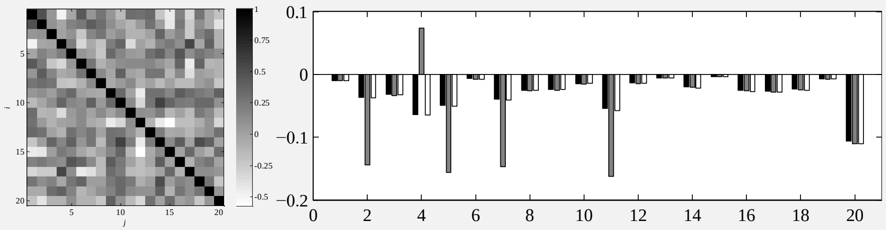
1. Introduction



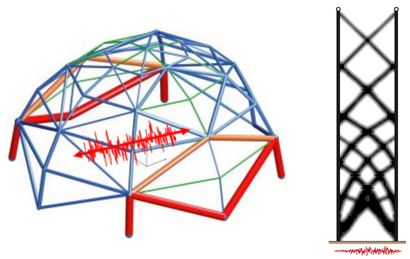
2. Discrete representation of stochastic process



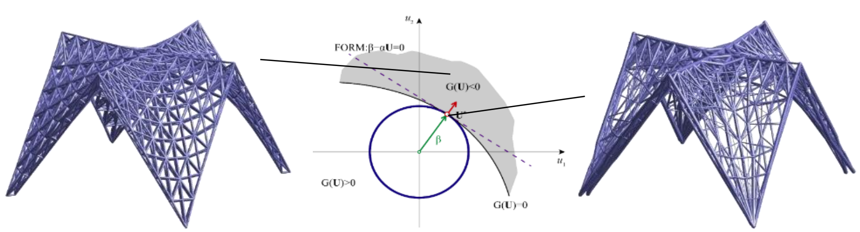
3. Parameter sensitivity of system reliability



4. Structural design and topology optimization under stochastic excitations

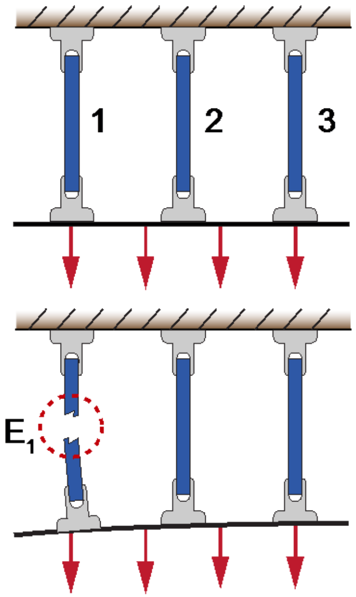


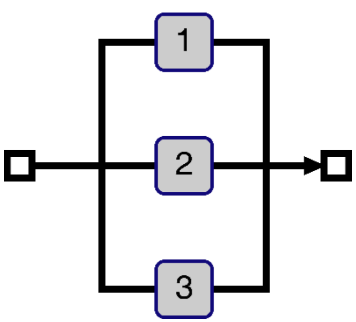
5. Reliability-based topology optimization by ground structure method



Reliability assessment needs to work with systems having components in parallel, series, and general

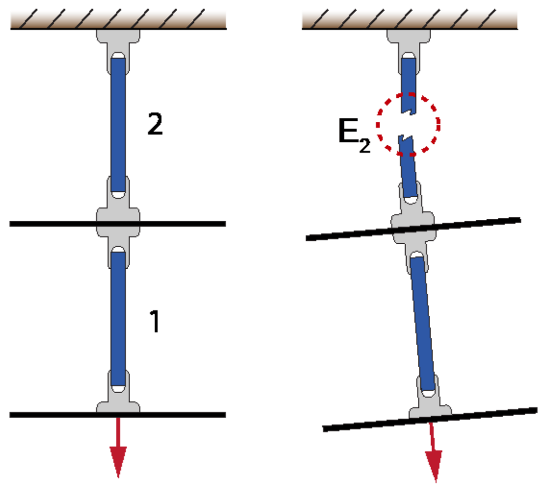
☐ Parallel System





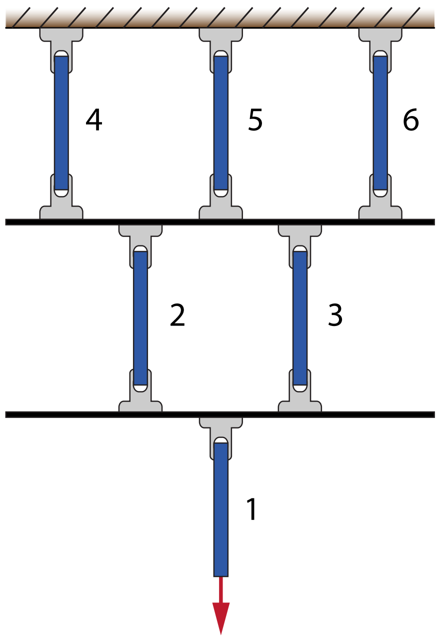
$$E_{parallel} = E_1 \cap E_2 \cap E_3$$

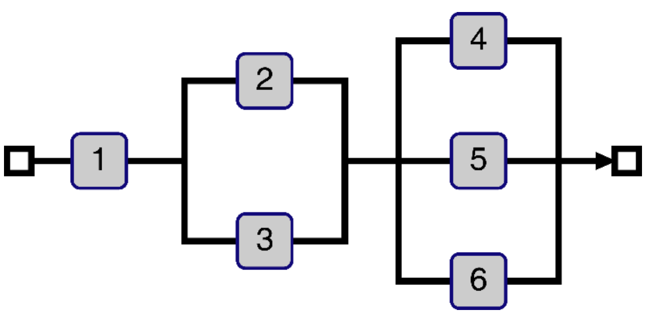
☐ Series System



$$E_{series} = E_1 \cup E_2$$

☐ General System



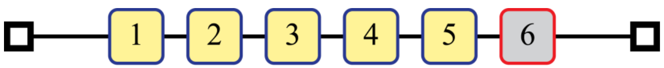


$$E_{general} = E_1 \cup E_2E_3 \cup E_4E_5E_6$$

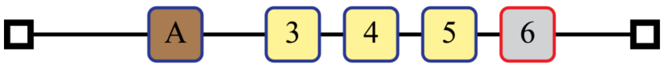
Sequential compounding method (SCM) is an efficient system reliability method

SCM compounds component events coupled by union or intersection sequentially until **a single** compound event **represents** all of the system events

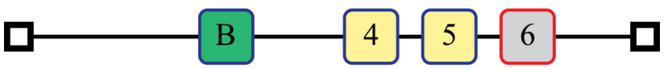
$$P(E_{sys}) = P(E_1 \cup E_2 \cup \cdots \cup E_6) = \int\int\int\int\int\int \varphi(\cdot) d\mathbf{z}$$



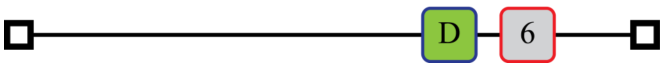
$$P(E_{sys}) = P(E_A \cup E_3 \cup \cdots \cup E_6)$$



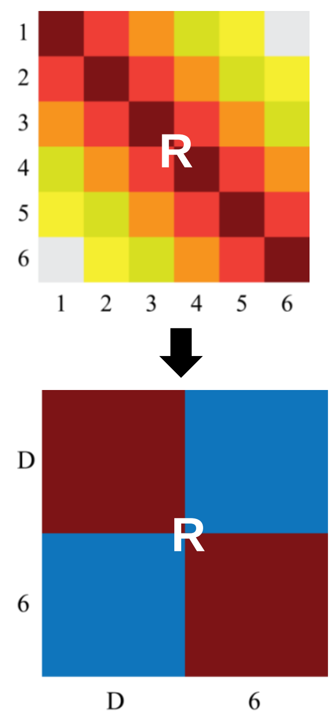
$$P(E_{sys}) = P(E_B \cup E_4 \cup E_5 \cup E_6)$$



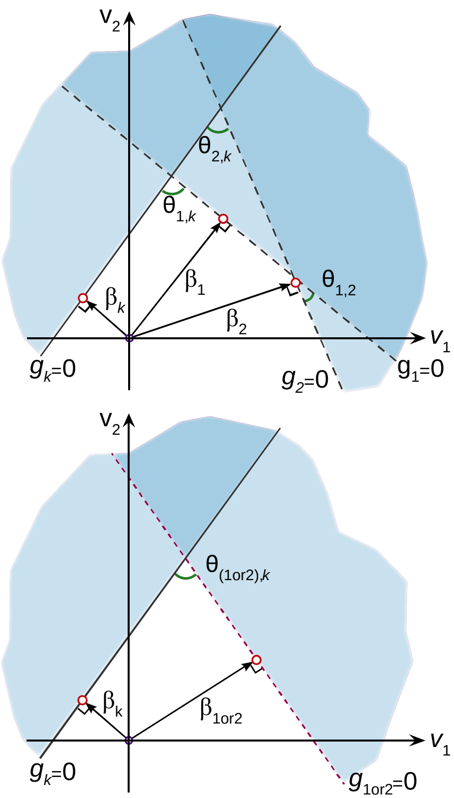
$$P(E_{sys}) = P(E_D \cup E_6)$$



SCM procedure



Correlation matrix



Kang, WH,, Song, J. (2010). Evaluation of multivariate normal integrals for general systems by sequential compounding. *Structural Safety*. 32(1): 35–41.

CSP is an efficient sensitivity method for various system problems using SCM

Sensitivity in Parallel System

$$P(E_{parallel}) = P\left(E_k \cap E_{P_k}\right) = \Phi_2(-\beta_k, -\beta_{P_k}; \rho_{k,P_k})$$

$$\frac{\partial P(E_{parallel})}{\partial \beta_k} = -\varphi(-\beta_k) \cdot \Phi\left[\frac{-\beta_{P_k} + \beta_k \rho_{k,P_k}}{\sqrt{1-\rho_{k,P_k}{}^2}}\right]$$

Sensitivity in Series System

$$P(E_{series}) = P(E_k \cup E_{S_k})$$

$$\frac{\partial P(E_{series})}{\partial \beta_k} = -\varphi(-\beta_k)\left\{1 - \Phi\left[\frac{-\beta_{S_k} + \beta_k \rho_{k,S_k}}{\sqrt{1-\rho_{k,S_k}{}^2}}\right]\right\}$$

Sensitivity in General System

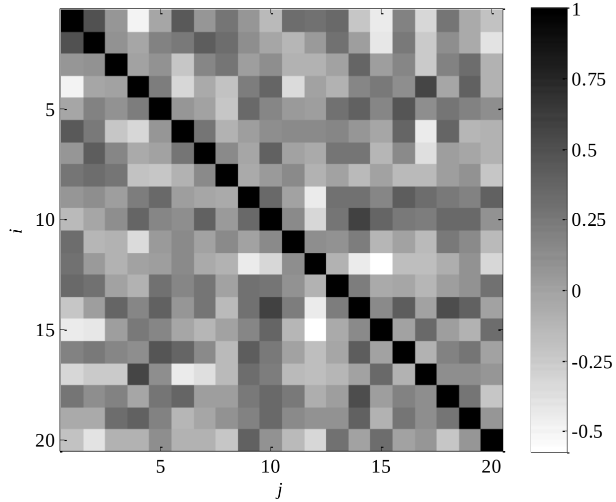
$$P(E_{cut-set}) = P\left(\bigcup_{m=1}^{n} E_{C_m}\right) = P\left[\bigcup_{m=1}^{n}\left(\bigcap_{j \in C_m} E_j\right)\right]$$

$$\frac{\partial P(E_{cut-set})}{\partial \beta_k} = \frac{\partial P(E_{cut-set})}{\partial P(E_{C_l})} \cdot \frac{\partial P(E_{C_l})}{\partial \beta_k} = -\frac{1}{\varphi(-\beta_{C_l})} \cdot \frac{\partial P(E_{cut-set})}{\partial \beta_{C_l}} \cdot \frac{\partial P(E_{C_l})}{\partial \beta_k}$$

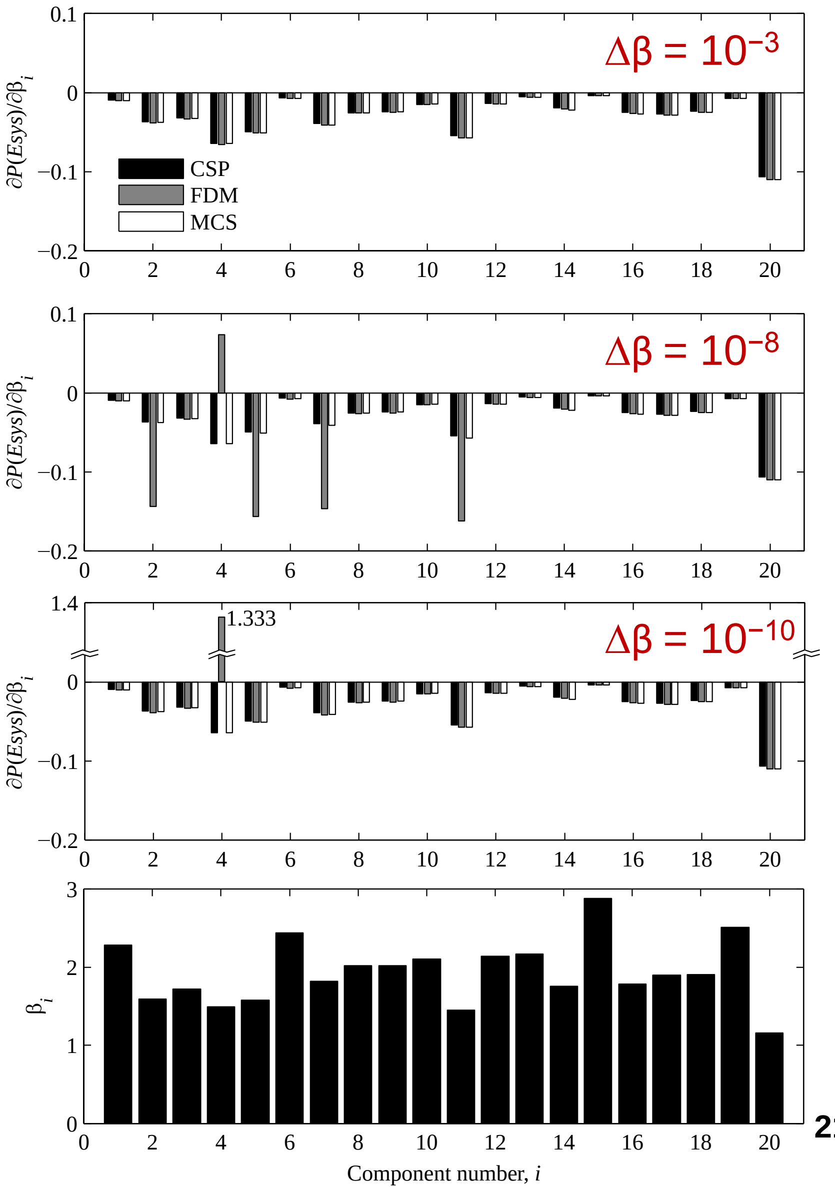
Chun, J., Song, J., Paulino, G.H. (2015) Parameter sensitivity of system reliability using sequential compounding method. *Structural Safety*. 55: 26–36

CSP method is tested for a series system consisting of 20 components

Sensitivity calculations by **FDM** may result in **large error** or even different signs depending on perturbation values

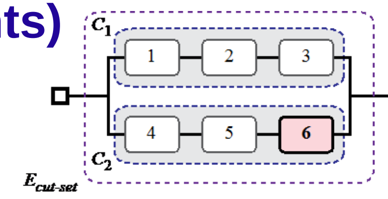


Correlation matrix



Illustrative example of the CSP method considers the cut-set system problem

General System (two cut-sets and six components)

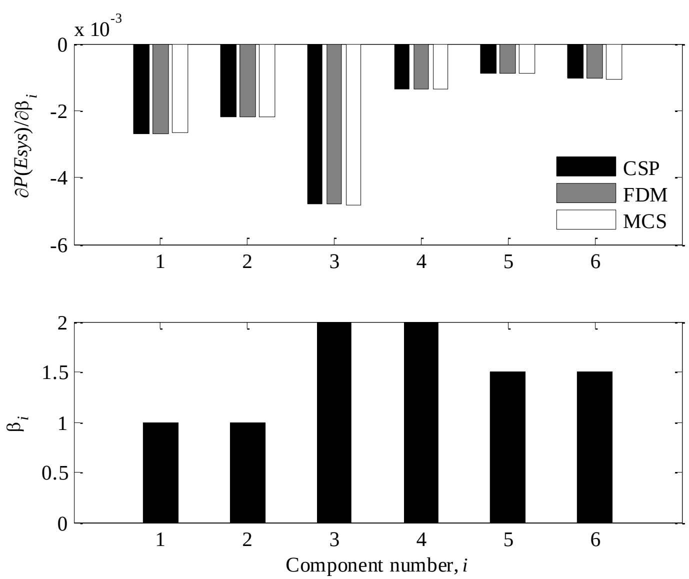


$$P(E_{sys}) = P(E_{C_1} \cup E_{C_2}) = P\left(E_1E_2E_3 \cup E_4E_5E_6\right)$$

$$\frac{\partial P(E_{sys})}{\partial \beta_6} = -\frac{1}{\varphi(-\beta_{C_2})}\cdot\frac{\partial P(E_{sys})}{\partial \beta_{C_2}}\cdot\frac{\partial P(E_{C_2})}{\partial \beta_6}$$

$$= -\frac{1}{0.002423}(0.002398 \times 0.001037)$$

$$= -0.001026$$



Sensitivities computed by the CSP method, FDM and MCS

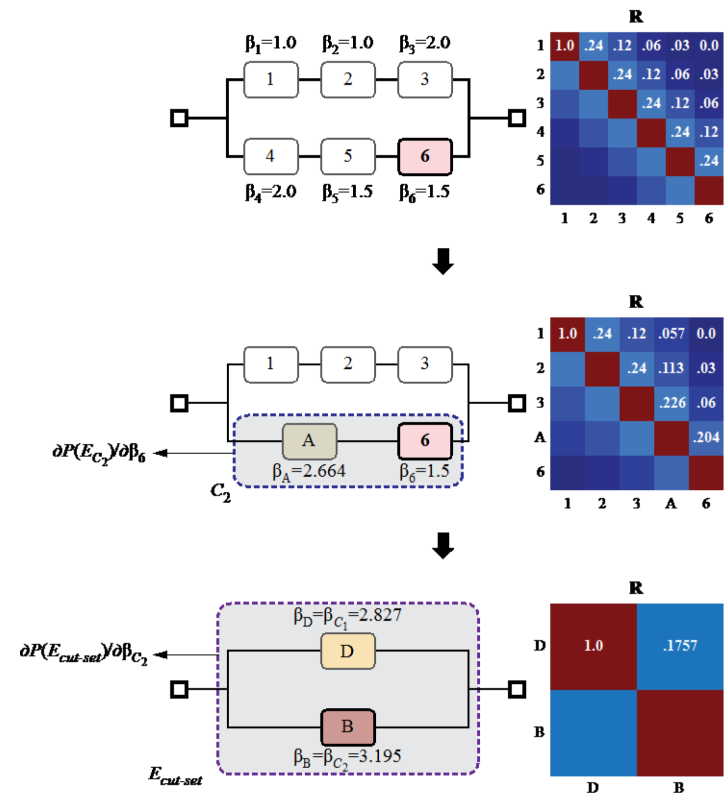


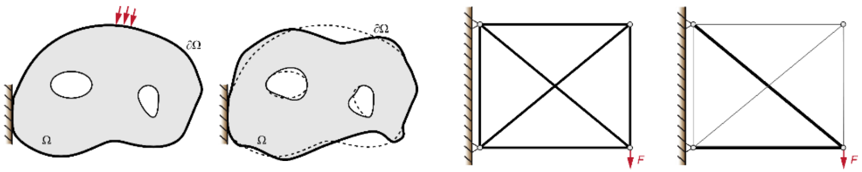
Illustration of the CSP method to compute sensitivity

CSP method remarks

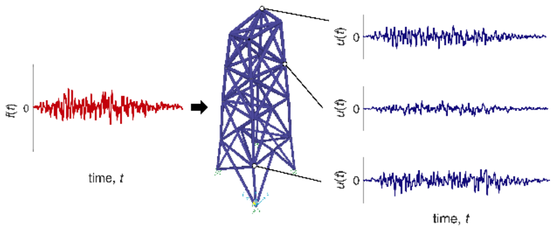
- CSP method can be used to facilitate efficient use of gradient-based optimization algorithms for design or topology optimization under constraints on system failure probability
- New sensitivity method, **CSP**, was developed to compute the parameter sensitivity of series, parallel and general system problems using **SCM**
- Sequential Compounding Method (**SCM**) was reviewed
- Sensitivity computed by CSP, FDM, and MCS were compared with respect to accuracy

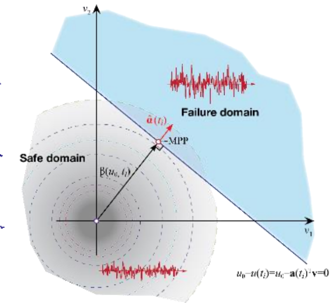
Presentation outline

1. Introduction

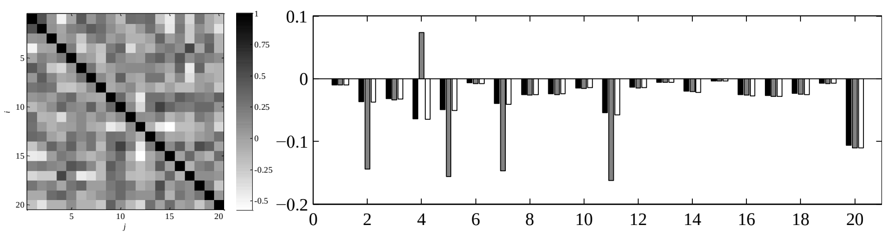


2. Discrete representation of stochastic process

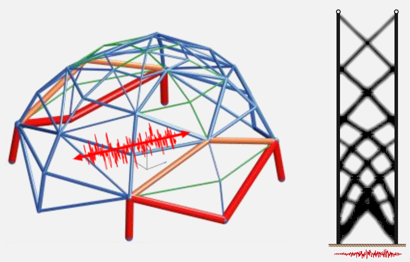




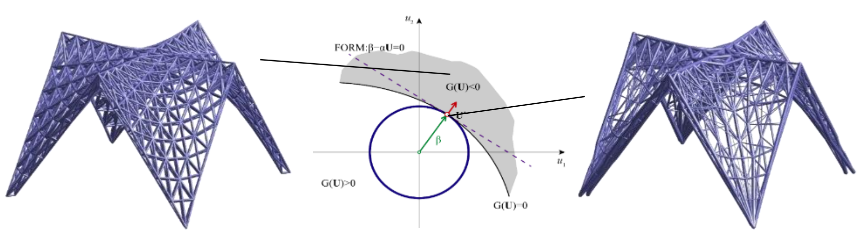
3. Parameter sensitivity of system reliability



4. Structural design and topology optimization under stochastic excitations



5. Reliability-based topology optimization by ground structure method



RBDO & RBTO aims to achieve the optimal design under probabilistic constraints on uncertain performance

System Reliability Based Design Optimization (SRBDO) problem

$$
\begin{aligned}
&\min_{\mathbf{d}} \quad f_{obj}(\mathbf{d}) \\
&s.t \quad P(E_{sys}^{i}) = P\left(\bigcup_{k=1}^{n} E_{f_i}(t_k,\mathbf{d})\right) = P\left(\bigcup_{k=1}^{n} \{g_i(t_k,\mathbf{d}) \le 0\}\right) \le P_{f_i}^{\text{target}}, \quad i = 1,\ldots,n_c \\
&\qquad \mathbf{d}^{lower} \le \mathbf{d} \le \mathbf{d}^{upper} \\
&\text{with} \quad \mathbf{M}(\mathbf{d})\ddot{\mathbf{u}}(t,\mathbf{d}) + \mathbf{C}(\mathbf{d})\dot{\mathbf{u}}(t,\mathbf{d}) + \mathbf{K}(\mathbf{d})\mathbf{u}(t,\mathbf{d}) = \mathbf{f}(t,\mathbf{d})
\end{aligned}
$$

Rayleigh Damping Model

$$\mathbf{C} = \kappa_0 \mathbf{M} + \kappa_1 \mathbf{K}$$

Earthquake Ground Excitation

$$\mathbf{f}(t,\mathbf{d}) = -\mathbf{M}(\mathbf{d})\mathbf{l}\ddot{u}_g(t) = -\mathbf{M}(\mathbf{d})\mathbf{l}f(t) = -\mathbf{M}(\mathbf{d})\mathbf{l}\cdot\left(\int_0^t h_f^{\mathrm{KT}}(t-\tau)W(\tau)d\tau\right)$$

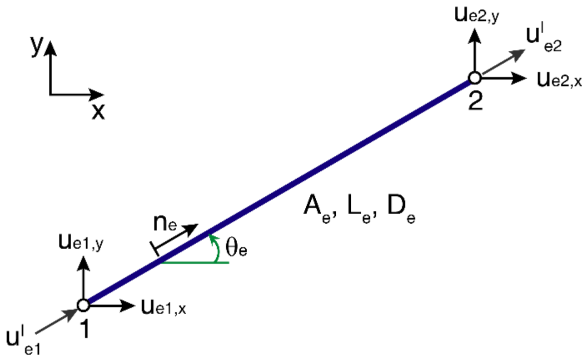
Kanai-Tajimi filter

$$h_f^{\mathrm{KT}}(t) = \exp(-\zeta_f\omega_f t)\left[\frac{(2\zeta_f^2-1)\omega_f}{\sqrt{1-\zeta_f^2}}\sin(\omega_f\sqrt{1-\zeta_f^2}\cdot t) - 2\zeta_f\omega_f\cos(\omega_f\sqrt{1-\zeta_f^2}\cdot t)\right]$$

Chun, J., Song, J., Paulino, G.H. System reliability-based design/topology optimization of structures constrained by first passage probability. To be submitted.

Various engineering constraints can be incorporated into RBDO&RBTO under first passage probability

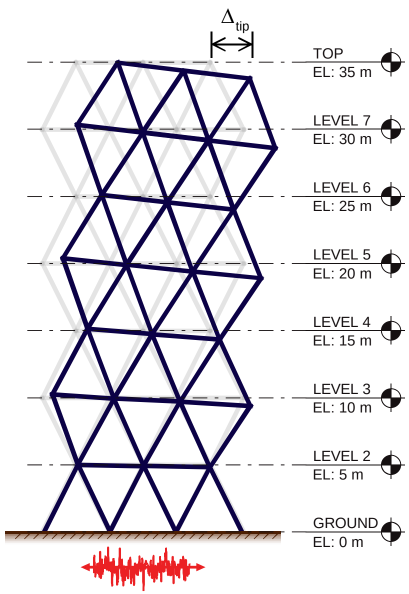
Stress in Bar



$$\sigma_e(t,\mathbf{d}) = \frac{D_e}{L_e}\mathbf{n}_e \cdot (\mathbf{u}^g_{e2}(t,\mathbf{d}) - \mathbf{u}^g_{e1}(t,\mathbf{d}))$$
$$= \frac{D_e}{L_e}(u^l_{e2}(t,\mathbf{d}) - u^l_{e1}(t,\mathbf{d}))$$

$$E_{fe}(t_k,\mathbf{d}) : \sigma_{oe} - \sigma_e(t_k,\mathbf{d}) \le 0$$

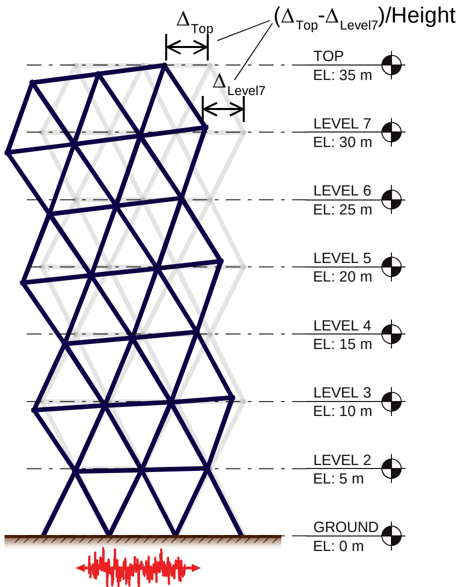
Maximum Displacement



$$E_{f_\Delta}(t_k,\mathbf{d}) : u_{o\Delta} - \Delta_{tip}(t_k,\mathbf{d}) \le 0$$

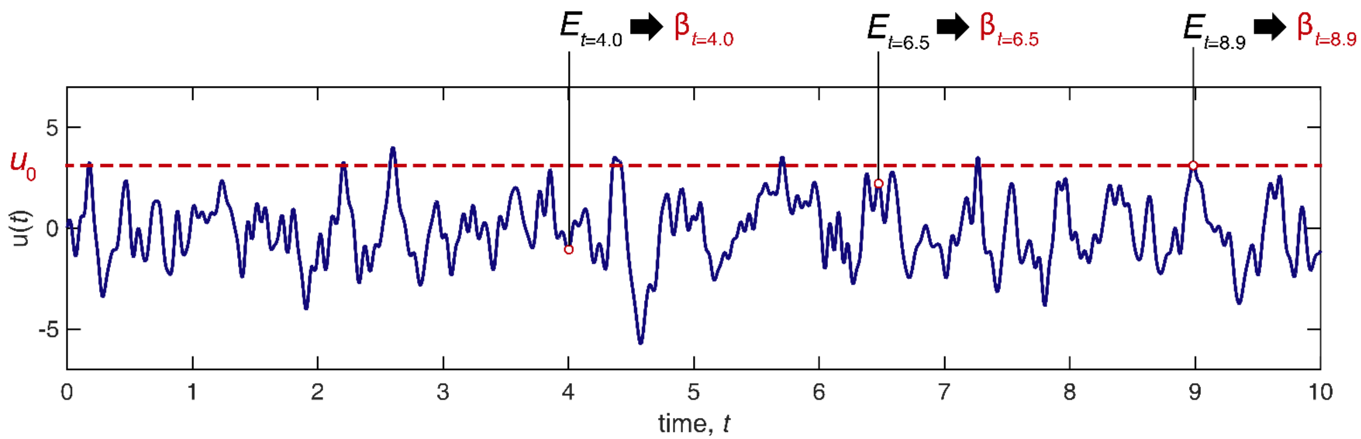
Hearst Tower (New York City)
http://www.sefindia.org/

Inter-story Drift Ratio



$$E_{f_{\Delta i}}(t_k,\mathbf{d}) : u_{o\Delta i} - \frac{\Delta_i(t_k,\mathbf{d}) - \Delta_{i-1}(t_k,\mathbf{d})}{H_i} \le 0$$

Computation of first passage probability of engineering constraint



$$
\begin{aligned}
P_{fp}(E_{sys}) &= P_{fp}\left(\bigcup_{k=1}^{n} E_{f_i}(t_k, \mathbf{d})\right) \\
&= 1 - \Phi_n\left[\beta_i(t_1, \mathbf{d}), \beta_i(t_2, \mathbf{d}), \ldots, \beta_i(t_{nt}, \mathbf{d}); \rho_{1,2}, \rho_{1,3}, \ldots, \rho_{n-1,n}\right] \\
&= 1 - \Phi_n\left[\boldsymbol{\beta}_i, \mathbf{R}\right]
\end{aligned}
$$

$$
\mathbf{R} = \begin{bmatrix} \rho_{1,1} & \cdots & \rho_{1,n} \\ \vdots & \ddots & \vdots \\ \rho_{n,1} & \cdots & \rho_{n,n} \end{bmatrix}
$$

□ Parameter sensitivity of first passage probability in RBDO&RBTO

$$
\frac{\partial P_{fp}(E_{sys})}{\partial d_i} = \frac{\partial\left(1 - \Phi_n\left[\boldsymbol{\beta}, \mathbf{R}\right]\right)}{\partial d_i} = \sum_{j=1}^{n}\left(\frac{\partial\left(-\Phi_n\left[\boldsymbol{\beta}, \mathbf{R}\right]\right)}{\partial \beta_j} \cdot \frac{\partial \beta_j(\mathbf{d})}{\partial d_i}\right) = \sum_{j=1}^{n}\left(c_j \cdot \frac{\partial \beta_j(\mathbf{d})}{\partial d_i}\right)
$$

$$
\frac{\partial \beta_j(\mathbf{d})}{\partial d_i} = -\frac{C_{cst} \cdot \left(\sum_{k=1}^{j}\left(a_k(t_j, \mathbf{d}) \cdot \frac{\partial a_k(t_j, \mathbf{d})}{\partial d_i}\right)\right)}{\left(\sum_{k=1}^{j} a_k(t_j, \mathbf{d})^2\right)^{3/2}}
$$

Implicit derivatives can be eliminated through adjoint method

Adjoint Method (AJM)

AJM introduces an adjoint system of equations so that computation of implicitly defined terms can be avoided. This results in significant reduction of computational cost.

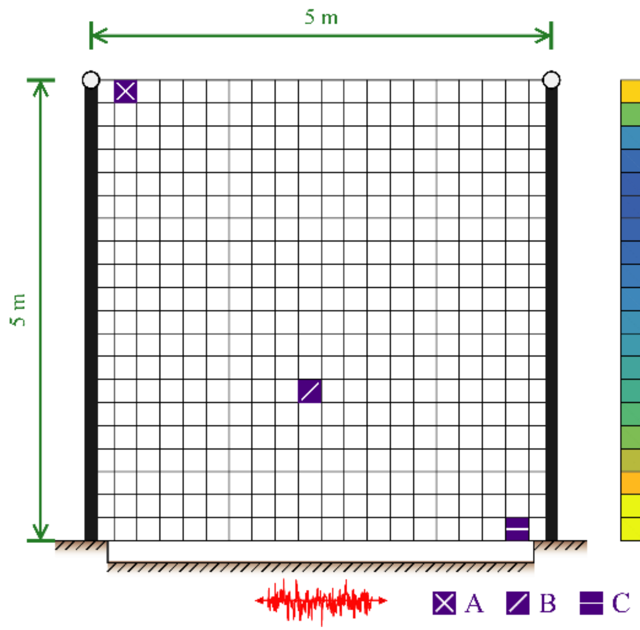
$$\frac{\partial P_{fp}(E_{sys})}{\partial d_i} = \sum_{j=1}^{n}\left(c_j \cdot \frac{\partial \beta_j(\mathbf{d})}{\partial d_i}\right) + \sum_{j=1}^{n} \boldsymbol{\lambda}_{n-j+1}^{\mathrm{T}}\left[\mathbf{M}(\mathbf{d})\ddot{\mathbf{u}}(t_j,\mathbf{d}) + \mathbf{C}(\mathbf{d})\dot{\mathbf{u}}(t_j,\mathbf{d}) + \mathbf{K}(\mathbf{d})\mathbf{u}(t_j,\mathbf{d}) + \mathbf{M}(\mathbf{d})\mathbf{I}f(t_j)\right]$$

$$\Downarrow$$

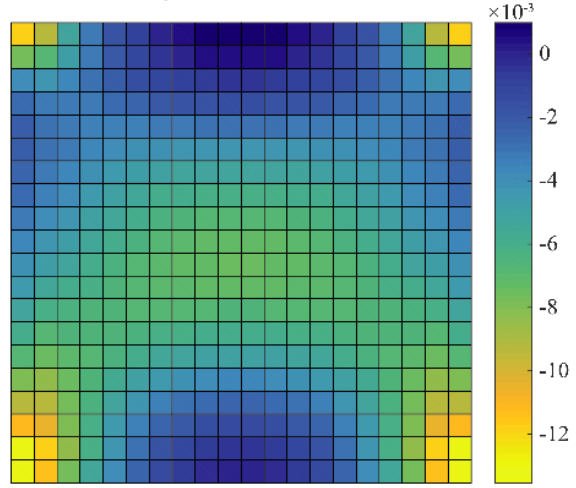
$$\begin{aligned}\frac{\partial P_{fp}(E_{sys})}{\partial d_i} = \sum_{j=1}^{n} \boldsymbol{\lambda}_{n-j+1}^{\mathrm{T}}\Bigg[&\frac{\partial \underline{\underline{\tilde{\mathbf{A}}}}(\mathbf{d})}{\partial d_i}\cdot \mathbf{u}(t_j,\mathbf{d}) - \eta\left(\Delta t\right)^2 \frac{\partial \mathbf{f}(t_j,\mathbf{d})}{\partial d_i} - (0.5+\gamma-2\eta)\left(\Delta t\right)^2 \frac{\partial \mathbf{f}(t_{j-1},\mathbf{d})}{\partial d_i} \\ &-(0.5-\gamma+\eta)\left(\Delta t\right)^2 \frac{\partial \mathbf{f}(t_{j-2},\mathbf{d})}{\partial d_i} + \frac{\partial \underline{\underline{\mathbf{B}}}(\mathbf{d})}{\partial d_i}\cdot \mathbf{u}(t_{j-1},\mathbf{d}) + \frac{\partial \underline{\underline{\mathbf{E}}}(\mathbf{d})}{\partial d_i}\cdot \mathbf{u}(t_{j-2},\mathbf{d})\Bigg] \\ &+\boldsymbol{\lambda}_{n}^{\mathrm{T}}\left[\underline{\underline{\mathbf{B}}}(\mathbf{d})\cdot \frac{\partial \mathbf{u}(0,\mathbf{d})}{\partial d_i} + \underline{\underline{\mathbf{E}}}(\mathbf{d})\cdot \frac{\partial \mathbf{u}(t_{-1},\mathbf{d})}{\partial d_i}\right] + \boldsymbol{\lambda}_{n-1}^{\mathrm{T}}\left[\underline{\underline{\mathbf{E}}}(\mathbf{d})\cdot \frac{\partial \mathbf{u}(0,\mathbf{d})}{\partial d_i}\right]\end{aligned}$$

Chun, J., Song, J., Paulino, G.H. System reliability-based design/topology optimization of structures constrained by first passage probability. To be submitted.

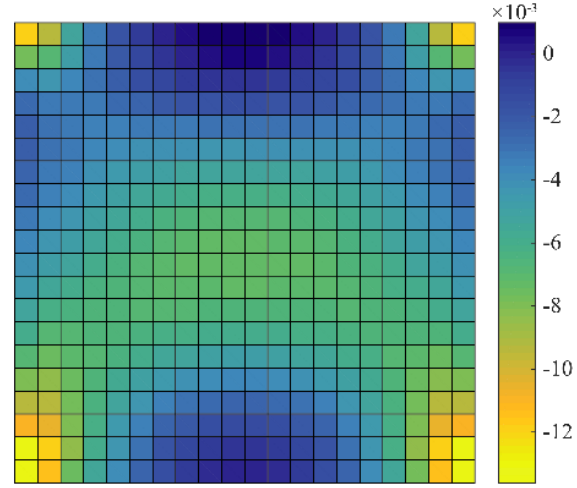
Sensitivities of first passage probability from AJM show a good agreement with those by FDM



Adjoint method

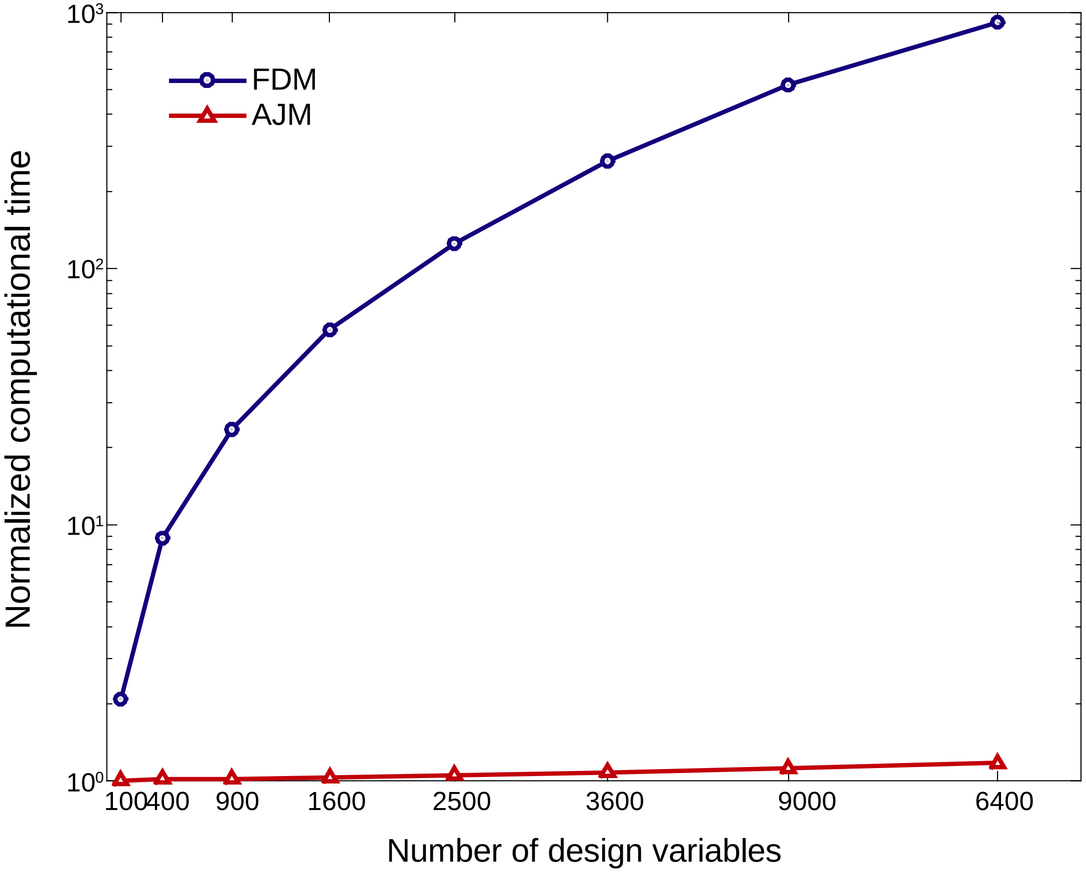


Finite difference method



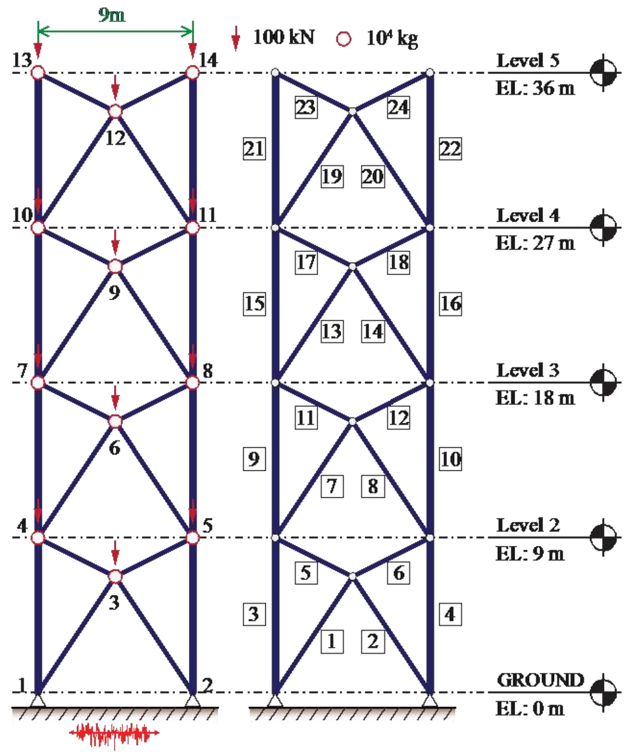
	FDM			AJM		
Δd	$\partial P_f/\partial d_A$	$\partial P_f/\partial d_B$	$\partial P_f/\partial d_C$	$\partial P_f/\partial d_A$	$\partial P_f/\partial d_B$	$\partial P_f/\partial d_C$
1×10^{-1}	-0.000452	-0.000293	-0.000569	-0.000486	-0.000312	-0.000599
1×10^{-2}	-0.000483	-0.000310	-0.000596			
1×10^{-3}	-0.000486	-0.000312	-0.000599			
1×10^{-4}	-0.000487	-0.000312	-0.000599			
1×10^{-5}	-0.000487	-0.000312	-0.000599			
1×10^{-6}	-0.000486	-0.000312	-0.000599			
1×10^{-7}	-0.000485	-0.000309	-0.000598			
1×10^{-8}	-0.000479	-0.000301	-0.000619			
1×10^{-9}	-0.000444	-0.000391	-0.000632			
1×10^{-10}	0.000250	0.000289	-0.000012			
1×10^{-11}	-0.000166	-0.001532	-0.004141			
1×10^{-12}	0.050625	0.025424	0.036526			

Numerical tests confirm significant reduction in computational time of proposed AJM compared to FDM



Optimization of a lateral bracing system subjected to earthquake ground motions

Image courtesy of SOM



E = 21,000 MPa
ρ_m= 2,400 kg/m^3

$$\min_{\mathbf{d}} \quad \text{volume}$$

$$s.t \quad P_{fp,i}\left(\text{stress}, \sigma_{oe} = 35\text{ MPa}\right) \le P_{f_i}^{\text{target}}, \quad i = 1,\ldots,n_c$$

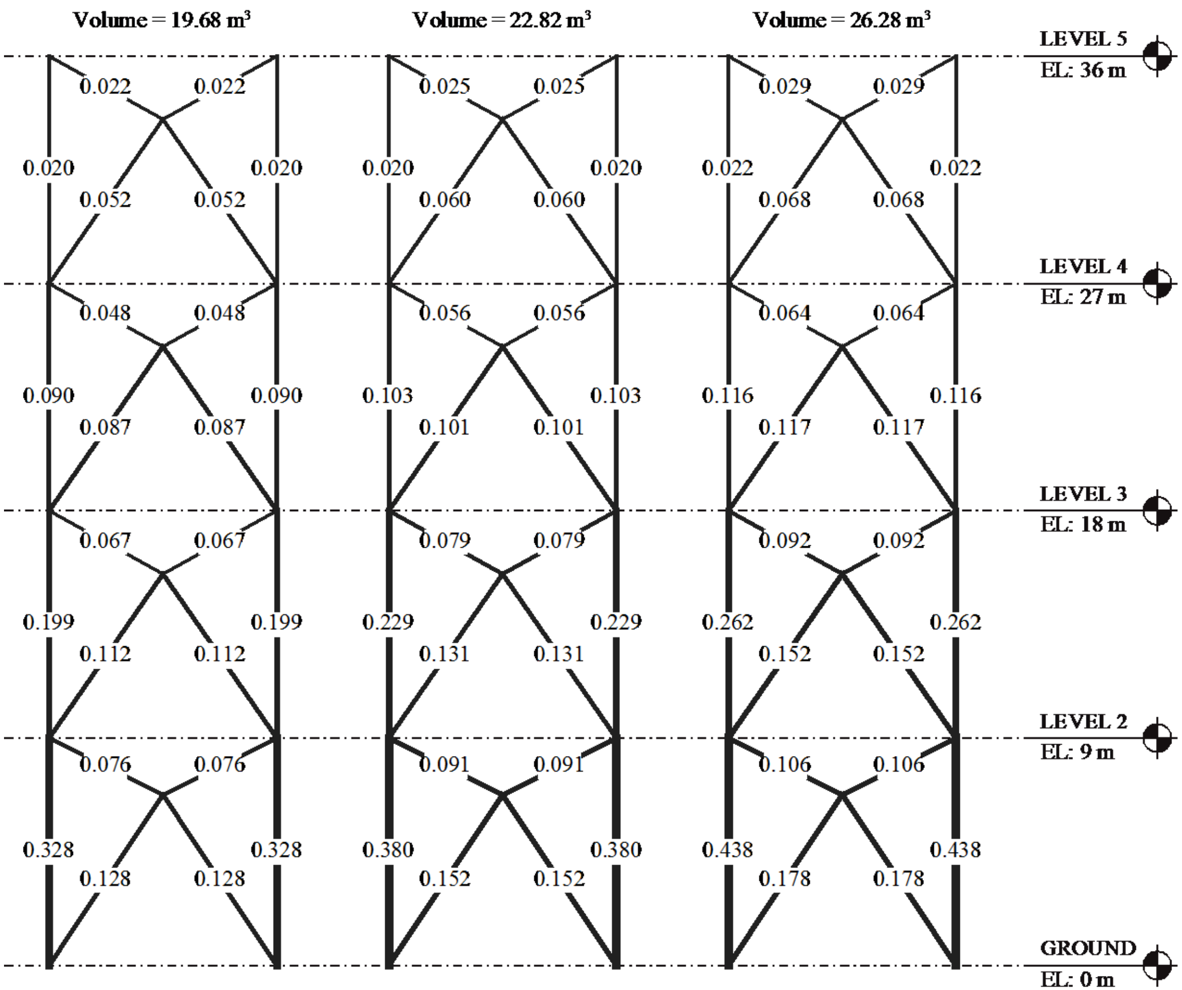
$$0.02\text{ m}^2 \le \mathbf{d} \le 1\text{ m}^2$$

$$\text{with} \quad \mathbf{M}(\mathbf{d})\ddot{\mathbf{u}}(t,\mathbf{d}) + \mathbf{C}(\mathbf{d})\dot{\mathbf{u}}(t,\mathbf{d}) + \mathbf{K}(\mathbf{d})\mathbf{u}(t,\mathbf{d}) = -\mathbf{M}(\mathbf{d})\mathbf{I}f(t)$$

L. L. Beghini, A. Beghini, N. Katz, W. F. Baker, and G. H. Paulino. (2014). Connecting architecture and engineering through structural topology optimization. *Engineering Structures.* Vol. 59, pp. 716-726.

Chun, J., Song, J., Paulino, G.H. System reliability-based design/topology optimization of structures constrained by first passage probability. To be submitted.

Proposed method enables optimization of elements for different target first passage failure probabilities

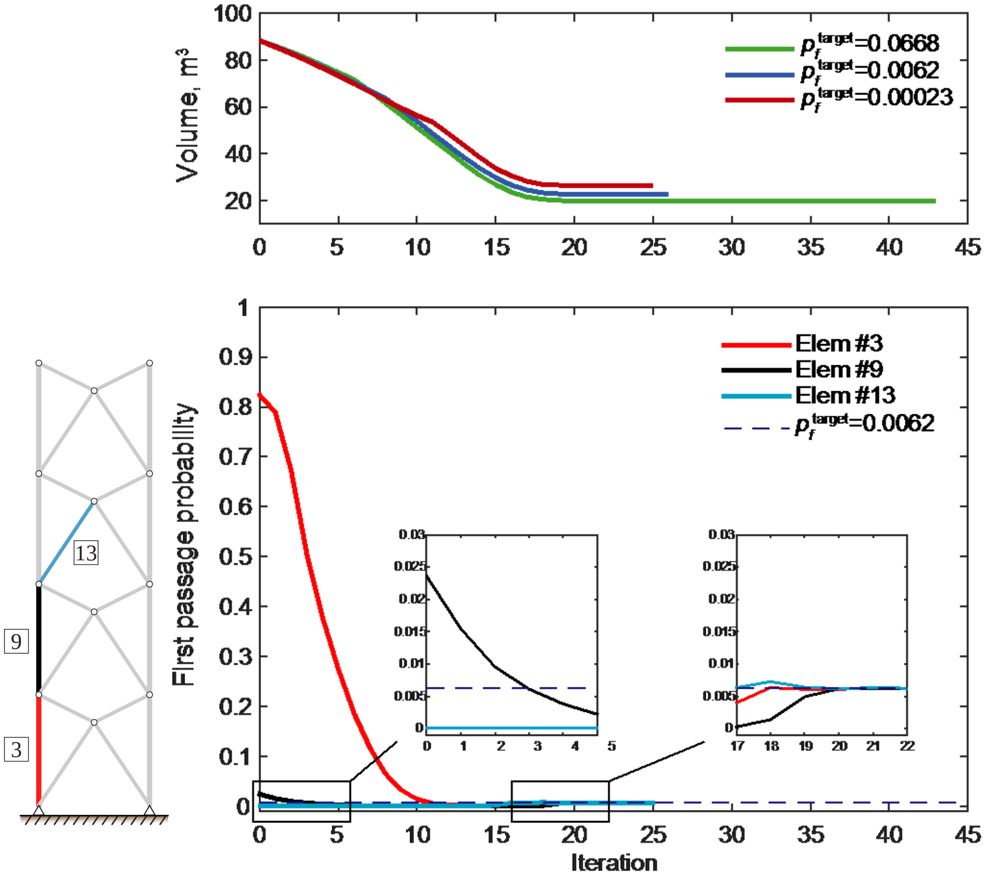


P_{fp}^{t}=0.0668 P_{fp}^{t}=0.0062 P_{fp}^{t}=0.00023

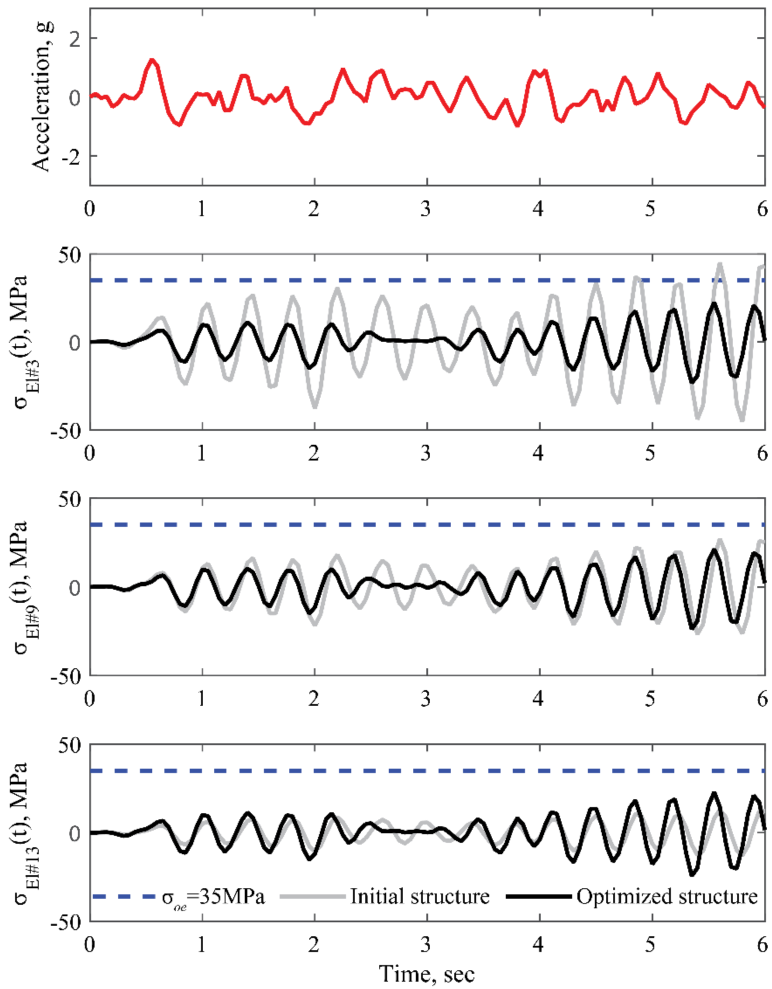
Chun, J., Song, J., Paulino, G.H. System reliability-based design/topology optimization of structures constrained by first passage probability. To be submitted.

Reduction of stress time histories decreases likelihood of exceeding the threshold value for P_f^{target}

Convergence History



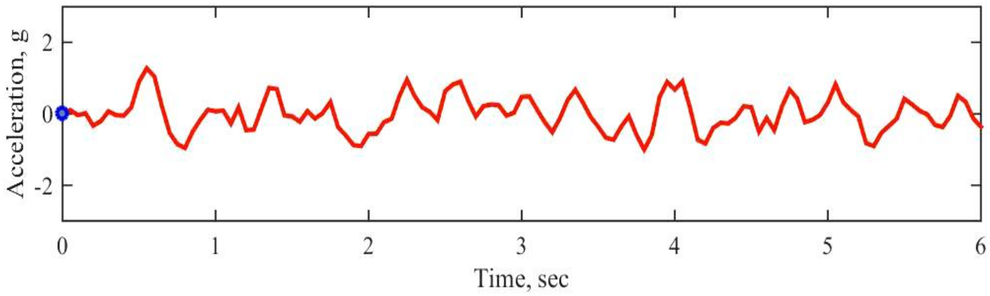
Dynamic Response Comparison

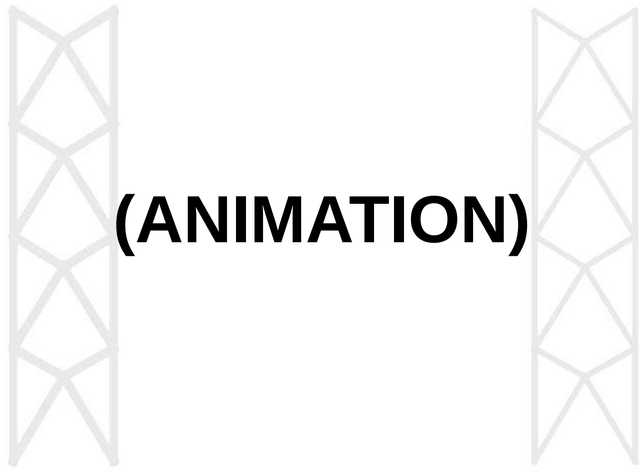


Chun, J., Song, J., Paulino, G.H. System reliability-based design/topology optimization of structures constrained by first passage probability. To be submitted.

Stress time histories are more uniform than stress levels shown in the initial structure

Dynamic Behavior (Stress)





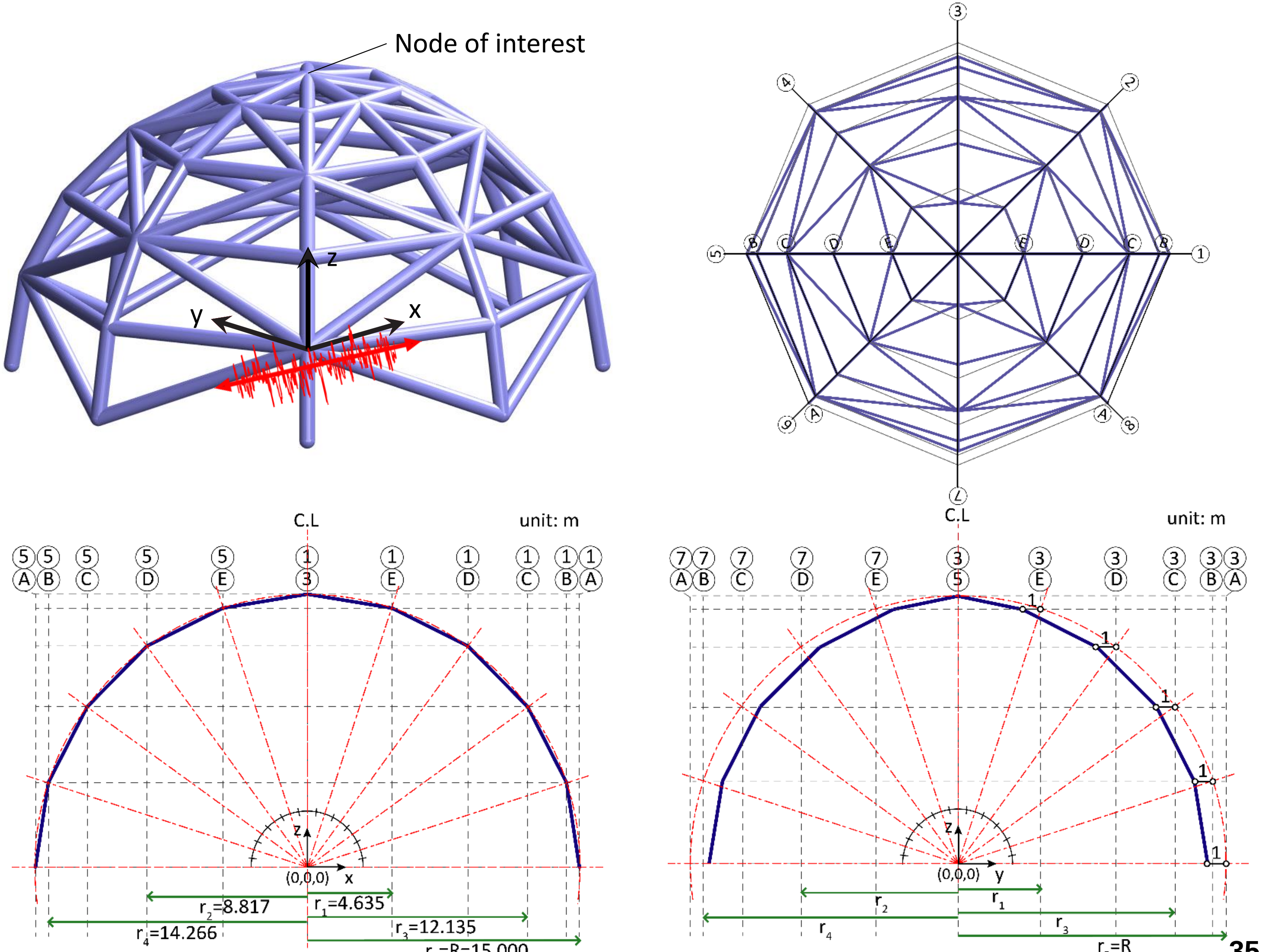
(ANIMATION)

Initial System **Optimized System**

Optimization of space truss dome subjected to stochastic excitations

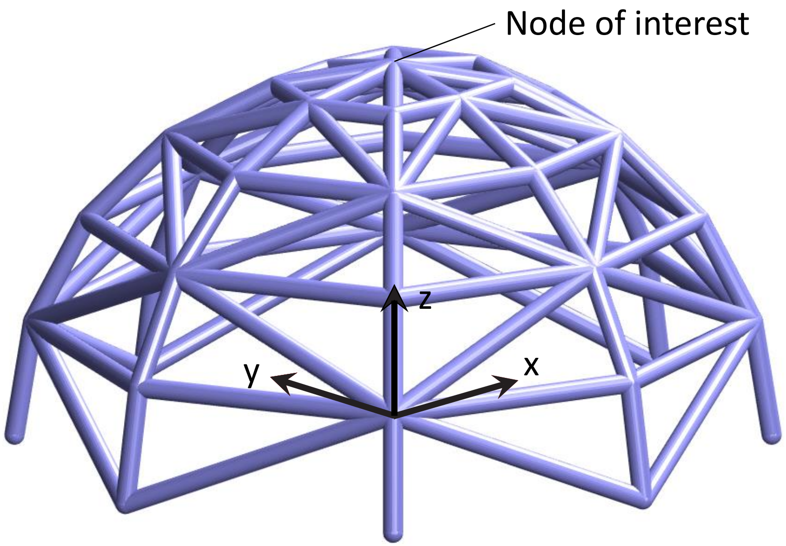
Space Truss Dome

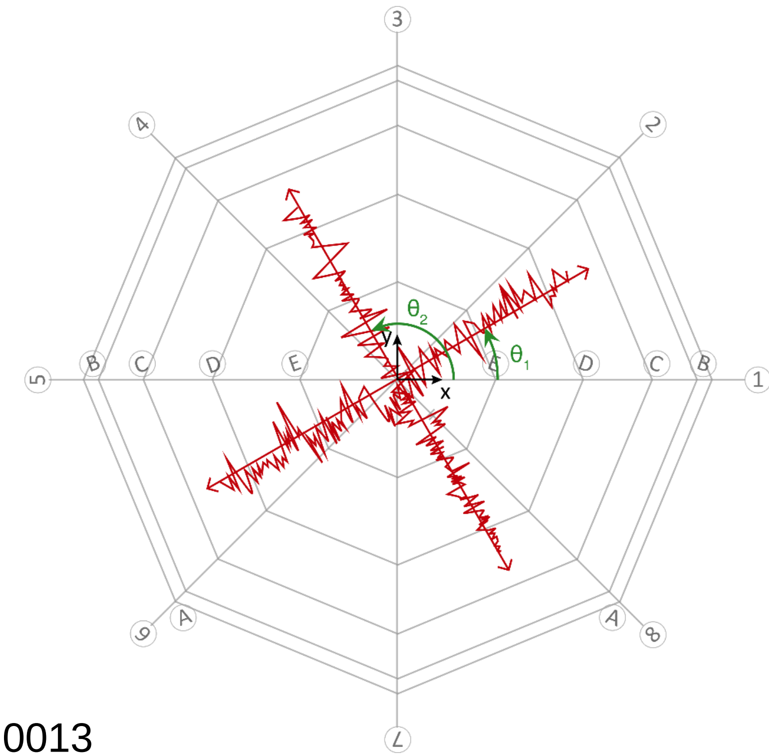
E = 210 GPa
ρ_m= 7,850 kg/m^3
2% damping ratio



Chun, J., Song, J., Paulino, G.H. System reliability-based design/topology optimization of structures constrained by first passage probability. To be submitted.

Simultaneous considerations of two direction components of earthquake ground excitations at different angles





$$\min_{\mathbf{d}} \quad \text{volume}$$

$$s.t \quad P_{fp}\left(\text{drift ratio in x-direction}\right) \leq 0.0013$$

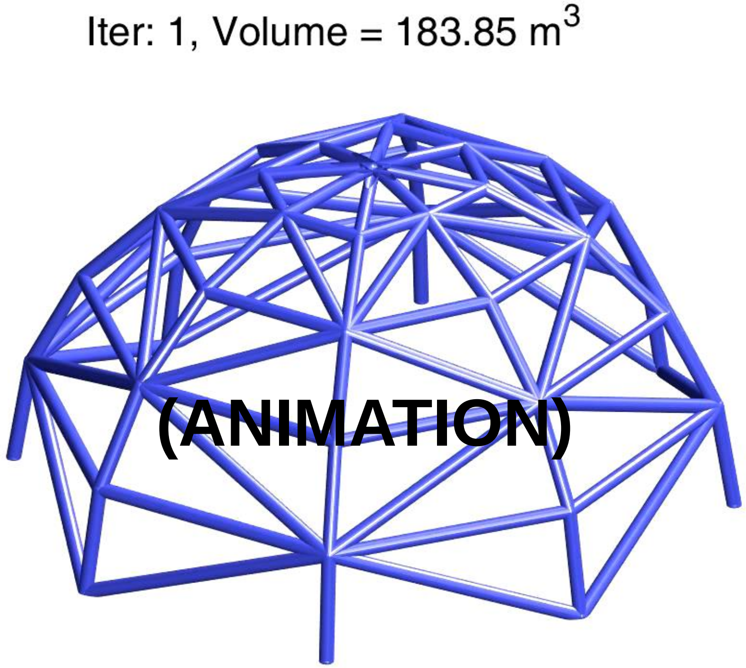
$$P_{fp}\left(\text{drift ratio in y-direction}\right) \leq 0.0013$$

$$0.02 \text{ m}^2 \leq \mathbf{d} \leq 1.5 \text{ m}^2$$

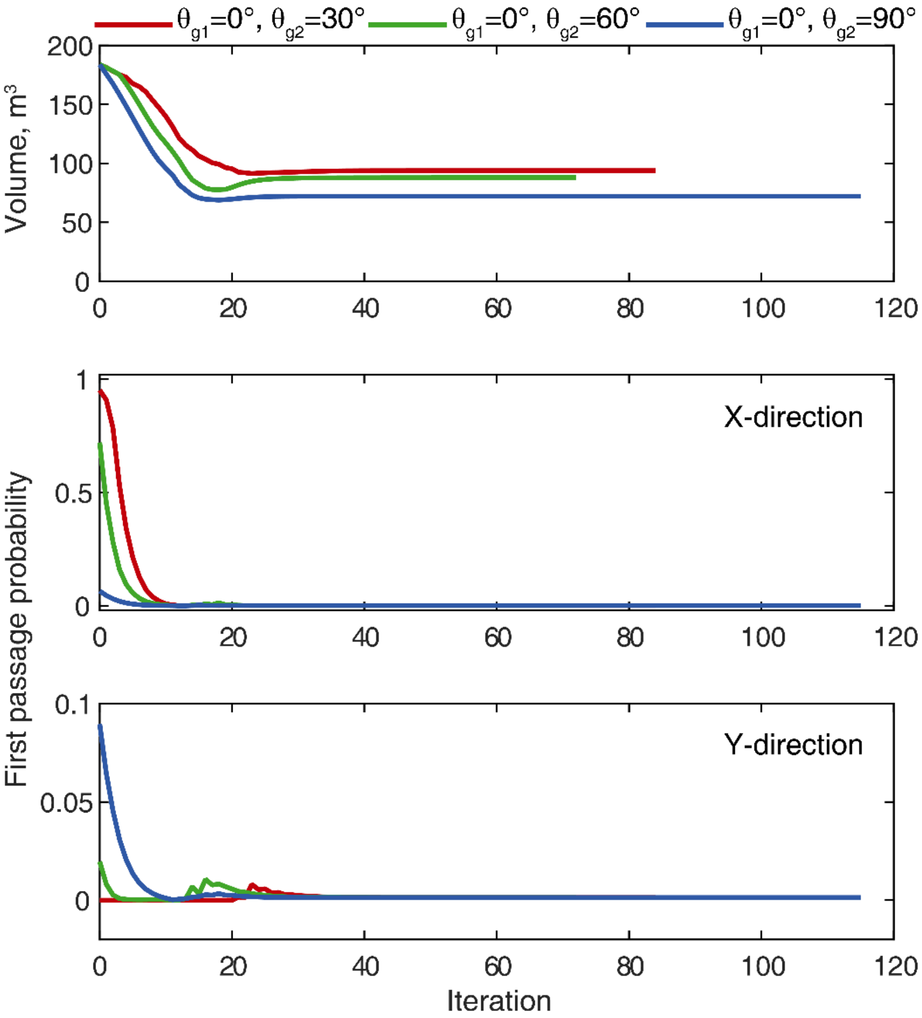
$$\text{with} \quad \mathbf{M}(\mathbf{d})\ddot{\mathbf{u}}(t,\mathbf{d}) + \mathbf{C}(\mathbf{d})\dot{\mathbf{u}}(t,\mathbf{d}) + \mathbf{K}(\mathbf{d})\mathbf{u}(t,\mathbf{d}) = -\mathbf{M}(\mathbf{d})\mathbf{I}(\theta_{g1})f_{g1}(t) - \mathbf{M}(\mathbf{d})\mathbf{I}(\theta_{g2})f_{g2}(t)$$

Φ_{g1o}	Φ_{g2o}	ω_f	ζ_f	t (sec)	Δt (sec)	Initial Area (m^2)	Threshold value
4.0	3.0	5π	0.4	6.0	0.06	0.25	$u_{o\Delta x} = 1/800$ $u_{o\Delta y} = 1/800$

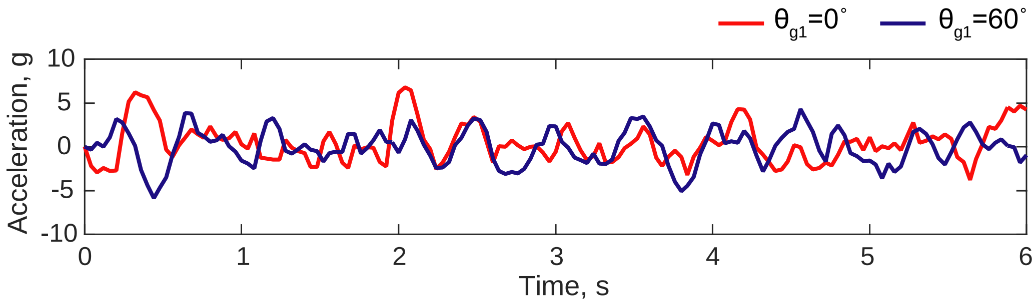
Optimization history: volume reduction while satisfying the target failure probability

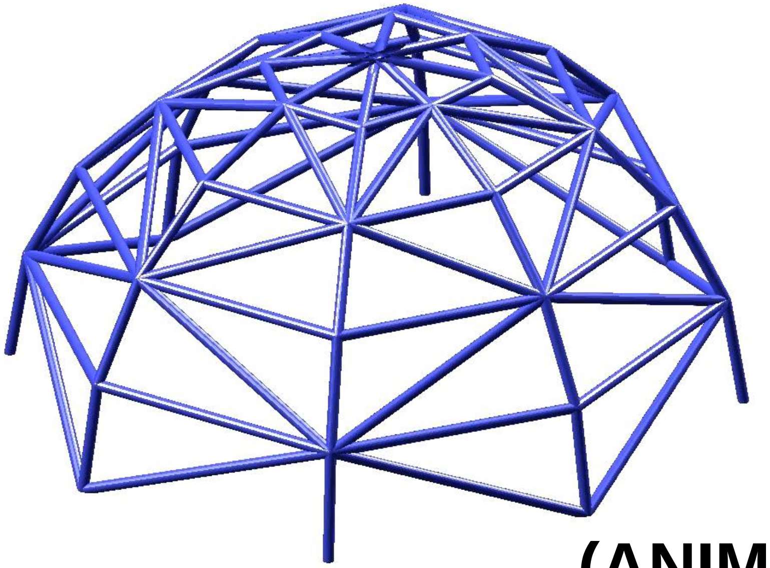


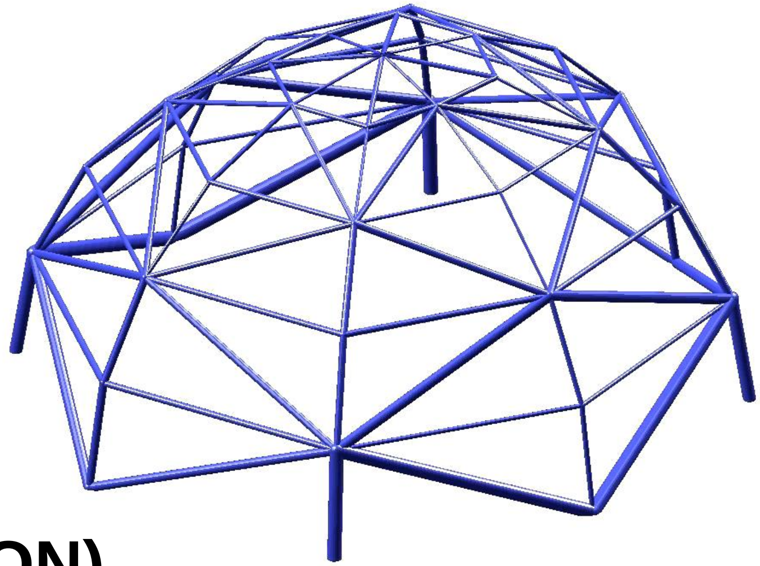
☐ Convergence History



Comparison of dynamic responses of the initial structure and optimized structures







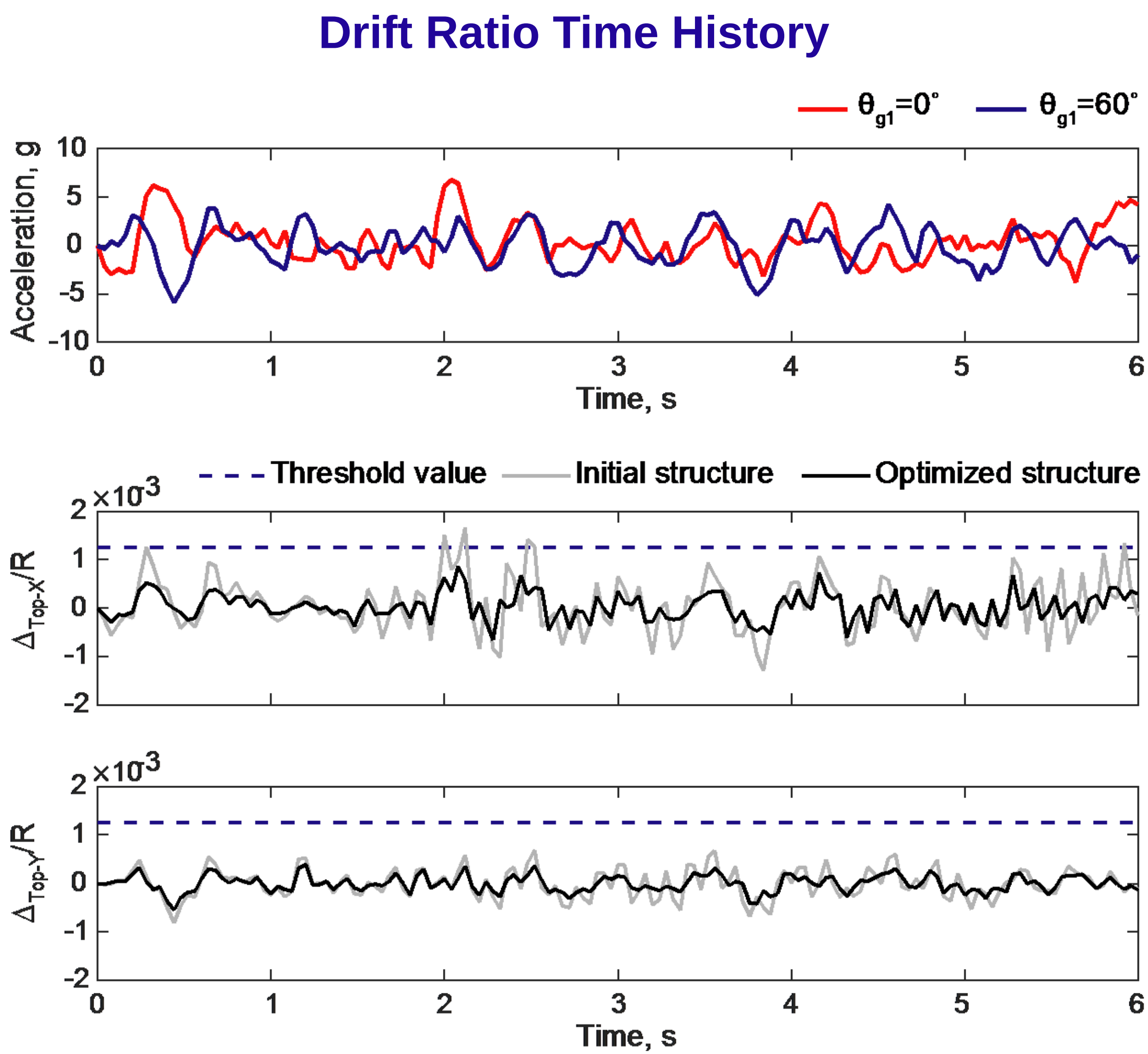
(ANIMATION)

Initial structure

Optimized structure

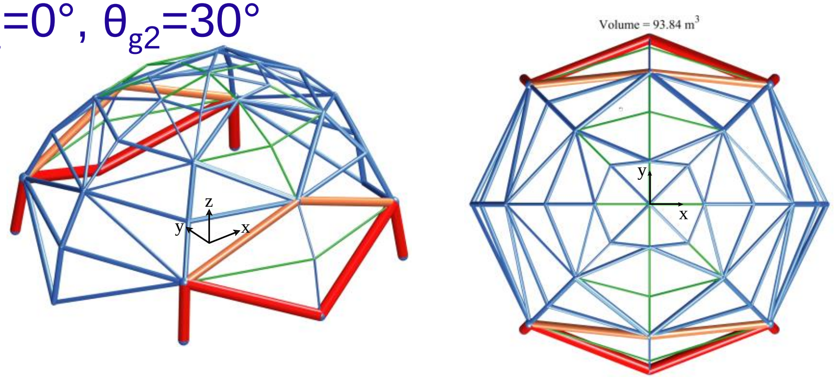
Scale Factor = 5

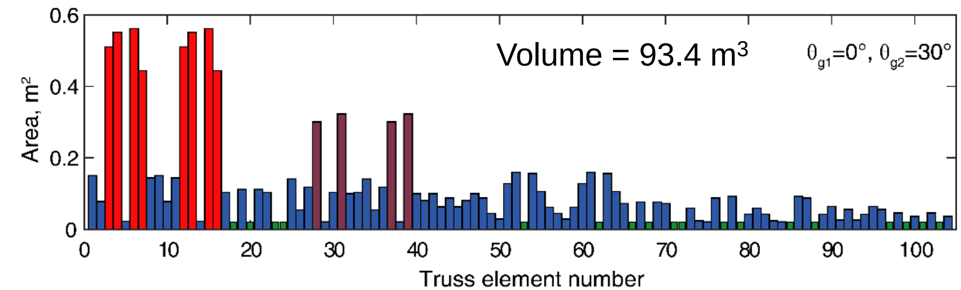
Reduced drift ratios of the optimized structure result in decreased likelihood of exceeding the threshold value



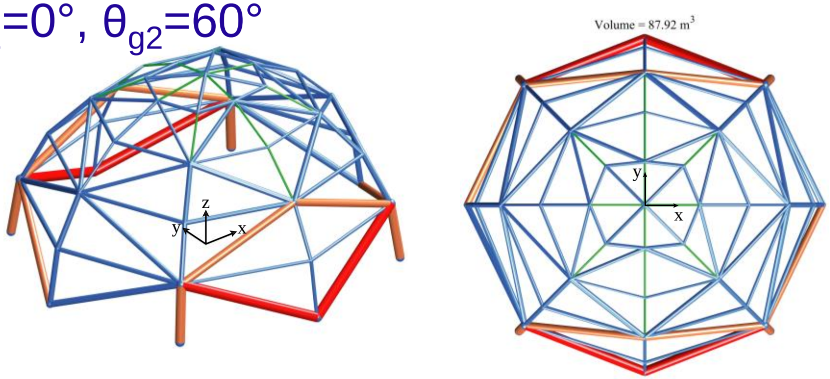
Two directional ground components of earthquake ground excitations: optimal bar areas

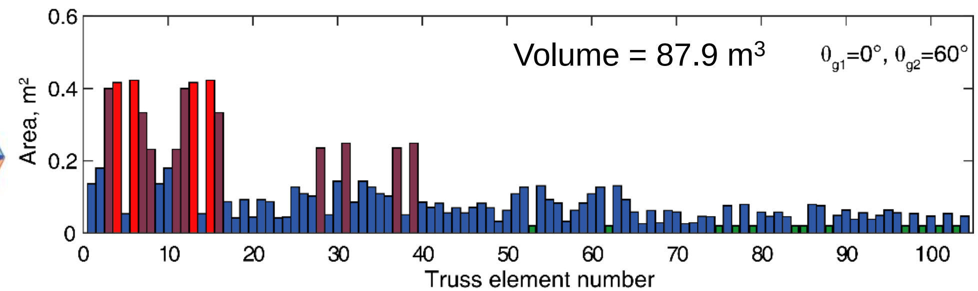
$\theta_{g1}=0°,\ \theta_{g2}=30°$



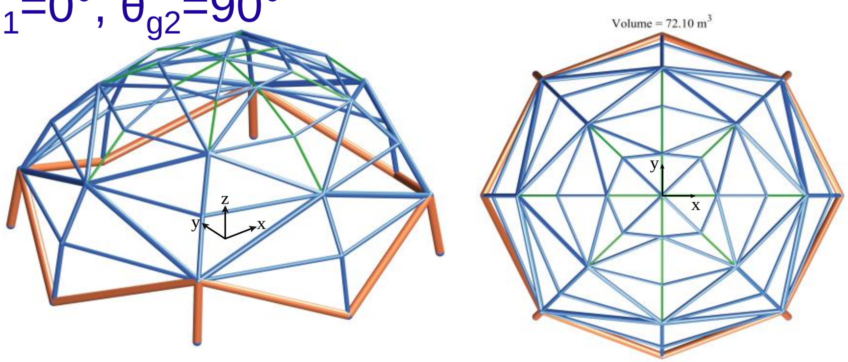


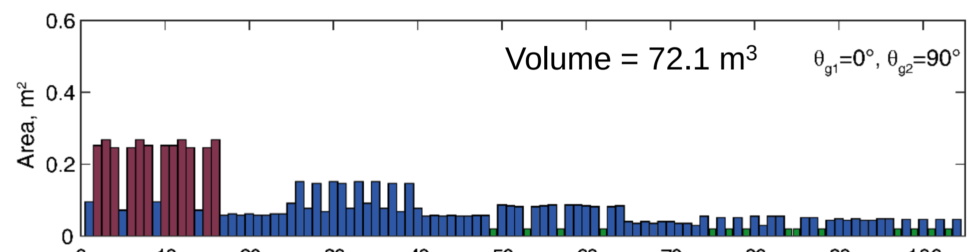
$\theta_{g1}=0°,\ \theta_{g2}=60°$



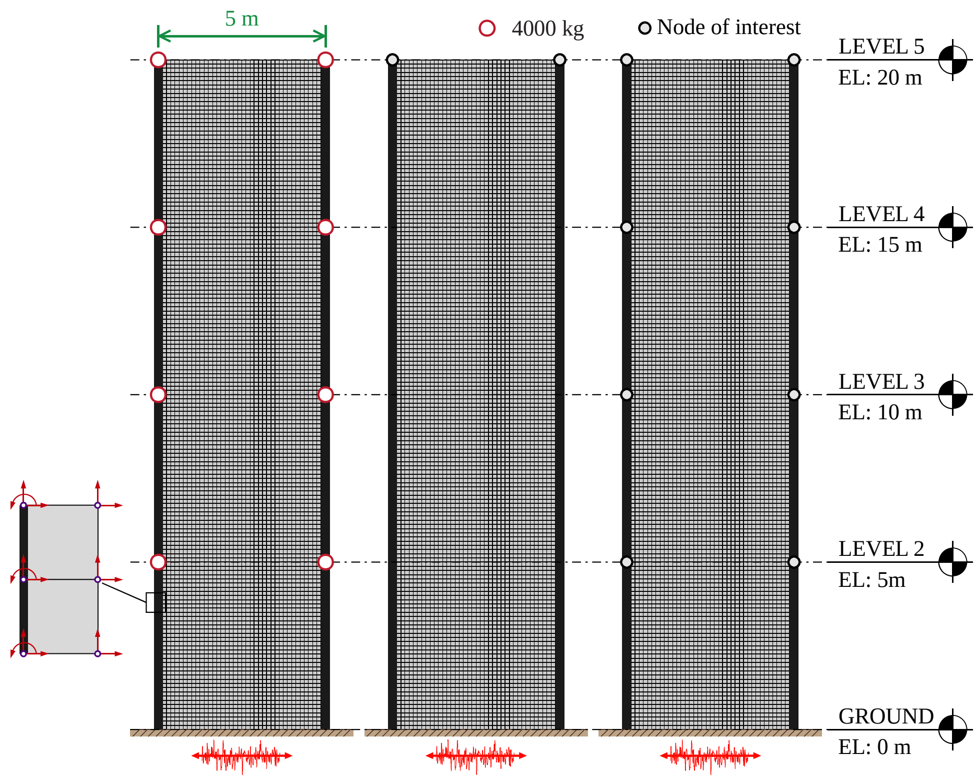


$\theta_{g1}=0°,\ \theta_{g2}=90°$





Topology optimization can identify the optimal bracing layout of a structure



E = 21,000 MPa
ρ_m = 2,400 kg/m^3
2% damping ratio
r = 0.25 m

$$\min_{\boldsymbol{\rho}} \quad \text{volume}$$

$$s.t \quad P_{fp}(\text{g}) \leq P_f^{\text{target}}$$

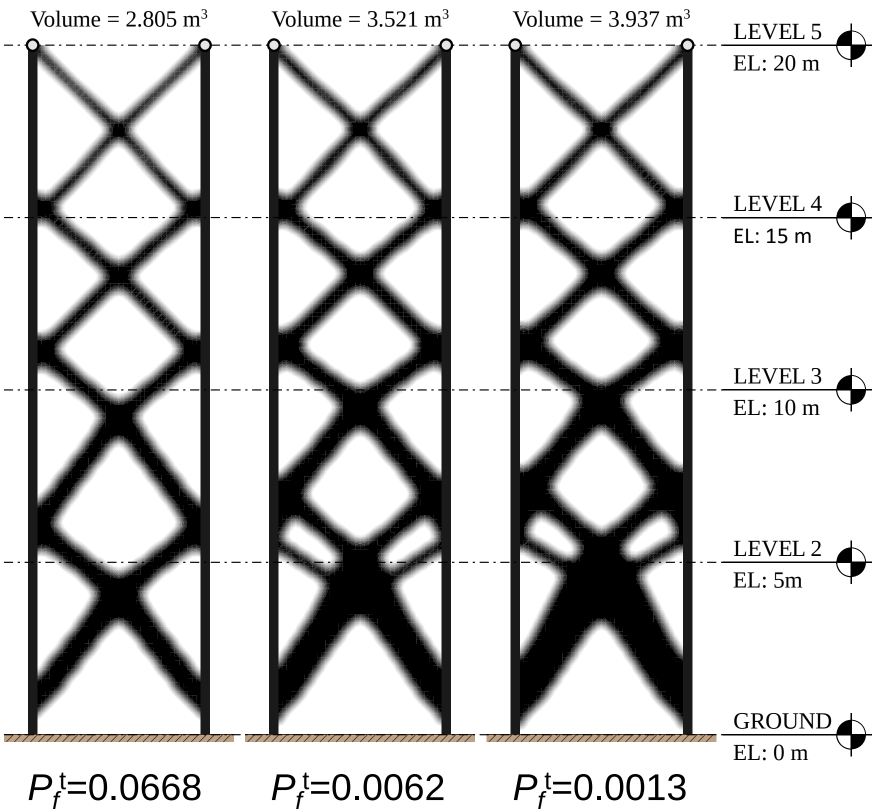
$$\varepsilon \leq \boldsymbol{\rho} \leq 1.0$$

$$\text{with} \quad \mathbf{M}(\boldsymbol{\rho})\ddot{\mathbf{u}}(t,\boldsymbol{\rho}) + \mathbf{C}(\boldsymbol{\rho})\dot{\mathbf{u}}(t,\boldsymbol{\rho}) + \mathbf{K}(\boldsymbol{\rho})\mathbf{u}(t,\boldsymbol{\rho}) = -\mathbf{M}(\boldsymbol{\rho})\mathbf{I}f(t)$$

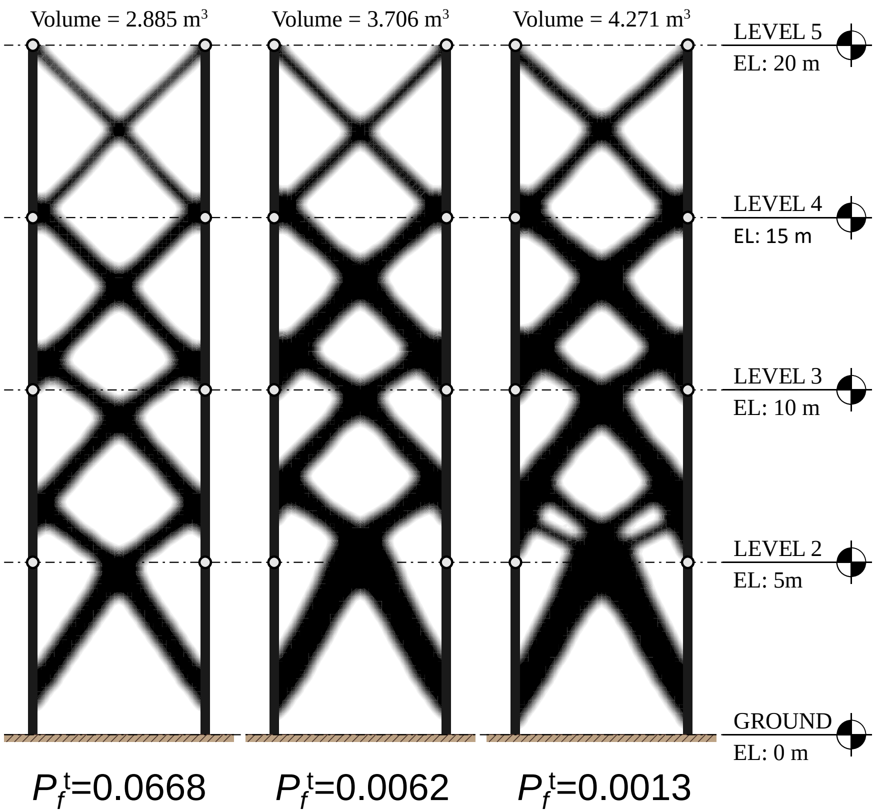
Chun, J., Song, J., Paulino, G.H. System reliability-based design/topology optimization of structures constrained by first passage probability. To be submitted.

Reliable bracing systems can be interpreted from optimal topologies

Tip Displacement Constraint



Inter-story Drift Constraint



Optimization shows that **reinforcing lower regions** will efficiently control the **tip displacement,** whereas adjusting **each bracing module** will lead to successful designs of structures fulfilling **inter-story drift** ratio criteria

Stochastic design and topology optimization remarks

- **New optimization framework** was proposed to incorporate the first passage probability into size optimization and topology optimization of structures

- **Parameter sensitivity formulation** of the probabilistic constraint on the first passage probability was derived

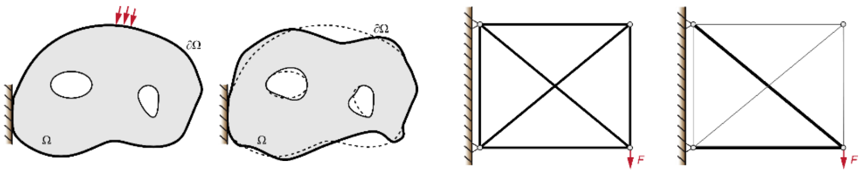
- **Lateral bracing system** of structures subjected to stochastic ground motions was optimized to identify optimal member sizes under engineering constraints associated with structural design criteria

- **Different types of failure events** such as different time points and locations as well as multiple design criteria can be considered

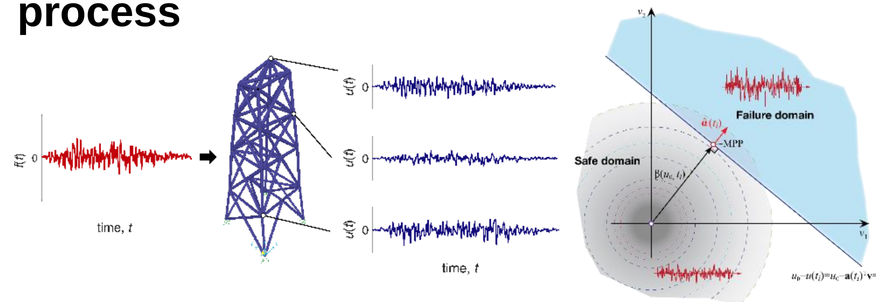
- Optimization frameworks under **non-stationary** stochastic processes in **time domain** as well as in **frequency domain** need to be further studied

Presentation outline

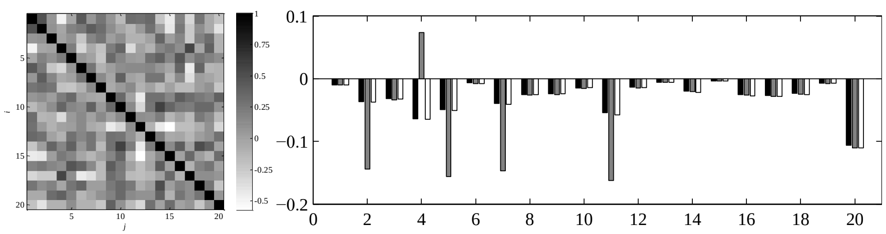
1. Introduction



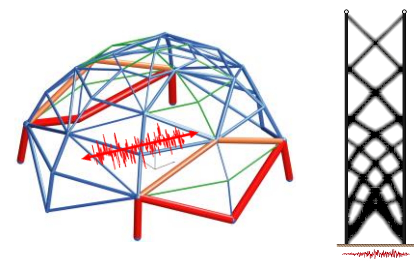
2. Discrete representation of stochastic process



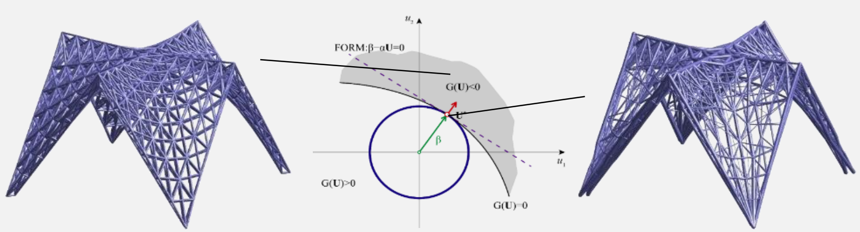
3. Parameter sensitivity of system reliability



4. Structural design and topology optimization under stochastic excitations



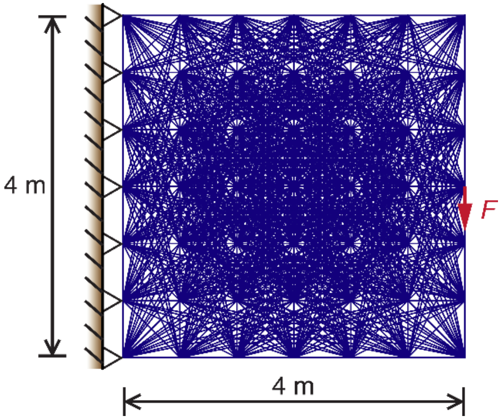
5. Reliability-based topology optimization by ground structure method



Determination of a proper cut-off value is ambiguous in conventional filter

Elastic formulation

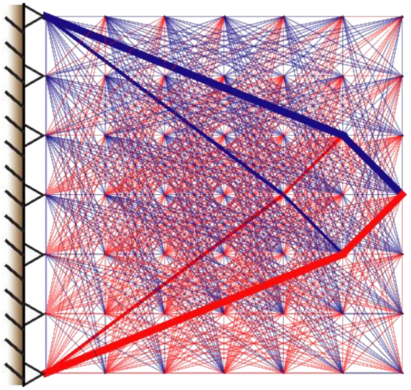
$$\min_{\mathbf{A}} \quad C = \mathbf{f}^{T}\mathbf{u}(\mathbf{A})$$
$$s.t \quad \mathbf{L}^{T}\mathbf{A} \le V_s$$
$$\mathbf{A}^{lower} \le \mathbf{A} \le \mathbf{A}^{upper}$$
$$\text{with } \mathbf{K}(\mathbf{A})\mathbf{u}(\mathbf{A}) = \mathbf{f}$$



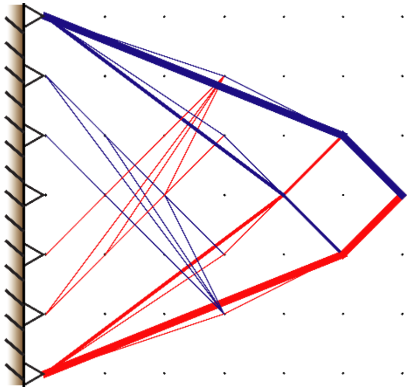
Ground structure

Filter

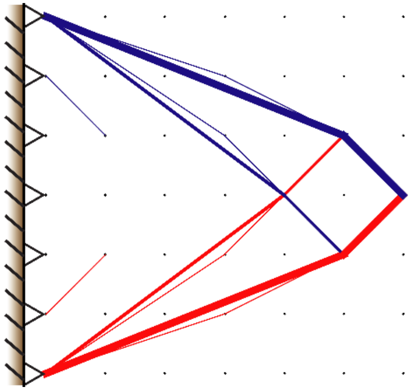
$$A_i = \begin{cases} 0 & \text{if } A_i / \max(\mathbf{A}) < \varepsilon_{\text{cut-off}} \\ A_i & \text{otherwise} \end{cases}, \quad 0 \le \varepsilon_{\text{cut-off}} \le 1$$


$\varepsilon_{\text{cut-off}} = 0.0$

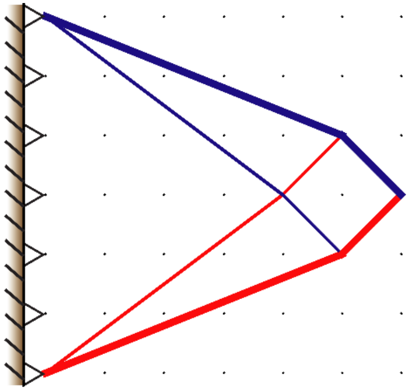
- Impractical structure


$\varepsilon_{\text{cut-off}} = 0.0001$

- Many thin bars


$\varepsilon_{\text{cut-off}} = 0.001$

- Hanging bars
- Internal mechanism

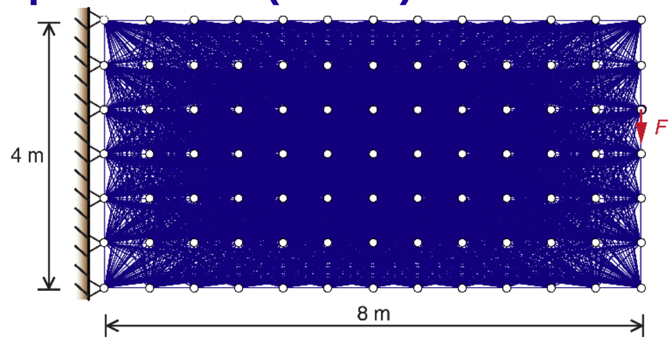

$\varepsilon_{\text{cut-off}} = 0.1$

- Feasible structure

Conventional filtering approach may lead to the violation of the prescribed target failure probability

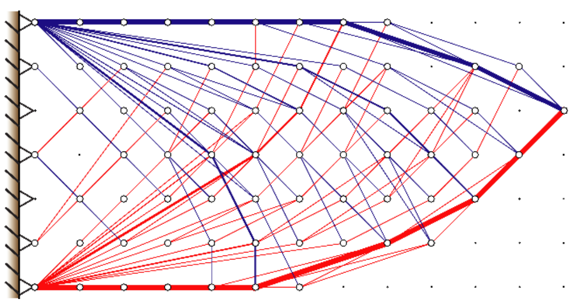
Reliability-based topology optimization (RBTO)

$$\min_{\mathbf{d},\boldsymbol{\mu}_X} \; f(\mathbf{d},\boldsymbol{\mu}_X)$$
$$s.t \quad P_f\left[g(\mathbf{d},\mathbf{X}) \le 0\right] \le P_f^{\text{target}}$$
$$\mathbf{d}^{lower} \le \mathbf{d} \le \mathbf{d}^{upper}$$
$$\text{with } \mathbf{K}(\mathbf{d})\mathbf{u}(\mathbf{d},\mathbf{X}) = \mathbf{f}(\mathbf{X})$$

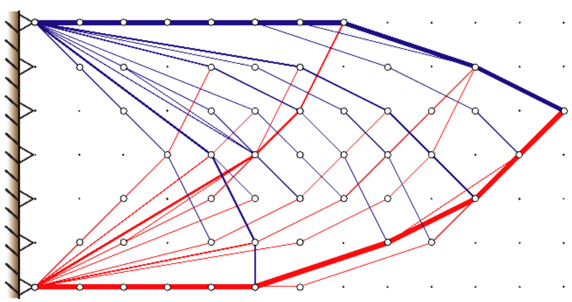


Ground structure

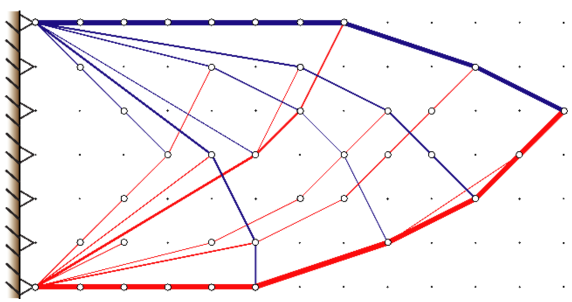
Objective: minimize volume, Constraint: compliance (P_f^{target}=0.005)



$\varepsilon_{\text{cut-off}}$ = 0.0001



$\varepsilon_{\text{cut-off}}$ = 0.01



$\varepsilon_{\text{cut-off}}$ = 0.05

	Conventional filtering method			
	$\varepsilon_{\text{cutoff}}$			
	0.05	0.01	0.0001	0.000001
Volume	369.777	384.323	386.480	386.489
$P_{f\,\text{FORM}}$	1.00	7.95×10^{-3}	5.01×10^{-3}	5.00×10^{-3}
$P_{f\,\text{MCS}}$	1.00	8.12×10^{-3}	5.09×10^{-3}	5.07×10^{-3}

Chun, J., Paulino, G.H, Song, J. Reliability-based topology optimization of truss structures using a discrete filtering technique. To be submitted for journal publication

Discrete filtering scheme enables filtering of well-defined structures from ground structures

Optimization formulation (compliance)

$$\min_{\mathbf{A}} \ \mathbf{f}^{\mathrm{T}}\mathbf{u}(\mathbf{A})$$
$$s.t \quad \sum_{i=1}^{ne} A_i L_i \le V_c$$
$$\mathbf{0} \le \mathbf{A} \le \mathbf{A}^{upper}$$

$$\text{with } \ \mathbf{A} = Filter(\mathbf{A}, \alpha_f) \ \text{ and } \ \min_{\mathbf{u}} \ \Pi(\mathbf{u}(\mathbf{A})) + \frac{\lambda}{2}\mathbf{u}(\mathbf{A})^{\mathrm{T}}\mathbf{u}(\mathbf{A})$$

Discrete filter approach

$$Filter(\mathbf{A}, \alpha_f) = \begin{cases} 0 & \text{if } A_i / \max(\mathbf{A}) < \alpha_f \\ A_i & \text{otherwise} \end{cases}, \quad 0 \le \alpha_f \le 1$$

- Filter application during optimization
- Removal of critical members for equilibrium
- Detection of global equilibrium of a filtered structure
- Solution for singular systems of equations

Regularized potential energy (Tikhonov regularization)

$$\min_{\mathbf{u}} \ \Pi(\mathbf{u}) + \frac{\lambda}{2}\mathbf{u}^{\mathrm{T}}\mathbf{u} \qquad \frac{\partial \Pi(\mathbf{u})}{\partial \mathbf{u}} = (\mathbf{K} + \lambda \mathbf{I})\mathbf{u} - \mathbf{f} = 0 \longrightarrow \mathbf{u}_p = (\mathbf{K} + \lambda \mathbf{I})^{-1}\mathbf{f}$$

Global equilibrium error

$$\frac{\left\| \mathbf{K}\mathbf{u}_p - \mathbf{f} \right\|}{\left\| \mathbf{f} \right\|} \le \gamma (= 10^{-4})$$

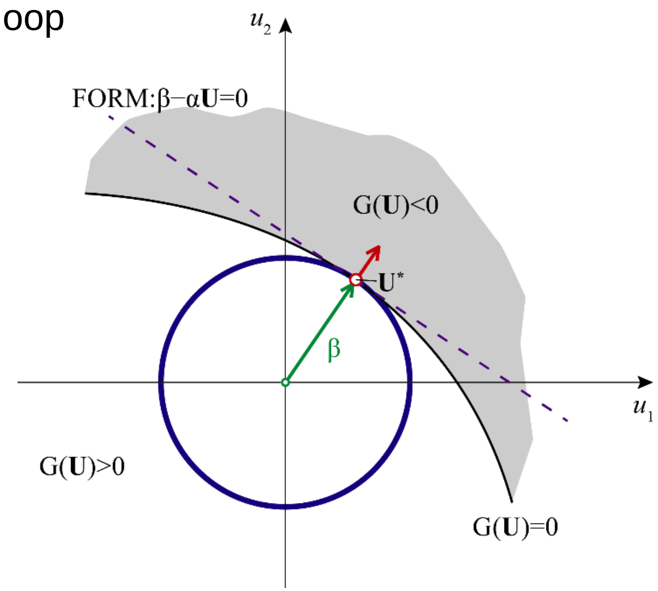
Ramos AS Jr., Paulino GH (2016) Filtering structures out of ground structures - a discrete filtering tool for structural design optimization. *Journal of Structural and Multidisciplinary Optimization.* Available Online. DOI 10.1007/s00158-015-1390-1.

Double loop RBTO: an outer loop for optimization and an inner loop for the reliability analysis

Double-loop RBTO

$$\min_{\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}}} \ f(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}})$$
$$s.t \quad P_f\left[g(\mathbf{d},\mathbf{X})\le 0\right]\le P_f^{\text{target}}$$
$$\mathbf{d}^{lower}\le \mathbf{d}\le \mathbf{d}^{upper}$$
$$\text{with } \mathbf{K}(\mathbf{d})\mathbf{u}(\mathbf{d},\mathbf{X})=\mathbf{f}(\mathbf{X})$$

Two nested optimization loops
1) Design optimization: outer loop
2) Reliability assessment: inner loop



FORM approximations for a component problem

Failure probability (FORM: Der Kiureghian, 2005)

$$P_f=\int_{g(\mathbf{X})\le 0} f(\mathbf{x})d\mathbf{x}=\int_{G(\mathbf{u})\le 0}\varphi_n(\mathbf{u};\mathbf{I})d\mathbf{u}\cong\Phi[-\beta]$$

Performance Measure Approach (PMA)

$$\min_{\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}}} \ f(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}})$$
$$s.t \quad g^{\text{PMA}}(\mathbf{d},\mathbf{U})\equiv g_{P_f^{\text{target}}}\ge 0$$
$$\mathbf{d}^{lower}\le \mathbf{d}\le \mathbf{d}^{upper}$$
$$\text{with } \mathbf{K}(\mathbf{d})\mathbf{u}(\mathbf{d},\mathbf{X})=\mathbf{f}(\mathbf{X})$$

$$g^{\text{PMA}}(\mathbf{d},\mathbf{U})=\arg\min_{\mathbf{U}}\left\{G(\mathbf{d},\mathbf{U})\ \middle|\ \|\mathbf{U}\|=\beta^{\text{target}}\left(=-\Phi^{-1}\left[P_f^{\text{target}}\right]\right)\right\}$$

Der Kiureghian A (2005) First- and Second-Order Reliability Methods. *Engineering Design Reliability Handbook*, CRC, Boca Raton, FL, Chap. 14.

Efficiency in RBTO can be improved by the reliability analysis loop by a non-iterative procedure

KKT optimality condition

$$\nabla_{\mathbf{U}} G(\mathbf{d},\mathbf{U}) + \lambda \cdot \nabla_{\mathbf{U}} \overbrace{\left(\|\mathbf{U}\| - \beta^{\text{target}}\right)}^{H} = \mathbf{0}$$

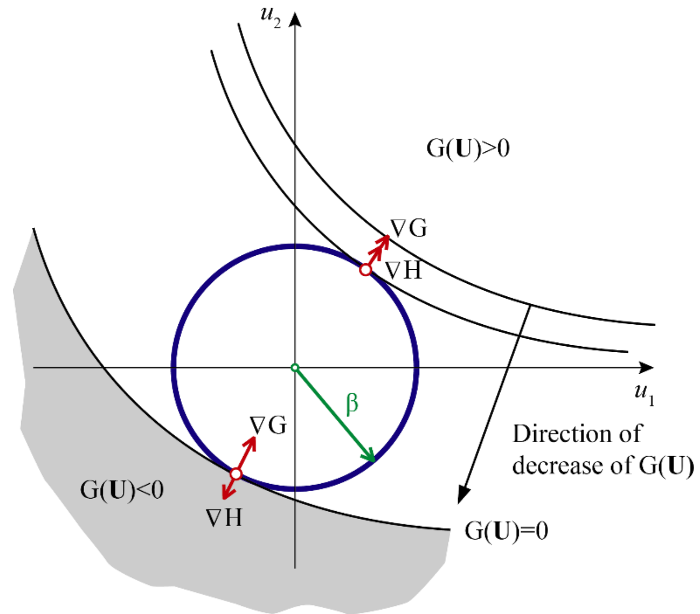
From the geometric interpretation

$$\lambda \geq 0$$

Assume $\nabla H = 2\mathbf{U}$

$$\mathbf{U}^* = \mathbf{U} = -\boxed{\frac{\|\nabla_{\mathbf{U}} G(\mathbf{d},\mathbf{U})\|}{2\lambda}} \cdot \frac{\nabla_{\mathbf{U}} G(\mathbf{d},\mathbf{U})}{\|\nabla_{\mathbf{U}} G(\mathbf{d},\mathbf{U})\|} \qquad \boxed{\beta^{\text{target}} = \frac{\|\nabla_{\mathbf{U}} G(\mathbf{d},\mathbf{U})\|}{2\lambda}}$$

Since $\|\mathbf{U}\| = \beta^{\text{target}}$



Geometric presentation of optimal solution (one active constraint).

Single-loop RBTO using discrete filter

$$\begin{aligned} \min_{\mathbf{A}} \quad & f(\mathbf{A}) \\ s.t \quad & g^{Single}\left(\mathbf{A}, \mathbf{X}(\mathbf{U}^t)\right) \geq 0 \\ & \mathbf{0} \leq \mathbf{A} \leq \mathbf{A}^{upper} \end{aligned}$$

with $\mathbf{A} = Filter(\mathbf{A}, \alpha_f)$ and $\min_{\mathbf{u}} \quad \prod(\mathbf{A}(\mathbf{d}, \mathbf{X}(\mathbf{U}^t))) + \frac{\lambda}{2} \mathbf{u}(\mathbf{A}, \mathbf{X}(\mathbf{U}^t))^T \mathbf{u}(\mathbf{A}, \mathbf{X}(\mathbf{U}^t))$

Liang J, Mourelatos Z, Tu J (2004) A Single-Loop Method for Reliability-Based Design Optimization. *Proceedings of the ASME Design Engineering Technical Conferences*.

Application of the proposed method in a large structure to identify the optimal topology with a desired reliability

Optimization problem

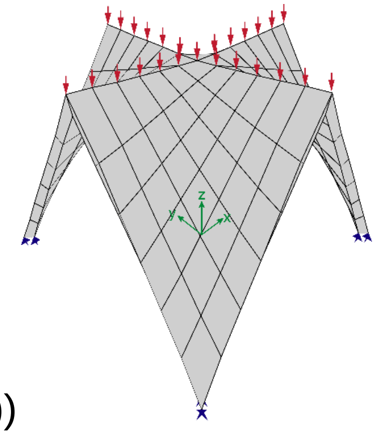
$$\min_{\mathbf{A}} \quad \text{Volume}$$

$$s.t \quad P_f\left(10 - \text{Compliance}\right) \le 0.0025$$

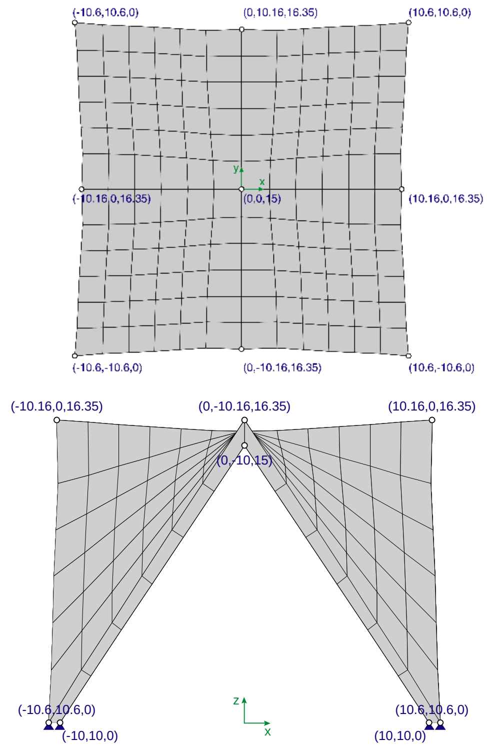
$$\mathbf{0} \le \mathbf{A} \le \mathbf{A}^{upper}$$

$$\text{with} \quad \mathbf{A} = Filter\left(\mathbf{A}, \alpha_f\right)$$

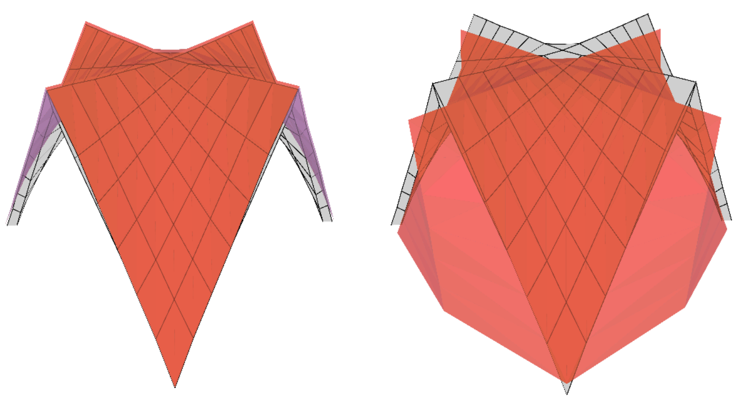
$$\text{and} \quad \min_{\mathbf{u}} \quad \prod(\mathbf{A}(\mathbf{d},\mathbf{X}(\mathbf{U}^t))) + \frac{\lambda}{2}\mathbf{u}(\mathbf{A},\mathbf{X}(\mathbf{U}^t))^T\mathbf{u}(\mathbf{A},\mathbf{X}(\mathbf{U}^t))$$



Design domain, loadings, and BCs



Plan & section view



Restrict zones (GRAND3)

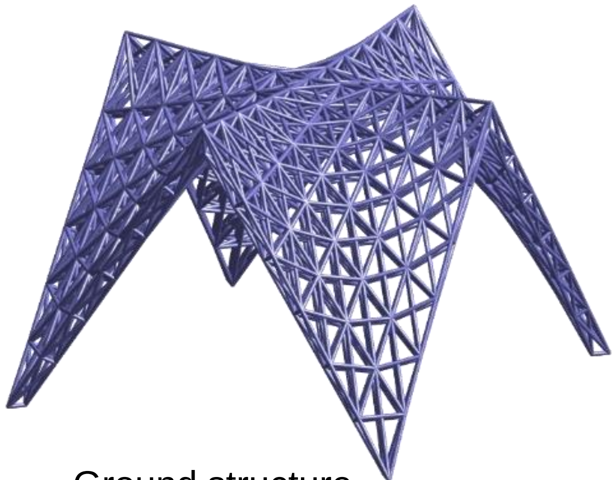
Random variable: Forces, Young' Modulus

E (GPa)		F (kN)		C_{max}	P_f^{target}	α_f	A^{upper}
μ_{mE}	σ_{sE}	μ_{mF}	σ_{sF}				(m^2)
200	40	100	20	10	0.0025	0.01	1.5

Zegard T, Paulino GH (2015) GRAND3 — Ground structure based topology optimization for arbitrary 3D domains using MATLAB. *Structural and Multidisciplinary Optimization* 52(6)
Chun, J., Paulino, G.H, Song, J. Reliability-based topology optimization of truss structures using a discrete filtering technique. To be submitted for journal publication

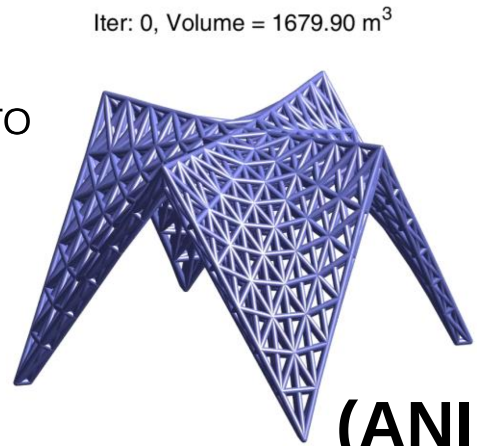
Different connectivity levels result in the different topologies in optimized solutions

- **Connectivity Level 1**

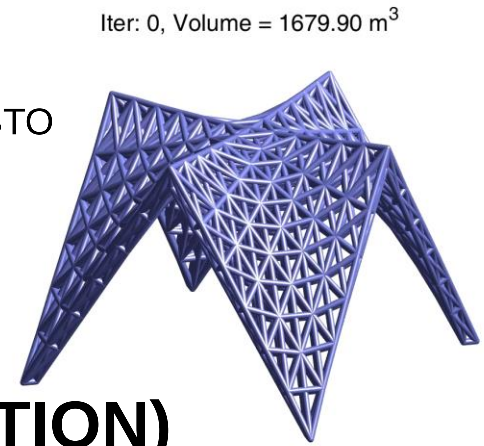


Ground structure

DTO



RBTO

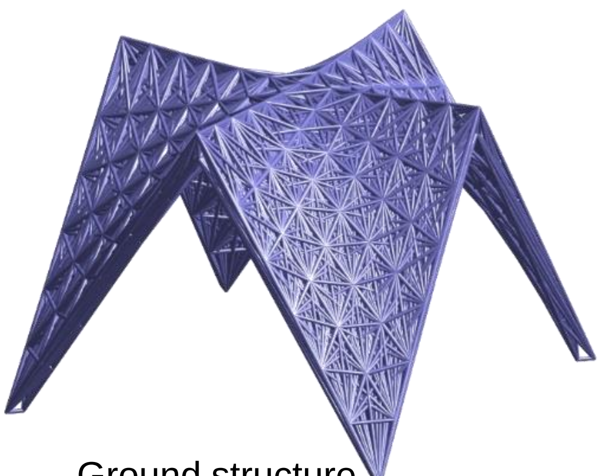


(ANIMATION)

$P_{f,\ \mathrm{FORM}}$ = 15.4%
Equilibrium Error=3.1e-05

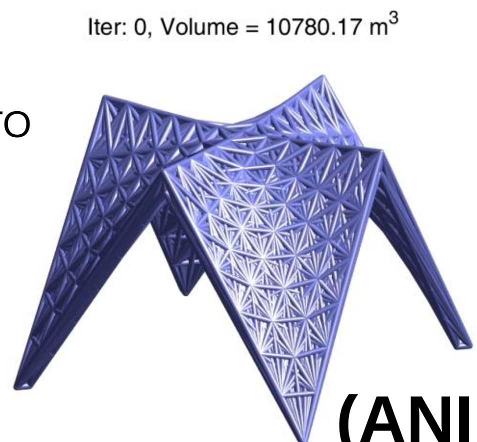
$P_{f,\ \mathrm{FORM}}$ = 0.0025%
Equilibrium Error=3.2e-05

- **Connectivity Level 4**

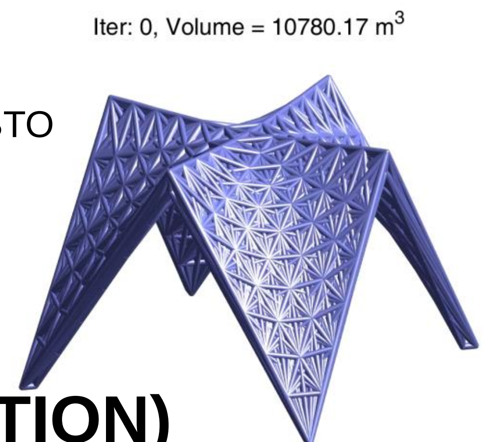


Ground structure

DTO



RBTO



(ANIMATION)

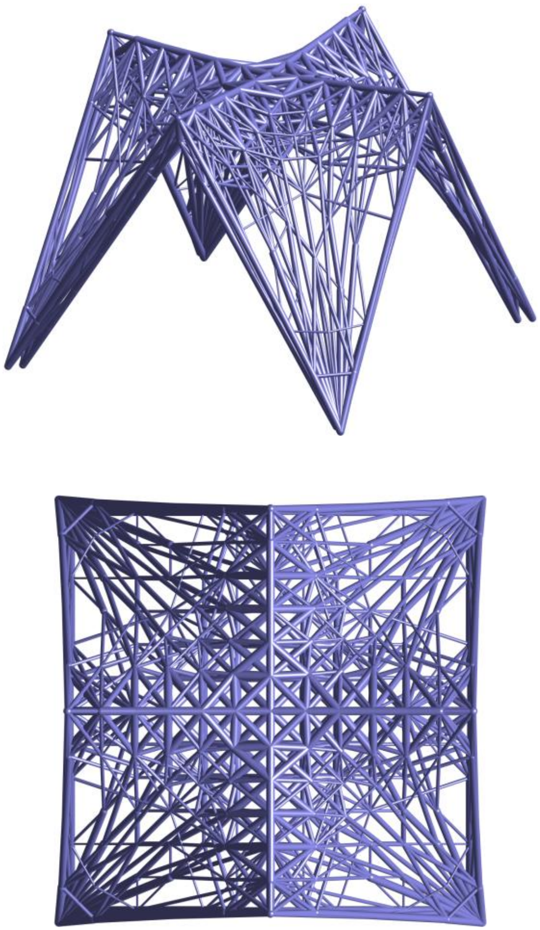
$P_{f,\ \mathrm{FORM}}$ = 14.4%
Equilibrium Error=3.4e-05

$P_{f,\ \mathrm{FORM}}$ = 0.0025%
Equilibrium Error=3.4e-05

Chun, J., Paulino, G.H, Song, J. Reliability-based topology optimization of truss structures using a discrete filtering technique. To be submitted for journal publication

Different filter parameters generate a variety of optimal solutions for application in engineering

Volume = 2579 m^3

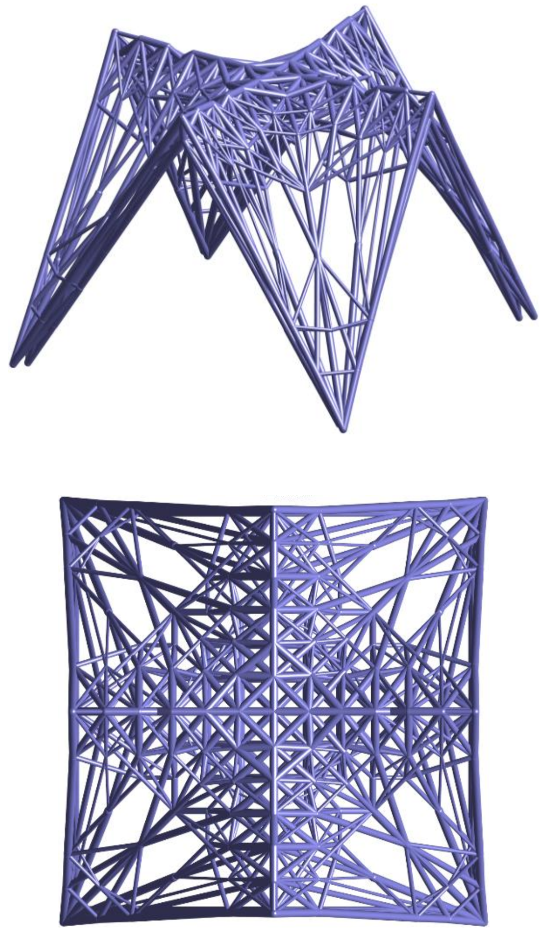


$\alpha_f = 0.05$

$P_{f,\ \mathrm{FORM}} = 0.0025\%$

Equilibrium Error=3.1e-05

Volume = 2670 m^3

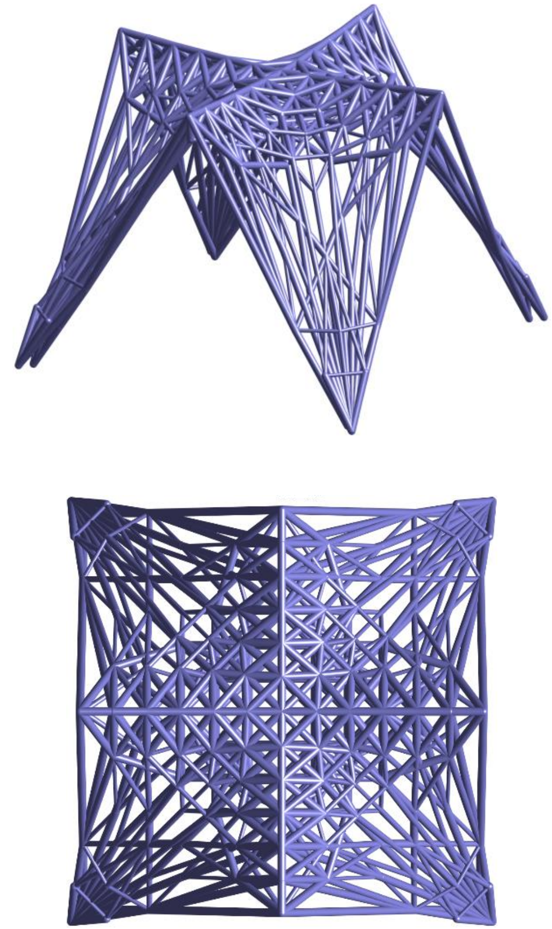


$\alpha_f = 0.075$

$P_{f,\ \mathrm{FORM}} = 0.0025\%$

Equilibrium Error=3.2e-05

Volume = 2897 m^3



$\alpha_f = 0.09$

$P_{f,\ \mathrm{FORM}} = 0.0025\%$

Equilibrium Error=3.2e-05

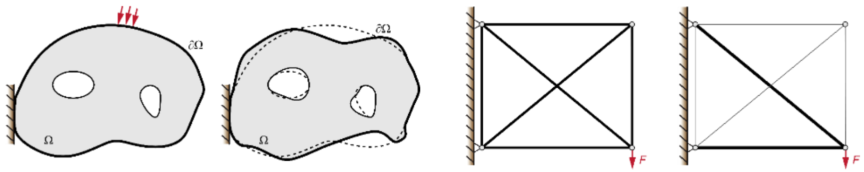
Chun, J., Paulino, G.H, Song, J. Reliability-based topology optimization of truss structures using a discrete filtering technique. To be submitted for journal publication

RBTO employing the discrete filter remarks

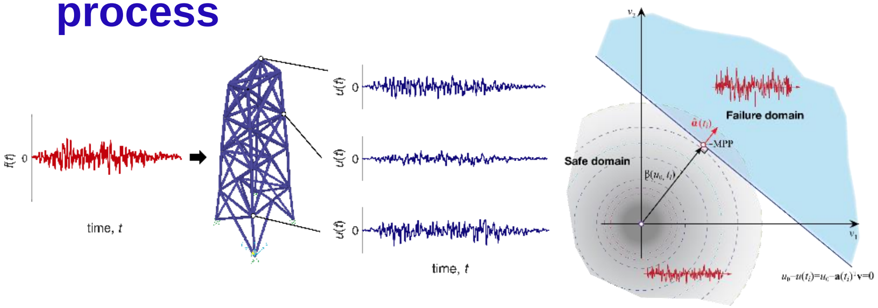
- The conventional filter and the discrete filter were reviewed
- The framework of single-loop RBTO employing the discrete filtering scheme was developed
- Optimal solutions satisfying the desired failure probability and global equilibrium were obtained from the proposed method
- Various optimal solutions can be delivered with the proposed method

Summary

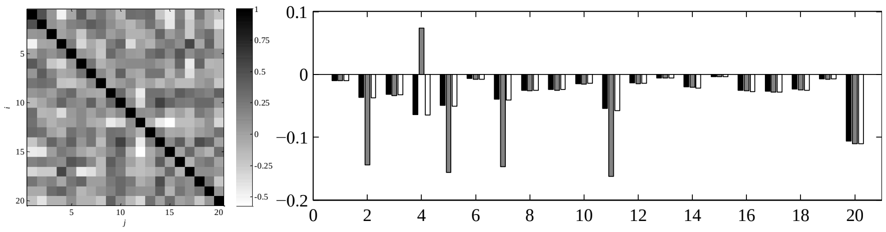
1. Introduction



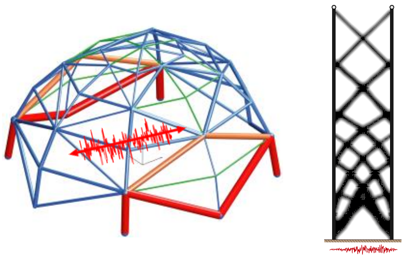
2. Discrete representation of stochastic process



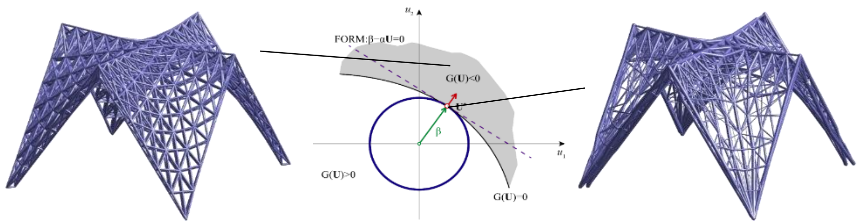
3. Parameter sensitivity of system reliability



4. Structural design and topology optimization under stochastic excitations



5. Reliability-based topology optimization by ground structure method



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Thank you